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Gillings, R.J.

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In: The Mathematical Gazette, XXXIX, 1955

p. 187 - 190

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GILLINGS, R.J.
1955

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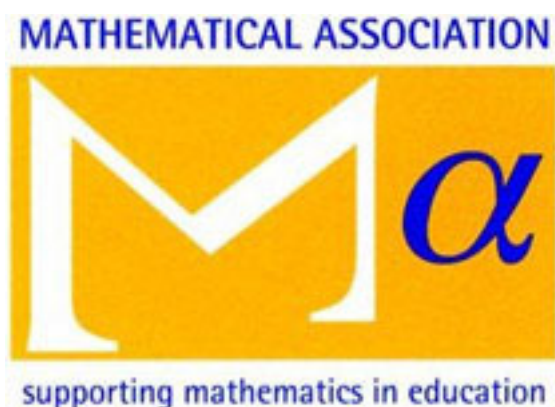
Math
Plimpton 322

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Mathematical Gazette
39 1955 187-190

VV1207
H Ref.

Subject: Plimpton 322.



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Reviewed work(s):

Source: *The Mathematical Gazette*, Vol. 39, No. 329 (Sep., 1955), pp. 187-190

Published by: [The Mathematical Association](#)

Stable URL: <http://www.jstor.org/stable/3608744>

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THE ORIENTAL INFLUENCE ON GREEK MATHEMATICS

BY R. J. GILLINGS.

The problem of finding rational integral numbers which could be made the sides of right triangles, is said by Proclus to have been solved by Pythagoras, by a method which is equivalent to the use of the formula,

$$m^2 + [\tfrac{1}{2}(m^2 - 1)]^2 = [\tfrac{1}{2}(m^2 + 1)]^2$$

where m is any odd integer.

Heath (1) quotes Proclus' statement as follows :

"The method starts from odd numbers. For it makes the odd number the smaller of the sides about the right angle ; then it takes the square of it, subtracts unity, and makes half the difference the greater of the sides about the right angle ; lastly it adds unity to this and so forms the remaining side the hypotenuse."

Another method attributed by Proclus to Plato depends on the formula

$$n^2 + [(\tfrac{1}{2}n)^2 - 1]^2 = [(\tfrac{1}{2}n)^2 + 1]^2$$

where n is any even number.

Again quoting Heath's (1) rendering of Proclus :

"But the method of Plato argues from even numbers for it takes the given even number and makes it one of the sides about the right angle : then bisecting this number and squaring the half, it adds unity to the square to form the hypotenuse, and subtracts unity from the square to form the other side about the right angle."

Coolidge (2) gives the Platonic formula as,

$$(2n)^2 + (n^2 - 1)^2 = (n^2 + 1)^2$$

remarking that there are few strictly mathematical passages in Plato's works, and that while Plato was certainly familiar with the Pythagorean theorem, "the only direct mention of it is in the special case where we have an isosceles right triangle, which appears at length in Meno 82-4."

How did the Greeks arrive at these formulae or methods?

From the sources, it is by no means clear to what extent the equivalent of what we may call theoretical algebra had developed with the Pythagoreans. Various conjectures, plausible enough no doubt, have been offered, as for example, Bretschneider (Heath I p. 358) (1), that Pythagoras had deduced his method inductively by observing the array of numbers,

	1	2	3	4	5	6	7	8	9	10	11	12	13...
Their squares	1	4	9	16	25	36	49	64	81	100	121	144	169...
Their differences	3	5	7	9	11	13	15	17	19	21	23	25...	

and noting which numbers in the difference row are perfect squares. Or again, Treutlein's suggestion (Heath I p. 358), (1) is that it came from the consideration of gnomons, and the arrays of dots commonly referred to as "figurate" numbers,

$$\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

where in the diagram, it is clear that for this case, the gnomon is a square number, and thus $4^2 + 3^2 = 5^2$. Similar explanations have also been made to

explain the source of the Platonic formula. Both are deducible as particular cases from Euclid's proposition 28 lemma 1 of Book X of his *Elements*, which may be written as

$$p^2q^2 + [\tfrac{1}{2}(p^2 - q^2)]^2 = [\tfrac{1}{2}(p^2 + q^2)]^2,$$

the proof of which is given by Euclid in geometrical form. It has also been maintained that Pythagoras obtained his theory from India, (Bürk, Heath, p. 360), (1) where the theorem was said to have been known and proved in its generality, as early as the sixth century B.C.

