

Michael C. Duddy

MDA Designgroup
International
19 Union Square West
New York, NY 10003 USA
MDuddy@mda-
designgroup.com

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Research

Roaming Point Perspective: A Dynamic Interpretation of the Visual Refinements of the Greek Doric Temple

Abstract. Writers, artists, and mathematicians since Vitruvius have attributed the use of the visual refinements as a means by which the Greek builders optically corrected the form of the Doric temple. This study proposes an interpretation in which the visual refinements of the Parthenon are considered from a non-stationary, or “roaming”, point of view. The mathematics of this type of visual space reveals a dynamic zone in which objects visually increase and decrease simultaneously, a behavior consistent with conditions addressed by the visual refinements of the Parthenon.

The Greeks created a plastic system by forcibly affecting our senses.They employed the most delicate distortions, applying to their contours an impeccable adjustment to the laws of optics.

Le Corbusier, *Towards a New Architecture*

Introduction

In the outline of his program for the classical orders in the *Ordonnance for the Five Kinds of Columns after the Method of the Ancients*, Claude Perrault attacks what he claims are the “abuses” of the orders, specifically the use of optical corrections or visual refinements. From the earliest Doric temples of ancient Greece, angular and curvilinear irregularities could be observed in the building forms that included the tapering of the columns, curvatures in the podiums and entablatures, inclinations of columns and friezes, and variations in column diameters and spacings. For Perrault, these adjustments opposed an otherwise rigorous system of logically proportioned elements, “distorting or spoiling proportions in order to prevent them from appearing distorted or for making something defective in order to correct it” [Perrault 1993: 162-163]. Perrault’s indictment references back to Vitruvius who, in setting out in Book III of the *Ten Books of Architecture* a detailed proportional system of the Classical orders, mentions that despite his exacting prescription, adjustments to the proportions of columns were necessary to compensate for optical distortions. “For the eye is always in search of beauty, and if we do not gratify its desire for pleasure by a proportionate enlargement in these measures, and thus make compensation for ocular deception, a clumsy and awkward appearance will be presented to the beholder” [Vitruvius 1960: 86]. Not only are the columns adjusted to account for such distortions, but “the level of the stylobate must be increased along the middle by the *scamilli impares*; for if it is laid out perfectly level, it would look to the eye as though were hollowed a little.”¹ In these recommendations, Vitruvius is accounting for the Greek and Roman convention of using the curved geometries in the columns, podiums, and entablatures was to “correct” optical distortions. Subsequent architects from Alberti to Le

Corbusier would note how the use of these “visual refinements” offered evidence of a higher order of visual understanding.²

This study began as an inquiry into the so-called “subjective” or curved model of visual space as a possible explanation for the visual refinements. Armed with an understanding of the non-Euclidean geometries of Bolyai and Lobachevski, writers from Hermann von Helmholtz in 1856³ to Rudolph Luneburg in 1947⁴ claimed that the geometry of visual experience actually behaved according to a hyperbolic construct. To substantiate their claims they would cite the subtle curvatures of the Greek visual refinements as evidence, noting that the visual refinements somehow acted as the inverse complement of the shape of visual space counteracting its curved geometry (fig. 1).

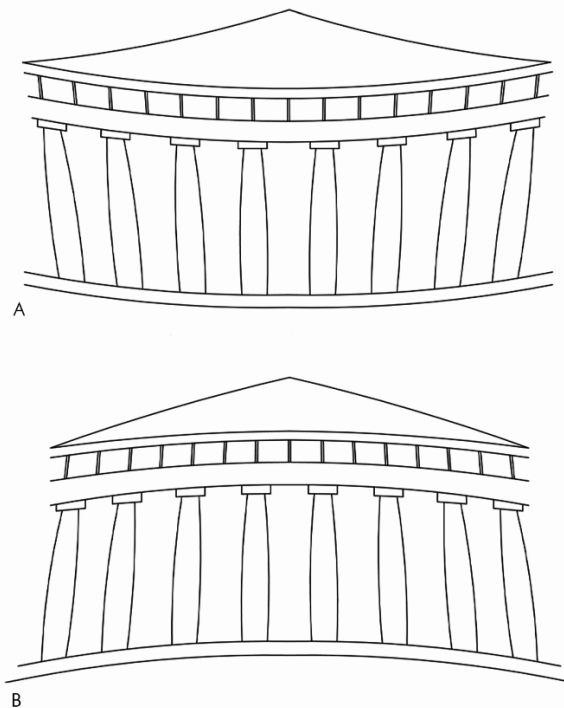


Fig. 1. When considered together the optical refinements of temple B would counter the apparent visual form of temple A to yield a temple in visual equilibrium.

