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CHAPTER 3

The Pythagoreans

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1. Introduction

The place of Pythagoras in the history of mathematics and the divine is, in a sense, well known. We may, for instance, follow Russell: “Pythagoras. . . was intellectually one of the most important man that ever lived. . . Mathematics. . . in him, is intimately connected with a peculiar form of mysticism. The influence of mathematics on philosophy, partly owing to him, has, ever since his time, been both profound and unfortunate. . . What appears as Platonism is, when analysed, found to be in essence Pythagoreanism. The whole conception of an eternal world, revealed to the intellect but not to the senses, is derived from him” [13, pp. 29, 37].

Much is told, in late sources, about a person from the sixth Century, called Pythagoras—in whom religion, mathematics and proto-Platonism come together. Of most of this, we are now less confident than Russell was in 1945. It appears that later antiquity had recreated the figure of Pythagoras and that later periods have contributed their own until the famous character of Russell’s description was finally formed.¹ Little, then, will be said here on Pythagoras himself.

There are other people, ‘Pythagoreans’, active in the late fifth and early fourth centuries, whom we need to investigate. The most important of these were, apparently, Philolaus and Archytas. They did not suffer the fame of Pythagoras and thus the truth concerning them is somewhat easier to sift from the accretion of later legends. Still, what we know is little, and is always dependent on later sources. And so we are driven back—as is so often the case in the study of early Greek philosophy—to Plato and Aristotle.

What I mean is not just that the later tradition concerning Pythagoras and Pythagoreanism was formed, in antiquity, through the double prismatic effect of Plato and Aristotle (this is the case for all Pre-Socratic thought). I also mean that the historical significance of Pythagoreans lies precisely in this: that they were important for Plato and for Aristotle. It is clear that Plato and Aristotle opportunistically changed the meaning of the Pythagorean sources available to them; in my view, the important thing is that they saw there an *opportunity*. Plato and Aristotle perceived an affinity between a group of thinkers whom they associated with Pythagoras. They further saw in this group something useful for their own philosophy. To Plato, the usefulness was in the encouragement of a kindred spirit; to Aristotle, the usefulness was in the warning of a philosophical pitfall.

Having said that, let us be clear: Philolaus and Archytas—indeed, even Pythagoras himself—were real historical figures. Plato could hardly have *invented* them. Certainly, he did not invent Archytas as, in all probability, the two even met in the flesh. Ever since antiquity, this meeting led to speculation and that is how a certain account of Plato’s growth is formulated:

Socrates dies when Plato is but twenty-nine. Plato, then, becomes the author of the early dialogues of Socratic refutation that prove the limits of everyday Athenian knowledge. Later, in his forties, he sails to the west; he stops at Tarentum and makes the acquaintance of Archytas—and soon thereafter the *Meno* is written. Now emerges a new, mathematical and Pythagorean Plato, with all that follows for western philosophy.²

¹On all of this, see [2].

²The best exposition of this account is in [16].

This is possible—though I find it difficult to believe that Plato, of all people, would have to be created from the outside, once by Socrates and then by Archytas. (He does not appear, from his dialogues, to have been easy to influence.) We just do not know who invented whom: did *Archytas invent Plato*—by making him a Pythagorean? Or did *Plato invent Archytas*—namely, did Platonist philosophy give, retrospectively, a meaning to the thought of so-called Pythagoreans?

Be that as it may: the truth is, to repeat, we do not know. From this basic ignorance, let us begin to look for Pythagoreanism and the divine. First, we look at the use made of the Pythagoreans by Plato and Aristotle (Section 2). Then we turn to the Pythagoreans themselves (Section 3). (Both Sections 2 and 3 are, of course, highly selective.) Section 4 offers a tentative explanation of why the Pythagoreans would initiate the history of the relation between mathematics and the divine.

2. Pythagoreanism in Plato and Aristotle

In a literary context, the mention of personal names would have had a certain jarring effect to the ears of ancient Greeks. Previous authors are typically merely alluded to, or their names are periphrastically suggested. Thus the name of Pythagoras is but infrequently mentioned in the writings of Plato and Aristotle. The most direct reference to Pythagoras in the writings of Plato is, at first sight, rather disappointing. Asking, in the last book of the *Republic*, whether Homer was indeed a great educator, Socrates asks if³ “[Homer was] a *private* educational guide during his lifetime to individuals who cherished him for his company and passed on for posterity a Homeric way of life, just as Pythagoras was himself exceptionally cherished for this reason, and his successors even now call their way of life ‘Pythagorean’ and are somewhat distinctive among other men?”

No mathematics or religion here; but this in itself is instructive. We see that the essence of Pythagoreanism for Plato is in some personal quality, a ‘way of life’. The basis of this in the historical Pythagoras—and in his memory in the 4th century B.C.—is clear: Plato thinks of the system of rules and taboos that governed the life of Pythagoras’ followers. But it is not the practice itself that is important (and it would appear that Plato does not necessarily assume that the ‘Pythagoreans’ of his day live quite the same life of Pythagoras’ immediate followers). What is important is the very distinctiveness of the practice, the willingness of Pythagoreans to see themselves as belonging to a ‘Pythagorean’ tradition, standing apart from their fellow-Greeks. The essence of Pythagoreanism, then, is in some self-imposed *difference*.

What about the philosophical views of Pythagoreans? Plato hints at them occasionally. Sometimes, indeed, they gain great significance. The ‘Pythagoreans’ are mentioned, explicitly, towards the end of the curriculum passage in the *Republic*:⁴ “It appears... that these sciences [astronomy and harmony] are sisters, as the Pythagoreans say and we, Glaucon, agree... [having agreed, we do keep our own principle:] that our students shall not try to study those things short of perfection... as we have just said for astronomy. For don’t

³Republic X 600a–b, translation from [9, p. 49].

⁴Republic VII 530d–531c, my abbreviation and translation.

you know that with harmony, too, they do this strange thing. . .—Yes, and absurd, too: they mention those so-called ‘quarter-tones’, and they stretch their ears [some claiming to hear, some not] . . . all of them put the ears before the soul. . .—You mean those worthy ones who approach the issue by the chords—torturing them on the rack. . . They do the same as in astronomy: They search the numbers in the audible harmonies, without passing onwards to abstract problems, studying just that—which numbers are harmonious and which are not”. This follows immediately upon the famous call to let the stars out of astronomy, and leads immediately to the culmination of Plato’s philosophy of the sciences in his description of supreme dialectics. Thus the curriculum appears, retrospectively, as a corrected version of Pythagoreanism. Since Archytas’ fr. 1 refers explicitly to the sciences of the curriculum as ‘siblings’, it appears that Plato had in mind a specific Pythagorean source (unless, of course, fr. 1 itself was written by a later Platonist, to construct the imputed source used by Plato. . .).

