



Highlights of Ancient Greek Philosophy

by Khan Amore

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Pythagoras

by Guy Murchie

Music of the Spheres, Volume II

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Pythagoras was without a doubt one of the most influential of the Greek philosophers, as well as one of history's most important mathematicians, and also perhaps history's first music theorist. Born in Samos, he was the founder of the Pythagorean school, a religious and philosophical school which exercised a lasting influence on the course of ancient science, philosophy, and theology in Greece, Alexandria, and elsewhere. Pythagoras was instructed in the teachings of the early Ionian philosophers, and in his subsequent travels he also became initiated into the doctrines of the Egyptian priests. He is said to have been driven from Samos by his disgust for the tyranny of **Polycrates** (see *The Amazing Story of Amasis and Polycrates*). About 530 B.C. he settled in Crotona, a Greek colony in southern Italy, where he founded the moral and religious school called by his name. To this school not only men, but women as well, flocked from all parts of southern Italy. Although the original purpose of the brotherhood was religious rather than political, the society became involved in the fierce struggle between the Aristocracy and the Democracy which was raging in southern Italy; when the democratic party gained the upper hand, it turned upon the aristocratic Pythagorean brothers in fury and burned them in their meeting places. It is not certain whether Pythagoras himself perished in this



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outbreak, or whether, as is claimed in one tale, he escaped only to starve himself to death at Metapontum. The Pythagoreans remained powerful in Italy until the middle of the Fifth Century B.C., when their order was violently stamped out. The exact nature of Pythagoras' own teachings is not known with great certainty. Like the legendary **Jesus of Nazareth**, Pythagoras committed nothing to writing and his disciples sought to gain credit for their own views by attributing them to their venerated master (*also* as in the case of Jesus, except in Jesus' case, the disciples may well have sought to gain credit for their own views by attributing them to a venerated master who may never have existed.) For this reason, it is generally wiser to speak of the theories of the Pythagoreans as a whole rather than to attempt a discussion of the views of Pythagoras. The Pythagoreans adhered to certain mysteries, which were similar in many respects to the *Orphic Mysteries*. An examination as to fitness was a qualification for admission into their number. Obedience and silence, abstemious eating habits, simplicity in dress and "external goods," and habitual introspection ("Know Thyself") were prescribed. The Pythagoreans believed in the immortality and transmigration of souls, including the reincarnation of human souls into animals. **Xenophanes** transmitted to us the tale of how, once, when a puppy was being whipped, Pythagoras, who was passing by, took pity on it, saying, "Stop! Do not beat it! It is the soul of a friend; I recognize his voice!" Clearly, whether it turns out to be what really happens to us after death or not, a belief in reincarnation makes for a kinder, gentler, more compassionate world. Although Pythagoras undeniably had strong leanings toward mysticism, he and his followers also carried on extensive mathematical investigations. From an early time, their attention was turned to odd and even numbers, and to prime and square numbers, and from this arithmetical standpoint they cultivated geometrical studies, with *number* becoming for them *the ultimate principle of the Universe*. According to the Pythagoreans, the proportion, order, and harmony of the Universe were all closely connected with *number*, which lay at the very foundation of existence. By making *number* the basis of their philosophical system, they raised mathematics to a science and laid the foundation for all later developments in geometry. The greatest of the mathematical advances of Pythagoras was the first proof of the hypotenuse theorem which still bears his name and is still taught today to every schoolgirl or boy — the **Pythagorean Theorem**. As the reader will no doubt recall, this proposition states that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides. Although the Babylonians had discovered this theorem a millennium earlier, Pythagoras is credited with being the first to *prove* it. **Pythagorean Triads** are numbers which are related to one another in this way, and represent the ratios of the three sides of a right triangle to each other; for instance (3,4,5) is a Pythagorean Triad because $3^2 + 4^2 = 5^2$; and because this relationship holds true we know that the 3-4-5 triangle (the famed "Egyptian Triangle" — the only Pythagorean triangle whose sides are in arithmetical progression, and the only one whose area is half its perimeter) *must* contain a right angle. The Astronomy of the Pythagoreans marked an important advance in ancient scientific thought, for they were the first to look upon the Earth as a *globe* revolving with the other