Drawing by the author after [Fletcher 1967: 95]

In the course of my research several points would direct my investigation: (1) The proponents of curved visual space never demonstrate how the refinements actually corroborate their model of curved visual space; (2) The refinements do not correspond to a concise mathematical model left to us by the Greeks; (3) The temple was never meant to be perceived as a static object from a fixed point of view. My premise, by extension, asks whether the refinements could qualify a different type of visual experience; that of a dynamic or roaming point of view. Perhaps, as opposed to the conventional linear and curved methodologies taken from a stationary view point where a three-dimensional form is

projected onto a two-dimensional surface, the visual refinements might account for an alternative condition in which a four-dimensional space-time is projected onto a three-dimensional form. What follows is a mathematically-based proposal for a dynamical consideration of the visual refinements as applied to the Parthenon, considered by most scholars as the exemplary Greek Doric temple.⁵

From a Fixed Point of View

Following Vitruvius, it is generally assumed that the visual refinements were employed to compensate for some type of optical effect present in the temple.⁶ “Dipping”, “bowing”, “sagging” are some of the ways this effect has been characterized, and explanations of optical effects typically rely on visual or psychological analyses derived from interpretations of subjectively-experienced phenomena. For example, the fact that straight parallel lines might appear bent when viewed in the context of adjacent skewed or curved lines provides a possible explanation as to why entasis was introduced. Similarly, if light-colored background fields can make foreground figures seem smaller, increasing the diameters of the corner columns should therefore offset their diminished appearance “because they are sharply outlined by unobstructed air round them, and seem to the beholder more slender than they are” [Vitruvius 1960: 84]. One approach to representing an aspect of the subjective condition is by linearly projecting an image onto a picture plane from the stationary view point of the observer. Linear perspective, as it is known, would eventually be generalized by Desargues to form the foundation of projective geometry.⁷

When the Parthenon is considered from a subjective point of view, we can first ask whether it behaves according to the logic of linear perspective as Perrault assumed.⁸ Had the Greeks intended that the Parthenon be perceived from a single position or from several specific positions according to the principles of linear perspective, it would follow that a fixed point or multiple points can be located from where the visual refinements, when considered collectively, would allow the temple to appear “correct”. These visual snapshots – singular views from multiple fixed points – are predicated on establishing specified vantage points from which visual space would be linearly constructed on essentially a two-dimensional pictorial surface in space. One can easily experience the effects of perspectival diminution, for example, by viewing Andrea Pozzo’s famous ceiling and “domed” crossing in Sant Ignazio from any point other than the center of the nave. Similarly, if in three-dimensional physical space, an optically adjusted form constructed from a single point only appears correct from a single station point, then that form would seemingly appear distorted from all other points. In the case of Bernini’s windows of the corridor leading to the Scala Regia that flank the piazzeta of San Pietro, the openings will appear correct from the ellipse of the Piazza San Pietro but are clearly “distorted” rhombic shapes when viewed from the piazzeta. Hence, following this reasoning, where might these points be located on the Acropolis where the Parthenon appears “correct”?

As with most of the acropoli of ancient Greece, the approach to the Parthenon is a sophisticated and elaborately choreographed procession (fig. 2).

Emerging from the east peristyle of the Propylaea, the processioner encounters the classic three-quarters view of the Parthenon sitting on the rise off to the right where both the north and western peristyles are in equal view. From here the path to the naos of Athena proceeds along the north side of the temple, bypassing completely its eastern front to a point near the northeast corner where it turns right toward the flight of stairs leading

to the platform on which the stereobate sits. At this point, the temple can again be perceived from a three-quarters view albeit from close proximity. From here the processioner continues to a position directly to the center of the east octastyle where upon turning right a frontal view of the east elevation is obtained at close range; the steps in the stereobate leading to the naos of Athena lies directly ahead.

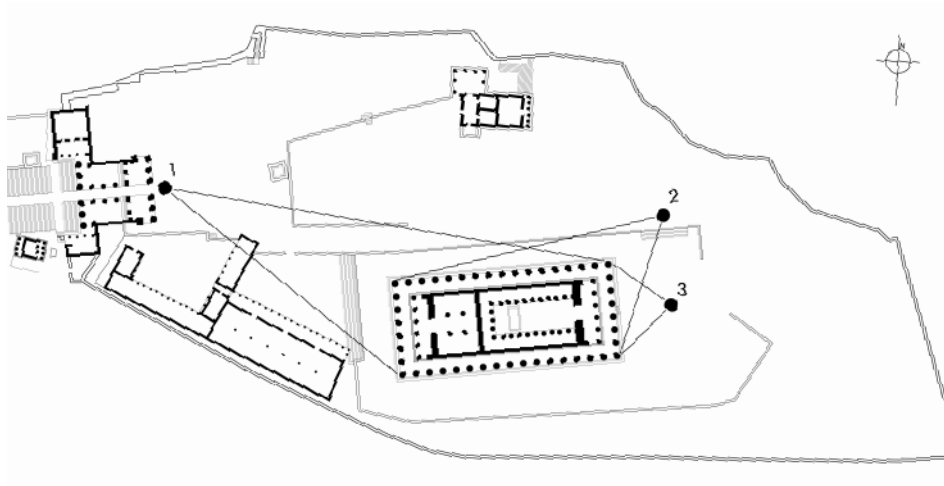


Fig. 2. Plan of the Acropolis showing the processional pathway from Propylaea to naos. Drawing by the author after the plan of the acropolis is [Scully 1991]