Another strategic reference to what may be a Pythagorean doctrine comes at the *Phaedo*, right before Socrates’ final great speech that introduces Platonic metaphysics at its fullness. Simmias, one of Socrates’ friends, has a worry about immortality:⁵ “One could surely use the same argument [as about the soul] as about the attunement of a lyre and its strings, and say that the attunement is something unseen and incorporeal and very lovely and divine in the tuned lyre, while the lyre itself and its strings are corporeal bodies and composite and earthy and akin to the mortal. . . If then, the soul proves to be some kind of attunement, it’s clear that when our body is unduly relaxed or tautened by illnesses and other troubles, then the soul must perish. . .” While this is often taken to represent Pythagorean doctrine, there is some debate as to the source of the theory.⁶ It is possible, indeed, that it was simply invented by Plato. But this debate may miss the point: there was no ‘doctrine of the Pythagoreans’ circulating in antiquity, and so, in the general context of what an ancient reader would expect of Plato and of his interlocutors, the mention of a theory having to do with music, which is held by friends of Socrates, but not quite by Socrates himself, would be taken in the same way as the ‘sibling sciences’ mentioned in the *Republic*. By the standards of ancient writing, this is a sufficiently explicit reference to ‘Pythagoreans’. So—following what, I admit, is a circular argument—we may put together these two key references to Pythagorean doctrine, from the *Republic* and from the *Phaedo*. The picture is consistent. The Pythagoreans have nearly reached the truth, and hence they can be mentioned at the very gates of Plato’s innermost philosophy. Their views are directly related to the Platonic interest in *duality*—the duality of the concrete and the abstract, the duality of body and soul. They sense somehow that there is an incorporeal realm, and connect it to the mathematical sciences; but they do not yet reach beyond the corporeal itself. The basic image is that of music. The Pythagoreans, alone of the Greeks, pursue it as mathematical; Plato goes further, and hears it as purely mathematical and abstract. We begin to see a connection between mathematics and the divine: a mathematical perception of music, serving as a stepping-stone leading from the perception of the world, to the perception of the otherworldly. The Pythagoreans lead in the right way (even if without reaching the goal).

⁵Phaedo 85e–86c, translated from [7, pp. 35–36].

⁶See, e.g., [8, pp. 306–319; 17, pp. 178–179].

Aristotle's attitude to the Pythagoreans is the mirror image of Plato's, and thus confirms it.

Of course, Aristotle was not the opposite of Plato. The distinction between them was much more fine-grained, and it appears that one of Aristotle's main questions was precisely that—"how am I different from Plato?". This is important to us, since one of Aristotle's strategies for distinguishing himself from Plato, was to distinguish himself from Pythagoreanism.

Let us consider the *Metaphysics*. In this work, Aristotle puts forward a world picture where reality ultimately depends on the perfection of divine stars, which are understood through an account put in the terms of mathematical astronomy. This is a bad start for anyone trying to distinguish himself from Plato or from the Pythagoreans. But then, Aristotle insists, it is a mistake to see reality in mathematical terms abstracted from their physical basis. This mistake he ascribes, specifically, to the Pythagoreans:⁷ "The Pythagoreans, as they are called, devoted themselves to mathematics; they were the first to advance this study, and having been brought up in it they thought its principles were the principles of all things. Since in of these principles numbers are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being... since, again, they saw that the attributes and the ratios of the musical scales were expressible in numbers... they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number... E.g., as the number 10 is thought to be perfect and comprise the whole nature of numbers, they say that the bodies which move through the heavens are ten". Notice that, like Plato, Aristotle associates Pythagoreanism primarily with a certain way of life. The main characteristic of the Pythagoreans is their having devoted, or attached themselves (*hapsamanoi*), to mathematics, or 'mathematical learnings' (*mathemata*). The context of the description is well known: Aristotle goes through the philosophies preceding his own, classifying them according to their account of the first principles. These are all material in a sense and, for the philosophers mentioned prior to the Pythagoreans, we can easily see the origins of the views, as described by Aristotle, in the ordinary reality of daily life. (It is here, for instance, that we have the famous reference to Thales who thought that all was made of water—perhaps, Aristotle suggests, because of his realization that nourishment is moist!⁸) In short, while ordinary people live a life of earth, air, fire and water, the Pythagoreans stand out, once again, in their very life. Living a strange life, 'attached to the mathematical learnings', an immediate consequence is that of a strange doctrine—"having been brought up in it they thought its principles were the principles of all things".

The place of this comment is significant: in Aristotle's account, the Pythagoreans immediately precede Plato, whose philosophy is explained through a mixture of influences from previous thinkers, mainly that of the Pythagoreans.⁹ Thus we see that the modern tendency to see Plato as an essentially *influenced* thinker begins with Aristotle. In this case, however, the motivation is clear. Aristotle asserts, effectively: "How do I differ from Plato? By not being influenced by the Pythagoreans".

⁷Metaphysics A 985b23–986a10, transl. Ross, in [1, p. 1559].

⁸Metaphysics A 983b22–23.

⁹Metaphysics A6.

Two books of the *Metaphysics*—M and N—effectively unpack this claim. Aristotle, once again, offers an account of Plato-under-influence. Plato (who is unnamed in this account) starts out from the Socratic search for the definitions of things; he then posits those definitions as possessing separate existence, apart from the things they define—following the example of Pythagoreans, who gave arithmetical definitions to things such as opportunity, justice, marriage and, apparently, saw those definitions as existing separately from any of the things they defined.¹⁰ In short, the Pythagoreans provided Plato with a way of thinking about the otherworldly. Here, for instance, is ‘justice’; and there is the number 4. The number 4 somehow underlies justice and is prior to it.¹¹ Thus the world is doubled, consisting of events of justice, on the one hand, and of the number 4, on the other hand. Aristotle’s main claim in the books MN is that such a doubling is unnecessary, even in the cases where, for epistemological reasons, it has some *prima facie* credibility. Thus, even the study of the mathematical sciences themselves does not call for the assumption of any reality over and above the ordinary physical one. The Pythagorean extrapolation of their way of life into an ontology was, therefore, unfounded, even given the way of life itself. Properly conceived, mathematical learnings do not assume the existence of any separate reality. As mentioned above, Plato uses the Pythagoreans to suggest his dualities—a duality of the concrete and the abstract, a duality of the body and soul. For Plato, the Pythagoreans are not dualistic enough; for Aristotle, they are too dualistic.

