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Further Notes on Aristoxenus and Musical Intervals

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FURTHER NOTES ON ARISTOXENUS AND MUSICAL INTERVALS.

THE *Ἀρμονικὰ Στοιχεῖα* of Aristoxenus, being the earliest treatise on Greek Music extant, have hitherto held an unchallenged position as the foundation of much of our knowledge of ancient musical theory. Mr. R. P. Winnington-Ingram's shrewd and critical examination (*C.Q.* XXVI, 195 ff.) of the many difficulties involved in Aristoxenus' treatment of subtleties of intonation is a very welcome contribution to a thorny subject; and it is in the hope of furthering our understanding that I venture to offer these comments on one or two points where alternatives or modifications may be suggested.

Aristoxenus did not employ ratios as a means of determining the value of musical intervals, but relied upon the judgement of his ear. He elected to use an imaginary unit of measurement, viz. one-twelfth of a tone, which he could not produce or teach with precision and accuracy in theory or practice by any method available in his day. He makes no mention of the monochord, which alone afforded an avenue to the scientific determination of intervals. Discrepancies in his findings are, therefore, a foregone conclusion.

It would seem advisable under these conditions to come first of all to a decision about the kind of tone Aristoxenus had in mind in his evaluation of intervals. The only tone clearly defined by him is the major tone of ratio $\frac{9}{8}$, and on this point his reiterations leave no possible doubt.

The following passage may be cited as an instance:

'A tone is the difference in compass between the first two concords, and may be divided by three lowest denominators, as melody admits of half-tones, thirds of tones and quarter-tones, while undeniably rejecting any interval less than these. Let us designate the smallest of these intervals the smallest Enharmonic diesis, the next the smallest Chromatic diesis, and the greatest a semitone.'¹

Such statements by Aristoxenus make it quite clear that it was the major tone ($=\frac{9}{8}$ or 204 cents)² that he had in mind: whatever discrepancies or errors may result from acceptance of the 204 cent tone as working basis must be faced. In his table (p. 198) Mr. Winnington-Ingram has taken a perfect Fourth of 498 cents as the basis of his interpretation and divided it into the Aristoxenian proportions as near as he could. I append his evaluations to those derived from the logical application of Aristoxenus' own unit:

TETRACHORDAL DIVISIONS, WITH EQUIVALENTS IN CENTS.

Enharmonic.

$$\frac{1}{4} + \frac{1}{4} + 2 \text{ tones} = \text{in cents } 51 + 51 + 408 = 510$$

$$50 + 50 + 398 = 498 \text{ (W.-I.)}$$

Chromatic μαλακόν.

$$\frac{1}{3} + \frac{1}{3} + 1\frac{5}{8} \text{ tones} = \text{in cents } 68 + 68 + 374 = 510$$

$$66 + 66 + 366 = 498 \text{ (W.-I.)}$$

¹ Macr., p. 180 (Mb. 21); cf. p. 199 (Mb. 46), p. 207 (Mb. 57), p. 211 (Mb. 62).

² Thus one tone=204 cents, the half-tone=102 cents, the quarter-tone=51 cents, the third-

tone = 68 cents, the sixth-tone = 34 cents, the eighth-tone = 25.5 cents, the twelfth-tone = 17 cents, the twenty-fourth tone = 8.5 cents.

Chromatic ἡμιόλιον.

$$\begin{aligned} \frac{3}{8} + \frac{3}{8} + 1\frac{3}{4} \text{ tones} &= \text{in cents } 76.5 + 76.5 + 357 = 510 \\ &75 + 75 + 348 = 498 \text{ (W.-I.)} \end{aligned}$$

Chromatic τριαῖον.

$$\begin{aligned} \frac{1}{2} + \frac{1}{2} + 1\frac{1}{2} \text{ tones} &= \text{in cents } 102 + 102 + 306 = 510 \\ &100 + 100 + 298 = 498 \text{ (W.-I.)} \end{aligned}$$

Chromatic (mixed, p. 52 Mb.).

$$\begin{aligned} \frac{1}{3} + \frac{2}{3} + 1\frac{1}{2} \text{ tones} &= \text{in cents } 68 + 136 + 306 = 510 \\ &66 + 133 + 299 = 498 \text{ (W.-I.)} \end{aligned}$$

Diatonic μαλακόν.

$$\begin{aligned} \frac{1}{2} + \frac{3}{4} + 1\frac{1}{4} \text{ tones} &= \text{in cents } 102 + 153 + 255 = 510 \\ &100 + 149 + 249 = 498 \text{ (W.-I.)} \end{aligned}$$

Diatonic σύντονον.

$$\begin{aligned} \frac{1}{2} + 1 + 1 \text{ tones} &= \text{in cents } 102 + 204 + 204 = 510 \\ &100 + 199 + 199 = 498 \text{ (W.-I.)} \end{aligned}$$

The vital difference between the literal interpretation of Aristoxenus' standard of valuation and the one given by Mr. Winnington-Ingram is that the former involves an error at the expense of the consonance of the Fourth: every tetrachord is found to be sharpened by 12 cents; in the latter the distortion of the Fourth has been avoided by assuming in its place the distortion, by tempering, of the smaller intervals. Now on the one hand it is incredible that the Fourths should have been distorted; on the other hand the tempering of the smaller intervals would imply the adoption of a standard tone of different value, one for which Aristoxenus provides no definition and (I believe) no implication.