Along this route we can identify three specific points of reorientation where the Greek builders might have intended for the traveler to pause and visually evaluate the Parthenon before proceeding. This suggests, of course, that these might be the likely points where it is essential that the temple appear “correct” and where the visual refinements fulfill their purpose. Nevertheless, can the Parthenon appear “correct” from all three points simultaneously? Can a three-quarters view appear equally correct from close proximity and from a point six times that distance? Can the frontal view from close proximity appear “correct” with the same adjustments that make the distant three-quarter view appear correct? According to the method of linear perspective, one station point necessarily has to prevail over the others; there can be only one point where the temple will appear “correct”. Perhaps the Greek builders intended that each individual refinement be specific to one position and not another, and that the cumulative effect of the visual refinements would be no more than an average of an optically adjusted temple where the correct and the non-correct interchange depending on position. Or, possibly, the Greeks may have devised an alternate method that unified the visual refinements, a construction that is not linear at all, but curvilinear.

Curved Considerations

While linear perspectival construction offers a reliable method to represent a three-dimensional space on a two-dimensional surface, it nevertheless proves to be inconsistent with the geometry of perceived reality, for even at the time when the methods of linear perspective were being perfected, artists were already confounded by the visual distortions

within the peripheral fields of their works, especially when projecting objects within a plane onto a parallel picture plane. Leonardo demonstrated in the *Codex Atlanticus* how these geometrical inconsistencies could be demonstrated by a row of equally sized and equally spaced columns projected onto a parallel plane from a central station point; the distant columns are rendered larger than those in the foreground (fig. 3).

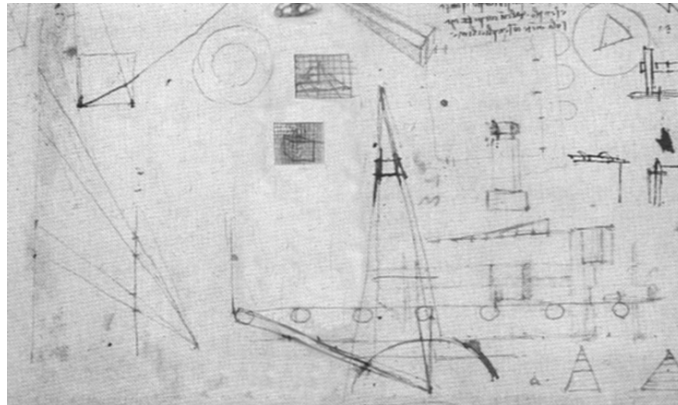


Fig. 3. Leonardo da Vinci, detail of a sketch from *Codex Atlanticus*, fol. 520 r showing the inconsistencies of linear perspective when a row of equally spaced columns are projected onto a parallel plane. The distant columns will appear larger than those in the foreground

The geometry of parallel plane projection could not reconcile these types of marginal distortions and brought into question whether distances projected onto the picture plane were isomorphic with the three-dimensional form.⁹ In response, some commentators considered instead the angular method where size and scale are determined by angular increments along a circular arc centered at the station point rather than relating lengths on a picture plane using the distance method of linear perspective (fig. 4).

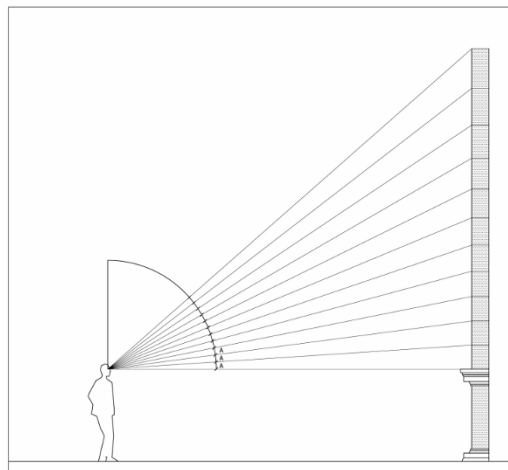


Fig. 4. Serlio's diagram of angle measure demonstrates that equal view angles correspond to increased intervals in a parallel plane projection. Drawing by the author based on [Serlio 1982]