In arguing against Pythagorean doubling, Aristotle uses two main strategies. One is serious and impersonal: the opponents are hardly alluded to, and the philosophical problem of the nature of mathematical abstractions is dealt with in a detailed philosophical analysis. The other strategy is much more *ad hominem* and rhetorical: Aristotle alludes to specific Pythagorean views, and makes fun of them. This is the climax of *Metaphysics N*:¹²

They even say that Ξ , Ψ and Z are concords, and because there are three concords, the double consonants are also three. They quite neglect the fact that there might be a thousand such letters; for one sign might be attached to $\Gamma\Phi$. These people are like the old Homeric scholars, who see small resemblances but neglect great ones. Some say that there are many such cases, e.g., that the middle strings are represented by nine and eight, and the epic verse has seventeen syllables, which is equal in number... no one could find difficulty either in stating such analogies or in finding them in eternal things, since they can be found even in perishable things.

Two points are interesting for us. First, Aristotle, in this critique, suggests that Pythagorean duality is a mere contrived analogy—they do not discover a real duality in the universe, but simply stitch up together two unrelated things. In other words, they engage in metaphorical language. Bear this in mind: this will be important for our discussion in Section 4. Second, notice the Aristotelian reference to the duality of the eternal and the perishable: this last point reminds us of the serious point of the joke. Aristotle, after all, subscribes to some dualist vision of the world. The shared assumption for everyone in the discussion—Pythagoreans, Plato and Aristotle—is that there is an eternal reality, to be distinguished from the reality of perishable things. Aristotle does not deny the duality: he merely grounds it in physical stars, and therefore can do away with intangible mathematical properties.

¹⁰*Metaphysics* M4.

¹¹The identification is provided by Alexander’s commentary to the *Metaphysics*, in *Met.* 38.10.

¹²*Metaphysics* N1093a20–b6, transl. Ross, in [1, pp. 1727–1728].

Let us see how Aristotle addresses the Pythagorean account of the stars themselves. Note the *ad hominem*—and comical note:¹³

It is clear that the theory that the movement of the stars produces a harmony, i.e., that the sounds they make are concordant, in spite of the grace and originality with which it has been stated, is nevertheless untrue. . . . Melodious and poetical as the theory is, it cannot be a true account of the facts. . . . Indeed, the reason why we do not hear, and show in our bodies none of the effects of violent force [that follows from large noises] is easily given: it is that there is no noise. . . . [and finally the authors of the view are identified] the very difficulty which made the Pythagoreans say that the motion of the stars produces a concord corroborates our view.

This is the famous harmony of the spheres, which Aristotle criticizes, typically, in a strictly physical way. Had the spheres produced a noise, Aristotle argues, we would expect certain physical consequences: the noise would be heard, and there would be other manifestations of that strong motion. Then what results is a ‘poetical’ theory that Aristotle would dismiss, once again, as mere metaphor. (Paradoxically, since the theory was taken literally, it is can now be read metaphorically only!)

Notice that Aristotle simply refuses to treat the theory as a more abstract metaphysical statement—e.g., that the true account of the motions of the stars is that they manifest the mathematical *structure* of musical harmony. According to that more abstract account, both music itself, as well as the stars, are explained through a more fundamental and abstract mathematical principle. This, probably, was Plato’s intention when, in both the *Timaeus* and in the *Republic*’s Myth of Er, he connected music and astronomy.¹⁴ I make this comparison between Aristotle and a possible Platonic view, because we reach here the difficulty of disentangling this complex melee of Pythagoreanism, Platonism and Aristotelianism. Who was the author of the theory of the harmony of the spheres? Our first evidence, in fact, comes from Plato himself, who has a version of the theory as part of the imaginary after-world of the Myth of Er. There is nothing to prove that Plato relied on a previous Pythagorean account. Aristotle’s criticism of the theory as Pythagorean might refer to authors who are contemporary or later than Plato himself; or it might be a characterization of the nature of the theory, rather than a description of its historical source. Let us assume for the sake of the argument that the theory is first put forward in Plato’s Myth of Er, and is then first characterized as ‘Pythagorean’ in Aristotle’s *De Caelo*. In this case, we see Pythagoreanism constructed by a tug-of-war between Plato and Aristotle. Plato, mathematicizing, has an astronomy anchored in the abstract properties of music; Aristotle, physicalising, gives Pythagoreanism its familiar shape, of the mathematical taken, concretely, to underlie the world. The music of the harmony of the spheres is taken literally, and so a strange, absurd world comes into being—an ever-present, never-heard soundtrack to accompany the universe. Aristotle’s Pythagoreans listen to the inaudible.

In this we have come full circle to Plato’s criticism of Archytas’ studies in harmony. For Plato, the mistake of the Pythagoreans is that, while tuning themselves to the more abstract study of music, they still pay attention to the audible—the concrete reality of strings stretched on the rack. For Aristotle, the mistake of the Pythagoreans is that, while engaged in beautiful and important studies, they go beyond the concrete reality itself, into a realm of an inaudible, otherworldly layer of existence: a sound that is never heard.

¹³ *De Caelo* II.9 290b12–291a9, transl. Stocks, in [1, p. 479].

¹⁴ *Timaeus* 34b–36d, *Republic* 616b–617d.

For both Plato and Aristotle, then, Pythagoreanism is almost the closest philosophy to their own, and yet to be distinguished from their own. Pythagoreans look for a reality that is eternal and superior to the one immediately surrounding us, and they find it through the study of such objects as music and astronomy. So far, Plato and Aristotle follow. There they diverge from both the Pythagoreans as from each other. In both cases the Pythagoreans are guilty, as it were, in producing *too much music*. For Plato, the mistake is in the attention to concrete, heard sounds; for Aristotle, the mistake is the attention to abstract, unheard sounds. In both cases, music is suggestive of the more basic principle, of *duality*—one thing being simultaneously something else. This is the principle Plato wishes to exploit, and Aristotle to deny and reduce to mere metaphor.

For both Plato and Aristotle, the Pythagoreans are somewhat admired, they are somewhat sublime—but, even more, made fun of. They display a certain absurdity, paying attention to the tiny details, whether these are the quarter-tones that Plato's Pythagoreans stretch their ears to hear, or the three consonants that Aristotle's Pythagoreans describe in musical terms. This special position—sublime, and ridiculous—is perhaps what makes them most 'Pythagorean'. After all, this duality of the sublime and the ridiculous is central to the tradition of Pythagoras himself, the prophet of metempsychosis—and the hermit of beans. Once again: it is not so much the contents of the Pythagorean life itself that is important, as its very otherness. The Pythagoreans insist on living differently, and, in this way, they reach a different reality. Plato wishes to go beyond them into that reality, Aristotle wishes to stay nearer to ordinary reality, but both sense that the way to eternal, higher metaphysical realms is through a certain distance from ordinary reality; and that mathematics, in particular music, may lead the way.

So here is the formula we gain from Plato's and Aristotle's reception (or, perhaps, construction) of the Pythagoreans. *Otherness (based on a mathematical duality of concrete and abstract, in particular the duality of mathematical music) leads to the otherworldly*. Let us see if this formula is corroborated by the little we know of the Pythagoreans themselves.

