

and in the harmonika of
Klaudios Ptolemaios

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contre Nicomache]; cf. N° 3593.

LEVIN F. R., *πληγή and τάσις in the Harmonika of Klaudios Ptolemaios* :
Hermes CVIII 1980 205-229. | Already in the 2nd cent. A.D. Ptolemy, on the
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πληγή and τάσις in the Harmonika of Klaudios Ptolemaios

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kumulierten »dati« und »notizie di ogni genere« zu kommen²⁴. Aber müssen wir denn überhaupt annehmen, daß etwa Epidem. II in der uns vorliegenden Form von einem »Verfasser« stammt? Können nicht, von wem und wann auch immer, die beiden »physiognomischen« Abschnitte in diese »Schrift« ohne Rücksicht auf deren sonstigen Inhalt hineingestopft worden sein? Ein Zeugnis für echte »Weiterarbeit« ist jedenfalls, für unser Thema, hier nicht gegeben.

Ganz anders in Morb. IV: Hier ist das archaische Ungerade-Prinzip fest in die allgemeinen Vorstellungen des Vfs. integriert (s. o.). Wenn wir nun GRENSEMANNS Spätdatierung dieser Schrift (s. o.) akzeptieren, dann hätten wir hier, von unserem Thema aus gesehen, einen eigentümlichen Fall: Ein Arzt, der eigentlich die knidisch-koische Schultradition, und damit auch sowohl das modifizierte Ungerade-Gerade-Prinzip in Epidem. I wie die ganz andere Krisenarithmetik des Prognostikon gekannt haben sollte, hätte dennoch das archaisch-exklusive Ungerade-Prinzip vorgezogen. Mir persönlich schiene dergleichen, bei diesem Vf., nicht völlig ausgeschlossen.

Kiel

FRIDOLF KUDLIEN

²⁴ Vgl. DI BENEDETTO a. O. 257.

πληγή AND τάσις IN THE HARMONIKA OF KLAUDIOS PTOLEMAIOS

The discovery that music is ruled by number, that its constituents – pitch and interval – can be accurately expressed by numerical ratios, was one of the most significant intellectual events of antiquity. It founded the science of acoustical physics; moreover, it gave rise to a period of speculation out of which issued important developments in mathematics, philosophy, cosmology and astronomy¹. According to general opinion no discovery of comparable magnitude was made until the seventeenth century, when MARIN MERSENNE, the French mathematician, philosopher and scientist, first explained in his 'Harmonie Universelle' the relations obtaining between tension and the frequency of vibration of a stretched string, relations subsequently codified in

¹ See, for instance, A. DELATTE, *Études sur la littérature pythagoricienne*, Paris 1915, 258 – 259; Sir TH. HEATH, *Aristarchus of Samos, The Ancient Copernicus*, Oxford 1913, 46 – 47; F. R. LEVIN, *Synesis in Aristoxenian Theory*, TAPA 103, 1972, 217 – 220.

'MERSENNE's Laws'². If this opinion is correct, then a period of some 2000 years intervened between the discovery of the musical ratios and that of the relation between tension and frequency of vibration. This paper is written in the belief, however, that already in the second century A. D. Ptolemy, on the basis of experiments that he conducted, was in the presence of, if not indeed possessed of, that knowledge which precipitated MERSENNE's discovery³. It will be argued that Ptolemy was led to this knowledge by his dissatisfaction with accounts of various Pythagorean experiments and, further, that the discussion in which this knowledge is evinced forms part of a polemic directed against Nicomachus of Gerasa⁴.

I. The Musical Ratios

According to numerous ancient authorities, the first one to have revealed the numerical ratios determining the consonant intervals of the musical scale was Pythagoras of Samos. Because of the deep and far-reaching implications which the discovery had for such fundamental branches of knowledge as mathematics, cosmology and astronomy – implications that extended far beyond its immediate utility in converting the sensory distinctions of pitch and interval into objective numerical form – it was treated by the ancients as a divine revelation⁵. The musical formulae vouchsafed to Pythagoras impressed the Pythagoreans as a key of universal application, leading them to postulate number as the element of all things and to promote the study of number per

² M. MERSENNE, *Harmonie Universelle*, The Books on Instruments, Paris 1635, trans. R. E. CHAPMAN, The Hague 1957. See note 52 below. The discovery of these relations was made independently by GALILEO, the contemporary of MERSENNE. See A. WOOD, *The Physics of Music*, London 1944, rev. J. M. BOWSER, London 1975, 90.

³ On the basis of astronomical observations made by him between 125 A.D. and 141 or 151, Ptolemy is assumed to have lived during the first three quarters of the second century A.D. Little is known of him otherwise. See Sir TH. HEATH, *A History of Greek Mathematics*, Oxford 1921, vol. 2, 273.

⁴ Nicomachus' *floruit* is judged to be c. 100 A.D. See L. TARAN, Asclepius of Tralles: Commentary to Nicomachus' Introduction to Arithmetic, *Transactions of the American Philosophical Society* n.s. 59. 4, 1969, 7–8; F. R. LEVIN, The Harmonics of Nicomachus and the Pythagorean Tradition, *American Classical Studies* 1, 1975, 8–10; W. C. McDERMOTT, Plotina Augusta and Nicomachus of Gerasa, *Historia* 26, 1977, 193. That Nicomachus was an older contemporary of Ptolemy is assumed in part from his apparent ignorance of Ptolemy's work on Harmonics. See K. VON JAN, *Musici Scriptores Graeci*, Leipzig 1895, 211; M. L. D'OGE, F. E. ROBBINS, L. C. KARPINSKI, *Nicomachus of Gerasa*, University of Michigan Studies, Humanistic Series 16, New York 1926, 71. The earliest reference to the Harmonika of Ptolemy appears in Nicomachus Exc. 4 (JAN 275, 7–9), but may be an interpolation by the excerptor. See I. DÜRING, *Die Harmonielehre des Klaudios Ptolemaios*, Göteborg 1930, lxxiii.

⁵ Nicomachus *Harmonikon Enchiridion* 6 (JAN 246, 6–7); Boethius *De inst. mus.* 1, 10 (FRIEDLEIN 197, 3–4).

se⁶. In its own turn, music was henceforward to be treated with the rigor of mathematics, its components – pitch and interval – being now susceptible of study through the mediation of number.

That the discovery of the numerical ratios was appreciated by the ancients for what it was – a monument of human genius – is evident. But that Pythagoras was in fact its author, as the ancient tradition maintains, has been called into question on two fundamental grounds: first, there exist no contemporary reports of Pythagoras's discovery, the earliest detailed accounts coming from the second century A. D. and later; second, the experiments described in these late accounts are mathematically so implausible that their association with Pythagoras can scarcely be credited; somewhere in the transmission the apocryphal must have made its appearance⁷.

The earliest reference to Pythagoras's discovery comes via a circuitous route from Xenocrates, who is cited by a certain Herakleides *apud* Porphyry In Ptol. Harm. (DURING 30, 1–8). Apart from the fact that the identity of this Herakleides is itself a disputed issue⁸, it is hard also to ascertain at a critical point in the passage who in fact is being cited by Porphyry⁹. Despite these difficulties, however, the passage is valuable for its implication that it was on the basis of experiments which he conducted that Pythagoras arrived at his discovery. At the same time, since the passage appears in a context which concerns speed of motion (ταχέα φορά) as a determinant of pitch (and not quantity), the reference to Pythagoras appears gratuitous and unmotivated. Herakleides writes about this [*sc.* ταχέα φορά] in his Introduction to Music as follows:

»Pythagoras, as Xenocrates says, discovered that the intervals in music have no origin apart from number. For there is intrinsic to them a comparative relation of one quantity to another (ποσοῦ πρὸς ποσόν). Ac-

⁶ Aristotle Met. 985b 31–986a 2. Cf. Aristoxenus (*fr.* 23 WEHRLI): Πυθαγόρας ... πάντα τὰ πράγματα ἀπεικάζων τοῖς ἀριθμοῖς. See H. CHERNISS, *Aristotle's Criticism of Presocratic Philosophy*, Baltimore 1935, 386.

⁷ See W. K. C. GUTHRIE, *History of Greek Philosophy*, vol. 1, *The Earlier Presocratics and the Pythagoreans*, Cambridge 1971, 223–224; W. BURKERT, *Lore and Science in Ancient Pythagoreanism*, trans. E. L. MINAR, Jr., Harvard: Cambridge 1972, 375–376.