On the basis of the angle method, subsequent writers were to argue that our visual field was in fact curvilinear, and that representing reality required acknowledging the subtle curves that describe our visual space where vertical and horizontal lines are represented as curves converging on distant points. Acknowledging the angle method proposed by Euclid in the *Optics*, these writers speculated that a rigorous system of curvilinear perspective could be constructed based on the notion that an object, when projected on the curved surface of our visual field, would occupy a portion of that surface determined by the angle limits as described by Euclid. The graphic approach, championed by Erwin Panofsky in his seminal essay, *Perspective as Symbolic Form* [1927], was first suggested by Herman von Helmholtz and was later developed by the mathematician Guido Hauck and others [Panofsky 1991: 27-36 and n. 12, 87-92]. In Hauck's method, where vanishing points were fixed on the horizon line and at points above and below the field of vision, vertical and horizontal lines would bow around the horizon line, converging on the vanishing points thereby creating a visual "shield" of convex space before the stationary station point of the observer. More recently, a rigorous analytic method was proposed by Rudolph K. Luneburg, who, in his investigation, *The Mathematical Analysis of Binocular Vision* [1947] set forth a methodology that mathematically described a non-Euclidean hyperbolic visual space based on the transformation of the visual shield as it moves between the near-space and the distant-space of the visual field. Nevertheless, in spite of these detailed graphic and analytic demonstrations, commentators continue to disagree as to whether the interpretation of the empirical results justifies the mathematical and graphic models proposed to represent curved visual space.¹⁰

While the science of curvilinear perspective is beyond the scope of this paper it should be emphasized that the proponents of curved visual space were consistent in citing the ancient Greek and Roman use of the visual refinements as evidence to support their claims. From Gombrich¹¹ to Heelan¹², writers contend that the ancient builders were the first to acknowledge the curvature of visual space, yet they never provide a demonstration nor offer a proof to support their claim. Kline attributes the curvilinear space directly to Euclid, claiming that "the Greeks and Romans had recognized that straight lines appeared curved to the eye. Indeed, Euclid said so in his *Optics*" ([Kline 1954] cited in [Brownson 1980: 181]). Based on the underlying premise that from a single point, visual rays can be projected to form a "cone of vision", Euclid's *Optics* describes the geometry of various conditions and relationships within this field of vision. Yet nowhere in the 58 propositions of the *Optics* does he state that straight lines appear curved. Perhaps the closest he comes is in his Proposition 8 where he brings into question the validity of the distance method of scaling, a key component of linear perspective. It is on the basis of this Proposition 8 that Panofsky would challenge the entire enterprise of linear perspective as nothing more than a convention for the representation of visual space, not its true geometry.¹³ Since Kline never substantiates his claim, he is most likely relying on the arguments proffered by Panofsky to support his bold statement.

Interestingly, there is another proposition in the *Optics* that states that "in the case of objects below the level of the eye which rise above one another, as the eye approaches the objects, the taller one appears to gain in height, but as the eye recedes, the shorter one appears to gain" [Brownson 1980: 177]. Proposition 15, Brownson notes, along with the associated Propositions 16 and 17 stand as unrelated oddities among the other Propositions, as they are predicated on "the special arrangement of objects which are not mentioned in the statement" [Brownson 1980: 177]. However, let us consider a stack

equally-sized objects one above the other in a position below the level of the eye. As we approach the stack we would anticipate that the ones at the top, closer to our eye, would increase to a greater extent than those lower in the stack because relatively the increase in their angular measure is greater. Similarly, when receding, we might anticipate that the inverse will occur; the lower objects would “appear to gain” since the angles of the upper objects are decreasing more rapidly. But in fact Euclid’s proposition only holds in certain positions of the viewer, for with a constantly moving view point, or roaming point, the rate of angular change varies. In some positions within what I will call the *dynamic zone*, some view angles are increasing while others are decreasing.

Roaming Point Perspective

To get a sense of a roaming point of view, consider the phenomenal effect of approaching a large upright planar surface such as a uniform brick wall. As we advance forward, the wall begins to fill our cone of vision as the view angles of the individual bricks increase. At the point where we enter the dynamic zone the wall expands beyond the limits of our visual cone and we begin to notice that the peripheral edges of the wall seem to snap back at a rapid rate in the direction of our movement. As our eyes near the surface of the wall the rate of “snapping” accelerates until the periphery settles into the plane of the wall as our eyes become coplanar with it. Reversing the movement produces the opposite effect; as we back away the periphery snaps forward to embrace us and as we reach the distant zone where the wall is comfortably contained in our cone of vision, the periphery again settles into a flat plane. The impression is that of a “visual shield” similar to that described by Panofsky that pulses in and out depending on the direction of our movement.

Consider a line m on which three distinct points O , a and b are equidistantly spaced at a distance D from each other (fig. 5).

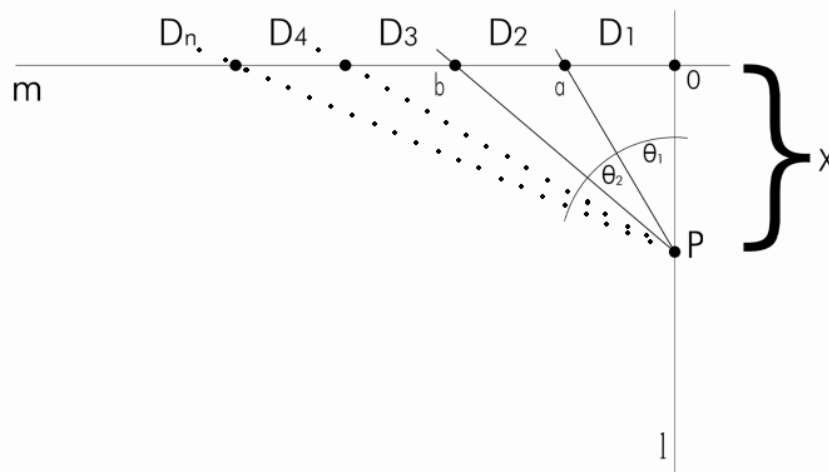


Fig. 5. As P moves away from O along l , θ_1 continuously decreases from 90° . At the same time, however, θ_2 increases until the point P reaches $x = \sqrt{2}D$ when it too decreases as x approaches infinity