3. Pythagoreanism: some evidence from the Pythagoreans

A number of people are mentioned in our sources as 'Pythagoreans', while others are identified as such by us, on the basis of what we know of their activity. For most of them, the little we know makes little sense. (Why, for instance, did Eurytus put pebbles together in the form of a human being?¹⁵) As a group, they belong to the late fifth to the early fourth century, but this again signifies little. The period begins where it does because only a few prominent thinkers are at all known from before the late fifth century (perhaps, not much intellectual activity, before that period, took the form of writing). The period ends where it does because, from the mid-fourth century onwards, the thinkers we would, coming from an earlier date, classify as 'Pythagorean', would instead be classified as 'Platonist'. (It is possible that Aristotle's 'Pythagoreans' included such later authors.) Thus the chronology and the very identity of the 'Pythagoreans' is a late construct made of the selection of the sources, and of our own nomenclature. There is a real identity of spirit between several

¹⁵[5], Vol. 1, pp. 419–420.

ancient authors, but this identity of spirit does not derive from anything like a 'Pythagorean school': the same is largely true for all such groups in early Greek thought. The ancient authors can be usefully classified, but they did not come together in clearly defined classes.

Working, then, with our own classification of 'Pythagoreans', we can see that two figures stand out. A little more is known about them than about their peers, and some sense of their intellectual personality can be formed. Philolaus lived in the late fifth century, Archytas in the early fourth century. Philolaus has been somewhat the more studied of the two (one can mention in particular the magisterial work, *Philolaus of Croton*, 1993, by Carl A. Huffman). This is in part because Philolaus' age makes him technically a 'Pre-Socratic' and thus of more interest to modern scholarship. There is another, more important reason for the relative neglect of Archytas by historians of philosophy. From their remaining fragments, Philolaus appears like a philosopher, while Archytas appears like a scientist. (That the term 'Pythagorean' can accommodate both is in itself significant.) To offer a general formula, Philolaus is the more suggestive and general author, while Archytas had produced accomplished and more detailed studies. Philolaus' metaphysics also appears, perhaps, to have been more speculative, while that of Archytas' might have been more grounded in the actual practice of science. To sum up the formula even more briefly, then, I suggest the following four-term proportion:

Philolaus : Archytas :: Plato : Aristotle

That is, the transition from Philolaus to Archytas may have been rather like the transition, a generation later, from Plato to Aristotle. It will also appear that Plato, when distancing himself from Pythagoreans, is thinking in particular of an Archytas-type interest in concrete reality; while Aristotle, when distancing himself from Pythagoreans, is thinking in particular of a Philolaus-type metaphysics.

Having said that, there are important continuities between the intellectual projects of Philolaus and of Archytas, which are then reflected later on in the main tradition of Greek philosophy preserved by Plato and Aristotle: in this lies their great historical significance. We shall concentrate here on those issues that have some bearing on the question of mathematics and the divine.

The best starting-point is Philolaus' fragments 6 and 6a, from which I quote following Huffman [11, pp. 123–124, 146–147]:

Concerning nature and harmony the situation is this: the being of things, which is eternal, and nature in itself admit of divine and not human knowledge, except that it was impossible for any of the things that are and are known to us to have come to be, if the being of the things from which the world-order came together, both the limiting things and the unlimited things, did not preexist. But since these beginnings preexisted and were neither alike nor even related, it would have been impossible for them to be ordered, if a harmony had not come upon them, in whatever way it came to be. Like things and related things did not in addition require any harmony, but things that are unlike and not even related nor of the [the same speed(?)], it is necessary that such things be bonded together by harmony, if they are going to be held in an order. ...

The magnitude of harmonia (fitting together) is the fourth, and the fifth. The fifth is greater than the fourth by the ratio 9 : 8. For from [lowest tone] to the middle string is a fourth, and from the middle string to highest tone is a fifth... [A brief discussion of the elementary structure of the octave follows.]

The system seems to run as follows. The world is made to be known—in a sense, is made to exist—by having some order imposed upon it. This order is made primarily of two main components (Philolaus speaks about them elsewhere and they seem to have been the foundation of his system), *limited* and *unlimited* things. These may be, say, some chance length of string—*unlimited*—and its being compared in length to some other string, which comparison then makes it *limited*.

How can the measure apply to the measured? Philolaus mentions harmony, which we may consider by discussing further the example of the string. Suppose the lengths of the two strings have, to each other, the ratio of, say, the side of the square to its diagonal (what we call the square root of two). This is not a harmony, and no special relationship of sound would form when the two are plucked in sequence. No knowable, fixed object will come to be. However, if the two strings are to each other as, say, 3 to 4, the relationship would be that of the fourth. When plucked in sequence, they would form a musical unit: a precisely given individual, both knowable and real. By belonging to this real musical structure, the two lengths of string acquire a meaning—they are no longer mere unlimited lengths. The harmony—the fitting together of the strings—is thus, in a reasonable sense, ontologically prior to each of the components of the harmony.

This then is a sober, if bold, theory in epistemology and in metaphysics. Lest the wrong impression of Philolaus be formed, I hasten to quote testimony 16 [11, pp. 237–238]:

Philolaus [says] that there is fire in the middle around the center which he calls the hearth of the whole and house of Zeus... And again another fire at the uppermost place, surrounding [the whole]. [He says] that the middle is first by nature, and around this ten divine bodies dance: heaven, planets, after them the sun, under it the moon, under it the earth, under it the counter-earth, after all of which the fire which has the position of a hearth about the center.

From such testimonia—and most testimonia are in this spirit—it appears that musical harmony is not, for Philolaus, a mere example of the more general principle of order. Rather, he develops a cosmology derived from numerical and harmonic principles: *ten* divine objects, moving in *dance*, surrounding a middle which is prior to them as *middle*—presumably, as being the middle term of some cosmic proportion. We can therefore see how both Plato and Aristotle could have used Pythagoreanism for their own purposes.

We also see that the sobriety of fragment 6 is misleading because—it appears—Philolaus deliberately aims at being *strange*. The cosmology is non-geocentric; a counter-earth is introduced. As usual, then, we see the Pythagorean emphasis on being different for difference's sake.

Now the following should be noted. It is likely that, as Philolaus brings forward the example of musical proportion, he uses a *recent* mathematical account of the nature of musical harmony. In other words, for Philolaus' audience there could have been a shock of surprise in the suggestion that the lower string to the middle string is as 3 to 4, rather like the shock of surprise in the suggestion that the heavenly bodies are to each other as ten dancing figures. We, moderns, are now familiar with the mathematical account of music. We take it to be straightforward science. We have thus lost the shock of the strangeness of mathematical music and, as we read Philolaus' fragment 6a, we find there a straightforward scientific example for a straightforward idea. On the other hand, we do not think of the heavens as participating in a dance and therefore, as we read testimony 16, we still feel the intended shock, coming across the idea of balletic astronomy. What I

suggest is this. To understand Philolaus, we should try not to rationalize balletic astronomy, so that we make it appear more natural to us; rather, we should somehow come to see the idea of mathematical music in its full strangeness. Fragment 6a should be as strange as testimony 16. An equation of strings, on the one hand, and numbers, on the other hand! The essence of Philolaus' system is in this, deliberate strangeness.

