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In: Centaurus, 5, 1958, p. 177 - 191

532 p. 177
WAERDEN, B.L. v.d.
1958

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Centaurus

Volume 5. 1956-58 p. 177 - 191

THE ASTRONOMICAL PAPYRUS RYLAND 27

by

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1. *Introduction and summary*

The Greek papyrus Ryl. 27 was first published in 1911 in Vol. I of the Catalogue of the John Rylands Library. In 1947, Knudtzon and Neugebauer¹ published a closely related Greek papyrus Lund 35 a, which shed a new light upon Ryl. 27 and which enabled Neugebauer to publish a commentary on both². This commentary made clear the main lines of the computation of the lunar apogee and longitude, but the computation of the latitude remained obscure.

The importance of Ryl. 27 lies in the fact that this treatise represents a phase of Hellenistic astronomy before Ptolemy. This phase may be described as *an elementary astronomy, based upon Babylonian methods but adapted to the Egyptian calendar*. "Elementary" means: without or with very little trigonometry.

From other sources little is known about this elementary astronomy. The demotic papyrus Carlsberg 9 teaches a rough method for finding the date of new moon, based upon mean motion and a 25-year cycle. The demotic papyrus Berlin P. 8279, the demotic Stobart tablets³ and the Greek papyrus fragments Teptunis 274 and Lund 35 b⁴ give dates of entrance of the five planets into zodiacal signs, but they do not tell us how the dates were computed. I have shown⁵ that Babylonian elementary (i. e. non-trigonometric) methods were used to compute these "eternal tables". Quite recently I have found (Bibliotheca Orientalia 13, p. 108) that the linear part of the Venus section in the Stobart tablets can be explained by rules described in Chapter XVIII of Varāha Mihira's Pañchasiddhântikā⁶, but a complete explanation of the planetary texts is still lacking.

Centaurus 1958: vol. 5: no. 3-4: pp. 177-191

The only texts giving a complete set of rules for computing one phenomenon, viz. the Moon's apogee, are the two related papyri mentioned before, viz. *L* (= Lund 35 a) and *R* (= Ryl. 27).

These rules are based upon the use of two periods *C* and *D*, of 248 and 3031 days. They contain approximately 9 and 110 anomalistic periods of the Moon. The first period is also known from cuneiform texts⁷. Both periods are mentioned by the 6th century Indian astronomer Varâha Mihira (Pañchasiddhântikâ II, 1-4 and VIII, 5) and used by Tamil calendar makers in the region of Pondicherry⁸. Both Varâha Mihira and the Tamil computers give rules for computing the moon's apogee by means of the periods *C* and *D*, and quite analogous rules are given in our papyri *L* and *R*.

These facts show once more the importance of our papyri. They supply the missing link between Babylonian and Indian astronomy. Between the cuneiform texts, written in the last 3 centuries B.C., and Varâha Mihira, who lived after 505 A.D., there is a gap of 5 centuries. Now a large part of this gap is bridged by our papyri *L* and *R*, written about 100 and 300 A.D. respectively.

In this paper the rules for computing lunar longitudes and latitudes used in *R* will be completely explained. It will be shown that *πλάτος* means: *argument of latitude, reckoned from the point of greatest latitude, divided by 15*. The division by 15 appears reasonable, because near the nodes a change of 15° in the argument of latitude implies a change of approximately 1° in the latitude.

The elements upon which the calculation of *R* is based are shown to be

(i) the daily motion of the moon's nodes:

$$0;3,10,49,15, \text{ i. e. } 0^{\circ}3'10''49'''15''''$$

(ii) the mean daily motion of the moon:

$$13;10,34,52$$

(iii) the anomalistic month

$$27;33,16,21,4 \pm 0;0,0,0,4 \text{ days}$$

(iv) the difference between the mean and the minimum daily motion

$$1^{\circ}17' \pm 1'$$

The epoch, from which the calculations start, is the midnight at the end of -31 June 30 (= Cleopatra 20, Epiphi 4). The midnight epoch explains an ambiguity in the counting of days: the midnight at the end of Epiphi 4 may be dated Epiphi 4 or 5, at will.

The longitudes were measured with respect to the fixed stars, just as in Babylonian and Indian astronomy. The zero-point of the sign Aries used in *R* coincides exactly with the zero-point of the Babylonian sidereal zodiac: it has longitude -4° on the tropical zodiac of the year -100 .