Construct a line l perpendicular to m at O . Now, place a point P on l through which lines extended from a and b will pass. Clearly the angles θ_1 and θ_2 formed at P will vary in size, with θ_1 being the larger and θ_2 the smaller. We can also surmise that as P moves along l , both the angle measure of θ_1 and θ_2 will vary as well depending on the position of P . For example when P lies at O , $\theta_1 = 90^\circ$ while $\theta_2 = 0^\circ$. Similarly, as P approaches infinity on l , angles θ_1 and θ_2 will approach zero. Between the two limits let us allow P to “roam” along l , maintaining a distance x from the point O . As P moves away from O on l , θ_1 diminishes continuously from 90° to 0° as x approaches infinity. However, angle θ_2 increases as x increases near O , and reaches a maximum value before it too diminishes to 0 as x approaches infinity. Using right triangle trigonometry, we see that $\tan \theta_1 = D/x$, and for $n=1,2,3 \dots$,

$$\theta_n = \tan^{-1}\left(\frac{nD}{x}\right) - \tan^{-1}\left(\frac{(n-1)D}{x}\right). \quad (1)$$

Taking the derivative $d\theta_n/dx$ and setting it equal to zero, we find that the maximum value of θ_n , for each $n = 2, 3, \dots$, is

$$\theta_n^{\max} = \tan^{-1}\sqrt{\frac{n}{n-1}} - \tan^{-1}\sqrt{\frac{n-1}{n}}, \quad (2)$$

which occurs at the x value

$$x_n^{\max} = \sqrt{n(n-1)}D. \quad (3)$$

As a result of our roaming view point, the apparent shifting of the visual shield between convex and concave according to the “snapping” of the peripheral limits of the wall follows this specific mathematical behavior. We can chart the values of the magnitude of the view angles θ_1 through θ_4 relative to the position of P according to the diagram in fig. 5 and plot the angle change as x increases (fig. 6a). Graphing these values enables us to readily observe how the view angle θ_1 continuously decreases as x approaches infinity, as we would expect, while at the same time θ_2 rapidly increases to a maximum at $\sqrt{2}D$ before it too decreases as x approaches infinity (fig. 6b).

Similarly angles θ_3 and θ_4 initially increase, albeit at slower rates and to lesser magnitudes than θ_2 , before they too decrease. Moving within the dynamic zone, the region in which some view angles are increasing while others are decreasing, accounts for the “shield” effect as experienced by the “snapping” of the periphery of the wall. In fact, when we approach the wall within the dynamic zone, it is not the periphery that is snapping back, but the center that is surging forward as its view angles open more rapidly compared to those on the periphery. The apprehension of the visual shield, convex on approach, concave when receding, disappears when we either move outside the dynamic zone or when the viewer stops and the view angles are no longer changing. Euclid correctly understood in his 15th Postulate that those objects closer to the viewer would appear to get larger more rapidly when approached. In the dynamic zone their view angles are actually increasing while those of the more distant objects are diminishing. What Euclid does not account for, however, is the fact that within the near zone, the closer objects are actually visually *decreasing* at a greater rate than the distant objects, and in the distant zone all the objects appear to gain at a perceptually equivalent rate.