It was indeed made sufficiently clear by Mr. Winnington-Ingram that he assumed tempering in this instance, not from the conviction that it was an established practice in the day of Aristoxenus, but rather from motives of expediency, in order to find a fitting basis of values that should not entail the violation of the consonance of the Fourth.

Now, while tempering on paper in cents may be adopted by some as a *pis-aller*, the suggestion that tempered intervals could have been used in the practical music of Ancient Greece must, I think, be rejected on several grounds. There is no possibility of finding a corresponding ratio of length of string or column of air to produce tempered intervals; there was no device in tuning, practicable among the Greeks, that would ensure judging tones of exactly 199 cents or tempered semitones of 100 cents; nor is there any assurance of being able to repeat such intervals at will on any degree of the scale¹; moreover, if we are to get to the root of the matter, no loose approximation has any *prima facie* claim to our consideration.

Tempering implies a departure from an established system. Before any suggestion for tempering can be seriously entertained,² it must be shown that some

¹ In relation to tempering, Erich M. von Hornbostel states 'that the most efficient of our piano tuners, making use of beats for the determination of correct tempering [an aid to the ear due to sympathetic resonance of the strings and their unisons, which is very powerful on the piano but very weak on the Kithara, if not altogether negligible.—K. S.], are wont to make errors of as much as four vibrations per second in the middle octave'—*Notiz über die Musik d. Einwohner v. Süd-Neu-Mecklenburg*. Abh. z. vergl. Musikwissenschaft. München, 1922, Bd. I., pp. 352-353.

A. J. Ellis has made similar statements, giving exact results of tests.

² Tempering in relation to Aristoxenus is a theory advanced by the foremost authorities of the French School, viz. A. J. Vincent, Théodore Reinach, Louis Laloy, etc., all of whom have been led to adopt tempering as a solution of the difficulties raised by the error of a comma involved in the method suggested by Aristoxenus (Macr., pp. 207-208; Mb., pp. 56-58) for verifying his assumption that the Fourth consists of two and a half tones.

powerful influence,¹ some urgent practical need or musical development had actually arisen to compel Aristoxenus and the Greeks generally to adopt the expedient of tempering or mistuning intervals, either consciously or unwittingly.

Equal temperament is a system which deliberately denatures and distorts all those intervals within the octave which man has absorbed into his inner consciousness through the ages as natural constituents of the physical basis of sound.

It has yet to be demonstrated that there was any powerful urge in the day of Aristoxenus to demand tempering. It seems evident that neither the strict application of Aristoxenus' own unit nor an interpretation dependent on a kind of tempering will provide a satisfactory account of the intonation in practical use during the period in question. Both those methods are therefore discredited.

The problem that faces us is really more fundamental than appears on the surface. We are certainly concerned with the fact which Mr. Winnington-Ingram points out that, since harmonic or vertical expansion was virtually unknown to the Ancients, the charm, interest and power of music made themselves felt mainly through subtleties of intonation; but such subtleties are meaningless until we apprehend the principles which determine their existence. Our proper quest is not the ability to identify one or another interval or division of the tetrachord, vaguely adumbrated by Aristoxenus, so much as to discover some underlying system to account for these subtleties and unusual intervals, and to trace such a system in place in the historical development of music in Greece. Happily we are not dependent solely upon the theorists for our knowledge of Greek Music: there are the few recovered fragments of hymns in alphabetical notation—the key to which is furnished by the Tables of Alypius—and there are the few surviving Auloi which preserve, in the disposition of their finger-holes, an imperishable record. No system, scale, or interval, can be deemed acceptable as a vital and significant fact in the history of Greek Music which is not corroborated by the Notation of Alypius, with all its implications, and by the testimony obtained from a practical study of the Aulos and its mouthpieces in surviving specimens. Carefully made facsimiles of the Auloi will, I believe, be found to provide a clue to unsolved problems presented by the Greek musical system.