However, the extent to which Greek mathematics was indebted to the work of the Babylonians over 1000 years earlier has yet to be adequately determined, and the problem of how the Babylonians determined these Pythagorean triads may shed some light on the question. A Babylonian clay tablet written in cuneiform, (-1900 to -1600) now in the Plimpton collection of Columbia University N.Y., referred to as Plimpton 322, contains a list of 15 triads, some of which are large and unusual numbers. The list is as follows, where l , b and d are connected by the relation $l^2 + b^2 = d^2$.

line	l	b	d
1	120	119	169
2	3456	3367	4825
3	4800	4601	6649
4	13500	12709	18541
5	72	65	97
6	360	319	481
7	2700	2291	3721
8	960	799	1249
9	600	481	769
10	6480	4961	8161
11	60	45	75
12	2400	1679	2929
13	240	161	289
14	2700	1771	3229
15	90	56	106

So far, no procedure texts have been found which give an indication of the method by which the Babylonian scribe arrived at these numbers, and the interest of the historians of mathematics has been excited to determine just how the Babylonian scribe accomplished it. Now, in addition to the numbers above, the tablet has a column of numbers giving the values of d^2/l^2 for each line, which suggests that the Babylonian Mathematician was concerned to list his triads in a certain order, an order which was determined by a regular changing of the shapes of the right triangles to which the numbers referred. By itself, this does no more than adumbrate the theory behind the method. But there are further indications which arise from the accidental circumstance that the scribe who wrote the tablet committed four errors. An examination of these errors is very enlightening. The first is a mere scribal error, where an extra wedge was inadvertently put in, a 9 written for an 8. The second error is the writing of 53 instead of 1,46 (106 in decimal notation), the scribe having omitted to multiply by 2, and the third error is the writing of 7, 12, 1 (25, 921 in decimal notation), instead of 2,41 (161 in decimal notation), that is, the scribe wrote the square of the number instead of the number itself. Thus to calculate the numbers of the tablet, the doubling of numbers, and the recording of squares, from their abundant table texts, must have been part of the

procedure. But the committal of the fourth error in line 2, where the scribe wrote 3, 12, 1 (11,521 in decimal notation), for 1, 20, 25 (4,825 in decimal notation), has given students of Babylonian mathematics much food for thought, and an intelligible explanation of how the scribe came to make this error must shed light on the ultimate algebraic theory by which the Pythagorean triads were determined. Error number four is nothing like as simple as the other two errors. Neugebauer had written of it (3) :

“ It seems to me that his error should be explicable as a direct consequence of the formation of the numbers of the text. This should be the final test for any hypothesis advanced to explain the underlying theory.”

The explanation or conjecture which follows here, and which has withstood examination by Neugebauer himself, shows that it is probable that the scribe was using the equivalent of the formula,

$$[2pq]^2 + [p^2 - q^2]^2 = [p^2 + q^2]^2$$

where p and q are integers prime to one another, are not simultaneously odd, and p is greater than q .