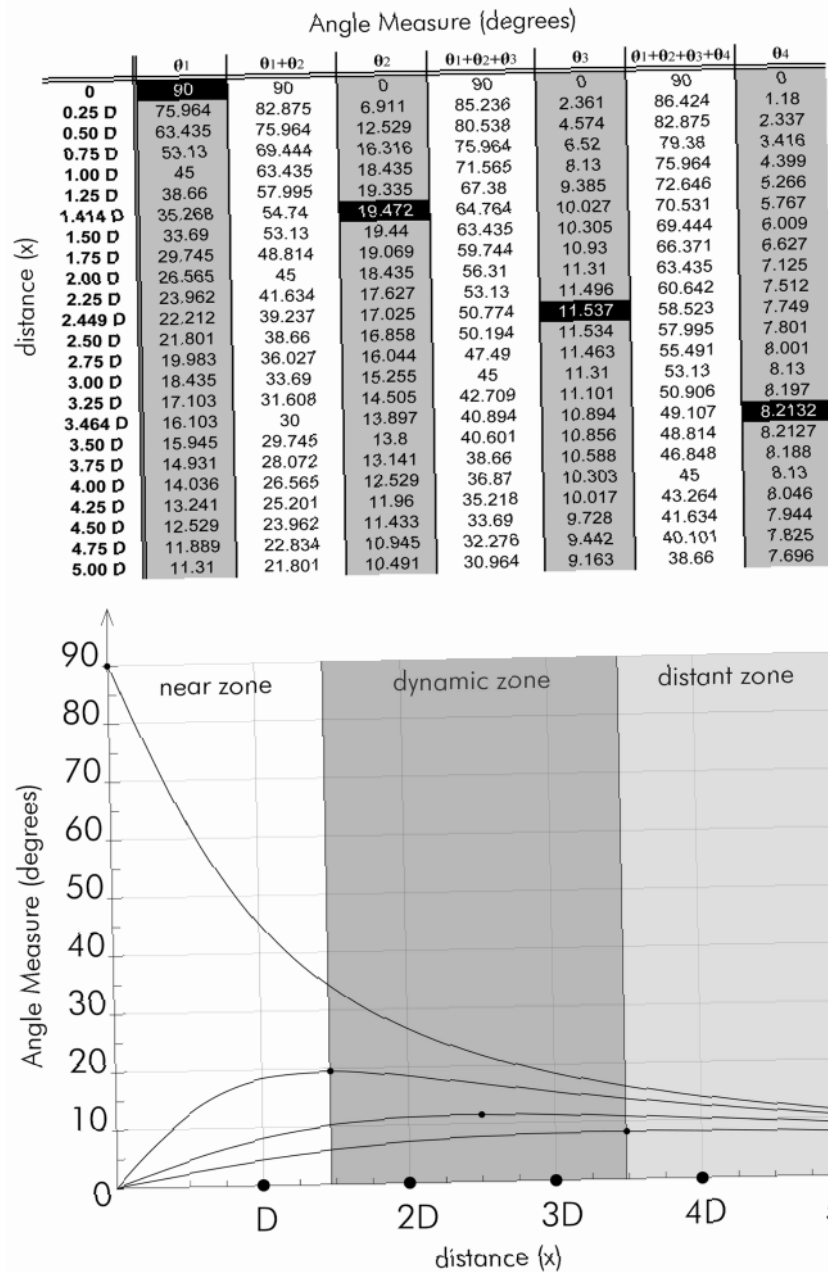


Fig. 6a (above) and 6b (below). Schedule of angle sizes for varying values of x ; $\theta_{\max}$ as a function of the position of P . In the dynamic zone some view angles are increasing while others are decreasing

Returning to the Parthenon, let us apply the mathematics of the dynamic zone to the visual refinements, keeping in mind that the procession from Propylaea to Naos allows for both a lateral and frontal apprehension of the temple's form. If we now consider figure 5 and locate the columns of the Parthenon tangent to points adjusted to the distances between the columns, we can intuit that the reducing of any of the intervals between the columns will exaggerate the convex shield effect as we approach the temple causing the periphery to recede more quickly (fig. 7).

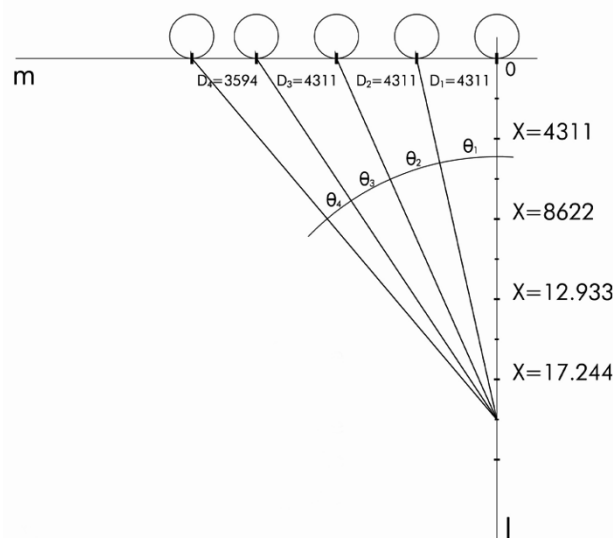


Fig. 7. Intercolumniation of the Parthenon: At $x = \sqrt{2} D$, θ_4 is at its maximum. With the shorter distance for D_4 , P will be closer to O when θ_4 is at its maximum compressing the dynamic zone

Mathematically, when an interval D_n is smaller, the $\tan^{-1} \theta_n$ becomes smaller placing P closer to O when $\theta_{\max}$ is reached as might be expected since the temple front is smaller. The intercolumniation intervals of the Parthenon are equal except at the corners, although variations can be observed particularly along the lateral elevations.¹⁴ These irregularities in the column spacing Dinsmoor attributes to construction error rather than deliberate intention [Dinsmoor 1974: 178]. However, if the incremental decreasing of the column spacing were employed, the net effect would be the compression of the dynamic and near zones and a shortening of the duration of the visual “snap”. The use of decreased spacing intervals within centralized colonnades would later become more common particularly in the orders with uniform friezes.

