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THE MINOR SIXTH (8:5) IN EARLY GREEK HARMONIC SCIENCE*

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These efforts to ground the progression 25, 40, 64 in Pythagoreanism are unavailing. W. A. Heidel ("The Pythagoreans and Greek Mathematics" in R. E. Allen, D. J. Furley, *Studies in Presocratic Philosophy*, I [London 1970]) has shown that there is no sound evidence that the early study of whole numbers and geometric proportionality was either characteristically or even essentially Pythagorean. This impugns the credibility of Mosshammer's denial that Diogenes

* I gratefully acknowledge the support of an Andrew Mellon Postdoctoral Fellowship (1977-78) from the University of Pittsburgh, which enabled me to prepare this paper.



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The following remarks are addressed to A. A. Mosshammer's "Geometrical Proportion and the Chronological Method of Apollodorus," (*TAPA* 106 [1976] 291-306). Mosshammer seeks to establish that Apollodorus used the progression 25, 40, 64 as a theoretical adjunct to his chronological method. His argument proceeds in two stages. It is sufficient to observe of the first that there seem to be only two instances in which Apollodorus *may* have synchronized ages of 25, 40 and 64 years. The second stage is a defence of the claim that the source of this progression is early Pythagoreanism. Mosshammer remarks that the study of square numbers and geometric proportionality lay at the heart of Pythagorean mathematics. Accordingly, since the sequence 25, 40, 64 is a geometric progression between the squares of 5 and 8, he reasons that it probably derives from the Pythagorean school (303-4). Furthermore, he conjectures that Apollodorus borrowed this particular sequence from the Pythagorean doctrine of the four ages of the complete life. This requires reconstructing the doctrine (80 years: 0-25, childhood; 25-40, youth; 40-64, maturity; 64-80, old age) and positing that Apollodorus' immediate source was Aristoxenus (302-5). In a footnote to his paper (305 note 42), Mosshammer also suggests that the ratio 8:5 (40:25, 64:40) may have been important in Pythagorean harmonic science.

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Laertius reports the true doctrine of the complete life (80 years: 0–20, childhood; 20–40, youth; 40–60, maturity; 60–80, old age) and his contention that Apollodorus knew it in its original form. Thus, connection of the sequence 25, 40, 64 with Pythagoreanism is possible only if the minor sixth (8:5) figures prominently in Pythagorean *ᾠμονικῇ*. But this interval is of peripheral importance in the early Greek science of music. Since the ratio (8:5) is not derivable from the *τετρακτύς* of the decad by composition and division of 2:1, 3:2 and 4:3, the original Pythagoreans did not even consider the minor sixth to be a melodic interval. When this interval does appear in musical theory, it is introduced as a consequence of an arithmetic division of the fifth or, assuming that Eratosthenes admitted it, a harmonic division. Before I demonstrate these claims about the minor sixth, I should like to correct the errors of long standing which Mosshammer repeats while inquiring about harmonic science.

Any who doubt that the musical ratios are all of greater inequality, i.e., that the antecedent or first term in each is greater than the consequent or second term, should consult Archytas *DK* 47 B 2. This Fragment, which begins *μέσαι δέ ἐντι τρις τᾷ μουσικᾷ*, requires that the ratios be of this form if the assertions about the three means are to be true. Accordingly, the ratios assigned to the octave, fifth, fourth and minor sixth, must be 2:1, 3:2, 4:3 and 8:5, and not 1:2, 2:3, 3:4 and 5:8, respectively, as Mosshammer and others would have them. The same conclusion is also evident in the very nomenclature and definitions of the musical ratios found throughout the ancient literature. For example, the ratio of the fourth is called *ἐπίτριτος* (sesquitertius), which signifies that the first term is four thirds the second.