Note further the following. Philolaus' account of epistemology and metaphysics may appear sober but it is still, in itself, counter-intuitive. This is because Philolaus suggests, in fact, that the intangible precedes, epistemologically and ontologically, the tangible. The more abstract structure of harmonies is prior to the more concrete matter that it informs. This fundamental idea will of course flourish, in different ways, in the philosophies of both Plato and Aristotle, and in fact many parallels to this can be found in many other Pre-Socratic philosophers. This however does not make Philolaus' philosophy any more intuitive: it merely reminds us that the counter-intuitive was valued by people other than Philolaus himself.

We start, then, with a tangible world, which is meaningless and lacking in value; we distance ourselves from it and perceive musical harmonies, and through them we come to see a structured reality, which also gains in value. As we move away from the tangible to the abstract, we are also led gradually from the mundane to the divine—the world of Zeus set in the center of a cosmic ballet.

Moving on now to Archytas, it becomes difficult to speak of a philosophical system in the same sense. The most substantial fragments and testimonies do not provide us with a philosophy at all, but with four, apparently unrelated pieces of mathematical science.¹⁶ I believe, however, that this mathematical science may form together a research program to complement Philolaus' metaphysics by offering a systematic, complete account of the science underlying it.

The first piece, which seems to come from a genuine fragment (number 1), is a physical theory of the origin of sounds.

The second piece, described (in somewhat different forms) in Ptolemy's *Harmony* and in Porphyry's commentary to the same work (testimonies 16–17, fragment 2), is a classification of ratios.

The third piece, reported only in Boethius (testimony 19), is a straightforward argument in number theory (probably genuine, for who would bother to forge an argument as straightforward as that?).

The fourth piece, very thoroughly and convincingly documented by the testimonies 14–15, is a brilliant solution to the problem of duplicating the cube.

The pieces fit together harmoniously, so to speak. They all belong to the study of proportions.

To begin with the first piece of the puzzle—fragment 1 and its acoustic theory—we should note that this study of the physical origin of sounds culminates, effectively, in a proportion. Archytas argues that pitch correlates to the speed of the motion of the air, and sums up fragment 1 in saying that 'it has been made clear to us by many [arguments, examples?] that the higher tones are moved faster, and the lower—slower'. While this does not yet have the mathematical form of proportion (i.e., Archytas does not offer the

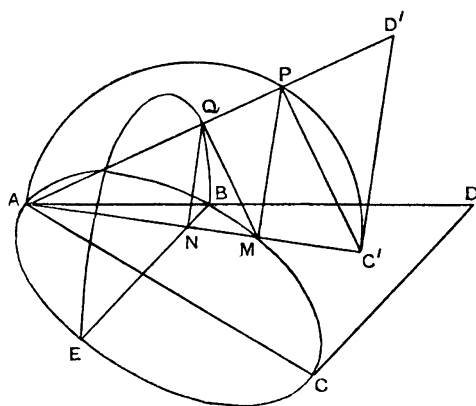
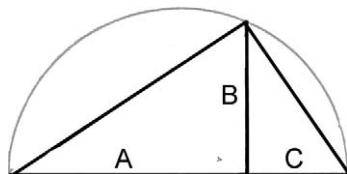
¹⁶For the fragments and testimonies concerning Archytas, see [5, Vol. 1, pp. 421–439].

mathematical statement that the ratio of the speeds is the same as the ratio of the sounds) it would have been amazing, in light of his further views on musical ratios (of which see below) if he did not believe in some such theory. In other words, if the musical theory available to Philolaus had two separate domains connected by a single proportion—length of strings, say, and pitch of sounds—the musical theory available to Archytas adds to this another domain, covered by the same proportion—the speed of motion of air. This is an important advance since this domain of air motion is *universal*. This domain is not tied to any particular musical instrument, but is coextensive with any sound whatsoever. Thus Archytas had achieved a complete system uniting a mathematical structure with a physical structure.

This is corroborated by the second piece, where we see Archytas dealing in much greater detail with this system: this time, not with its physical–mathematical interface, but with the mathematical structure alone. Once again, Archytas’ aim seems to have been the *completion* of the system. Thus he goes on to classify kinds of numerical ratios, corresponding to different musical relations. The details, especially as reported by Ptolemy, are quite complicated and reveal Archytas’ computational fluency. The principle is simple: from the basic assumption that the octave is correlated to a 1 : 2 ratio, divided into, e.g., 2 : 3 and 3 : 4 (the fifth and the fourth) one can derive the entire system going as far as the quarter-tones (the debate as to their existence, we recall, was evoked, to comic effect, by Plato). One indeed reaches quickly pure numerical constructs whose auditory correlates are probably impossible to judge (how about the interval of 256 to 243?). A strange, purely numerical structure takes over the auditory world of music.

Even more abstract—but necessary for the sake of the completion of the system—is the third piece in the puzzle: the study in number theory reported by Boethius. Archytas proves that a ratio in integers of the form $(n + 1) : (n)$ cannot serve as the extreme terms in a continuous proportion in integers $(n + 1) : (k) :: (k) : (n)$. (This is not the trivial result that between two consecutive integers another integer cannot be found: note that the ratio $(n + 1) : (n)$, e.g., 9 : 8, is equivalent to many other ratios between non-consecutive integers, e.g., 18 : 16.) This proposition has immediate consequences on the possibility and impossibility of musical proportions, and in fact it appears as the third proposition of the musical treatise ascribed to Euclid, the *Sectio Canonis*. (Boethius, of course, quotes it in the course of his own musical treatise.) However, it does not feature any particular musical ratios. This study—no doubt, a fragment of a larger study in number-theory—moves beyond the actual numbers put forward by the facts of music, to a study of the possibility and impossibility of proportion in integers as such.

The same kind of extension, finally, may be true for the fourth and final piece of the puzzle—Archytas’ solution of the problem of duplicating the cube. Both the problem, and Archytas’ approach to its solution, are closely related to issues in proportion theory. We can see this as follows. How can one find, given two lengths, their geometrical mean? E.g., given the lengths 18 and 16, how can we find the length B that satisfies $18 : B :: B : 16$? As we have seen, there is no purely numerical solution to the problem; however, it can be solved geometrically. In particular, we can fit together right-angled triangles in a circle so that, by triangle similarity, the triangles will display the geometrical progression $A : B :: B : C$ (Figure 1). By producing A and C as 18 and 16, we find the desired B. Thus, by considering ratios in a more abstract way—moving beyond the ratios between integers to



research—just as it was the key to Philolaus’ metaphysics—and that, in both cases, the interest is not in the fact that mathematics has concrete manifestations but that, on the contrary, beneath the world of concrete sounds there is another world, richer and more abstract, of mathematical relations. Notice that when the theory of proportion reaches its mathematical culmination—we are likely to have Archytas’ best work in his preserved solution of the problem of duplicating the cube—it becomes fantastically complicated and abstract, involving a cosmic dance no less remarkable than that of Philolaus’ stars. The world of swirling semicircles, studied by Archytas, already belongs to a realm of pure mathematics that exists apart of any ordinary Greek experience.

