

An Enigma Resolved

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AN ENIGMA RESOLVED

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Excavations in Ur by Sir A. Woolley showed that the theorem of Pythagoras was known in Babylonia at a very early date. The Dutch professor F. Schuh gives in his book on numbers, their scientific and mystical significance (F. Schuh, *De macht van het getal*), an account of this excavation.

The gist of the information obtained by Sir A. Woolley is, that the theorem and some of the several proofs were known a considerable time before Pythagoras.

It is therefore not easy to understand, why the name of Pythagoras was attached to the theorem; he evidently did not propose the theorem, nor did he give the first proof. It is not understandable why the Pythagoreans, with their aversion of bloodshed and violence, and their stressing of the due measure of all things, should sacrifice a hekatombe of bulls, according to tradition, in connection with this theorem, a long known and proven theorem. It is perhaps easier to understand why such a tradition came into existence, when we assume that Pythagoras (or perhaps his pupils) used this long known and proven theorem in elucidating something fundamentally new and of great mathematical importance.

We know that traditionnally the Pythagoreans occupied themselves very much with the theory of numbers. Of course I must remind my hearers that the present communication is spoken in 20th century language. No classical Pythagorean would say of himself that he was busy with the theory of numbers; he had no such understanding of his activities.

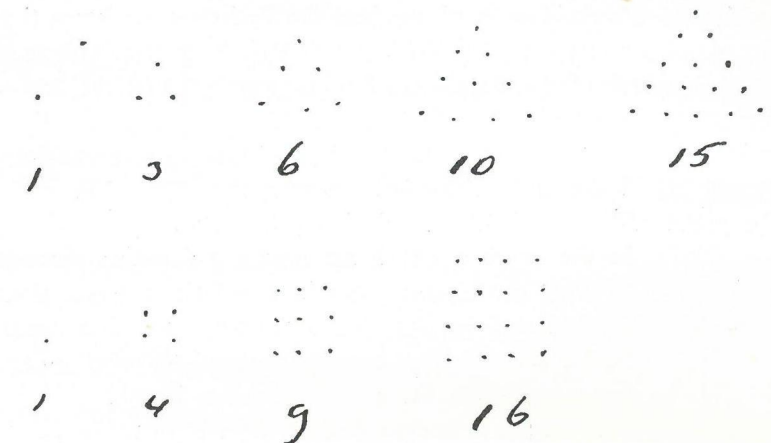
The principal technique of the Pythagoreans was that they placed the number in a row of numbers.

For instance :

The row of rational numbers : 1 2 3 4 5 6 7 8 a.s.o. till infinity.

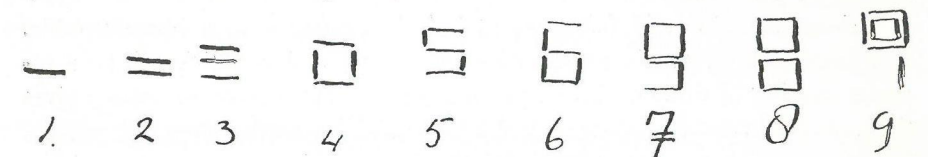
The row of triangular numbers : 1 3 6 10 15 21 28 a.s.o.

The row of square numbers : 1 4 9 16 25 36 49 a.s.o.



Perhaps it is interesting to note, that the classical Hindoo mathematician denoted the numbers not as a collection of points in a geometrical figure but as a collection of *lines* in a geometrical figure.

We can easily recognize a pattern of lines in our present-day ciphers (derived from Hindoo ciphers).



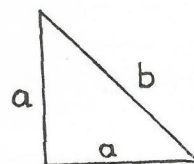
It must be said, that it is very astonishing that the classical Pythagoreans were able to work on pure mathematics which were only given an application about 2000 years later (in probability calculus). Perhaps the lack of application gave rise to the unsound practice of attributing qualities to numbers. For instance, active or passive. Perhaps it is possible to say that passivity is dual, or the second (or we define a certain row of numbers as passive analogous with the row of triangular numbers). But you can never say that two is passive or has the quality of passivity.

(Note : The following quotation was not included in the communication. Numbers seemed to the Pythagoreans to be first things in the whole of nature, and they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number. The Pythagoreans treated the elements of numbers *not* as priorities of certain other substances, but as the reality of everything (Aristoteles *Meta. A.* 986a, 587).