In attempting to identify the Chroai there is an alternative course available as a substitute for the two evaluations mentioned above, which does not entail the violation of the purity of any of the intervals, and which further commends itself by being not merely speculative but a practical reality. This alternative consists in rescuing from oblivion the Harmoniai or Aulos-scales mentioned by Aristoxenus in the slighting terms he adopts toward all that emanates from the Harmonists.² Concerning these scales, he says: 'Others again, having regard to the boring of finger-holes on the flutes (αὐλοί), assume intervals of three-quarter-tones (τρισι διέσειςιν³) between the three lowest Keys, etc.'

If it may here be assumed that the Harmoniai of Greek Music were derived from the Aulos-scales, it is seen that these depart from the tetrachordal system of Aristoxenus, Ptolemy and the Graeco-Roman theorists in that the unit is not the tetrachord but the octave, a fact emphasized on several occasions by Aristoxenus himself. Some of the formulae of the Chroai recorded by Ptolemy are in fact tetrachords of Aulos-scales separated from their context.

¹ When equal temperament was adopted in the eighteenth century there was a strong inducement or necessity as driving power: it was in order to satisfy the desire for modulation into various tonalities in the face of the prohibitive technical exigencies of musical instruments.

² The system of the Harmonists and the ratios of the Aulos-scales have been identified and

established by well-authenticated evidence in a work which I have in preparation for the Press.

³ P. 193 (37 Mb.). The translation by Macran of *dieisis* as 'quarter-tone' is not a happy one here, for the diesis had no definite magnitude; it was valued by Aristoxenus merely as something less than a semitone.

The ratios of the modal system of the Harmoniai and the modifications incurred during the Graeco-Roman period suggest that the most important of the shades of the genera, as recorded by Aristoxenus and Ptolemy, were segments of the Perfect Immutable System, and not merely varieties for occasional use. Thus the alternative course in the valuation of the intervals of Greek Music consists in a plea for recognizing in the Chroai of Aristoxenus his attempt to express in his own terms some of the more familiar of the Aulos-scales of the Harmonists, on which were based the ancient ἀρμονίαι.¹ Aristoxenus has selected those which have a perfect Fourth and Fifth on the Tonic.

These scales were evidently in high favour with an important section of the musical world of his day, judging from the violence of the polemic which Aristoxenus directs against the Aulos (Mb., pp. 39-43; Macr., pp. 194-197) and against the claim of the Harmonists that this instrument embodies the τὴν τοῦ ἡρμοσμένου φύσιν, which in an important sense it certainly does, in spite of anything Aristoxenus—only superficially acquainted with their teachings—may say to the contrary.

The suggested identification of the Chroai of Aristoxenus with the Harmoniai or Aulos-scales follows:

SUGGESTED IDENTIFICATION OF THE CHROAI OF ARISTOXENUS AND OTHERS
WITH TETRACHORDS DERIVED FROM THE AULOS-SCALES (OR HARMONIAI)
RECONSTRUCTED BY K. S.

Enharmonic Genus.

- | | | | | |
|--|--------|---|--------------------------|-------|
| 1. Aristoxenus | | $\frac{1}{4} + \frac{1}{4} + 2$ | = in cents 51 + 51 + 408 | = 510 |
| 2. Eratosthenes. Aulos-scale (Hypolydian according to K. S.) | } | $\frac{40}{39} \times \frac{39}{38} \times \frac{19}{18}$ | = in cents 44 + 45 + 409 | = 498 |
| 3. Didymus. Aulos-scale (Hypodorian K. S.) | | $\frac{32}{31} \times \frac{31}{30} \times \frac{5}{4}$ | = in cents 55 + 57 + 386 | = 498 |

Enh.-Chromatic Genus of Four Quanta to the Fifth (Arist., p. 72 Mb.).

- | | | | |
|--|---|--|--------------------------------------|
| 4. Aulos-scale (Phrygian reconstructed by K. S.) | $\frac{24}{23} \times \frac{23}{22} \times \frac{11}{9} \times \frac{9}{8} = \frac{3}{2}$ | | = in cents 74 + 77 + 347 + 204 = 702 |
|--|---|--|--------------------------------------|

Chromatic Genus.