Of course, it is impossible for us to tell what were Archytas’ precise metaphysical views. Clearly, an admirer of Philolaus could have been excited by the science itself; while an admirer of Archytas could have enjoyed the science while criticizing the metaphysics. I have suggested above that these two routes were taken by Plato, and Aristotle, respectively. At any rate, we can now sum up the evidence provided by the four main protagonists—Plato, Aristotle, Philolaus and Archytas. We may recall the formula gained at the end of the previous section: *Otherness (based on a mathematical duality of concrete and abstract, in particular the duality of mathematical musical) leads to the otherworldly*. This is, indeed, corroborated by what we know of Philolaus’ metaphysics and Archytas’ science. In this section, though, we have noticed the predominant role of a special mathematical concept, that of *proportion*. Proportion, apparently, served the Pythagoreans to support their dualities of concrete and abstract. So let us reformulate this even more briefly:

Otherness, based on proportion, leads to the otherworldly.

In the next section, we shall try to explain how this formula came to be characterized as ‘Pythagorean’ and, finally, how the Greeks first formulated the question of mathematics and the divine.

4. Mathematics and the divine in the Pythagoreans: a suggestion

What made the Pythagoreans *Pythagorean*? That is, what made the ancients perceive a link between the thought of, say, Philolaus and Archytas, and the stories attached to person of Pythagoras? This question involves many unknowns. It does appear that, to have been called a ‘Pythagorean’, an ancient author had to come from the Greek West (where most of Pythagoras’ activity took place). But does that mean that there was a historical continuity between the Pythagoreans of the sixth century, and those of the fourth? We simply do not know. It is better to concentrate, then, on the pattern of beliefs and practices that the name ‘Pythagoras’ evoked to the Greeks. Somehow, Philolaus and Archytas fitted that pattern. Why was that so?

On this question, too, one has to be speculative. Speaking somewhat dogmatically, then, we may offer the following account of ‘Pythagoras’ as present to the Greek imagination.

It appears that Pythagoras was primarily remembered as a charismatic religious leader of a special kind. He offered a way of life guaranteeing some form of release from pain and mortality. If this is indeed the case, then Pythagoras had embodied a Greek *type*—usually, however, fulfilled by mythical figures. Orpheus was another such savior-figure and so, in other ways, were other heroes and gods, such as Dionysius himself. Around such

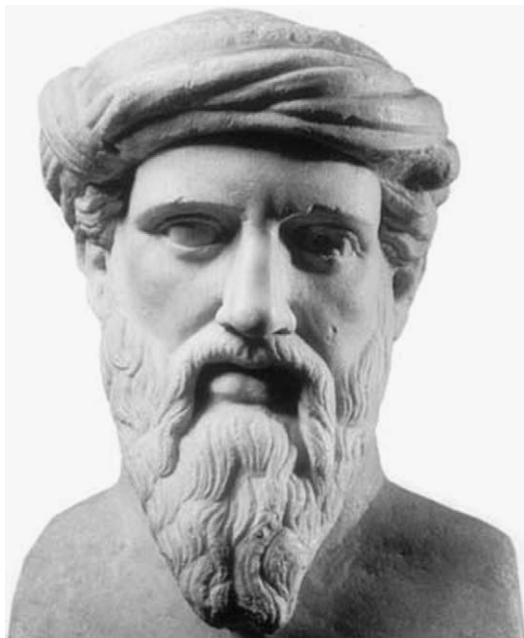


Fig. 3. Pythagoras. Courtesy of the Museo Capitolino, Rome.

savior-figures, special practices formed, the *mystery cults*.¹⁷ These had a powerful role in the Greek imagination. It is therefore, probably, in relation to such mystery practices that we should understand what the term ‘Pythagorean’ evoked.

To follow mystery practices would be to stand out from the public Greek religion of the body of citizens in a city-state. There is something exclusive about being saved: as it were, the idea of one’s being saved implies that there are others, who are not. There is only so much room in the boat. A distinction is thus made between those who belong to a saved inner circle, and those who do not. The transition into the inner circle—the initiation—occurs through participation in an activity peculiar to the group, whether in the ascetic ritual of the Orphics or the Pythagoreans or in some special rite performed in a ritual center.

Briefly put, the core of such practices is the transition from the mundane to the divine through a process of becoming-other. This is precisely the formula we have gained for the Pythagoreans of the fifth and fourth century (minus the role given to mathematics and in particular music and its proportions as an agent in the initiation.) This then would be a reason for Greeks in the fourth century to conceive of Philolaus, for instance, as a Pythagorean. Authors such as Philolaus offered a way reaching from the *here* of the everyday to the *there* of the divine, passing through a strange form of life.

¹⁷See [3, pp. 296–304] and, in general for Greek mysteries, see [4].

How do Greeks bridge the gap between the mundane and the divine? We can follow a structure, whereby the bridge comes more and more to be, so to speak, under human occupation.

Religious ritual as such aims at bridging the gap between the mundane and the divine: but the traditional ritual of the City—by being ordained as ritual—belongs, in a sense, to the gods. Therefore a further, more personal ritual is called for, the mystery cult. Here, the individual makes a choice to perform the rite and thus assumes a control over his or her own destiny, beyond that provided by the ordained ritual of everyday religion. Pythagoras, by explicitly instituting a *new* mystery cult, brought the divine even nearer: the claim was that a practice, instituted by a mortal, could reach beyond mortality. Even so, the irrationality of the original Pythagorean practice implies that the practice is either worthless or divinely inspired. To make this claim plausible—of transformation to the transcendent, through human means—the transformative power should be given some rational basis. Thus the Pythagoreans of the fifth and fourth centuries went on, to *rationalize* mystery—to produce a systematic philosophical counterpart of the experience of mystery. There are important differences between the four stages—traditional ritual, mystery cult, Pythagoras and Pythagorean philosophy—but there is a basic continuity which explains, I suggest, why the Pythagoreans were called ‘Pythagoreans’. In short, of all the intellectual variety on offer at the late fifth century, the Pythagoreans offered a life most closely resembling that of the mystery cult. Hence their special absurdity, as well as their special promise.