Nevertheless, without application mathematics is not able to go very far; the thinking is bound to become very muddled when there is no practical application as a guide.

Besides the aforesaid rows of numbers the Pythagoreans knew the row of the irrational roots: $\sqrt{2}$ $\sqrt{3}$ $\sqrt{5}$ $\sqrt{7}$ $\sqrt{11}$ $\sqrt{13}$ $\sqrt{17}$. They were able to prove without making use of the definition of prime number, that the first seven members of the row of primes 2 3 5 7 11 13 17 19 23 29 31 a.s.o. had an irrational root. [(Plato, Theaitetos 147c - 148b.) (A. E. Taylor, Zahl und Gestalt bei Platon und Aristoteles. Gnomon II (1926), 396 - 405.)]

The proof of the irrationality of $\sqrt{2}$ is given now.



In the figure a and b are numbers denoting the number of times the biggest possible comparison length gives the determined lengths. In consequence a and b cannot be both even, for in that case the comparison length (common measure) is not the greatest possible length.

Propose; b is uneven (odd), then b^2 is uneven, however this is impossible, for b^2 is the sum of two equal parts and as a consequence of that b^2 is even.

Let us therefore propose: b is even; in this case b^2 is the sum of two even parts, a^2 is therefore even and a also, for alone an even number has an even square. This is however in contradiction with the observance that a and b cannot be both even.

From the aforesaid follows that b is in respect with a immeasurable. It is impossible to obtain any length comparing a and b, which will go a rational number of times on both lengths a and b. The number $\sqrt{2}$, which gives the numerical relation between a and b ($b = a\sqrt{2}$) is neither even nor uneven, but *irrational* as the row of rational numbers consists of the even and the uneven numbers.

In the same way the classical Pythagoreans (Theodoros) proved that $\sqrt{3}$ $\sqrt{5}$ $\sqrt{7}$ $\sqrt{11}$ $\sqrt{13}$ $\sqrt{17}$ were irrational ($\alpha\lambda\omicron\gamma\omicron\nu$).

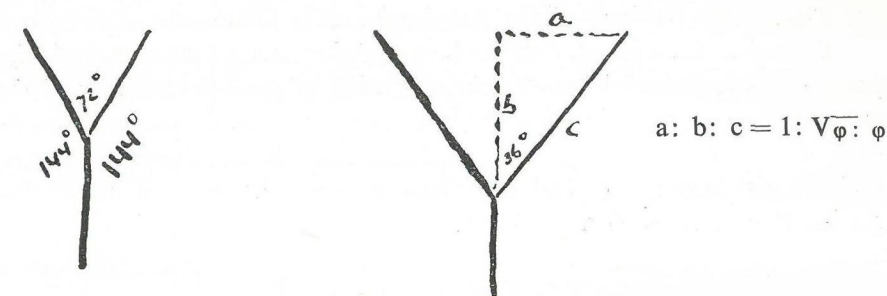
The conception of irrationality is not the same thing as the conception of prime numbers (naturally the root of a prime number is irrational) — for there are several other irrational numbers; for instance

$$\begin{aligned}\pi &= 3, 141592653589 \dots\dots \\ e &= 2, 7182818 \dots\dots\dots \\ \phi &= 1, 618034 \dots\dots\dots\end{aligned}$$

(Note, the following observations were not included in the communication). The golden proportion ϕ is derived from the observance that it is the only proportion which can be stated with a minimum of numbers involved.

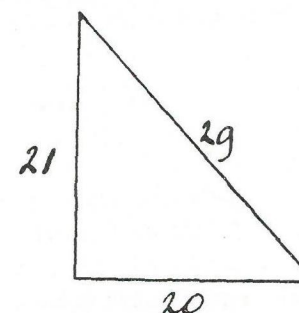
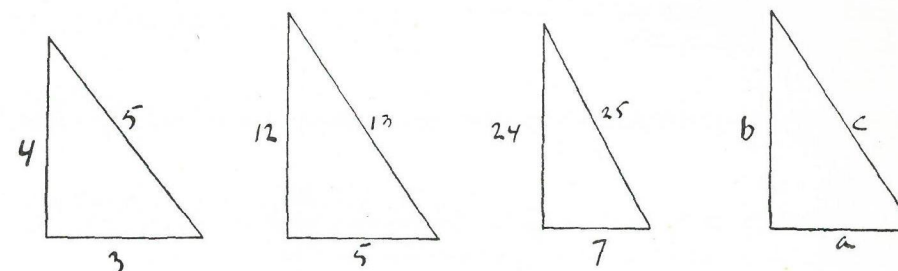
$$a : b = b : (a + b) = 1 : \phi$$

The letter of Pythagoras (Ypsilon) contains the golden proportion, as is elucidated by the following figure:



The principal interest of the classical Pythagoreans in irrationality was the connection with immeasurability of actual lengths. This fact made a tremendous impression on them, as it doubtless will make on you if you realise fully its significance.