- | | | | | |
|--|---|---|------------------------------|------------------|
| 5. Aristoxenus μαλακόν | ... | $\frac{1}{3} + \frac{1}{3} + 1\frac{5}{6}$ | = in cents 68 + 68 + 374 | = 510 |
| 6. ἀρμονία (Mixolydian K. S.) | } | $\frac{28}{27} \times \frac{27}{26} \times \frac{26}{25}$ | = in cents 63 + 65 + 370 | = 498 |
| 7. Aristox. ἡμιόλιον | | $\frac{9}{24} + \frac{9}{24} + 1\frac{1}{2}$ | = in cents 76.5 + 76.5 + 357 | = 510 |
| 8. ἀρμονία (cf. 4) (Phrygian K. S.) | } | $\frac{24}{23} \times \frac{23}{22} \times \frac{11}{9}$ | = in cents 74 + 77 + 347 | = 498 |
| 9. Aristox. τριαλὼν | | $\frac{6}{12} + \frac{6}{12} + 1\frac{8}{12}$ | = in cents 102 + 102 + 306 | = 510 |
| 10. Eratosthenes. Aulos-scale (Hypolydian according to K. S.) | $\frac{20}{19} \times \frac{19}{18} \times \frac{6}{5}$ | | = in cents 89 + 93 + 316 | = 498 |
| 11. Aristox. (mixed Chrom., p. 52 Mb.) | $\frac{1}{3} + \frac{2}{3} + 1\frac{1}{2}$ | | = in cents 68 + 136 + 306 | = 510 |
| 12. ἀρμονία (Lydian K. S.) | $\frac{27}{26} \times \frac{13}{12} \times \frac{6}{5}$ | | = in cents 65 + 138 + 316 | = 520 |
| 13. Aristoxenus: Enh.-Chrom. unnamed of Four Quanta to the Fifth (p. 72 Mb.), identified with an Aulos-scale (Hypodorian) according to K. S. (No. 14). | | | | |
| 14. { Pyknon of unequal parts— | | | | |
| { The least as between Hyp. and Parh. } | ... | ... | $\frac{16}{15}$ | = in cents 112 |
| { The next as between Parh. and Lich. } | ... | ... | $\frac{15}{14}$ | = in cents 119.4 |
| { The third smallest, a tone (as between Mesê and Para-mesê—K.S.) } | ... | ... | $\frac{9}{8}$ | = in cents 204 |
| The largest, as between Mesê and Lichanos | ... | ... | $\frac{7}{6}$ | = in cents 267 |

Sequence from Hypatê Mesôn to Paramesê $\frac{16}{15} \times \frac{15}{14} \times \frac{7}{6} \times \frac{9}{8} = \frac{3}{2}$ (K. S.) 702

¹ It is recognized that an acceptance of this statement involves a leap in the dark, but this paper anticipates the publication of a detailed work on the subject.

Diatonic Genus.

15. Aristoxenus *μαλακόν* ... $\frac{6}{12} + \frac{9}{12} + \frac{15}{12} =$ in cents 102 + 153 + 255 = 510
16. *ἄρμονία* (modified Dorian, as used in the Tonoι of the Phrygian and Lydian groups.—K. S.) $\left\{ \frac{21}{20} \times \frac{10}{9} \times \frac{9}{8} = \right.$ in cents 85 + 182 + 204 = 471
(i.e. 27 cents flat)
17. Cf. Ptolemy's *μαλακόν* ... $\frac{21}{20} \times \frac{10}{9} \times \frac{8}{7} =$ in cents 85 + 182 + 231 = 498
18. Aristoxenus *σύντονον* ... $\frac{6}{12} + \frac{12}{12} + \frac{12}{12} =$ in cents 102 + 204 + 204 = 510
or in cents 90 + 204 + 204 = 498
19. Didymus *σύντονον ἄρμονία* (Hypodorian according to K. S.) $\left\{ \frac{16}{15} \times \frac{10}{9} \times \frac{9}{8} = \right.$ in cents 112 + 182 + 204 = 498
20. Eratosthenes *ἄρμονία* (Hypolydian according to K. S.) $\left\{ \frac{20}{19} \times \frac{19}{17} \times \frac{17}{16} = \right.$ in cents 89 + 192·3 + 216·3 = 498
21. Aristoxenus (mixed with Chrom. Parh., pp. 27 and 52 Mb.) $\left\{ \frac{3}{8} + 1\frac{1}{8} + 1 = \right.$ in cents 68 + 238 + 204 = 510
22. Archytas ap. Ptolemy ... $\frac{28}{27} \times \frac{8}{7} \times \frac{9}{8} =$ in cents 63 + 231 + 204 = 498
23. *ἄρμονία* (Mixolydian according to K. S.) $\left\{ \frac{28}{27} \times \frac{9}{8} \times \frac{8}{7} = \right.$ in cents 63 + 204 + 231 = 498
(cf. No. 27)
24. Aristoxenus (mixed with Chrom. Parh.) $\left\{ \frac{3}{8} + 1\frac{1}{8} + 1 = \right.$ in cents 76·5 + 229·5 + 204 = 510
25. *ἄρμονία*, artificial formula (Phrygian according to K. S.) $\left\{ \frac{24}{23} \times \frac{23}{20} \times \frac{10}{9} = \right.$ in cents 74 + 242 + 182 = 498
26. *Diatonic, Aristoxenus, unnamed of Three Quanta to the Fifth* (p. 72 Mb.).
27. Diatonic with lowered Parh. interval less than a semitone $\left\{ \frac{28}{27} = \right.$ in cents 63 $\left. \begin{array}{l} \text{Interval of a tone} \dots \dots \dots \frac{9}{8} = \text{in cents } 204 \\ \text{Interval greater than a tone} \dots \dots \frac{8}{7} = \text{in cents } 231 \\ \text{Interval of one tone} \dots \dots \dots \frac{9}{8} = \text{in cents } 204 \end{array} \right\}$ Identical with *ἄρμονία* of Archytas. (Mixolydian according to K. S.)
702
28. *Diatonic, Aristoxenus, unnamed of Four Quanta to the Fifth* (p. 72 Mb.). No definition.
29. *ἄρμονία* according to K. S. $\left\{ \frac{21}{20} \times \frac{10}{9} \times \frac{9}{8} \times \frac{8}{7} = \right.$ in cents 84·4 + 182 + 204 + 231 = 702
See No. 16 above
30. *ἄρμονία* Dorian, *σπονδείον* according to K. S. The scale of the Elgin Aulos and of the Bucheum flute from Armant $\left\{ \frac{11}{10} \times \frac{10}{9} \times \frac{9}{8} \times \frac{8}{7} = \right.$ in cents 165 + 182 + 204 + 231 = 782
(i.e. augmented Fifth)