Why mathematics? That is, why did the Pythagoreans take *mathematics* as the agent for transformation to the transcendent? We can, I suggest, account for that as well. To outline this suggestion, let us look more closely at the mystery cult practice.

In a mystery cult, say the famous one of Eleusis, the initiate would follow a long and special rite: sacrifices, processions, fasts, all leading to a special ceremony conducted in an atmosphere of secrecy, terror and ecstasy; secrets lead on to secrets until the highest secrets are revealed. All of this, finally, is supposed to endow the initiate with a better after-life and to revoke the otherwise expected punishments of hell. The secrets, under close inspection (they were sometimes revealed in antiquity, despite the harsh rules against their being divulged) appear disappointing. What is the height of the Eleusis mystery? A priest announces “The mistress has given birth to a sacred boy, Brimo to Brimos”, and then an ear of corn is displayed and cut in silence.

Any concrete representation of ecstasy is bound to appear disappointing and, with the appropriate preparation and staging, the rite would no doubt have been effective enough. But to concentrate on the staging effects is to miss a more important point, namely, that *any* rational statement would have been inappropriate at this context. Every text can be staged but some texts are more appropriate for ecstasy than others. The very irrationality of the Brimo-to-Brimos text endows it with a transformative power. The text cries out to be interpreted (Who is Brimo? Is she perhaps the goddess of Eleusis, namely Demeter? And who is her boy?). Such irrational texts provide us, then, with a verbal counterpart to the main theme of the ritual, which is that of becoming-other. The text itself is caught, so to speak, in the act of becoming-other: it is a metaphorical statement, whose non-metaphorical significance is left opaque. Precisely the same holds of the cut ear of corn: we immediately see the act as rich in metaphorical significance, but we are not given the indications which would specify a unique literal interpretation of the metaphor, so that the cut corn remains, as it

were, a metaphor hanging in mid-air. (This, incidentally, is why the secrecy surrounding the mysteries would have been so important: with the ritual revealed, the assertions and acts would have acquired standard interpretations and would have thus turned into *dead* metaphors.)

The principle, then, is as follows. A certain ‘mystery’ statement—or act—is meaningless on its own, but it does suggest a meaning which however is different from its surface meaning. If we call the statement P, then the form of the expression is ‘P is not P’. By immersing yourself in a system of expressions of this kind, you are made to become non-yourself, and this is the basis for the transformative power of mystery practice. If identity is put into question, then so is self-identity.

The transition accomplished in a mystery practice is mediated through strangeness. We can therefore begin to apply what we have seen from the mystery cults, to the Pythagoreans themselves. We have noted the aiming at difference for difference’s sake, characteristic of both the early Pythagoras and the Pythagoreans themselves. We have also noted the sense of the sublime and the ridiculous, somehow combined, attaching to both. Pythagoras’ abstaining from beans, or the Pythagoreans’ interest in the three characters Ξ , Ψ and Z , are both as absurd and as meaningful as the ear of corn: they suggest ways of transformation, whether of the self into a reformed, ascetic person, or of the world into a musical structure.

We can be even more specific. Let us consider again the task as it faces the Pythagoreans. They are looking for a rational grounding for the mystery experience: that is, they can no longer rely on the Brimo-to-Brimos type of expression. They need some kind of literal statement that keeps the sense of the metaphorical, of a thing-being-something-else. We have offered above a formula to describe Pythagoreanism: *Otherness, based on proportion, leads to the otherworldly*. I now suggest that this can be derived from the task of Pythagoreanism: to rationalize mystery.

The best starting point for the rationalization of mystery would be a thing that *literally is something else*: an object that is simultaneously two radically separate things, so that for which, the paradoxical statement ‘X is not X’ could be literally valid.

Music, under its mathematical interpretation, offers just that, and in a peculiarly appropriate form: it is a concrete thing, the musical instrument, and it is simultaneously, under its mathematical interpretation, an abstract thing—a system of proportions. There is a single formula underlying both so that one can literally say ‘the ratio gave birth to the harmony’—Brimo gave birth to Brimos—and so the world of strings and instruments (the mortal world in which we live) is simultaneously something else: another, intangible world. Here then is one reason why music would be an obvious mode for a systematic Pythagoreanism: it embodies the continuity of the worldly and the otherworldly.

This can be generalized. For what makes this continuity at all possible? How can it be possible to say, literally, that ‘X is not X’? This is because we are dealing with proportion statements, ‘this string is to that string as 3 is to 4’. This is proportion: the most basic way of saying, in a literal way, that two separate domains are, in some defined way, the same. They are different; and yet they embody the same relations. Thus proportion-statements are the most natural route to be taken by the Pythagoreans. Their project was to offer an intellectually systematic correlate of a mystery practice—as it were, to literalize metaphor without losing its metaphorical power. This is precisely what proportion statements are. We recall Aristotle complaining about the Pythagorean tendency to ground everything, in-

appropriately, in a numerical and musical realm:¹⁸ ‘They even say that Ξ , Ψ and Z are concords, and because there are three concords, the double consonants are also three. . . . Some say that there are many such cases, e.g., that the middle strings are represented by nine and eight, and the epic verse has seventeen syllables, which is equal in number. . . . no one could find difficulty either in stating such analogies or in finding them in eternal things, since they can be found even in perishable things’. From such criticisms, it is clear that the Pythagorean speculation was expressed by statements such as ‘as X in domain A , so Y in domain B ’—the general form of proportion or analogy statements or, indeed, the general form of metaphor. Aristotle criticizes Pythagoreanism for its metaphorical language, but metaphors are sometimes valid—when they are true analogy statements, that is proportions. Thus, for instance, if we take the harmony of the spheres as a proportion statement (as the motions of the stars to each other, so the motion of the strings on the octave) we obtain a statement which in principle could be simply *true*. At the same time, it is still—as Aristotle would put it—‘poetical’. Indeed, the statement has the appropriate ‘mystery’ effect of alienating us from the mundane world around us and making it appear rather more ‘divine’, simply by virtue of its possessing a duality or metaphor inherent to its mechanism. If we are surrounded simultaneously by stars, and by musical harmonies, then the mundane world is not just mundane.

To sum up, then, there are two properties of Greek mathematics—and, in particular, of Greek musical theory—that would have made it appropriate for the Pythagorean project. First, the correlation of *the concrete and the abstract*. (This is most obvious in mathematical musical theory, but it is also a wider feature of Greek mathematics with its equation of a concrete diagram and a general, intangible theorem.) Second, Greek mathematics essentially relied on the tool of *proportion*, which is the general tool of correlating separate domains. Thus these two seemingly irreconcilable domains, too—Mathematics, and Mystery—could be brought together. Greek mathematics could have functioned as mystery, made literal.

The above account, of course, was merely a dogmatic statement of a speculative suggestion. This is perhaps inappropriate for a historical study and so I shall conclude by restating the suggestion made here as a philosophical claim concerning mathematics and the divine.