⁸ The identification of this Herakleides with the celebrated Herakleides Pontikos has been assumed by numerous scholars, the most recent of whom are E. A. LIPPMAN, *Musical Thought in Ancient Greece*, New York and London 1964, 142 and G. H. JONKER, *The Harmonics of Manuel Bryennius*, Groningen 1970, 381, note on 94. However, F. WEHRLI, *Die Schule des Aristoteles*, vol. 2, *Aristoxenus*, Basel 1945, 102, comm. on fr. 122, and 113, argues that this identification is without any real basis in fact. The various arguments for and against the identification are reviewed by BURKERT (n. 7 above), 380–381, who himself subscribes to WEHRLI's position.

⁹ DURING 30, 6–7: καὶ ἀνελθὼν ἐπὶ τὴν γένεσιν τῆς φωνῆς ἔφη in which the speaker can be either Xenocrates or Herakleides. On the difficulty of interpretation, see GUTHRIE (n. 7 above), 222.

cordingly, he proceeded to investigate under what circumstances the intervals become consonances and dissonances, studying as well the whole question of attunement and discordancy». And returning to the origin of sound, he said: »There must be movement of some sort, if the interval meant to be heard is a consonance such as results from an equality of proportion.«

The reference to Pythagoras in the above passage is thus interposed between two parts of a discussion whose subject is motion, not quantity; and the pertinence of quantity (ποσός) to the context is not clear. However, the reference to quantity and the subsequent discussion suggest that there was available in Xenocrates's account a description of experiments conducted by Pythagoras¹⁰. This description, to judge from the discussion, presumably dealt not with speed of motion but with quantity as the determinant of pitch. Furthermore, if as Xenocrates states, the relation that Pythagoras discovered was one holding between quantity and pitch, it is reasonable to suppose that Pythagoras's experiments were conducted upon the Monochord-Canon, an instrument on whose single string the »comparative relation of one quantity to another« could be accurately demonstrated¹¹.

Assuming then that Pythagoras had first surmised that string length was the quantitative factor that determined the pitch of the sound, a logical procedure would have been to section off various lengths of the string and to correlate them with the different pitches thus produced¹². This would have revealed the relations of quantity to quantity, as Xenocrates said, or one string length to another. Correlating these lengths with the various pitches produced would have demonstrated that the longer the string, the lower the pitch; and, conversely, the shorter the string, the higher the pitch. Translating these quantitative relations into numbers would then have shown that an Octave was described by two string lengths in the double ratio, 2:1; a Fifth in the hemiolic

¹⁰ DURING 30, 4: ἐσχοπεῖτο τοῖνυν. See BURKERT (n. 7 above), 376, who believes it to be unlikely that Xenocrates failed to mention how Pythagoras arrived at his discovery or that he omitted reference to the instrument on which Pythagoras conducted his experiments, namely, the κανών, the Pythagorean name for Monochord. See Nicomachus Harmonikon Enchiridion 4 (JAN 243, 14–15).

¹¹ Significantly, Pythagoras on his death-bed is said by Aristides Quintilianus De mus. 3, 2 (WINNINGTON-INGRAM 97, 3–7) to have advocated experimentation on the Monochord (μονοχορδίξειν) to his disciples in order to study the relation between pitch and quantity (ἀκρότητα τὴν ἐν μουσικῇ . . . δι' ἀριθμῶν).

¹² This is the method recommended by Ptolemy Harm. 1, 8 (DURING 17, 27ff.) for determining the relative string lengths necessary for the production of the consonant intervals. And in fact Gaudentius Harmonike Isagoge 11 (JAN 341, 12ff.) explains that Pythagoras conducted experiments of this sort on the Canon because he was not satisfied with the single procedure thus far adopted by him involving the use of weighted strings.

ratio, 3:2; a Fourth in the epitritic ratio, 4:3. It is possible that in Xenocrates's original attribution of the discovery to Pythagoras he included a description of the experiment that Pythagoras conducted in arriving at his results – perhaps one similar to that outlined above. Unfortunately, no such description appears in the passage cited. If it existed in the Xenocrates source, it was presumably omitted by Herakleides (whose words Porphyry reports). Such an omission would account for the sense of incoherence in Porphyry's citation and would suggest a lacuna of indeterminate length in line 6 of DÜRING's text.

If a description of the experiments with string lengths had been included in Xenocrates' account, the authority of the ancient tradition ascribing to Pythagoras the discovery of the musical ratios would have been rendered secure in its foundations¹³. Since such a description is missing from Xenocrates's account, however, we are forced to rely on those that have come down to us. Unfortunately, in these accounts the descriptions of Pythagoras's experiment encourage little credence, contravening, as they do, the laws of physics and mathematics¹⁴. This is especially true of the account recorded by Nicomachus, *Harmonikon Enchiridion* 6 (JAN 245, 18–248, 26), in which the celebrated tale of Pythagoras and the blacksmith's hammers makes its first appearance¹⁵.

Nicomachus begins his account with Pythagoras pondering the problem of how to translate the sensory testimony of the ear into some accurate and visible measure such as is available in the case of the other sensory perceptions¹⁶. According to Nicomachus, Pythagoras, while thus deliberating, chanced to walk by a smithy, where by a fortuitous circumstance he heard the smith's hammers beating out upon the anvil a medley of sounds. These sounds registered upon his unaided ear as the consonances – Octave, Fifth and Fourth – while between the latter two intervals he heard a single dissonance – the whole-tone. Amazed at this god-given phenomenon, Pythagoras rushed

¹³ Of the findings based upon string length dimensions, it is observed by J. BURNET, *Early Greek Philosophy*, London 1930, 107: »On the other hand, the statement that he [sc. Pythagoras] discovered the 'consonances' by measuring the lengths corresponding to them on the monochord is quite credible and involves no error in acoustics.«

¹⁴ BURNET (n. 13 above), 106–107 argues that while it would be a waste of time to rationalize these stories, their absurdity demonstrates the existence of a real tradition in that »they are not stories which any Greek mathematician could have invented«.

¹⁵ See BURKERT (n. 7 above), 376; LEVIN (n. 4 above), 69–70; H. CHERNISS, *Plutarch's Moralia*, xiii, part 1, Loeb Classical Library: Harvard 1976, 306–307, n. a.

¹⁶ The problem is discussed by Plato *Philebus* 56b 3–c 6, in which music is judged by Plato to be of all the *technai* the least susceptible of accurate and systematic study precisely because it lacks the proper tools of measurement. For inasmuch as it is dependent upon the guesswork of the ear for proper tunings, it is in Plato's estimation paradigmatic of the imprecise *technai*. The need for instruments to serve the senses is discussed at length by Bryennius Harm. 2, 415 (JONKER 174, 12ff.); cf. Harm. 2, 404 (JONKER 150, 20ff.) on the ἀλῶγος of the senses.

into the smithy, where he determined by various tests that it was hammers of different weights that were responsible for the sounds he had heard. After ascertaining the weights of the hammers, he hurried home to duplicate by a controlled experiment the results obtained in the smithy. His method for setting up the experiment is explained by Nicomachus as follows (JAN 246, 22–247, 7):

He planted a single stake diagonally in the walls in order that no difference might arise from this implantation, or, in brief, that no variation might be detected from the use of several stakes with their own peculiar properties. From this stake he suspended four strings of the same material, made of an equal number of strands, equal in thickness and of equal torsion. He attached a weight to the bottom of each, suspending each by each in succession. And he contrived that the lengths of the strings should be exactly equal. Then, plucking two strings simultaneously in alternation, he found the aforementioned consonances, one pair producing one consonance, another pair producing another consonance.

Nicomachus goes on to explain that the suspended weights, which had been arranged in a graduated series, were of the values: 6, 8, 9, 12; that the string suspended by the weights of 6 and 12 units produced when plucked an Octave, those suspended by weights 8 and 12 produced a Fifth, those suspended by 9 and 12 units produced a Fourth. In addition, the pairs 6 and 8 were found to produce a Fourth, while 6 and 9 produced the Fifth. And the dissonant whole-tone was discovered to be produced by the pair, 8 and 9. Thereby it was demonstrated that (JAN 248, 1–6):

The Octave is a compound either of the Fifth and Fourth in conjunction, just as the double ratio (2:1) is a compound of the hemiolic (3:2) and the epitritie (4:3), as, for example, 12, 8, 6; or conversely, it is a compound of the Fourth and Fifth, just as the double ratio is a compound of the epitritie and the hemiolic, as, for example, 12, 9, 6, in such order.