With the aid of the periods of 248 and 3031 days the date and position of the apogee can be found, but this is not sufficient to calculate the moon's position at any given date. This position may be computed either by elementary or by trigonometric methods. The elementary methods, used by Babylonian and Indian sources, are based upon the use of arithmetical series of order 2 or, what amounts to the same, quadratic functions of time. However, it seems that the methods taught in papyrus *R* were meant to be combined with a trigonometric table for the day-by-day motion of the moon during 248 days, of the same kind as the tables used in Tamil astronomy.

The following table summarizes the methods used in lunar theory in the Babylonian, Greek and Indian sources here considered:

Source	Periods used	Method for day-by-day motion
Cuneiform texts	248	elementary
Papyrus <i>R</i>	248 and 3031	probably trigonometric
Pañchasiddhāntikā	248 and 3031	elementary*)
Tamil astronomy	248, 3031 and 12 372	trigonometric

* The method explained in Chapter II, stanza 2-6 is based upon the use of a quadratic function $at \pm bt^2$. I intend to give a detailed explanation of the method on a later occasion.

2. The date of the apogee

Section I (lines 1-14) deals with the computation of the apogee. Besides the periods *C* and *D*, of 248 and 3031 days, two derived periods

$$B = D - 11C = 303^d$$

and

$$A = 3C = 9093^d = 25 \text{ Egyptian years} - 32^d$$

are used. The starting point of the calculation is an apogee of the moon, 61 days before the New Moon of Thoth 1 of the year Cleopatra 21

(= Augustus - 1). From this starting point, periods D (as many as possible) and C (as few as necessary) are counted off until one finds an apogee in the desired year. The whole computation is described in a few lines in the following style:

"Moon. To the total of years add 2, divide by 25, (multiply) the rest by 365, the cycles of 25 by 32, then add 61. The total number divide, if possible, by 3031, and the remainder by 248 and the remainder subtract in the case of nodes from 303, in the case of no nodes from 248 ...".

For a detailed explanation, see Neugebauer's paper².

3. Longitudes and latitudes

According to section 2 (lines 15-31) the motion in longitude and latitude for the periods A , D , C and B is

Period	Longitude	Latitude
$A = 9093^d$	$a = 292;33,57,21$	$a' = 6;38,11,24,45$
$D = 3031^d$	$d = 337;31,19,7$	$d' = 9;12,43,48,15$
$C = 248^d$	$c = 27;43,24,56$	$c' = 2;43,28,34,0$
$B = 303^d$	$b = 32;[33,4]4,5[1]$	$b' = [13;14,2[9,34,15]$

The figures for longitudes are certainly correct, for a is equal to $3d$ but for a multiple of 360° , and b is equal to $d - 11c$. Moreover, the figures have the right order of magnitude. If the mean daily motion of the moon is assumed to be $13;10,34,52$ (Ptolemy's value is $13;10,34,58, \dots$) we obtain for the mean motion during $C = 248$ and $D = 3031$ days

$$\bar{c} = 27;44,6,56 \quad \text{and} \quad \bar{d} = 337;31,20,52$$

The values are slightly larger than the text values. The agreement is better in the case of d , which was to be expected, because the period D is more accurate than C .

For latitudes, the results are less satisfactory. The expected relations

$$\begin{aligned} a' &= 3d' \\ d' - 11c' &= b' \end{aligned}$$

are fulfilled for the sexagesimal parts, but not for the integers. Neugebauer has already given the correct explanation, viz. that the numbers were reduced by subtracting multiples of an unknown integer m . We shall see presently that the "modulus" m is 24, and that we need only multiply the numbers a' , d' , c' , b' by 15 in order to obtain the "argument of latitude" in the sense of Ptolemy, i. e. the distance from the North point of the moon's orbit to the actual position of the moon.

4. *Explanation of the latitudes*

The draconic month does not differ much from the anomalistic month. This implies that the change in latitude of the moon during one anomalistic month is only small, and hence the change in latitude in 11 anomalistic months cannot be much larger than the change in latitude in 9 such months, i. e. b' cannot be much larger than c' . Therefore let's try the possibility⁹

$$b' = 3;14,29,34,15$$

From the relation

$$b' = d' - 11c' + km$$

where km is an integer multiple of the "modulus" m we now find

$$km = 24$$

hence m is a divisor of 24. Since d' cannot exceed m , the only possibilities are

$$m = 12 \quad \text{and} \quad m = 24$$

We shall see that $m = 24$ makes sense. If 12 or 24 is the correct modulus, the integer part of a' must be $27 - 24 = 3$ (not 6). As Neugebauer told me, this correction is graphically possible.