It must be confessed on examination of the Table that it would be unreasonable to cavil at Aristoxenus for accepting as equal dieses intervals differing merely by 1 or 2 cents in the Pykna of the Enharmonic genus, and by 2 to 4 cents in those of the Chromatic: i.e. differences equivalent to ratios $\frac{1731}{1730} = 1.002$ cents; $\frac{864}{863} = 2$ cents; $\frac{513}{512} = 3.7$ cents; $\frac{384}{383} = 4.5$ cents; while the half-comma excess constant in what has been termed the strict interpretation of the Aristoxenian intervals may be expressed by ratio $\frac{144}{143}$. A sensitive ear would experience no difficulty in distinguishing the intervals $\frac{12}{11}$ and $\frac{11}{10}$ when starting from the same note, but the ear does not readily estimate such differences when the intervals follow one another in melodic succession, ascending or descending in pitch according to the *ῥημοσμένον*. It will at once be apparent that the formulae of Archytas, Eratosthenes and Didymus are claimed as Aulos-scales; nevertheless, there is no attempt in this identification merely to go over the ground already covered by the previous writer: the proposition to be elicited is quite a different one.

It is possible by reliable evidence¹ to establish the facts that these formulae are

¹ Which the scope of this little paper does not allow the writer to produce.

isolated statements by Ptolemy of parts of a homogeneous system based upon the embodiment of a natural law in the Aulos, and not merely cleverly devised divisions of the tetrachord. It seems important at the present juncture to recognize the fact that the Ditonal scale which forms the theme of the treatise of Aristoxenus plays only a very small and unimportant part in the development of music among the Greeks. For instance, it necessarily excludes the immensely significant system of Notation in which lies latent, as a final appeal, a test for the many problems and speculations which still agitate the minds of students of this subject.

A careful scrutiny of the scheme discloses the plan on which the allotment of the symbols has been made, the subtlety of which arouses the profound admiration of the present writer, whereas it provoked derision in Bellermann, who saw in it merely an ill-constructed and clumsy device, which he was at great pains to improve upon. According to the Tables of Alypius it is an indisputable fact that *not one single Tonos* among the fifteen can be produced as evidence of the use of the Ditonal scale, and only two Tonoi, the Lydian and the Hypolydian, support the accepted theory of a Perfect Immutable System consisting of five tetrachords identical in structure (but of a formula which excludes the Ditonal described by Aristoxenus). It is true that the Ditonal scale is written large over the treatises of the Graeco-Roman theorists—a fact further emphasized by the Arabs, who, after studying and translating these writings, bore the scale and its theory along their victorious path through Persia, India, North Africa, Spain and Sicily. The inference is that the scheme of Notation was the affair of the Harmonists. Ptolemy neither mentions Notation nor makes any use of it: that he was not instructed in their lore is obvious from his *Harmonics* even on the testimony of Lib. I. c. 16 alone, in which he describes the Homalon Diatonic tetrachord with a condescending indulgence as a kind of rustic yet sweet sounding *rara avis*, while using, or misusing, the ratios of that Harmonia in his Chromatic σύντρονον.

In short the Tables of Alypius provide entirely satisfying evidence that the scheme of Notation was devised for the Harmoniai, and that it had no connection with the Ditonal scale. The Perfect Immutable System must only be considered as a frame, consisting of a series of recognized steps or degrees, the nomenclature of which, indefinite as to the pitch and function of notes, bore only an indication of the relative sequence of positions, as demonstrated by Ptolemy in explaining the use of the *ὀνομασίαι κατὰ θέσιν καὶ κατὰ δύναμιν*.¹

The Perfect System was, therefore, equally adapted for the Ditonal and for the Harmonia. Taken at its face value it merely postulates foreknowledge of a certain scale, a precedent followed also in the system of Neums, which can be read with certainty only when the ratios of the mode indicated by the Martyria are known.