After applying these findings in tests upon various other instruments, such as Monochords (JAN 248, 16), Pythagoras correlated the numerical values thus adduced with the pitches he heard (JAN 248, 18–23):

Hypate (E)	6
Mese (A)	8
Paramese (B)	9
Nete (E')	12

Inasmuch as the experiment described by Nicomachus could not possibly yield the results that he reports, it is clear that he did not himself attempt to verify it. Instead, he seems to have transmitted a tale long since absolved of absurdities by the weight of ancient tradition¹⁷. The initial absurdity takes the form of an interchanging of the active and the passive in the process leading to pitch variation. According to Nicomachus (JAN 246, 7 – 8), the musical consonances heard by Pythagoras were produced by four hammers of different weights striking upon a single anvil (ἐπ' ἄκμωνι). Moreover, he asserts that Pythagoras ascertained this fact after having studied four factors – the weights of the hammers, the dimensions of the hammer-heads, the force of the hammer blows, and the anvil (on which the iron was being forged; JAN 246, 16 – 20). And from this investigation Pythagoras concluded that it was the different weights of the hammers that determined the variation in the pitches produced. This account is preposterous, however, since pitch variation produced by percussion of this primitive sort depends not on the object used in striking the blow, but on the object affected by the blow. Thus, even though the affected object is one of the four factors among which the pitch determinant was to be located by Pythagoras, and even though it is this factor which in actuality does determine the pitch, Nicomachus has Pythagoras rejecting it (JAN 246, 19 – 20: οὐδὲ παρὰ τὴν τοῦ ἐλαυνομένου σιδήρου μετάρθεσιν) and selecting rather the irrelevant factor of the hammer weights. In neglecting the role played by the affected object (the anvil) in determining pitch, and in focussing on that of the hammer weights, Nicomachus appears to have been ignorant of a basic fact: four hammers of different weight striking an anvil at the same point would have produced identical pitches – pitches which would vary in loudness only, depending upon the force of the hammer blows¹⁸. In this respect an anvil, when set into vibration by the hammer blow, would function no differently than a bell, cymbal, tuning-fork or a single bar of a xylophone, which similarly give off each a particular pitch irrespective of the object with which they are struck¹⁹. In order for four different pitches to have been produced, it would have been necessary that a (single) hammer strike four anvils differing in either their size, their shape or their material composition. Differences in any of these factors or combination of factors would influence the pitch produced by each anvil.

Indeed, an experiment in which the aforementioned principles are brought to bear is said by Aristoxenus (fr. 90 WEHRLI = Schol. Plato Phaedo 108 d) to have been conducted by Hippasus of Metapontum. Using four bronze discs of identical diameter, the comparative thicknesses of which were in the pro-

¹⁷ Cf. LEVIN (n. 4 above), 73 – 74.

¹⁸ Cf. GUTHRIE (n. 7 above), 223.

¹⁹ See WOOD (n. 2 above), 147 – 148; 150 – 155.

portions, 4:3, 3:2 and 2:1, Hippasus is reported to have found that these discs produced when struck the consonances, Fourth, Fifth and Octave, respectively, this enabling the musician Glaukos of Rhegium to play a tune upon them. Were the tradition which makes Hippasus a contemporary and dissident disciple of Pythagoras more reliable, it could be argued that Hippasus's experiment was conducted along lines proposed by Pythagoras²⁰. As matters stand, however, the uncertainty regarding Hippasus's date and position among the Pythagoreans does not permit us to refer Aristoxenus's evidence even inferentially to Pythagoras. Nonetheless, it does demonstrate that the ancients understood certain basic principles of sound production which are found utterly perverted in Nicomachus's account²¹.

It remained for Ptolemy to incorporate the primitive causes of sound – percussion (πληγή), substance (οὐσία) and motion of air (τῶν ἀέρων κίνησις) – into a comprehensive theory, one in which the attributes of sound – pitch and timbre – are referred respectively to the appropriate categories of material quantity and quality²². As Ptolemy demonstrates, the role of percussion in the production of sound is far more complex than had hitherto been realized. Furthermore, although this was not its intent, Ptolemy's exposition of the several factors involved in percussion effectively disposes of Nicomachus's misrepresentation of the facts, and in this sense it may be regarded as a critical commentary on Nicomachus's text.

II. πληγή

Like Archytas before him, Ptolemy Harm. 1, 3 (DÜRING 6, 14ff.) refers the cause of sound to percussion (πληγή) occasioned by the impact of one object against another²³. An example of such a (primitive) type of percussion is the blow of a hammer against an anvil²⁴. But Ptolemy recognized that percussion is also manifested – if less obviously – in the operation of string and wind instruments. Ptolemy thus proposed that there are various means by which percussion is induced and that these means, in connection with other factors, account for differences in sound.

In his discussion of this passage DÜRING argues that Ptolemy intends by πληγή the characteristic sound produced by diverse instruments, as for exam-

²⁰ The problems surrounding Hippasus are discussed by J. A. PHILIP, *Pythagoras and Early Pythagoreanism*, Toronto 1966, 26 – 27.

²¹ Cf. BURKERT (n. 7 above), 377.

²² Ptolemy's contribution to musical theory is acknowledged by MERSENNE (n. 2 above), *Second Book of String Instruments*, Prop. IV, 84: »Ptolemy is the most knowledgeable of all those who have taught us Greek music«.

²³ Archytas Vors. 47 B 1 (DIELS-KRANZ 432, 10 – 433, 6).

²⁴ See note 44 below.

ple, the »gong« (Schlag) of a metal plate, the »twang« (Ausschlag) of a plucked string or the »blast« (Stoß) of a wind instrument. For this reason he finds it difficult to express the sense of *πληγή* by a single word (»Ferner ist die Bedeutung von *πληγή* schwer mit einem Wort zu fixieren«)²⁵. But DÜRING's interpretation of *πληγή*, apart from raising the difficulty noted by him, does not do justice to the line of investigation adopted by Ptolemy. For Ptolemy treats *πληγή* not as the (characteristic) sound (»die Reaktion«, as DÜRING has it) but as the immediate cause of the sound²⁶. By doing so, he orders the acoustical facts in a manner identical to that advanced by modern acousticians. This ordering provides the researcher with the means of studying percussion and its reaction as two separate events. By distinguishing then between the active and passive agents involved in percussion, acousticians refer the initiator of sound to the former, and to the latter the pitch and timbre of the sound produced. This analysis makes it possible to comprehend under percussion not only the primitive type of percussion, but also the methods by which strings and air-columns are activated: plucking of strings by fingers or *plectra* and stimulation of air-columns by the action of mouth-pieces²⁷. Ptolemy anticipates this scientific method of dealing with acoustical phenomena when he construes the active percussive agent in wind instruments to be the mouth-piece: *ὕφολμιου, τουτέστι τοῦ πλήττοντος* (DÜRING 9, 2–4), and that in strings to be the act of plucking the string: *κρουόντων* (DÜRING 7, 4)²⁸. Each of these mechanisms is understood by him to be the initiator which sets into motion (*ἀρχὴν τῆς κινήσεως*; DÜRING 6, 22) the entire vibrating system, be it air-column or string, and which results in the production of the characteristic sound.

In Harm. 1, 3 (DÜRING 6, 14–22), Ptolemy associates with these several types of percussion three possible causes of pitch variation: 1) force exerted by the active percussive agent (*τὴν τοῦ πλήττοντος βίαν*); 2) distance (*τὴν ἀποχὴν*) between the active and passive entities; 3) corporeal composition of the passive receptor (*τὰς σωματικὰς συστάσεις τοῦ ... πληττομένου*) and of

²⁵ I. DÜRING, Ptolemaios und Porphyrios über die Musik, Göteborg 1934, 149.

²⁶ *Loc. cit.*: »für die Reaktion im Aulos oder in der Luftröhre würde man eher 'Stoß' sagen«. By referring *πληγή* to the active principle in percussion, Ptolemy evidently followed the analysis of Aristotle *De an.* 419b 10–12: *πληγή γάρ ἐστιν ἡ ποιούσα. διὸ καὶ ἀδύνατον ἑνὸς ὄντος γενέσθαι ψόφον· ἕτερον γὰρ τὸ τύπτον καὶ τὸ τυπτόμενον*.

²⁷ See C. A. TAYLOR, *The Physics of Musical Sounds*, London 1965, 89; A. H. BENADE, *Horns, Strings and Harmony*, New York 1960, 39f.; 62; 142f.; 165f.