Inclining the peristyles provided another method by which the Greek builders could compress the duration and magnitude of the dynamic and near zones. Here they could retain constant vertical measurements, yet each visual angle measure would appear decreased in comparison to its complement on a perpendicular flat surface (fig. 8).

The slight backward tilting of the columns and entablature leads to a reduction of both the near and distant visual angles and causes the maximum view angles to be reached closer to the façade, thus shortening the duration of the shield effect (fig. 9).



Fig. 8. The inclination of the peristyle of the Parthenon. Photo by the author

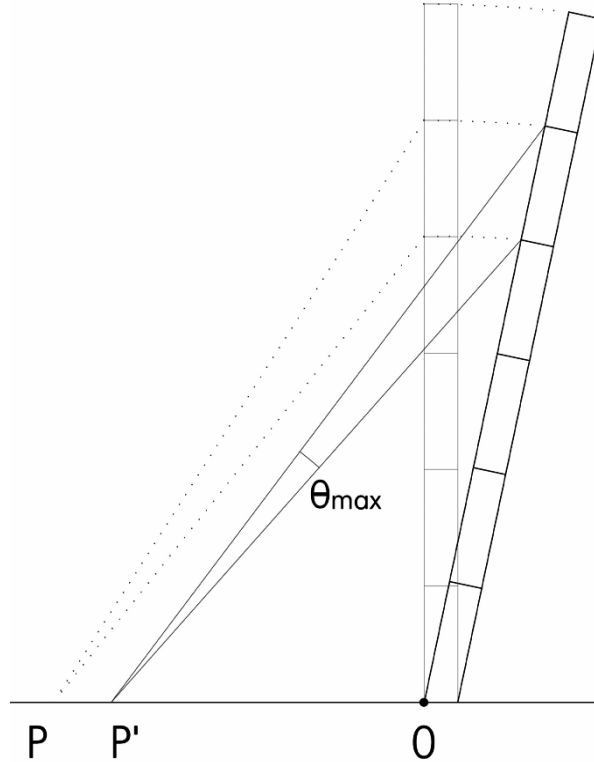


Fig. 9. With the inclination of the peristyle, $\theta_{\max}$ is reached closer to O . As a result, the dynamic zone is compressed

By drawing the far zone closer to the façade, the impression of the snapping back of the entablature is compressed. As with intercolumniation where the intervals D_n are incrementally decreasing from the center, or when the line m is inclined, as in the Parthenon, the mathematical expressions are more complex than (2) and (3) above. In both cases, however, the position of P at $\theta_{n(\max)}$ moves closer to O reducing the domain and magnitude of the dynamic zone, which thereby reduces the acceleration of the snap.

Entasis accomplishes a variety of visual purposes and has, through the centuries, been accomplished in a number of ways from a simple tapering of the column shaft to an exaggerated bottle-shape found in Renaissance and Baroque buildings. The entasis of the typical Parthenon column is a simple linear taper from a base to a point about one-fifth the 10.5M height of the column at which point a gentle curve blends a slightly more inclined taper to the necking ring beneath the fillets (fig. 10).



Fig. 10. Column entasis of the Parthenon in the context of adjacent columns. Photo by the author

From any distant station point, especially from the most prominent view at the Propylaea, the entasis of the columns in the west and north peristyles is quite pronounced. The increased visual space between the columns at the top lightens the visual mass of the entablature while the complementary increase of the girth at the base firmly anchors them to the stylobate.

From a roaming point of view, entasis accomplishes two visual purposes, one involving perpendicular movement and the other parallel movement. Approaching the east peristyle perpendicularly, the upward taper of the columns facilitates the visual inclination of the façade, and helps to compress the dynamic zone. Ascending the stereobate and passing between the columns involves a movement where the viewer's position approaches and then recedes from the line of the columns. The impression is one of column sway. This phenomenon is especially pronounced along the extended path past the north peristyle where the processioner is moving parallel to the elevation. Here, the sway of the columns is directional, leaning forward as they are approached, swaying back into the vertical when alongside, and continuing their forward sway as they are passed. Again, their upward taper compresses the period and magnitude of the dynamic zone, buffering their visual movement, the "pushing" and "pulling" of the capitals.

Perhaps the most dramatic effect can be experienced in the downward curvature of the stylobate which is noticeably curved even when standing at a stationary position beside it (fig. 11).



Fig. 11. The curvature of the stylobate of Temple of Apollo at Selinute, Sicily. Photo by the author

The stylobate of the Parthenon rises 60mm at the center along the east and west fronts, and 110mm along the lateral faces so that from the center of the façade; the distant view angles are smaller as a result of the gentle downward curvature. In ascending the stereobate to enter the Pronaos, the upward “snap” of the corners is affectively countered, the result of a concave visual shield since in this case the viewer’s eye is moving away from the surface plane of the stylobate. Simultaneously, the downward curvature of the entablature, most likely is the consequence of the curved stylobate supporting columns of equal length, counters the convex shield as the viewer ascends to the stylobate, slowing down the upward snap of the peripheral columns.