Aristoxenus likewise entirely ignores *modality*, the dominant factor in the formation of the Perfect Immutable System and of the Tonoi; the latter are also left unexplained, as Aristoxenus openly acknowledges (Mb., p. 37; Macr., p. 192), in spite of the fact that '*all that relates to the theory of Scales and Keys* was promised as part of his program' (Mb., pp. 1 and 2; Macr., p. 165). The result is that the real connection of the Tonoi with the Modes and species has remained a first-class theme for speculation.

For the most part the formulae of modal scales or Harmoniai quoted in the Table are those of scales widely used in musical circles, taught in the schools of the Harmonists and traditional in Hellenistic Asia among Greeks, Arabs and Persians.

It will be remembered that the formula of the Enharmonic Harmonia of Eratosthenes is reminiscent of the frets of the Tanbur of Bagdad according to

¹ *Harm.*, Lib. II., cap 5.

Al-Farabi¹, who also describes the borings of pipes with their Arabic notation. It is at once patent that these borings of the finger-holes indicate the ratios of other Aulos-scales of the Greeks. The three formulae ascribed to Eratosthenes have a homogeneous structure from a basis common to the three genera; moreover, they are all playable upon an Aulos bored to give that Harmonia. A light is thrown upon his formula for the Diatonic genus by a passage from the *Harmonics* of Nicomachus of Gerasa (Mb., p. 24), wherein it is stated that the Diatonic tetrachord, according to the Pythagoreans, is based upon the section of the canon, 'not as erroneously expressed by Eratosthenes or Thrasyllus, but as given by Timaeus the Locrian, whom Plato followed. . . .' The implication is that Eratosthenes was using the formula $\frac{2}{1} \times \frac{9}{8} \times \frac{1}{2} \times \frac{1}{2} =$ in cents $89 + 192 + 216 = 498$ belonging to the Aulos-harmonia, which is practically equivalent to that of the Ditonal scale of the *Timaeus*, of ratios $\frac{2}{1} \times \frac{5}{4} \times \frac{3}{2} \times \frac{2}{1} = \frac{15}{4} =$ in cents $90 + 204 + 204 = 498$; the substitution in Ptolemy's formula must be laid at the door of Pythagorean enthusiasts. This Harmonia, which I identify as Hypolydian, actually exists as a record on the straight Elgin Aulos (British Museum) when played from Hole 2, and on a flute of Roman provenance recently discovered in Egypt,² from Hole 1.

The genesis of the three formulae of Didymus, which has followed an analogous procedure, must likewise be referred to an Aulos-scale, which I identify as Hypodorian. The different respective positions of the ratios $\frac{1}{2}$ and $\frac{9}{8}$ in the Syntonon Diatonic of Ptolemy merely signify, according to the present writer's opinion, that Ptolemy has based his procedure in the matter of ratios upon the ascending Harmonic Series.³ The formulae of Archytas as recorded by Ptolemy certainly appear to be indicated by Aristoxenus (as pointed out by Mr. Winnington-Ingram). The Diatonic of Archytas (Ptolemy's version) is practically equivalent to the Diatonic with Chromatic Parhypatê (op. cit. Mb., pp. 27 and 52). This tetrachord of Archytas—unnamed but an unmistakable fit—is described by Aristoxenus in the following proposition (Mb., p. 72). 'It is required to prove that the Diatonic genus is composed of two or of three or of four simple magnitudes' ($\mu\epsilon\gamma\acute{\epsilon}\theta\eta = \text{quanta}$, Macr.). As an example of the Fifth of two magnitudes Aristoxenus mentions the Syntonic Diatonic, a proof that his Ditone has the meaning of two equal tones, the ratio of which would depend upon the value he had in mind for his semitone. Next he states that through the lowering of Parhypate two intervals remain equal and two become unequal, so that 'there will be three quanta constituting the Diatonic genus, namely, an interval less than a semitone, a tone, and an interval greater than a tone'—it is assumed that it is the interval of one tone that is duplicated. Aristoxenus continues: 'Again, if all the parts of the Fifth become unequal, there will be four quanta comprised in the genus in question.' Since the magnitude of the diesis and of the third interval is left undefined, the tetrachord of Archytas undoubtedly fits in, while other solutions are not necessarily excluded: thus $\frac{2}{2} \times \frac{8}{7} = 63$ cents, the $\frac{9}{8}$ tone = 204, and the $\frac{8}{7}$ tone = 231, and again the $\frac{9}{8}$ as tone of disjunction, total 702. In this order the $\frac{9}{8}$ tone is placed next the diesis and the Lichanos is at an interval of a

¹ A translation into French of Al-Farabi's treatise (*Grand Traité de la Musique*) *Kitabū L-Mūsīqī Al-Kabīr* by Baron Rodolphe d'Erlanger, as the first volume of a projected series to be published under the general title of 'La Musique Arabe.' Paris, Librairie Orientaliste, Paul Geuthner, 1930. See section on Flutes, with diagrams, pp. 263 sqq. Those who are versed in the acoustic properties of reed-blown pipes and flutes will be able to distinguish erroneous from true statements in this section. For the Tanbur of Bagdad see pp. 218 sqq. Al-Farabi, born

A.D. 872, died A.D. 950.