²⁸ K. SCHLESINGER, *The Greek Aulos*, London 1939, 71, believes that the mouth-piece, *ὕφολμιου*, described by Ptolemy as *τοῦ πλήττοντος*, must refer to the single »beating-reed mouthpiece of the Aulos«. This interpretation of *τοῦ πλήττοντος*, »the beating thing«, as she translates it, is then claimed to constitute the only literary reference to the existence of a single reed Aulos. Since SCHLESINGER's proposal rests upon a forced interpretation, the passage in Ptolemy can hardly be treated as the evidence SCHLESINGER alleges.

the active agent (τοῦ δι' οὗ ἡ πληγή)²⁹. In characteristically difficult language, Ptolemy approaches the problem of pitch variation as follows³⁰:

A difference obtains between sounds, as between all other things, which is qualitative and quantitative. It is impossible properly to demonstrate under which of these aforesaid categories highness and lowness should be subsumed until we examine the causes of such an occurrence. These causes seem to me to be factors which are somehow integral to the differences arising from percussion of diverse types. For the sensations resulting from these various types of percussion differ with respect to the force of the agent executing the blow, the corporeal composition of the item struck and of that by which the blow is executed, and, further, the distance of the item struck from the point of origin of the motion.

Of the three factors adduced by him for their possible bearing upon pitch variation, Ptolemy rejects force on the grounds that it produces changes in loudness only, having no effect upon the pitch at all (DURING 6, 27–7, 1): βία μεγέθους ἂν εἴη μόνον αἰτία καὶ οὐκ ὀξύτητος ἢ βαρύτητος. Further, the corporeal composition of the agent of excitation is mentioned by Ptolemy only insofar as it concerns the quality of the sounds produced by wind instruments. For inasmuch as mouth-pieces of wind instruments are reckoned by him to be percussive agents³¹, he ascribes differences in sound quality to differences in the mouth-pieces employed – reeds and lips, for example (DURING 7, 11): οἷον τῶν γλωσσῶν καὶ τῶν στομάτων. Apart from this reference to mouth-pieces, the composition of the active percussive agent (the initiator) enters not at all into Ptolemy's analysis. The remaining factors – composition of the passive agent and distance – are thereby determined as responsible for pitch variation.

The factor, distance (ἀποχή, διάστασις), is treated by Ptolemy only as it pertains to string and wind instruments; that is, in speaking of the distance between the object struck and the point of origin of the motion, Ptolemy is not considering the distance obtaining in the case of simple or primitive percussion, as, for example, that occurring on the impact of a hammer upon an anvil. For distance in a percussion of this type would have no influence upon

²⁹ The material of the active agent in percussion is treated as a causative factor inasmuch as it must be rigid enough for sound to be produced at all. Cf. Aristotle *De an.* 419b 6–8; Porphyry *In Ptol. Harm.* (DURING 39, 4–5). In discussing this problem we follow Porphyry *In Ptol. Harm.* (DURING 38, 23–24) in treating the active agent in conjunction with the passive receptor, thus as constituting a single causative factor.

³⁰ The infelicities in Ptolemy's style throughout the treatise appear to result more often than not from his grappling with the inadequacies of the language. Cf. R. P. WINNINGTON-INGRAM, *Mode in Ancient Greek Music*, Cambridge 1936, 67.

³¹ See p. 213 above.

the pitch produced, but only upon the force exerted³². Rather, Ptolemy intends distance in the case of plucked strings, where the distance extends between the fixed bridge and the movable bridge of a Monochord, for example. In this situation, the lesser the distance between these bridges, the higher is the pitch produced (DURING 8, 27 – 9, 2)³³. In respect to wind instruments, the same principle obtains between the mouth-piece and the finger holes. For the sounds thus produced grow progressively higher in pitch as the finger holes are successively opened in the direction of the mouth-piece (DURING 9, 1 – 4), this operation being equivalent to shortening the length of the air-column³⁴. On this basis, the factor, distance, and hence measurable length, is determined by Ptolemy to be an attribute of quantity, and this quantity is specifically correlated with pitch, whose highness and lowness is found to be reciprocally proportional to the distance (DURING 8, 15 – 21)³⁵:

On this basis, it appears that the difference between sounds with respect to highness and lowness is a species of quantity (ποσότητος εἶδος εἶναι τι) and, moreover, dependent upon the inequality of the distances between the item affected by the percussion (τοῦ τε πληττομένου) and the agent executing the percussion (τοῦ πλήττοντος). For quite clearly, the difference in pitch is implicated in the size of these distances. Highness of pitch is consequent on the lesser distances owing to the vigor (of motion) issuing from a more limited dimension; lowness of pitch is consequent on the greater distances owing to the slackening (of motion) issuing from a larger dimension. The result is that sounds are in a reciprocal proportion to the distances.

Ptolemy next considers percussion in its relation to the factor, corporeal composition (DURING 7, 5 – 8):

The difference between sounds which proceeds from the means by which percussions are executed depends then on the primary substance of the corpus (τὰς πρώτας τοῦ σώματος συστάσεις), that is, on those elements according to which each thing is accounted to be rare or dense, thin or thick, smooth or rough.

³² See Porphyry In Ptol. Harm. (DURING 53, 11 – 14), who explains that distance, as it pertains to simple percussion, is a factor inseparable from force and hence has no bearing upon pitch variation.

³³ Cf. Porphyry In Ptol. Harm. (DURING 54, 15 – 21).

³⁴ Cf. Porphyry In Ptol. Harm. (DURING 54, 21 – 26; 55, 20 – 22). See SCHLESINGER (n. 28 above), 43 – 44.

³⁵ At the same time the sounds are inversely proportionate to the speed of motion. Cf. Nicomachus Harmonikon Enchiridion 4 (JAN 244, 6 – 11). See DURING (n. 25 above), 151 on 8, 17.

Accordingly, Ptolemy argues that the corporeal material of τὸ πληττόμενον ultimately determines not only the timbre of the sound produced, but the pitch as well. Thus the substance of the item struck is referred by him to the categories of quality and quantity alike³⁶. For the material attributes of τὸ πληττόμενον are, he says, not only responsible for the characteristic sounds (timbres) produced, as for example (DURING 7, 13): πάταγοι καὶ δοῦποι καὶ φωναὶ καὶ κλαγγαὶ καὶ μύρια ὅσα τοιαῦτα – but also for their inherent pitch range. He thereupon explains (DURING 7, 29) that striking upon metal produces a higher pitch than striking upon wood.

As we have seen, the material attributes which bear upon the pitch and timbre of the sounds produced are recognized by Ptolemy to be density and rarity, thickness and thinness, roughness and smoothness. Of these attributes, roughness and smoothness are referred by him to the category, quality, inasmuch as their effect on the sounds produced is specifically qualitative and immaterial to pitch change (DURING 7, 17): ὅτι καὶ αὐταὶ ποιότητές εἰσι κυρίως. But since density and rarity, thickness and thinness, admit of quantitative differences, and these differences have a direct bearing on the highness or lowness of the sound, it is to these quantitative factors that pitch must be referred (DURING 7, 22 – 31)³⁷:

For that which is denser contains a greater mass of substance distributed within the same radius; and that which is thicker contains a greater amount of the same material distributed over the same length. The more compact and the thinner the item, the higher is the pitch produced; the more rarified and the thicker the item, the lower is the pitch produced. In other words, the higher is said to be such by virtue of its association with the thinner item, just as the duller is associated with the thicker item. For the thinner items produce a more concentrated percussion in proportion to their ability to be penetrated more quickly; while the denser items produce a more concentrated percussion in proportion to their greater mass. For this reason bronze produces a higher pitch than wood; and a lyre string produces a higher pitch than a thread, for the former items are of a denser material.

In the foregoing passage, Ptolemy refers the pitch difference that obtains between two objects to quantitative factors – greater and lesser density and thickness – in the material of which those objects are composed. Thus, the greater the density of the material, the higher is the pitch produced; on the other hand, however, the greater the thickness of the material, the lower is the

³⁶ Cf. Porphyry In Ptol. Harm. (DURING 44, 5 – 9).

³⁷ Thus Porphyry In Ptol. Harm. (DURING 44, 9 – 10): δῆλον οὖν ὡς ἡ βαρύτες τῶν φθόγγων καὶ ἡ ὀξύτες εἰς τὴν ποσότητα ἀναχθήσεται.

pitch produced³⁸. In this analysis pitch is shown to be a function of factors that inhere in the material properties of an object. Later in the chapter (DURING 7, 30–8, 8), Ptolemy shows that there is another respect in which quantity affects pitch. As we shall see, this respect concerns not only the nature of the material of which an object is composed, but also the size and shape which that material assumes or is made to assume.