We might perhaps be surprised that there is any relationship between the two. This is because we often think, today, of mathematics as the domain of the *literal* par excellence. We feel that metaphor is more appropriate for discussing the divine—and that mathematics provides us with no metaphors. In mathematics—we tend to think—everything is just what it is and the only allowed relation is that of logical entailment, that is, identity. Thus mathematics is conceived as the domain of the statements of the form ‘ X is X ’. How can it guide us, then, into the otherworldly—the domain, so to speak, of the not- X ?

But, after all, this is not the only way in which to see mathematics and perhaps not even the most natural way to see it. After all, why did Russell so dislike Pythagoras? Not for his absurdity, but for his persuasiveness: Russell, himself, saw his own philosophical development as leading away from Pythagoreanism.¹⁹ Russell, by his own account, had once been a Pythagorean.

¹⁸Metaphysics N1093a20–b6, transl. Ross, in [1, pp. 1727–1728].

¹⁹See [14].

By describing his philosophical growth in such terms, Russell meant the following. In his early, Logician period, Russell believed that mathematics was informative, that is, it discovered non-obvious truths. Later on, based on what he understood to be the argument of Wittgenstein's *Tractatus*, Russell came to think of mathematics as a mere system of tautologies. That is, it was no more than a system of useful shorthand expressions of the form 'X is X'. Information could be gained through empirical investigation only, and, to such an investigation, mathematics gave no more than shape.

This later view was one of the standard accounts of the relationship between mathematics and science in the 20th century, and it may inform our own surprise when encountering the relationship between mathematics and the divine. It was based on such a view that, in his *History of western Philosophy*, Russell said that Pythagoreanism had an influence on philosophy 'both profound and unfortunate'. Pythagoreanism, to Russell, was the origin of his own youthful mistake—the view of mathematics as going beyond tautology, that is, asserting that 'X is not X'.

Now whether mathematics is informative or not is a question I shall not enter here—but the very fact that the youthful Russell Russell's believed it was, is of interest. This was the position held by Russell *while still engaged in mathematics* (if, that is, this is how we should call the writing of the *Principia Mathematica*). To the practitioner of mathematics, this intuition—that mathematics deals with 'X is not X'—is overwhelming. Quite simply, 'X is X' is never asserted in any mathematical text. That is: there are no statements in mathematics of the form 'The squares on the sides of the right-angled triangle are equal to the squares on the sides of the right-angled triangle'. Mathematical statements are always of the form 'The squares on the sides of the right-angled triangle are equal to the square on the hypotenuse'. Mathematics asserts the identity of the different, not of the same. This is what it most essentially is: the tool for making *valid* assertions of the apparent form 'X is not X'. Whether the deep form of such assertions may turn out, upon logical analysis, to be 'X is X', is a separate question: the fact remains that, at its surface, mathematics asserts the paradoxical identity of the different. At its surface, then, mathematics is a mystery, that is, an assertion of the identity of the different—an assertion which, however (unlike standard mystery assertions) also happens to be true.

This fact about mathematics—that it is, in a real sense, a mystery—is, I suggest, of historical significance. Mathematics deals with metaphor, in a rational way. It follows that those who care about both rationality and metaphor would, naturally, appeal to it. This, perhaps, may serve to explain the historical phenomenon of Pythagoreanism.

Notice for further reading

The fundamental study of Greek religion is Burkert [3].

The fundamental study of Pythagoras is Burkert [2]. Its deflation of Pythagoras-the-scientist has been followed here and is challenged in, e.g., Zhmud [18].

There are many accounts of the Pythagorean influence on Plato, of which I took Vlastos [16] as an example. The Pythagorean influence on Aristotle is less well known: a brilliant introduction is Sorabji [18].

As mentioned in the text, Huffman [11] is an essential study of Philolaus. There is nothing as useful on Archytas, whose achievement can be pieced together from such standard sources as, e.g., Heath [10].

An account of philosophical Pythagoreanism must be based on some picture of the history of Greek mathematics. In the view of this author, the scholarship on Greek mathematics underwent a sea change in the late twentieth century, rendering much of earlier scholarship obsolete (perhaps everyone thinks this way about his or her discipline). In particular, during the twentieth century, much of the literature was influenced by a belief in a late fifth century philosophical crisis, caused by the discovery of irrationality, leading to the formation of historical Pythagoreanism (as distinct from that of Pythagoras). Very few historians of Greek mathematics still believe in this story: see Knorr [12] for one of the first studies to doubt the traditional story, and Fowler [6] for a recent summary of the problem.

An account of philosophical Pythagoreanism that brings together the lessons of Burkert [2] with those of the recent research into the history of Greek mathematics, remains to be written.

References

- [1] J. Barnes (ed.), *The Complete Works of Aristotle*, Princeton, 1984.
- [2] W. Burkert, *Lore and Science in Early Pythagoreanism*, Cambridge, MA, 1972.
- [3] W. Burkert, *Greek Religion*, Cambridge, MA, 1985.
- [4] W. Burkert, *Ancient Mystery Cults*, Cambridge, MA, 1987.
- [5] H.A. Diels, *Die Fragmente der Vorsokratiker, Griechisch und Deutsch*, 9. Aufl. Hrsg. Von Kranz W., Berlin, Weidmann, 1959–1960, 3 vols.
- [6] D.H. Fowler, *The Mathematics of Plato's Academy*, 2nd edition, 1999.
- [7] D. Gallop, *Phaedo/Plato*, Oxford, 1983.
- [8] W.K.C. Guthrie, *A History of Greek Philosophy, Vol. I: The Earlier Presocratics and the Pythagoreans*, Cambridge, 1967.
- [9] S. Halliwell, *Republic 10/Plato*, Warminster, 1988.
- [10] T.L. Heath, *History of Greek Mathematics*, 2 vols., Oxford, 1921.
- [11] C.A. Huffman, *Philolaus of Croton*, Cambridge, 1993.
- [12] W.R. Knorr, *The Evolution of the Euclidean Elements*, Dordrecht, 1975.
- [13] B. Russell, *History of Western Philosophy*, New York, 1945.
- [14] B. Russell, *My Philosophical Development*, London, 1959.
- [15] R. Sorabji, *Aristotle, mathematics and colour*, *Classical Quarterly* **22** (1972), 293–308.
- [16] G. Vlastos, *Elenchus and mathematics: a turning point in Plato's philosophical development*, *American Journal of Philology* **109** (1988), 362–396.
- [17] C. Witt, *Dialectic, motion and perception: De Anima, Book I*, *Essays on Aristotle's De Anima*, M.C. Nussbaum, A.O. Rorty, eds., Oxford, 1992, pp. 169–183.
- [18] L. Zhmud', *Wissenschaft, Philosophie und Religion im fruhen Pythagoreismus*, Berlin, 1997.