The invariancy of the form of the musical ratios is established without reference to what these ratios represent. In general, those who write the ratio of the fourth as 3:4 or 3/4 maintain that all the musical ratios are of lesser inequality because they represent the relative lengths of vibrating strings or sonant pipes and because variation in the pitch of the sound produced by a given string or pipe requires division of its length (other factors remaining constant). This inference, however, is a non sequitur: the inverse variation of the pitch of a sonant string or pipe and its effective length

is an empirical principle which enables the interpretation or determination of musical ratios; it makes no requirement of their form. One should also notice that the assignment of the ratios used in harmonic theory to the lengths of strings or pipes finds no support in ancient testimony.¹ The Greek *ἀρμονικοί* were unanimous in contending that the ratios be assigned to sound, specifically, to melodic intervals; the only controversy in this was whether to correlate the greater of the numbers in the ratio with the higher or with the lower of the pitches defining the interval.

When musical intervals are represented by means of ratios, the addition and subtraction of these intervals corresponds to the composition (*σύνθεσις*) and division (*διαίρεσις*) of their ratios, respectively. For purposes of rough calculation, these operations may be likened respectively to the multiplication and division of fractions. Accordingly, the interval that lies midway between the octave and the fourth is specified by the equation

$$(2/1)/(x/y) = (x/y)/(4/3)$$

which, in modern notation, has the simpler form

$$(x/y) = (2\sqrt{2}/\sqrt{3}).$$

Since the ratio of this interval, $x:y$, is not of whole numbers, no theorist who analysed musical intervals by means of *λόγοι τῶν ἀριθμῶν* would admit that there is a melodic interval that halves the difference of an octave and a fourth, i.e., the fifth. Indeed, there is early proof deriving from the Pythagorean school that intervals, such as the fifth, which are represented by superparticular ratios cannot be partitioned into any number of equal subintervals because the terms of these ratios admit no number of geometric means (see Archytas *DK* 47 A 19; Euclid, *Sectio Canonis*, [ed. K. von Jan], Prop. 3; = Porphyry, *In Ptolemaei Harmonica Commentaria*, [ed. I. Düring], 99, 15-26). Mosshammer, therefore,

¹ The use of the monochord with movable bridge does not of itself afford evidence that the musical ratios represent the effective lengths of vibrating strings. Measurements on the monochord served only as an empirical basis for the assignment of numerical quanta to melodic relations, i.e., to musical sound. The justification for this use of the *κανών* was developed in Greek acoustical physics.

should not confuse the interval sounded by the string whose length is the arithmetic mean of those lengths required to produce the octave and the fourth with the interval that lies half-way between the octave and the fourth. The latter interval is impossible in Pythagorean musical science. The former interval is the minor sixth (8:5); it extends a minor third (6:5) beyond the fourth and falls short of the octave by a major third (5:4).

Consider now the question of the status of the ratio (8:5) in the Pythagorean harmonic science that dates from the late fifth century B.C. to the time of Apollodorus. One should not expect that this ratio was recognized as melodic by every school of Pythagorean musical theory. For example, those who sought to derive all the musical ratios from the *τετρακτύς* of the decad by compounding and dividing the ratios of the primary and most familiar intervals, the concords of the octave, fifth and fourth, would find the minor sixth unascertainable. The derivation of the minor sixth from the octave, fifth and fourth—a process of deduction which is the true significance of the often repeated claim that Pythagoras and his immediate followers studied only these three basic concords—requires additional musico-mathematical techniques. There is reason to believe that these were supplied by Archytas in the early fourth century B.C.