Before explaining how changes in the size and shape of an object may yield variation in pitch, Ptolemy first considers the basic problem of inherent pitch, i. e., that pitch produced on percussion of a single object without any alteration in its size or shape. Such an object, Ptolemy argues, produces when struck a single and undeviating pitch, which is itself a consequence of the continuous and unchanging compression and rarefaction generated in the surrounding air by the percussion (DURING 8, 12–14)³⁹. The inherent pitch of an affected object is described by him accordingly (DURING 6, 24–27):

The difference between sounds construed with respect to the composition of the object struck would be negligible (οὐδόλως) or at least imperceptible (οὐκ αἰσθητή γε) owing to the fact that the change in air motion remains constant in its progress to the ear.

By attributing the inherent pitch of an object to its material composition and to the unchanging periodic motion produced in the air on its percussion, Ptolemy expresses what modern acousticians term free vibration. Thus, WOOD⁴⁰:

Any source of sound if set into vibration and left to itself vibrates in its own natural frequency, producing a note which gradually dies away as the vibrations decrease, *but remains constant in pitch* [italics mine].

In order for an individual object to produce on percussion a change in its natural pitch, it is required that some alteration be made in its corporeal composition. As Ptolemy explains it, a deviation in any one of the three factors adduced by him will cause a consequent change in the sound (DURING 6, 22–24)⁴¹:

³⁸ MERSENNE (n. 2 above), Third Book of String Instruments, Ninth Rule, 178, demonstrated this effect on pitch using strings of brass, gut, steel, gold and silver. Applying the same amount of tension to strings of different density, he found that the string of less dense material (as, e.g., gold) produced a lower pitch than that of more dense material (as, e.g., steel). To bring such strings into unison he computed the amount of tension that had to be exerted on the string of lesser density. The effect of thickness on pitch is explained by him in his Fifth Rule (177), in which he computes the amount of tension required to bring strings of unequal thickness into unison.

³⁹ On this physical definition of sound, see DURING (n. 25 above), 150–151 on 8, 12. Cf. Aristotle De an. 419b 21–27.

⁴⁰ WOOD (n. 2 above), 23.

⁴¹ See p. 214 above.

For clearly, assuming other things to be equal, each of the aforesaid factors imparts its own peculiar property to the sensation, whenever and in whatever respect it may deviate (ὅταν αὐτὸ διενέγκῃ καθ' ὄντινα οὖν τρόπον).

Thus, he says, a deviation in force produces a change in loudness (DURING 7, 1–5); a deviation in distance, as in string length or air-column, produces a change in pitch (DURING 8, 18 ff.); and a deviation in the density or thickness in the substance of τὸ πληττόμενον also produces a change in pitch (DURING 7, 5 ff.). This latter type of change and its effect on pitch is described by WOOD in connection with the single bar of a xylophone⁴²:

The bars are made of hard wood, and are hit with hammers . . . If we lessen the stiffness we tend to slow the vibrations and flatten the pitch of the note. On the other hand, if we thin the bar at its ends we reduce the mass which the stiffness has to move, and so speed up the vibrations and sharpen the pitch.

This same effect is described by Ptolemy with surpassing clarity (DURING), 7, 30–8, 8):

In the case of bronze items of similar and equal density, that which is thinner produces the higher pitch; while in the case of strings of similar density and equal length, the thinner produce the higher pitches; hollow vessels produce higher pitches and, moreover, the denser and the thinner the air channels, the higher are the pitches. Each of these instances of higher pitch occurs not specifically because of the density and thinness itself, but because of the elasticity (διὰ τὸ εὐτόνον), seeing that items of such composition happen to be more elastic⁴³. And the more elastic the item, the more agitated it becomes on percussion and this produces a greater condensation and a higher pitch. Wherefore, if something is otherwise more elastic, as, for example, if it be stiffer or generally more (compact), it produces a higher pitch, since, inasmuch as both attributes are able to produce a similar result [*i. e.*, higher pitch], an excess on either standard of proportion is the prevailing factor.

By referring the origin of sound to πληγή in its active and passive aspects and proceeding therefrom to an analysis of the properties of vibrating systems, Ptolemy accurately determined the principles underlying all vibration, and therefore all sound – means of excitation (τὸ πληττον), comprehending striking, blowing and plucking, and motion of the affected object conditioned by its material properties. His approach to the study of sound is one that is

⁴² WOOD (n. 2 above), 148.

⁴³ On the meaning of εὐτόνον, see DURING (n. 25 above), 150 on 8, 2.

shared by modern theorists, who commence their analyses from an identical point of departure⁴⁴:

What kind of bodies vibrate? Let us start with solid matter and consider first of all 'three-dimensional' objects, that is, objects in which no one dimension is of a different order of magnitude from the others ... If they are made of hard material – glass or steel – it is possible that a note of specific pitch may be emitted if the object is struck with a hammer; the characteristic 'ring' of an anvil is perhaps an example. The notes of such objects are usually of relatively high pitch and, since it is not easy to change either the shape or the state of stress, the pitch is fixed.

Ptolemy's recognition of the principal factors in the production of pitch by percussion – τὸ πληττόμενον and its material properties – testifies not only to his own insight into the nature of sound; it reveals also the fallacy underlying the account in which Nicomachus describes the activities of Pythagoras in the smithy⁴⁵. From this initial fallacy there proceed the further absurdities in Nicomachus's account. On one issue only is Nicomachus in agreement with Ptolemy – the negligible effect on pitch produced by the factor force⁴⁶. For he has Pythagoras reject force (Harmonikon Enchiridion 6; JAN 246, 11–18), presumably because it affects only the loudness of the sound (*cf.* Harmonikon Enchiridion 4; JAN 243, 5–8). On the other hand, however, that factor which scarcely enters into Ptolemy's analysis of sound generation, namely the agent of percussion or τὸ πλήττον⁴⁷, becomes in Nicomachus's account the source from which Pythagoras derived the pertinent numerical data. Thus the smith's hammers or τὰ πλήττοντα, whose function, as initiators of sound, is purely trivial in percussion, are advanced by Nicomachus in place of the anvil or τὸ πληττόμενον, the actual source of the sound.

III. τάσις

On the basis of the data obtained from the smith's hammers, Pythagoras, we are told by Nicomachus, devised the ingenious experiment on strings which led him to postulate that the musical ratios were directly proportionate to the weights of the hammers striking upon the anvil⁴⁸. The fact is, however, that

⁴⁴ TAYLOR (n. 27 above), 8–9.

⁴⁵ See pp. 210–212 above.

⁴⁶ See pp. 214 above.

⁴⁷ See p. 209f. above.

⁴⁸ See p. 210 above.

the experiment described by Nicomachus would not produce the results he adduces, since strings stretched by weights in the proportions of 6:8:9:12 would not yield the consonant intervals at all. Inasmuch, therefore, as Nicomachus' account of this experiment with weighted strings was accepted without demur by the numerous ancient authorities who transmitted it – whether verbatim or with minor variations – down to the time of Boethius⁴⁹, it is a safe inference that none of these writers attempted to reconstruct the experiment; had they done so, they would have been unable to duplicate the findings reported by Nicomachus⁵⁰. For because the frequency of vibration productive of a string's pitch is directly proportional to the square root of the tension, the tension would have to be quadrupled (not doubled) in order to raise the pitch to the higher octave note. Thus the fact that none of the writers who transmitted the story of Pythagoras's experiment with weighted strings expressed even a modicum of skepticism at the results purportedly derived from it has led scholars to conclude that the ancients were ignorant of the behavior of strings under tension⁵¹.