Conclusion

In our study we have assumed that the visual refinements were intended to counteract the geometry of visual space, acting as its inverse complement. We have noted some of the deficiencies of the linear and curved systems of perspectival representation to account accurately for the refinements, emphasizing that each of the methods rely on a stationary point to project a three-dimensional form onto a two-dimensional surface. As the formal

system of the visual refinements was reinterpreted over the centuries, artists and commentators took often opposing positions as to the mathematical explanation of the tapers and curves that account for the optical corrections. While it is certain that the Greek builders intended to correct optical inconsistencies, and that the refinements provide evidence of a curved visual space is justified, the mathematics of that space remains speculative. As the art historian James Elkins writes, “Subjective curvature may be unique to the history of art in that it is a genuine unsolved scientific problem. It is the only case I know in which real, ongoing science enters into art history. The analytic question of subjective curvature is an unsolved scientific problem” [Elkins 1994: 186]. As with the many other accounts of visual space, Roaming Point Perspective offers a possible explanation for the geometry of visual space, and it too has its foundations in the Greek visual refinements.

Notes

1. [Vitruvius 1960: 89]. The curvature of the stylobate was accomplished by the use of leveling blocks of varying dimensions that incrementally decrease in “small steps” (*scamilli impares*) toward the center to generate the upward curvature of the surface; see [Panofsky 1991: fig. 11, 89].
2. [Alberti 1988: 189-218]. Throughout the first nine chapters of Book VII, Alberti prescribes many of the same refinements that were discussed by Vitruvius as necessary for visual harmony. Some 400 years later, Le Corbusier would cite the refinements of the Parthenon as the ultimate expression of a “plastic” architecture – a “pure creation of the mind” [Le Corbusier 1970: 185-207].
3. Helmholtz was especially critical of entire a priori justification of Euclidean geometry claiming it was inconsistent with the actual facts of perception; see [Körnigsberger 1906: 254-266].
4. For a concise explanation of Luneburg’s mathematics of hyperbolic visual space, see [Heelan 1983: appendix, 281-319].
5. Scholars have not adopted universally accepted measurements for the Parthenon. Accordingly, I have relied on the measurements referenced in [Dinsmoor 1974].
6. Scholars have ascribed the origin of curvature of the stylobate with the functional necessity of accommodating the drainage of rainwater. This interpretation, while plausible, is not consistent with all temples, however. For example, to the east porch of the Erechtheum does not exhibit any curvature at all [Rhodes 1995: 76]. Another interpretation provided by Mavrikios [1974] claims the downward pitch at the corners of the stylobate and the entasis of the columns as a means of accommodating the visual weight of the temple. As he sees it, the Greeks employed principles of empathy to visually respond to the transference of the temple’s gravitational weight to its foundation..
7. For a summary of the development of projective geometry from perspective, see [Klein 1970: ch. 14, 285-301].
8. [Perrault 1993: 162]. Perrault aims his criticism at the fact that distortion can only be corrected from a single point of view, leaving all other viewpoints compromised. “Even when the judgment of sight might not be able to prevent the distance and position of objects from deceiving us, the alteration of proportions is still not a good remedy for this supposed effect, because the effect of alteration is only at a given distance and only if the eye does not change position. There are optical figures whose proportions are modified in such a way that their effect is only favorable only if they are viewed from a specific location, these proportions, like those of optical figures, will also appear totally defective as soon as the viewer changes place, because an aspect that is oblique when a person is near becomes progressively less so as he moves away.”

9. It should be noted that the most exaggerated visual distortions in perspectival projection occur outside the normal field of vision and are therefore not readily detectable. Nevertheless, when following the methods of linear perspectival construction, these inconsistencies will appear within the field of vision as well as Leonardo shows.
10. [Elkins 1994: 181-216]. Elkins provides a good commentary of the controversy surrounding curved perspective which includes an historical overview and examples of models that have been proposed.
11. [Gombrich 1960: 258]. Gombrich states, "It is perhaps significant that the prime argument for this claim of a curvilinear world is taken from architecture and not painting. The Greeks allegedly introduced the so-called 'refinements' of deviation from rectangularity in their temples to correct the distortions of vision."
12. [Heelan 1983: 29]. According to Heelan, "the relationship of the [Greek] refinements to the near zone of hyperbolic visual space is an intriguing one."
13. [Panofsky 1991: 35-36]. Euclid's Proposition 8 states that "equal and parallel magnitudes at an unequal distance to the eye are not seen proportionally to the distances." Accordingly, Panofsky argued that Euclid recognized that scale is determined by view angle and not by distance thus countering the central premise of linear perspective and thereby providing support for curvilinear constructions.
14. [Lawrence 1973: 173]. Lawrence includes a measured sketch of the north peristyle.

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About the author

Michael C. Duddy is a founding principal of Designgroup International, an architectural practice with offices in New York and Shanghai specializing in urban mixed-use buildings. He is currently researching aspects of the history of the relationship between the built environment and the disciplines of epistemology and metaphysics. He is a guest critic at the School of Visual Arts in New York City.