The classical Pythagoreans were interested in seeing if all hypotenuses were in regard to the other sides immeasurable. Now it is an interesting thing to note that the very few exceptions to the rule that the hypotenuse is in regard to the other sides immeasurable, are called: «Pythagorean triangles». I will give the following examples:



This procedure for obtaining Pythagorical triangles is mentioned in Proclus Diodochus «In primum Euclides elementorum librum commentarii».

The triangles 3 - 4 - 5 and 20 - 21 - 29 were known in Egypt.

My conclusions are therefore, that the theorem of Pythagoras was the theorem used by Pythagoras (or his pupils) to demonstrate the principle of immeasurability.

This principle was so difficult to understand by the commentators, who were not initiated, that it gave rise to false legends in which nevertheless is contained an awe for the findings of the philosopher. I will give two examples:

First : The tradition of the slaughtering of a hekatombe of bulls.

Second : (Jamblichos, Euclides). A tradition that a Pythagorean (Hip-
pasos) was swallowed by the sea in retaliation of the fact that he told the
world the secret of irrationality (or in other variants the existence of the re-
gular dodecahedron). (E. Sachs, Die fuenf Platonischen Koerper. Zur Ge-
schichte der Mathematik und der Elementenlehre Platons und der Pytha-
goreer. Berlin. 1917. 37).

INTRODUCTION à LA SCIENCE PYTHAGORICIENNE DES NOMBRE

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Pythagore, touché par un rayon d'indicible lumière, a entrevu que d'une Cause première créatrice, sont émanés tous les êtres vivants, de même que du premier nombre, sont sortis tous les autres en lui empruntant leurs qualités et leurs vertus respectives.

Il a vu comment le nombre 1 présente son épanouissement complet dans la tétrade 1, 2, 3, 4 ; comment les trois premières tétrades se sont organisées pour former la décade, véritable octave se reproduisant pour porter à l'infini les lois de la tétrade sacrée, les pouvoirs de l'Unité.

Nous avons voulu essayer de contrôler, de vérifier par nous-mêmes cette vision grandiose. Depuis, nous sommes certains de son exactitude.

L'étude que nous présentons est principalement d'ordre méditatif. En dépit de ses apparences, elle n'est pas d'ordre mathématique. C'est par nécessité que nous avons adopté un peu le langage mathématique : les symboles $G_0, G_1, G_2, \dots, T_0, T_1, T_2 \dots$ qu'on y rencontre sont simplement les abréviations des expressions « Groupement initial », « Groupement du 1^{er} ordre », « Groupement du 2^e ordre », « Tableau d'origine », « 1^{er} Tableau », « 2^e Tableau » Sans ces abréviations, notre texte eût été considérablement alourdi et nous nous y serions perdus nous-mêmes. Nous prétendons que toute personne qui possède les premiers éléments de l'Arithmétique peut nous suivre à condition d'apporter à nous comprendre la même volonté que nous avons déployée dans l'espoir de nous rendre intelligibles.

Nous devons reprendre la voie que Pythagore a ouverte il y a 25 siècles et qui mène à la Vérité.

Nous ne concevons plus qu'on puisse aborder aucune étude transcendente, dans un domaine quelconque : Religieux, Moral, Philosophique, Politique, Économique, Social ..., sans avoir devant les yeux la figure pyramidale de la Tétrade sacrée.

Dans tous ces domaines, interrogeons-la comme on interroge une personne savante : posons-lui des questions, des sous-questions. Demandons-lui des précisions. Nous n'aurons pas à craindre qu'elle ne nous réponde pas. Elle nous parle même quand nous n'écoutons pas.

Nous avons ébauché l'étude de la famille (page 176). Voulons-nous savoir si la femme est inférieure, égale ou supérieure à l'homme ? Si elle doit rempla-