If, as Nicomachus has it, two strings are suspended by weights whose ratios are 2:1, they would be subjected to tensions in a double ratio. But the interval yielded by their respective pitches would not only be less than an Octave, it would also be less than a Fifth and greater than a Fourth. Thus, if one were to reconstruct the experiment described by Nicomachus, he would be struck by the incompatibility between the intervals produced and the ratio of the weights. It was not until the seventeenth century, however, that the factor, tension, was shown, by MERSENNE, to be the source of this disparity. MERSENNE traced the disparity to the peculiar relation obtaining between tension

⁴⁹ The account of Iamblichus De vita Pyth. 115 – 120 (DEUBNER 66, 12 – 69, 5) is taken *verbatim* from Nicomachus. An abridged version appears in Iamblichus In Nic. Arithm. Intro. 171 – 176 (PISTELLI 121, 15 – 125, 13). Similar versions are given by Gaudentius Harmonike Isagogae 11 (JAN 340, 4 – 341, 25); Censorinus De Die Natali 10 (HULTSCH 17, 19 – 19, 2); Macrobius Comm. in Somnium Scip. 2, 1, 8 – 14 (WILLIS 96, 16 – 97, 25); Fulgentius Mitologiae 3, 9 (HELM 75); Chalcidius Comm. in Platonis Tim. 45 (WASZINCK 93 – 94); Boethius De inst. mus. 1, 10 – 11 (FRIEDLEIN 196 – 198). Cf. Plutarch De an. proc. in Tim. 1021a; Aristides Quintilianus De mus. 3, 1 (WINNINGTON-INGRAM 94, 11 – 95, 7); Porphyry In Ptol. Harm. (DÜRING 119, 29 – 120, 7); Isadore Etym. 3, 16, 1 (LINDSAY); Hagiopolites (VINCENT, Notices, 266 – 268).

⁵⁰ As numerous scholars have observed of this experiment, the ratio 6:12 does not correctly express the relation between a string's pitch and the amount of tension required to produce the higher octave note. See TH. H. MARTIN, Études sur le Timée de Platon, Paris 1841, vol. 1, 391; A. E. CHAIGNET, Pythagore et la philosophie pythagoricienne, Paris 1873, vol. 2, 134; JAN (n. 4 above), 141; BURNET (n. 13 above), 106; B. L. VAN DER WAERDEN, Die Harmonielehre der Pythagoreer, Hermes 78, 1943, 179ff.; GUTHRIE (n. 7 above), 223 – 224; BURKERT (n. 7 above), 375 – 376; CHERNISS (n. 5 above), 307, note b.

⁵¹ See BURKERT (n. 7 above), 376, note 24.

and frequency, where the latter is the square root of the former⁵². This relation requires that the Octave be expressed by the ratio 4:1, the Fifth by 9:4, the Fourth by 16:9. In order, therefore, to raise the pitch of a string to an Octave, the tension exerted upon it must be four times greater than that productive of the lower note; to produce a Fifth, the tension must be $2\frac{1}{4}$ times greater; to produce a Fourth, it must be $1\frac{7}{9}$ times greater⁵³. Thus, the experiment of Pythagoras, in which the tension is applied to four strings in the proportions of 6:8:9:12, would produce a series of nondescript intervals.

In light of the unanimity with which the ancients accepted the results derived from Pythagoras's experiment with weighted strings, Ptolemy's unequivocal rejection of these same results must be accounted as singular. No less remarkable is the censure which he applies to those scholars who accepted the results of such experiments (Harm. 1, 8; DURING 16, 33 – 17, 2):

Representations of the aforesaid [sc. ratios] using auloi and syringes or strings suspended by weights must be rejected. For such demonstrations cannot attain to complete accuracy, but rather introduce grounds for indicting the investigators (ἀλλὰ διαβολῆς μᾶλλον ἐμποεῖν ἀφορμὰς τοῖς πειρωμένοις).

Ptolemy's reasons for rejecting the wind instruments, auloi and syringes, as proper vehicles for experimentation are the same as those lodged against weighted strings – both methods introduce so many variables into the experiments that accurate results cannot be expected⁵⁴.

Even though Ptolemy did take this strong exception to the experiment with weighted strings as described by Nicomachus, and even though he was the only ancient authority to have done so, his criticism is not considered by scholars to reflect any advanced knowledge on his part respecting the behavior of strings under tension. Rather, his rejection of this particular experiment is judged important only as implying the priority of Nicomachus' account⁵⁵. Setting aside the question of priority, however, there is noticeable in the language which Ptolemy uses a vehemence that is disproportionate for the mere expression of dissatisfaction with the experiment. For Ptolemy not only repudiates the findings adduced; he also directs his attack *ad homines*. That Ptolemy would have expressed himself in this peremptory manner without good and sufficient reasons seems unlikely. It will be argued here that lying behind

⁵² MERSENNE (n. 2 above), First Book of String Instruments, Prop. XVI, 59; Third Book of String Instruments, Prop. VII, Sixth Rule, 177.

⁵³ See WOOD (n. 2 above), 91.

⁵⁴ For experiments conducted by the Pythagoreans on wind instruments see Plutarch De an. proc. in Tim. 1021a; Porphyry In Ptol. Harm. (DURING 119, 13 ff.). Cf. Athenaeus 4, 184e.

⁵⁵ Cf. BURKERT (n. 7 above), 376.

the reasons he in fact offers was a greater knowledge of the problem than has hitherto been attributed to him.

Of the various extant accounts of Pythagoras' experiment with weighted strings, only two can be placed in the second century A. D., the period of Ptolemy's activities – those of Adrastus the Peripatetic *apud* Theon of Smyrna (HILLER 57, 1–4; *cf.* 60, 7–8; 66, 22–23) and of Nicomachus Harmonikon Enchiridion 6 (JAN 245, 21–248, 26). To these accounts should be added the comments of Theon on the factor tension (HILLER 65, 10–24). Since other versions now lost may have been current at the time of Ptolemy and hence known to him, it is possible also that his polemic was directed against them. As matters stand, however, the only available evidence against which Ptolemy's arguments may be weighed is that provided by Adrastus and Nicomachus, other accounts coming from later than Ptolemy's era⁵⁶.

Adrastus omits the story of Pythagoras's observations in the smithy, referring only to Pythagoras's »rather well-known experiment with weighted strings« (HILLER 57, 4): γνωριμώτερον κατὰ τὴν ἐξάρτησιν τῶν βαρῶν. Apart from this reference, Adrastus offers no details about the manner in which the experiment was conducted, remarking only that tension was introduced, as it was in the case of an experiment in which tuning pegs were used (HILLER 57, 2–3): τῆς τάσεως γινομένης κατὰ τὴν στροφὴν τῶν κολλάβων⁵⁷. This leaves Nicomachus' detailed account as a likely source of information for Ptolemy. In fact, the various points of contact between Nicomachus's account and Ptolemy's criticism suggest very strongly that Nicomachus may indeed have provided Ptolemy with the basis on which to frame his argument.

In the passage cited above (pp. 210), Nicomachus described in detail the experiment of Pythagoras in which four weighted strings were suspended from a single stake. The purpose of this single stake was, he explained, to provide against the introduction of variables which would derive from the use of multiple stakes (JAN 246, 23–247, 1): ἵνα μὴ χάκ τούτου διαφορά τις ὑποφαίνεται ἢ ὅλως ὑπονοῆται πασσάλων ἰδιαζόντων παραλλαγῇ. Accordingly, a single stake was implanted in the walls⁵⁸, from it the four strings were suspended each by each (ἐκάστην ἐφ' ἐκάστης), and a weight was attached to the free-hanging end of each string. Evidently, Nicomachus was under the impression that the behavior of the strings thus weighted would have been in-

⁵⁶ See note 49 above.

⁵⁷ Such an experiment is mentioned also by Nicomachus Harmonikon Enchiridion 6 (JAN 248, 10–13).

⁵⁸ The stake was presumably implanted into the opposite angles formed by the four walls, thus occupying the position of an hypotenuse. Accordingly, M. MEIBOM, *Antiquae Musicae Auctores Septem*, Amsterdam 1652, *Notae ad Nic.*, 47; *Ab uno angulo transversum ad alium*. *Cf.* Aristides Quintilianus *De mus.* 3, 3 (WINNINGTON-INGRAM 99, 1–7).

fluenced variably if suspended from multiple stakes. Therefore, a single stake, by ruling out this variability, would guarantee the validity of the final results. The fact is, however, that the single stake neither provides against variables nor influences the behavior of the strings at all. It merely affords a rigid support for the strings. In order properly to conduct such an experiment, the researcher must focus upon the strings themselves and not upon the stake, for it is exclusively from the weighted strings that the results must be ultimately derived. Significantly, Ptolemy commences his critique by arguing this very point; moreover, his statement to this effect appears to address the text of Nicomachus (DURING 17, 7 – 12):

In the case of the experiment in which weights are attached to strings, there is no way at all to maintain the strings in a relation to one another in which there are no variables (μη διασφζομένων ἀπαραλλάκτων ἀλλήλαις παντάπασι τῶν χορδῶν); when indeed the task is to discover how each individual string behaves by itself (ὁπότε καὶ πρὸς αὐτὴν ἐκάστην οὕτως ἔχουσιν εὐρεῖν ἔργον). Nor will it be possible any longer for the ratio of the weights to harmonize with the sounds produced by the strings (οὐκέτι δυνατόν ἐσται τοὺς τῶν βαρῶν λόγους ἐφαρμόσαι τοῖς γινομένοις δι' αὐτῶν ψόφοις) even by having denser and thinner strings under the same tensions produce higher sounds.