Both Tannery (*Mémoires Scientifiques* [Paris 1915] III, 78-81, 110-4, 234-37) and Winnington-Ingram ('Aristoxenus and the Intervals of Greek Music,' *CQ* 26 [1932] 206-7) have argued persuasively that Archytas defined the genera of the tetrachord by a procedure involving the division of the fifth (3:2) into a minor third (6:5) and a major third (5:4), and of the fourth (4:3) into a septimal third (7:6) and a major tone (8:7). Though this is not what Ptolemy reports (*Harmonicorum Libri Tres*, [ed. I. Düring], I.13), it is a superior account for this reason: the divisions of the concords of the fifth and fourth follow immediately as applications of Archytas' definitions of the musical means (*DK* 47 B 2). The fifth is partitioned according to an arithmetic mean ($3:2 = 6:5:4$) and the fourth, according to a harmonic mean ($4:3 = 28:24:21$). Since the octave exceeds the fourth by a fifth [$(2/1)/(4/3) = (3/2)$], this means that the minor sixth (8:5) may be derived from the fifth by transposing the arithmetic division

of the fifth downwards by a fourth if the greater number in each ratio is assigned the higher pitch, or upwards by a fourth if the greater number is correlated with the lower pitch. In other words, the minor sixth is derivable from the fifth because it is a major third less than an octave $[(2/1)/(5/4) = (8/5)]$ and a minor third more than a fourth $[(8/5)/(6/5) = (4/3)]$.

This account of the minor sixth finds support in Ptolemy's description of Archytas' enharmonic division of the octave (*Harmonicorum*, II.14). The division, which is given in the sequence of ratios

$$5:4 \quad 36:35 \quad 28:27 \quad 9:8 \quad 5:4 \quad 36:35 \quad 28:27$$

[where $(4/3) = (5/4) (36/35) (28/27)$], presents the minor sixth as that melodic interval composed of the subintervals represented in the sequence

$$36:35 \quad 28:27 \quad 9:8 \quad 5:4 \quad 36:35 \quad 28:27$$

[where $(6/5) = (36/35) (28/27) (9/8)$].

Ptolemy, apparently, knew of no other theorist who lived prior to the time of Apollodorus and explicitly recognized the minor sixth as a melodic interval. This does not mean that Archytas and his followers were unique in their acceptance of this interval. Ptolemy restricts his attention to the octave divisions of his predecessors. Such a restriction, however, is in all likelihood inaccurate historically for those theorists who lived after the fourth century B.C. For it was during this time that scales of a double octave magnitude, i.e., the Greater Perfect System, were constructed to facilitate the analysis of melody. Thus, if one takes the chromatic division of the octave that Ptolemy attributes to Eratosthenes (*Harmonicorum*, II.14)

$$6:5 \quad 19:18 \quad 20:19 \quad 9:8 \quad 6:5 \quad 19:18 \quad 20:19$$

[where $(4/3) = (6/5) (19/18) (20/19)$, and $(5/4) = (19/18) (20/19) (9/8)$], and extends it in the pattern of the Greater Perfect System by adding a tetrachord (6:5, 19:18, 20:19) to the left and a pentachord (6:5, 19:18, 20:19, 9:8) to the right, the minor sixth is readily ascertained in the octave

$$6:5 \quad 19:18 \quad 20:19 \quad 6:5 \quad 19:18 \quad 20:19 \quad 9:8.$$

Inspection of the pentachord (6:5, 19:18, 20:19, 9:8) discloses that this time the minor sixth is derived from a harmonic division of the ratio of the fifth ($3:2 = 15:12:10$).

The minor sixth (8:5) is of little importance in the history of early Greek harmonic science. The original Pythagoreans did not admit it into their analysis of musical relations because the ratio (8:5) cannot be ascertained by manipulating the ratios of the elementary concords of the octave, fifth and fourth. There was, in their view, no way of connecting this interval with the *τετρακτύς* of the decad; hence, it could not be melodic. When the minor sixth is recognized as an interval that might serve in proper or tuneful melody (*ἡρμοσμένον μέλος*), it is explained as the consequence of an arithmetic division of the fifth or, if Eratosthenes recognized it, a harmonic division. This acceptance of the minor sixth was a lasting innovation in musical theory. Both Didymus and Ptolemy (*Harmonicorum*, II. 14, 15) retain it in their analyses of melodic relations.

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