Assuming $m = 24$, we may change the scale, multiplying the numbers c' and d' by 15. Thus we obtain

$$\begin{aligned} c'' &= 15 c' = 40;52,8,30 \\ d'' &= 15 d' = 138;10,57,3,45 \end{aligned}$$

The modulus now becomes $15m = 360$, which is the usual reduction modulus for the argument of latitude. So we may conjecture that c'' and d'' represent the motion in latitude in the usual sense during the periods C and D .

If this is correct, we may calculate the motion of the nodes, by subtracting from c'' and d'' the motion in longitude c and d respectively. Thus we obtain for the nodes

$$\begin{aligned}c^* &= c'' - c = 13;8,43,34 \\d^* &= d'' - d = 160;39,37,56,45\end{aligned}$$

Dividing c^* by 248, and d^* by 3031, we obtain in both cases exactly the same figure for *the daily motion of the nodes*, viz.

$$(i) \quad e^* = 0;3,10,49,15$$

The order of magnitude is right, for, according to section 5, line 54 of the same text, the daily motion of the nodes is 0;3,10. Ptolemy's value is 0;3,10,41, So (i) is very good.

This explains the formation of the "latitudes" completely. They are found simply by adding to the figures for "longitudes" the motion of the nodes during the periods A , D , B , C and dividing by 15.

5. The motion in longitude

The fact that the motion during the periods C and D is not proportional to the number of days might be explained in two ways, viz.

1) the figures might represent the motion during 9 and 110 exact anomalistic periods;

2) the figures might represent the true (not the mean) motion during 248 and 3031 days.

In the first case the ratio of the motions ought to be as 9 to 110. This is not the case, so only the second possibility remains. Let's follow this path.

Let the mean daily motion be

$$13;10,34,52 + x$$

Let the anomalistic period be

$$27;33,16,22 + y$$

y is small. If the period of 3031 days is assumed to be exact, we should have $y = 0$. If the Tamil period $R = 12\ 372^d$ is assumed to be exact, y becomes $+4'''$. If the Babylonian period 27;33,16,27 is adopted, y is $+5'''$. So we may expect y to lie between (say) $-2'''$ and $+6'''$.

Let the difference between the mean daily motion and the slowest daily motion be z . We may expect z to be less than 2° .

9 anomalistic periods are

$$9(27;33,16,22 + y) = 248 - (0;0,32,42 - 9y) = 248 - r$$

days. So 248 days are 9 periods plus a small remainder of r days. The motion during this time is equal to the mean motion minus rz . The mean motion is

$$248(13;10,34,52 + x) = 9 \text{ rotations} + 27;44,6,56 + 248x$$

The motion according to the text is, disregarding complete rotations, 27;43,24,56. So we must have

$$27;44,6,56 + 248x - rz = 27;43,24,56$$

or, substituting $r = 0;0,32,42 - 9y$,

$$(1) \quad + 248x + 9yz - 0;0,32,42z = - 0;0,42,0$$

In the same way, 110 anomalistic periods are

$$110(27;33,16,22 + y) = 3031 + (0;0,0,20 + 110y) = 3031 + s$$

days. So 3031 days are 110 periods minus s days, where s is very small. The motion during this time is equal to the mean motion plus sz . The mean motion according to the text is 337; 31, 19, 7. So we must have

$$337;31,20,52 + 3031x + sz = 337;31,19,7$$

or, substituting $s = 0;0,0,20 + 110y$,

$$(2) \quad + 3031x + 110yz + 0;0,0,20z = - 0;0,1,45$$

Thus we have 2 equations with 3 unknown quantities x , y , z . Of course, we cannot solve these without additional assumptions. However, it so happens that z can be solved without ambiguity.