The beginning of Ptolemy's statement above apparently has reference to the claim of Nicomachus that the use of a single stake in the experiment results in the elimination of variables. Ptolemy points out that no such result is achieved. For the fact is that when strings are stretched perpendicularly by weights, as Nicomachus described, their reaction is not a function of the support from which they are suspended but, rather, of the weights by which they are stretched. And since the diverse weights affect each string in a different way, the experiment is influenced at the very outset by the introduction of variables. These variables account for the inconsistency between the intervals produced by the strings and the ratio of the weights. In order to isolate the factor responsible for this inconsistency, it is necessary, according to Ptolemy, to study the behavior of each individual string. Testing whether the inconsistency could be due to the dimensions of the strings, Ptolemy first varied the thickness and density of the strings under the same tensions and found that the inconsistency persisted. Ptolemy therefore concluded that the pitches produced must be referred to some factor other than the dimensions of the strings. In discrediting the experiment with weighted strings by showing that variables are indeed introduced, Ptolemy was in effect countering Nicomachus's claim concerning the elimination of παραλλαγή with his own μη . . . ἀπαραλλάκτων, while at the same time treating Nicomachus's subordination

of each individual string (ἐκάστην ἐφ' ἐκάστης) to the domination of the supporting stake as wholly inappropriate (ἐκάστην ... εὐρεῖν ἔργον). These correspondences in language between Ptolemy's argument and the text of Nicomachus seem too pointed to be fortuitous.

Ptolemy next appears to address himself to Nicomachus's stipulations concerning the lengths of the strings used by Pythagoras. While Nicomachus asserts that the lengths were identical, it is hard to determine from his account whether they were equal before or after the application of the weights (JAN 247, 1 – 7):

[From the stake] he suspended four strings of the same material, made of an equal number of strands, equal in thickness and of the same torsion. He hung them up each by each in succession, having attached a weight to the bottom end of each. And having contrived that the lengths of the strings were in every respect absolutely equal, he thereupon plucked simultaneously two strings in alternation and found the aforementioned consonances.

Nicomachus appears to be saying that the four free-hanging strings, each weighted by a different unit of measure, were somehow equalized in length after the attachment of the weights. It is possible, however, that Nicomachus means that the strings were to be equal in length before the attachment of the weights – the order of the steps in his account being non-significant. In this case, however, the problem arises that four strings of equal length would, when submitted to four different weights, be stretched to four different lengths⁵⁹. If, on the other hand, Nicomachus indeed meant that the strings turned out to be equal in length after the attachment of the weights, this would require that they were initially unequal in length and, moreover, that their inequalities were in the correct proportions so as to allow for their all being pulled by the different weights to the same length. It is unlikely, however, that Pythagoras had determined the lengths of string required for this result before commencing the experiment. For in order to do this he would have had to know those facts which could only have been derived from the experiment itself. In any case, Ptolemy's critique makes it evident that the experiment was to have commenced with strings of equal length (DURING 17, 12 – 16):

Even if one were to assume a priori that this were possible and, further, that the lengths of the strings were equal, the heavier the weight, the more will it increase the extension of the string attached to it owing to

⁵⁹ The elongation would be proportional to the downward force exerted by the weight, according to Hooke's Law, for which see M. Y. COLBY, *Sound Waves and Acoustics*, New York 1934, 60 – 62.

the greater tension (τῇ πλείονι τάσει); and the more dense will it make the string (πυκνώσει μᾶλλον). The result of this experiment therefore is that the rise in the sounds turns out to be inconsistent with the ratio of the weights (παρὰ τὸν λόγον τῶν βαρῶν).

This statement, by assuming that the strings were of equal length before the application of the weights, is highly useful for the interpretation of Nicomachus's text. Of more significance, however, is the basis upon which it refutes the claims made by Nicomachus. For to argue, as Ptolemy does, that strings of equal length cannot remain equal after the attachment of the weights, is to challenge Nicomachus's representation of the experiment at its outset. And to assert, as Ptolemy does, that the pitches produced by such weighted strings do not harmonize with the ratio of the weights is to repudiate the final results of the experiment. The burden of Ptolemy's argument then was to substantiate this proposition, namely, that the ratio of the weights does not accurately express the intervals produced by the strings (DÜRING 17, 9–11).

The inconsistency noted by Ptolemy between the ratios of the weights (6:8:9:12) and the intervals produced by the four strings derives from Nicomachus's assumption that the musical ratios which are embodied in the weights are necessarily paralleled by the tensions on the strings. In reality, of course, these ratios express string length proportions, not tension⁶⁰. The assumption is evident in the following passage, where Nicomachus says (JAN 247, 7–9): »He [sc. Pythagoras] found that the string stretched (τεινομένην) by the greatest weight produced, when compared with that stretched by the smallest, an Octave.« On the basis that the heaviest weight exerted a tension on the string equivalent to the unit of weight (12), thereby producing the highest note, the largest number in the series is assigned to the higher note of the Octave, Nete, the smallest number assigned to the lower note, Hypate (JAN 248, 19–23). This is in effect to assume that the rise in pitch is consistent with the ratio of the weights and the ratio of the weights is in direct proportion to the tension on the strings. Therefore the ratios 6:8:9:12 are determined by Nicomachus to represent the tension required to produce the rise in pitch from Hypate through the consonances, Mese and Paramese, culminating in Nete⁶¹.

A similarly misleading assumption is made by Theon who, in rationalizing these numerical assignments in which the larger number is allotted to the higher note, evidently intuited that a difference obtains between the ratio of the weights and the force of the tension. But by arguing that this difference was somehow mutually compensatory, he convinces himself that the final re-

⁶⁰ See p. 208 above.

⁶¹ See p. 210 above.

sults of the experiment with weighted strings are consistent with those derived from experiments based upon string length dimensions (HILLER 65, 10–24):

He [sc. Plato] says that it is proper to assign the larger numbers to the lower notes even though this appears not to accord with certain cases involving tension, as, for example, when tension is introduced by the suspension of weights. For when two strings are equal in length and thickness and are alike in other respects as well, the greater weight will produce a higher note because of the greater tension. For since the greater weight produces a greater tension, it adds extrinsically a greater force to the higher note, which note embodies a force peculiar to itself that is less than the weight. Conversely, it is clear that a lower note, by possessing a force peculiar to itself that is greater than the weight, suffices to preserve the proper harmonia and consonance. So that the larger number must be assigned to the greater force. And there is agreement between these and other factors.

Theon here apparently means that the two weighted strings produced an Octave (ὀκτάβια) in a double ratio (cf. HILLER 58, 3–4), the higher note of which he refers to the heavier weight⁶². He then proceeds to argue that an increase in the other factors – length and thickness (HILLER 65, 4–66, 5) – produces a result that is the reciprocal of that induced by tension. Accordingly, he says that the assignment of the larger number to the higher note must be based upon the added force exerted by tension to the intrinsic force of the higher pitch, while the assignment of the larger number to the lower note is based upon the concomitantly greater intrinsic force of the lower pitch over the increase in length and thickness. The illogicality of Theon's argument derives from his according to pitch an intrinsic force (τὴν ἰδίαν ἰσχύον), which he treats as independent of the factors producing it.