For the numbers of the second line the scribe chose $p=1,4$ (64 in decimal notation), and $q=27$, and the values for l and b in the equation, $l^2 + b^2 = d^2$, become,

$$\begin{aligned} l &= 2pq &= 2 \times 1, 4 \times 27 \\ & &= 57, 36 \\ & &= 3456 \text{ in decimal notation.} \\ b &= p^2 - q^2 = (1, 4)^2 - 27^2 \\ & &= 1, 8, 16 - 12, 9 \\ & &= 56, 7 \\ & &= 3367 \text{ in decimal notation.} \end{aligned}$$

For d , the substitutions give,

$$\begin{aligned} d &= p^2 + q^2 = (1, 4)^2 + 27 \\ & &= 1, 8, 16 + 12, 9 \\ & &= 1, 20, 25 \\ & &= 4825 \text{ in decimal notation. (4)} \end{aligned}$$

Thus the scribe obtained the primitive triad 3456, 3367, 4825; truly a remarkable performance, over 1000 years before Pythagoras is said to have celebrated the discovery of the similar relation between the numbers 3, 4 and 5, by the roasting of an ox.

The explanation of the error (4) is that the scribe, in determining the value of $d = p^2 + q^2$, reversed the order of squaring and adding and thus evaluated $(p + q)^2$ instead of $p^2 + q^2$. Thus he would have obtained $(1, 4 + 27)^2 = (1, 31)^2 = 2,18,1$ merely by reading from his table of squares. Having made this very commonplace error, (as has been repeated by many a careless schoolboy since), he would have corrected it by subtracting $2pq$, which he should have determined by evaluating,

$$\begin{aligned} &2(1, 4) \times 27, \\ &\text{or } 2(1, 0 + 4) \times 27, \\ &\text{or } 54(1, 0 + 4). \end{aligned}$$

Then he adds, $54 \times 1, 0 = 54, 0$ (omitting the portion of the product 54×4 , by

error), and writes the sum,

$$\begin{array}{r} 2, 18, 1 \\ 54, 0 \\ \hline 3, 12, 1 \end{array}$$

The omission of the 54×4 , as part of the scribe's mistake is the suggestion of O. Neugebauer, and is plausible enough as suggested by other errors found in extant texts. Until another explanation of the commission of the error of writing 3, 12, 1 for 1, 20, 25, by the scribe is discovered, which is more plausible than the foregoing, we may accept it as explaining the underlying theory, and it leads us to accept, what is already pretty well indicated by Plimpton 322, namely, the fact that the Babylonian scribe was familiar with the algebraic determination of the integral numbers representing the three sides of right angled triangles, in the millenium before the Greeks provided a theoretical proof of the relation. Is it beyond the bounds of reasonable possibility that the Babylonians transmitted their mathematical knowledge to the Greeks? The following passage from Heath's *Mathematics in Aristotle* (Oxford, 1949, page 18), refers to the Babylonians as the "barbarians".

"Eclipses had been observed 'by certain barbarians', but the knowledge of the eclipse came to the Greeks later, through Thales. This is of course an allusion to the story that Thales predicted a solar eclipse which took place during a battle between the Lydians and the Medes, (probably the eclipse of May 585 B.C.) The explanation of the prediction is no doubt that Thales had learnt directly or indirectly from the Babylonians, the period of 223 lunations after which eclipses recur, which period had been discovered by the Babylonians as a result of observations of eclipses through long centuries. (It is curious that in the passage in Simplicius about 'certain barbarians', it is eclipses of the moon which are spoken of as having been known to them, but unknown to the Greeks before Thales)."

And Neugebauer says of the Oriental influence of scientific Greek Mathematics (3), (p. 141),

"The theory of irrational quantities and the related theory of integration are of purely Greek origin, but the contents of the 'geometrical algebra' utilize results known in Mesopotamia." And he goes on to say that the traditional stories of discoveries made by Thales or Pythagoras must be discarded as totally unhistorical. As time goes by, and more mathematical texts are translated, one does not doubt that the debt which the Greeks owed to the Babylonian culture of over 1000 years earlier will be shown to be greater than has hitherto been thought. And the emphasis given by certain historians, to the Egyptian influence on the origins of Greek geometry, from the necessity of land measurement resulting from the annual flooding of the Nile river, should shift to the algebraic and arithmetic influence of the earlier and far superior mathematics of the Babylonians, as revealed by their cuneiform clay tablets.

R. J. G.

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