We eliminate x by multiplying (2) by $\frac{248}{3031}$ and subtracting (1). The coefficient of yz in the resulting equation is

$$\frac{27280}{3031} - 9 = \frac{1}{3031}$$

Now yz is small (less than $12''$), and $\frac{yz}{3031}$ is quite negligible. Hence yz cancels out and we obtain an equation for z :

$$(3) \quad (1962 + \frac{248}{3031} \cdot 20) z = 2520 - \frac{248}{3031} \cdot 105$$

The solution of (3) is

$$(4) \quad z = 1;16,44$$

The 44 minutes in (4) cannot be trusted, because the figures $-42''$ and $-1''45'''$ on the right of (1) and (2) may have been rounded off to seconds and to quarter-seconds respectively.

If the computation of z is repeated with $0;0,41,31$ or $0;0,42,59$ instead of $0;0,42,0$, we find $z = 1;16$ and $z = 1;18$ respectively. So it seems safer to write instead of (4)

$$(5) \quad z = 1^{\circ}17' \pm 1'$$

Having determined z , we may substitute into (1) or (2) and thus obtain an equation for x and y . We begin by considering the figures on the right of (1) and (2) as exact; then we have to substitute z from (4) and obtain two equations for x and y . Of course, the two equations are dependent: we have in reality only one equation for x and y . This equation is most readily obtained by multiplying (2) by 3 and substituting (4):

$$(6) \quad 9093x = -422y - 0;0,5,32$$

We have already seen that y lies between $-2'''$ and $+6'''$. It follows that x lies between $+4$ and $-20'''$. Now the latitudes in our text are given with 4 sexagesimals, not rounded off, whereas the longitudes have only 3 sexagesimals. If we adopt the most probable assumption that the longitudes were not rounded off either, it follows that the mean daily motion underlying the calculation had only 3 sexagesimals. On this assumption, $x = 0$ is the only possibility.

Moreover, $x = 0$ is the only assumption which leads to the round figures $0;0,42,0$ and $0;0,1,45$ for the corrections from mean motion to true motion. So there is every reason to suppose $x = 0$. Hence the *mean daily motion* is

$$(ii) \quad 13;10,34,52$$

The equations (1) and (2) now simplify to

$$(7) \quad (-9y + 0;0,32,42)z = 0;0,42,0$$

$$(8) \quad (110y + 0;0,0,20)z = -0;0,1,45$$

We may eliminate z by division and solve for y . This gives

$$(9) \quad y = -0;0,0,56$$

The maximal error in y resulting from the rounding off of the figures 42,0 and 1,45 on the right of (7) and (8) is $4'''$. So the *anomalistic month* is

$$(iii) \quad 27;33,16,21,4 \pm 0;0,0,4$$

6. Sidereal or tropical longitudes?

As we have seen, Papyrus *R* assumed a mean daily motion of the moon

$$(10) \quad v = 13;10,34,52$$

and a retrograde daily motion of the nodes

$$(11) \quad n = 0;3,10,49,15$$

Were these motions reckoned with respect to the fixed stars or with respect to the equinoxes?

The Babylonian Systems I and II both used a sidereal zodiac. This means: longitudes were measured from the fixed stars. It does not imply that the authors of systems I and II were aware of the precession of the equinoxes. From the constants of system I, Kugler¹⁰ computed the following mean sidereal motion

$$v_s = 13;10,34,51, \dots$$

Hipparchus and Ptolemy used a tropical zodiac. This means: they measured longitudes from the spring equinox. Their figures for tropical motion are

$$v_{tr} = 13;10,34,58, \dots$$

$$n_{tr} = 0; 3,10,41, \dots$$

If Ptolemy's precession ($6'''$ a day) is subtracted from v_{tr} and added to n_{tr} , we obtain the sidereal motions according to Hipparchus and Ptolemy

$$v_s = 13;10,34,52, \dots$$

$$n_s = 0; 3,10,47, \dots$$

These figures, which agree well with modern observations, are nearly equal to (10) and (11). This is a strong argument in favour of the assumption that R is based upon sidereal motion. Once more, this does not imply that the author of the system of R knew about precession.