² The flute was found in a Roman dump during the excavation of the Bucheum by Dr. Robert Mond at Armant, with Mr. Oliver Myers as Director of the Expedition sent out by the Egypt Exploration Society.

³ Ptolemy thus uses the ratios of the Harmonic Series while demonstrating the practice by means of lengths of string, a contradiction which would account for certain difficulties in interpretation encountered throughout his treatise.

septimal tone from Mesê, an arrangement which is in accordance with the Aulos-scale (Mixolydian according to K. S.), from which the formula of Archytas undoubtedly derives; it likewise accords with Ptolemy's division of the Fourth into two ratios, $\frac{7}{8} \times \frac{9}{8}$, whereas the tones, reversed in position as in his formula, divide the Fourth into the ratios $\frac{3}{2} \times \frac{9}{8}$, a Ditonal combination which is out of place with a Parhypate lowered to $\frac{2}{3}$. The objection to the lowering of the Diatonic Lichanos below the tone postulated between it and Mesê may be met by drawing attention to Bellermann's interpretation of Notation, according to which the interval of a $\frac{9}{8}$ tone between Lichanos and Mesê, and in analogous positions in the other four tetrachords, only occurs, unnoticed by him, in forty-six out of the seventy-five Diatonic tetrachords belonging to the fifteen Tonoï.

If we now turn to the Fifth of four magnitudes it is found that there is no indication of values of any kind given; the gates are left wide open to speculation. One fact, however, emerges, i.e. that tones of three different ratios must be found which with the addition of Parhypate will comprise a Fifth. It will be difficult to escape the admission here of the $\frac{1}{9}$ tone, unless it be assumed that Aristoxenus had in mind the $\frac{1}{9}$ tone of 192 cents occurring in the Aulos Diatonic of Eratosthenes [q.v.] as an alternative to the tone of 204 cents, a difference of half a comma. What then is this Fifth of four quanta? If the formula of the Diatonic *μαλακόν* of Aristoxenus be suggested, viz. $\frac{1}{2} + \frac{3}{4} + 1 + 1 =$ in cents $102 + 153 + 255 + 204 = 714$, which actually has four quanta, why was he silent concerning this example? Was he conscious of the excess of half a comma over the Fifth? This would be inconsistent with his acceptance of the same excess for the tetrachord interpreted literally. Aristoxenus may have had in mind a scale almost identical with Ptolemy's soft Diatonic, of ratios $\frac{2}{3} \times \frac{1}{9} \times \frac{9}{8} \times \frac{8}{7} = \frac{3}{2}$ in cents $85 + 182 + 204 + 231 = 702$, which it fits exactly. This is an Aulos-scale (see Table Nos. 16 and 29), restated by Ptolemy, possibly out of regard for the sanctity of the perfect Fourth, but which entails a violation of the principle underlying the boring of finger-holes on the Aulos. The statement by Aristoxenus of the Fourth and Fifth by quanta suggests the lower or conjunct part of the Perfect System, viz. from Hypatê Hypatôn to Paramesê. There are other Aulos-scales of four quanta which might legitimately be cited here, such as the *ὁμαλὸν διάτονον* recorded by Ptolemy and regarded by the previous writer as 'a curious scale' and 'a peculiar looking tetrachord' (pp. 201 and 204); that it was certainly not an invention of Ptolemy's is clear from his description and from the epithets *ξενικώτερον μὲν πως καὶ ἀγροικώτερον* he bestows upon it (Lib. I. C. 16). This tetrachord of ratios $\frac{1}{11} \times \frac{1}{10} \times \frac{1}{9}$, duplicated, has continued in use in Asia Minor to the present day in some few of the Greek churches¹ and among the Eastern Arabs.²

It may be of interest to note at this point that the *σπονδείον*, according to Plutarch, was a Libation Hymn played on the Aulos, and that it was duly characterized by the interval of three dieses known as *συντονώτερος σπονδειασμός*.³ Aristides Quintilianus⁴ adds that the *σπονδειασμός*, *ἐκλυσις* and *ἐκβολή* were intervals used by the Ancients in the differentiations of their *ἁρμονίαι*: the Spondeiasmos as a rise of three dieses, the Eklysis as a fall of three dieses. The whole question bristles with controversial points which cannot be discussed here, but the epithet *συντονώτερος* applied to the *σπονδειασμός* gives the clue to the strange fact that two unusual intervals of the same magnitude were known by different names, according to their use in

¹ *Über die altgriech. Musik in der griech. Kirche*, by Dr. Joh. Tzetzis, München, 1874. See pp. 30, 52, 77, 83, 93, etc.