planets, including the Sun, around a central fire. They explained the harmonious arrangement of things as that of bodies in a single all-inclusive sphere of reality, *moving according to a numerical scheme* (an assumption which is still held dear in present-day science, for the trajectories of planets and spacecraft do indeed seem to obey mathematical laws). Because the Pythagoreans thought the heavenly bodies to be separated from each other by intervals corresponding to the harmonic lengths of strings, they held that the movement of the celestial spheres gave rise to a musical sound, which they called the “harmony of the spheres”. In the mysteries celebrated by the Pythagoreans (as in the orgiastic Eleusinian Mysteries scene of Khan Amore’s *HYPATIA*) a sacred contrivance — somewhat similar to the *bull-roarer* of the Australian Aborigines — was employed to create an other-worldly sound which suggested this mystical music of the spheres. According to Archytas, the Pythagorean, “The ‘*rhomboi*’ which are whirled about in the mysteries produce a low note when whirled gently, but a high one when whirled vigorously.” It was because of this ancient contrivance that the Pythagoreans made the connection between mathematics and music and astronomy. The planets, too, whirled about the Earth at different distances, as if at the ends of strings of different lengths. Should they not, too, make a harmonious sound like the strings of an aeolian harp, vibrating in the wind? The whole Universe must then be a cosmic symphony of order and harmony, based on number as all music ultimately is. In the history of human thought, this was man’s first attempt to give a mathematical description of the Cosmos. And how did the Pythagoreans account for the fact that we do not hear the harmony of the spheres? They claimed that this sound is with us since birth so that we are unable to distinguish it from its opposite, silence; for sound and silence are only known by contrast. Consequently, we are like workers in bronze, who are so used to noise that they do not notice it. It’s similar to why we don’t know the smell of oxygen — we only noticed that smell once, at the moment of our birth, when we took our first breath. Since then, we haven’t ever *not* smelled it, and so we are unable to recognize its smell at all anymore (although, from chemical considerations, I suspect it has a sweet, fresh smell similar to that of ozone, only less pungent.) The Pythagorean world-view was based upon the principle that *number is the basis of all things*, and their conception of a measure or proportion or harmony in all things was a salient characteristic of all Greek thought which was to follow. In a similarly Pythagorean vein, in addition to inventing *Lyrics* in general, the ancient Greeks also advanced the development of Music, as they were probably the first people to make a conscious use of combined tones of different pitch in order to produce *harmony*. The Greeks often accompanied a melody with its octave above, probably as the result of combining boys’ voices with men’s; the former, unable to sing in the low register of the men, adjusted their pitch to an octave above. The practice of combining such tones was known as *magadizing*, a term derived from the harp-like instrument the *magadis*, which, by means of frets, was capable of producing octaves. (The theoretical writings of the Greek musicians disclosed that they considered the octave, fifth, and fourth as being concordant; and the thirds and sixths as discordant.) For a long time, the teachings of Pythagoras were kept secret. Although the