The epoch of R is Cleopatra 20 (= Nabonassar 716) XI 5 (midnight, morning or noon). To decide between these three possibilities, we compare with Ptolemy's *Almagest*. Neugebauer has already made the comparison for longitudes at noon, but here we are hampered by the fact that we do not know whether the sidereal initial point of Aries coincides with Ptolemy's. We therefore compare not only the longitudes, but also the arguments of latitude. According to Ptolemy, we have for the noon epoch

longitude	73;20
anomaly	354;28
argument of latitude	185;51

According to R we should have

longitude	70;2,16,10
anomaly	360 (apogee)

The argument of latitude would be, according to line 31 of the text,

$$(12) \quad 24 - 12;12,39,19,15$$

However, Neugebauer has shown that (12) has to be corrected, most probably, into

$$(13) \quad 24 - 12;10,39,19,15$$

We shall adopt the latter figure (13). Our conclusions would be the same if we had adopted the former one. To compare (13) with Ptolemy's value, we have to multiply (13) by 15. This gives, rounded off to minutes:

$$\text{argument of latitude } 360 - 182;40 = 177;20$$

The difference from Ptolemy's argument of latitude is $8^{\circ}31'$. This would imply a difference of $43'$ or nearly $1\frac{1}{2}$ lunar diameters in the latitude. This is certainly too much if the latitudes are derived from

eclipse observations. Therefore, I am inclined to believe that the epoch is neither morning nor noon, but midnight.

The assumption of midnight epoch also explains why the papyrus Lund 35a associates the same longitudes with a date 1 day earlier. Midnight may be reckoned to the following or to the preceding day. Papyrus *R* apparently uses in section 1-2 the first convention and in section 3-4, where a "shorter method" is taught, the second convention. Since it is practically impossible to attach the same lunar longitudes to two different dates, the assumption of a midnight epoch seems to be the only possible explanation.

For midnight, calculation according to Ptolemy and modern tables would give the following results

year -31 July 1 midnight Alexandria	longitude	mean anomaly	arg. of latitude
Ptolemy (mean position) . . .	66;43	347;56	179;14
Modern (mean position) . . .	67;5	348	179;20
Modern (true position)	68;45		181;0
Papyrus <i>R</i>	70;2,16,10	360	177;20,10,11,15

The error in the argument of latitude would be $3^{\circ}40'$ if the figure of *R* were compared with the *true* position. However, since the argument of latitude cannot be measured with any accuracy near the south point of the orbit, we must consider the text value as computed and therefore compare it with the *mean* value according to modern computation. Hence, the error is only 2° , which implies an error of only $10'$ or $\frac{1}{3}$ lunar diameter in the latitude.

The error in the anomaly is 12° , but this is not serious, because the exact determination of the apogee is a very difficult matter.

We now consider the longitudes. It seems impossible that the longitude 70;2,16,10 is a result of direct observation. It has seconds and thirds and hence it must have been computed from other observations. If so, it must have been computed as a mean position, because the moon was supposed to be at its apogee. Therefore, it must be compared with the modern mean position. The difference may be partly due to observational errors, but mainly to the difference between the sidereal and the modern tropical zero point of the zodiac.

In previous investigations of Kugler and myself about the zero point

of the sidereal zodiac¹¹, all modern longitudes were reduced to the vernal point of -100 . To obtain the reduced "Kugler longitudes", we have to subtract $58'$ from the modern longitudes for the year -31 , i. e. we have to add $58'$ to the nearly 3° previously found. This gives us 3.9 as an estimate of the difference between R longitudes and Kugler longitudes.

From Babylonian tables of ecliptical stars and from planetary tables I have found a mean difference of 4.1 degrees between Babylonian longitudes and Kugler longitudes (l. c.¹¹), p. 223). Hence, *the zero point of the zodiac of R coincides exactly with the zero-point of the Babylonian sidereal zodiac.*

7. The day-by-day motion of the moon in cuneiform texts

Our papyrus R gives rules for computing the position of the moon at apogee only. Babylonian and Indian sources give fuller information: they also teach us how to compute the motion of the moon during a period of 248 days following the apogee or perigee. We shall explain the principle of the Babylonian method, and return to the Indian sources on another occasion.

The Babylonians first compute the daily motion of the moon as a "linear zigzag function", i. e. as a function increasing and decreasing by constant differences. By adding the daily motions to the initial longitude, the daily positions are found. The following example from Neugebauer's text No 191 (Astronomical Cuneiform Texts III, p. 132) will illustrate this.

Date	Position		Difference*
IX 1	7;9,30	Capricorn	13;23,10
2	20;32,40		13;41,10
3	4;13,50	Aquarius	13;59,10
4	18;13		14;17,10
5	2;30,10	Pisces	14;35,10
6	17;5,20		14;53,10
7	1;58,30	Aries	15;11,10
8	17;9,40		15
9	2;9,40	Taurus	14;42
10	16;51,40		14;24

* The differences are not given in the text.