Theon's tortuous attempt to provide a theoretical basis for numerical assignments derived from tension proceeds on the implicit acceptance of the experiment with weighted strings. And it is evidently against construals of this sort that Ptolemy levelled the charge of διαβολή (DURING 17, 2). For by so expressing himself, Ptolemy was in effect taxing the experimenters with a total disregard for scientific procedures, the primary task of which was to provide against the introduction of uncontrolled variables (DURING 17, 19–20): ἐργώδους ὄντος πάνυ τοῦ τηρεῖν ... ἀπαραλλαξίαν. Thus he argued that there was no way in which strings of equal length and density could be made

⁶² According to Nicomachus Harmonikon Enchiridion 9 (JAN 252, 5–6) ὀκτάβια was the Pythagorean term for Octave (τὸ διὰ πασῶν). Cf. Philolaus Vors. 44 B 6 (DIELS-KRANZ 409, 10) = Nicomachus Harmonikon Enchiridion 9 (JAN 252, 17–18).

to remain identical in these respects once the different weights were attached to them. For in every case these dimensions were increased as the weights became heavier (DURING 17, 13 – 15). Therefore, although the difference in weights played a role in producing the pitch variation, that difference, in view of the inconsistency between the weight ratios and the pitch intervals, could not be the determining factor. For the ultimate and real cause of the inconsistency was that the tension exerted on the strings by the weights in these ratios was insufficient to produce the postulated intervals. Faced with this inconsistency, which he attributed to the as yet undetermined properties of tension, Ptolemy advocated the use of the Canon⁶³ in preference to experiments with weighted strings (DURING 17, 20 – 24):

A string stretched over a so-called canon will demonstrate to us more accurately and with greater accessibility the ratios of the consonances, not by admitting tension on a random basis (οὐ μὴν ὡς ἔτυχε λαβοῦσα τὴν τάσιν) but first by providing through preliminary tests against the anomaly that would result from the preparation of the experiment.

The inference from this statement is that Ptolemy conducted the experiment with weighted strings and was unable to duplicate the findings advanced by Nicomachus; that in place of the expected consonances asserted by Nicomachus, he found an ἀνωμαλία.

The consonances which Ptolemy expected to be produced by the weighted strings are described thus by him in Harm. 1, 7 (DURING 15, 26 – 16, 6):

Of the homophones⁶⁴, the fairest and the most unified is the Octave; so that with this homophone the double ratio is in harmony (ἐφαρμόζειν) ... After the homophones, the first consonances which most nearly divide the Octave in two are the Fifth and the Fourth; so that the Fifth is assumed by the hemiole and the Fourth by the epitrite.

Ptolemy then proceeded to carry out the experiment in accordance with the dictates of Nicomachus⁶⁵. He suspended four strings of identical length, thickness and density. (These strings, were they to be held stationary at both ends, would produce unisons or ἰσότονοι)⁶⁶. To these free-hanging strings he

⁶³ See note 12 above.

⁶⁴ Homophones are defined by Ptolemy Harm. 1, 7 (DURING 15, 10 – 12) as intervals which when sounded simultaneously create in the ear the impression of one note. Such intervals are Octaves and combinations of Octaves (ὡς οἱ διὰ πασῶν καὶ οἱ ἐξ αὐτῶν συντιθέμενοι).

⁶⁵ See pp. 222 – 223 above.

⁶⁶ Isotones are defined by Ptolemy Harm. 1, 4 (DURING 10, 1 – 2) as sounds »which are indistinguishable in pitch« from one another.

attached weights in amounts analogous to the musical ratios. When plucked each string indeed produced a pitch which rose in proportion as the weights increased. But the rise in pitch was, as Ptolemy observed, »inconsistent with the ratios of the weights« (DURING 17, 15 – 16). Thus while there was a rise in pitch (ἐν τοῖς ψόφοις ὑπεροχὴν) from string to string, it was not sufficient to produce the Octave, Fifth and Fourth prescribed by these ratios. Since the intervals thus produced were anomalous (DURING 17, 24), Ptolemy evidently employed strings of different thickness and material (density) in an attempt to correct this deficiency. As expected, the thinner and the denser the strings, the higher were the pitches produced⁶⁷. But varying these factors did not eliminate the anomaly. For submitting the strings to the same tensions (i.e., weights) as before, Ptolemy found the same anomalous intervals occurring, only at a higher pitch register. Thus he concluded that no accord was possible between the intervals produced and the ratios of the weights even when the pitch range was heightened by the use of thinner and denser strings (DURING 17, 11 – 12): τῷ καὶ πυκνοτέρας καὶ λεπτοτέρας ἐν ταῖς αὐταῖς τάσεσιν ὀξύτερος φθόγγος ποιεῖν. Having eliminated the factors, thickness and density, as responsible for the deviant results, Ptolemy reached a critical stage in the experiment. For the only recourse left to him was alteration of the tension exerted by the weights. But this would have required him to depart from the prescribed ratios. Since, however, he had already observed that the weights as prescribed increased the lengths and density of the strings (DURING 17, 13 – 15), it is a fair assumption that he saw the solution as implicit in the relationship between the weights and the pitches. Thus the problem became for him how to determine what increase of tension and consequent elongation of the strings had to be applied in order to raise the pitches to the required height. Having arrived at this stage in the experiment, Ptolemy was on the brink of discovering the true proportions between tension and pitch variation.

The problem facing Ptolemy was the obverse of that dealt with by MERSENNE. For while Ptolemy commenced the experiment with strings of equal length (thus with potential unisons) which he sought to convert into consonances by using weights in the musical ratios, MERSENNE commenced the experiment with strings whose lengths differed in proportion to the musical ratios (thus with potential consonances) which he sought to convert into unisons. Thus the problem for Ptolemy consisted in determining what weights were necessary to make strings of equal length be at the different tensions required for the production of the consonances. For MERSENNE »the difficulty consists simply in knowing what weight is necessary to make the double string be at a tension equal to that of half the length«⁶⁸. In other words, MERSENNE's

⁶⁷ See pp. 215 – 218 above.

⁶⁸ MERSENNE (n. 2 above), First Book of String Instruments, Prop. XVI, 59.

task was to raise the pitch of the longer string an Octave in order to produce a unison with the shorter string. He discovered that a fourfold weight was necessary to put the string of double length (productive of the lower octave note) in unison with the shorter string (productive of the higher octave note). He thus concluded⁶⁹:

When, first of all, the strings are double in length, the force or weight which draws the double string tight must be four times the weight which draws the half length tight, when one wishes to put them in unison; and if the string were quadrupled in length, there would be necessary a weight sixteen times the size, that is to say, which would weigh sixteen times as much as that which tightens the string one-fourth the length. Thus the length follows the single ratio of lines and of the roots, and the size of the weights follows the double ratio of planes or squares.

Despite the differences in their approaches, the issue for both Ptolemy and MERSENNE was the same: how to raise the pitch of a string an Octave. For to convert an Octave into a unison, as with MERSENNE, and to convert a unison into an Octave, as with Ptolemy, the same solution obtained: the pitch of one string had to be raised an Octave by a requisite increase in the tension⁷⁰. Whether Ptolemy in fact produced the desired results must remain an open question. The fact that he had isolated the problem, however, places him at the very threshold of discovery⁷¹.

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⁶⁹ Loc. cit.

⁷⁰ See MERSENNE (n. 2 above), *Third Book of String Instruments*, Prop. VII, First Rule, 176, which pertains to the experiment as conducted by Ptolemy with strings of equal length and thickness.

⁷¹ References are made to the following texts: L. DEUBNER, *Iamblichi De Vita Pythagorica Liber*, Leipzig 1937; H. DIELS and W. KRANZ, *Die Fragmente der Vorsokratiker*¹², Dublin/Zürich 1966; I. DÜRING, *Die Harmonielehre des Klaudios Ptolemaios*, Göteborg 1930; I. DÜRING, *Porphyrus Kommentar zur Harmonielehre des Ptolemaios*, Göteborg 1932; G. FRIEDLEIN, *Anicii Manlii Torquati Severini Boetii De Institutione Musica*, Leipzig 1867; R. HELM, *Fulgentii Mitologiarum Libri Tres*, Leipzig 1898; E. HILLER, *Theonis Smyrnaei Expositio rerum mathematicarum ad legendum Platonem utilium*, Leipzig 1878; FR. HULTSCH, *Censorini De Die Natali Liber*, Leipzig 1867; G. H. JONKER, *The Harmonics of Manuel Bryennius*, Groningen 1970; H. PISTELLI, *Iamblichi In Nicomachi Arithmetica Introductionem Liber*, Leipzig 1894; A. J. H. VINCENT, *Notices et extraits des manuscrits de la bibliothèque du Roi et autres bibliothèques*, vol. XVI, part 2, Paris 1847; J. H. WASZINK, *Chalcidii Commentarii in Platonis Timaeum*, London and Leiden 1962; J. WILLIS, *Macrobian Commentarii in Somnium Scipionis*, Leipzig 1963; R. P. WINNINGTON-INGRAM, *Aristidis Quintiliani De Musica*, Leipzig 1963; W. M. LINDSAY, *Isidori Hispalensis Episcopi Etymologiarum sive Originum Libri XX*, Oxford 1911.