disciples of Pythagoras maintained a remarkable silence, it became known that Pythagoras held that the soul is immortal; that it transmigrates into other kinds of animals; that the same events repeat in cycles, nothing being truly new; and, finally, that *all things with souls should be regarded as akin* (a particularly noble sentiment.) Pythagoras may not have originated these teachings himself, but may have learned at least some of them in the East; on the other hand, it is also possible that he arrived at these conclusions independently, for they offer a logical resolution of the conflict between the Ionian belief in an immortal soul and the reality of a body which is subject to decay and dissolution. Whatever the case, Pythagoras seems to have been the first to introduce these beliefs into Greece. Pythagoreanism really begins with the question of how an immortal soul can possibly be related to an ephemeral body, and Pythagoras' answer to it was momentous for the history of western thought. According to Philolaus, Pythagoras maintained that "For the sake of punishment the soul is yoked to the body and buried in it as in a tomb." Mind you, this was momentous for the history of Western thought not because it was a *beneficial* development, but because Plato later seized upon this idea, and it was transmitted through various other doctrines — including Neo-Platonism — to the Christians, who twisted it into a justification for all manner of atrocities (for example, the killing of heretics in order to "save their immortal souls," etc.) and who used it as a "logical" basis for pleasure-hating prudery, as well as a philosophical prop for the dogma, still beaten into Christians by their moralists today, that pleasure is evil, that the body is evil, and especially, that *sex* is evil, for it is the cause of the enslavement of the immortal soul in the mortal body. For the most part, Pythagorean cosmological views differed little from those of **Anaximenes**, but the Pythagorean departure from this view occurred when it was noted that the bodies which inhabit the upper air — the sun, moon, and stars — move eternally; thus, they were said to be "immortal and divine." But here, below, all is subject to corruption and decay, and so it was assumed that different laws apply in the heavens and on Earth. The difference between the two realms amounts to a radical breach in the natural order (before this, it was assumed that "as above, so below.") For the history of Western thought this development was unfortunate; for in the form which **Aristotle** gave to it, this view prevented the development of a rational system of celestial mechanics down to the time of Galileo (who proceeded once more from the ancient assumption that the same laws of Nature apply above, as those that apply here below.) [Of course, Aristotle cannot be blamed for having been elevated to the status of God-like authority by later people who were trained to accept everything on authority, without question. He was a brilliant natural philosopher who made great contributions to the ascent of man, but *brilliance does not equal infallibility*. No authority must ever be beyond question if the ascent of man is to continue, and throughout the millennium of ignorance known to us as the *Dark Ages* and *Middle Ages* nearly all further human advancement was held in stasis, not by Aristotle's supreme authority (as has been suggested by Christian historians) but by a Church-enforced total lack of original thinkers who dared to openly question all arguments, even those which issue from supreme authority. That is, after all, how the

ascent of man works: each thinker gives it his best shot, but he is not infallible, so it is up to those that follow to study his methods and conclusions, and sift the few precious gems of truth from the nonsense, and then finally, having rejected the nonsense, to use these few hard-won truths as a starting point in their own *further* search for truth. In this way, each generation builds upon the work of the last, instead of re-inventing the wheel every generation. “If I have seen further,” said Isaac Newton, “it is by standing on the shoulders of giants.”]

In the Pythagorean view, the division between heavenly and earthly realms corresponds to the division between soul and body, and soul is distinct from life, and is immortal. It is not affected by the corruption which overtakes the body, but stands apart from it even in life — a detached portion of the divine in exile from its native land. Because it is in exile it yearns to return to that upper region whence it came, to be released from the prison house of the body, and to rejoin once more to the company of the gods. The effect of this new conception of the soul was to undermine the traditional view of man’s place in the Cosmos, which was quite separate from the gods, who alone possess everlasting life, while men are but creatures of a day:

“In brief space the joy of mortals waxes;
In brief space it falls to the ground,
Stricken by an adverse fate.
We are but creatures of a day.”

— Pindar, *Pythia*

“If a man having wealth surpass all others in beauty,
displaying his strength by victory in the games,
let him remember the limbs he arrays are mortal
and that he will come to the end that all men come to,
clothing himself with earth.”

— Pindar, *Nemea*

The teachings of Pythagoras ran directly counter to this humble traditional view. Man is *not* the creature of a day; he has in him that which is immortal and makes him akin to the gods. And therefore, as Aristotle put it:

Highlights of Ancient Greek Philosophy
PYTHAGORAS

“We ought not to obey those who tell us that a man should think a man’s thoughts, and a mortal the thoughts of a mortal. On the contrary, we should endeavor as far as possible to become immortal, and to do all that we can to live in accordance with what is highest in us.”