Each difference exceeds the preceeding one by 18'. In line 8, the addition of 18' to the preceding difference would give 15;29,10, which is 0;14,35 more than the "ideal maximum" 15;14,35. By subtracting the excess from the ideal maximum the next difference 15;0,0 is obtained. From now on 18' is subtracted for every day, until the difference would become less than the "ideal minimum" 11;6,35, and so on. The mean daily motion is 13;10,35. The difference between the mean and the minimum is 2°4', much larger than in *R*. The anomalistic month would be, according to this text,

$$27;33,40 = \frac{248}{9} \text{ days}$$

but the Babylonians and Hipparchus also knew the more accurate value

$$\frac{251}{269} \cdot 29;31,50,8,20 = 27;33,16,26,57$$

which is slightly larger than the value of *R*.

8. *The day-by-day motion of the moon in elementary Hellenistic astronomy*

As we have seen papyrus *R* teaches the computation of the lunar apogee and node and of the longitude and (argument of) latitude of the moon at its apogee. It does not teach the computation of the longitude and latitude for an arbitrary day.

This is very strange, for the whole treatise is eminently practical. It is not a theoretical investigation like the *Almagest*, but it gives practical computation rules for astrologers just like Ptolemy's *Handy Tables*. Now what is the practical use for the moon's apogee? The apogee may be a good starting point for the computation of longitudes and latitudes, but in itself it has no importance for the astrologer.

For this reason it seems probable that *R* was only an incomplete copy or an excerpt from a more complete treatise which also contained rules for computing longitudes and latitudes at any given moment.

We do not know whether these rules were elementary or trigonometrical. From the relation of *R* to Babylonian methods one might be inclined to conclude that in *R* too the daily motion was assumed to be a linear

increasing and decreasing function of time. However, this assumption leads to very improbable consequences.

We have seen that in R the difference between minimum and mean daily motion is $77'$. Now, if the moon's velocity were assumed to be a linear increasing function of time during the first half of the anomalistic period, the maximum correction from mean to true longitude would be

$$\frac{1}{2} \frac{T}{4} (v_{\text{mean}} - v_{\text{min}}) = \frac{1}{2} \frac{62}{9} \cdot 77' = 4^{\circ}25'$$

This is an extremely low figure, much below the true value 6° and the Babylonian 7° .

On the other hand, if we adopt a trigonometric formula for the correction from mean to true motion, e. g.

$$\omega = b \sin at$$

and if we determine b in such a way that the difference $v_{\text{mean}} - v_{\text{min}}$ is $77'$, we obtain for the maximum correction the very reasonable value

$$b = \frac{2}{\pi} \cdot \frac{62}{9} \cdot 77' = 5^{\circ}38'$$

The combination of the periods of 248 and 3031 days with a trigonometric table for the day-by-day motion during 248 days would yield a quick and accurate method for computing the moon's position. The same combination is also used in Tamil astronomy¹².

NOTES

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3. O. Neugebauer: Egyptian planetary texts, Trans. Amer. philos. soc. N.S. 32: 2, p. 209 (1942).
4. See footnote 1.
5. B. L. van der Waerden: Egyptian "eternal tables", Proc. Kon. Ak. Wet. Amsterdam.
6. G. Thibaut and M. S. Dvivedi: The Panchasiddhāntikā of Varāha Mihira, Benares 1889. B.

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O. Neugebauer, *Astronomical Cuneiform Texts* (London 1955) I, p. 77 and 178-183.
8. O. Neugebauer: *Tamil Astronomy*, *Osiris* 10, p. 252 (1952).
B. L. van der Waerden: *Tamil Astronomy*, *Centaurus* 4, p. 221 (1956).
9. I am indebted to O. Neugebauer for communicating this conjecture to me.
10. F. X. Kugler: *Babylonische Mondrechnung* (Freiburg 1900) p. 110.
11. B. L. van der Waerden: *History of the Zodiac*, *Archiv für Orientforschung* 16, p. 216 (1953).
12. I. M. V. Krishna Rav: *The Motion of the Moon in Tamil Astronomy*, *Centaurus* 4, p. 198 (1956).