² Ascertained from an Arabian Professor of Music in Cairo by means of monochord tests by M. F. Grant.

³ *De Musica*, ed. Weil and Reinach, C. 11, p. 1135, pp. 42-51, §§ 108, 114-117; and C. 19, pp. 72-77, §§ 168-177. Cf. Aristoxenus, Mb., p. 37, and Macr., p. 193.

⁴ *De Musica*, Lib. I., p. 28 Mb.

ascending or descending passages respectively; the necessity for such a distinction is difficult to realize. The explanation is supplied by the fact that the epithet *συντονώτερος* is applied to the first step in the scale and draws attention to the effort required by the Aulete to produce, instead of the diesis or semitone, the larger interval of three dieses (the present writer has reason to interpret this as of ratio $\frac{11}{10} = 165$ cents), which necessitates a tightening of the muscles controlling the glottis, in order to effect the greater compression of the breath needful to produce the higher Parhypatê. On the other hand, the *ἐκλυσίς*, indicating feebleness, relaxation, bears an analogous reference to the slackening of the muscles of the glottis that takes place when the Aulete is descending to a lower note. The fact that the *σπονδειασμός* is said by Plutarch to occur in the Dorian Tonos and that the *ἐκλυσίς* is attributed to Polymnestos,¹ together with the Hypolydian Tonos, in which Tonoi (read as Aulos-scales) the intervals in question are actually found in the positions specified, corroborates the statement of Aristides that these intervals were used by the Ancients in the Harmoniai. The Spondeion is the scale for which the Elgin Aulos at the British Museum was bored, starting from finger-hole 1; and it is also the scale given by the Roman flute mentioned above when played from the exit; the Spondeiasmos syntonoteros is thus obtained on both pipes and the Eklysis from the harmonic register of the flute in the descending Hypolydian Harmonia.

Thus Aristoxenus reveals the fact—hitherto overlooked—that two rival systems, both using the *σύστημα τελείον ἀμετάβολον* for the exposition of their theories, were contending for the mastery in musical matters. The Harmonists, by far the better equipped by reason of their practical application of ratio for the exact determination of intervals,² by their use of certain Katapyknotic diagrams, expressed in the *παρασημαντική* notation to which they justly assigned great importance,³ were sure of their ground. The probability is, moreover, that the monochord—the custodian of the ratios of the Harmoniai—was used in the schools, as enjoined by Pythagoras.⁴ This was the modal system of the Ancients, parts of which have survived in folk music at the present day.

The Aristoxenian non-modal system founded on the pan-pipe scale derived from a cycle of seven ascending Fourths (*not Fifths*, as commonly asserted) was later known through the theorists as ditonal. Musical opinion, misled by the sanction bestowed by the *Timaeus* of Plato upon the scale resulting from a geometrical progression by three (= cycle of perfect Fifths), failed to notice the difference in practice between the two.

The cycle of Fourths⁵ produces a conjunct scale having a limma as first step and a perfect Fourth on the Tonic; the cycle of Fifths has a tone as first step and an augmented Fourth on the Tonic—i.e. a Tritone.

In terms of melodic material for the making of music, the struggle was between the tetrachordal aggregates—as voiced by Aristoxenus and later by Ptolemy—and the octave unit of the Harmonists with their seven octave scales which they called Harmoniai.⁶ These Harmoniai were not at this time independent scales produced by the haphazard boring of finger-holes along the length of the pipe. The seven Harmoniai were kindred modal scales, produced by the operation of one law which is embodied in every pipe or flute, and which in application results in a definite series of intervals, bearing ratios conditioned by the position of the finger-holes. To the primitive pipe-maker the process is simplicity itself, but to the theorist, who is invariably constrained to begin at the wrong end, it was, and still remains, a sore puzzle; it has in consequence been left severely alone.

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¹ *De Musica*, pp. 112-113, C. 29, § 287.

² *Macr.*, pp. 188-189 (Mb., p. 32).

³ *Ibid.*, p. 194 sq. (Mb., pp. 39-41).

⁴ *Arist. Quint.*, Lib. III., p. 116 Mb.

⁵ Cf. Aristoxenus, *Macr.*, p. 206 (Mb., p. 55), in which the process is given for the Ditone, and by implication for the scale.

⁶ *Macr.*, p. 192 (Mb., p. 36)