— Aristotle, *Nicomachean Ethics*

If this statement is interpreted to encourage us to be the best that we can be, it is a noble sentiment, but this line of thought has unfortunately been used as the theological starting point for religious grandiosity, arrogance, and intolerance. The traditional humble and compassionate view that we are all but creatures of a day who know nothing for certain — creatures who rejoice but little and suffer much before clothing ourselves in earth — is replaced by the arrogant stance of the clergymen, who claim to speak for God, and who claim to be able to confer godlike immortality on themselves and their followers, so long as they attempt to adhere to some inhuman and sexless conception of what a human being *ought to be*.

Another distinguishing feature of the Pythagoreans was that they *ate no meat*. This was almost certainly connected with their belief in the transmigration of souls, for if the souls of men enter into the bodies of animals, it follows that we must view all creatures as kin. The eating of animal flesh, being a kind of cannibalism, ought then to be forbidden, and the killing of animals for food will be considered murder, incurring the same blood-guilt as the slaying of one man by another. According to Diogenes Laertius:

“Pythagoras forbade the killing, let alone the eating, of animals which share with us the privilege of having a soul.”

They also ate no beans, perhaps in an attempt to minimize the rank disruptions of the cosmic harmony engendered by flatulence, but there was more to the Pythagorean way of life than the mere observance of dietary rules. This way of life sprang from a desire for purity which ultimately expressed itself in the form of asceticism. Although Pythagoras had a wife (named Theano), she spoke of love as “the sickness of a longing soul,” and so it seems likely that theirs was not a passionate union, but a pragmatic — and ascetic — one. Pythagoreanism was not simply the *desire* for purity, though, it was the conception of *philosophy* as a way of *achieving* it — of bringing the soul into harmony with the divine. Not only the conception, but even the very *word* “Philosophy” was Pythagorean. According to Diogenes Laertius,

“The first to use ... [the word, ‘Philosopher’] and to call himself a philosopher [i.e., a “lover of wisdom”] was Pythagoras.”

In the Pythagorean view, the lover of wisdom can never possess the object of his desire; but the pursuit of it becomes for him a way of life, and a source of happiness which is

pure and unalloyed. To be ever engaged in intellectual inquiry, then, was the Pythagorean prescription for happiness:

“Pythagoras, son of Mnesarchus, practiced inquiry beyond all men ...”

— Heraclitus

The practice of “inquiry” was a prominent feature of the Pythagorean way of life, and it was considered beneficial to the welfare of the soul. It may therefore be surprising for us to learn that:

“ ‘Inquiry’ was the name which Pythagoras gave to geometry.”

— Iamblichus, *Vita Pythagoras*

Although the idea may seem startling to us today, to the ancient Greek philosophers (particularly the Pythagoreans) the “love of wisdom” was the equivalent of the “love of the divine.” To them, a life devoted to inquiry (i.e., *questioning* all, and striving to find provable answers) was a pure and happy one. The study of mathematics and geometry — the derivation of theorems — was their method of bringing the soul into harmony with the divine. Indeed, it might be said that the Pythagoreans raised mathematics to the level of a religion. If the Pythagoreans were correct in their assertion that intellectual inquiry is the way to happiness, then Pythagoras must have been a happy man, for:

“It was [Pythagoras] who brought geometry to perfection ... Pythagoras worked very hard at the arithmetical side of geometry, and discovered the musical intervals of the *monochord*. Nor did he neglect even medicine.”

— Diogenes Laertius

The earliest Greek mathematical work that we have is Euclid’s *Elements* (which, it should be recalled, has come down to us through the Theonine/Hypatian recension — in other words, we have it only because Hypatia preserved it for us). Composed in the Fourth Century B.C., the *Elements* was a compilation of the works of earlier geometers, some of whom were certainly Pythagoreans. It is perhaps amazing that the ancient Greeks were able to bring geometry to its perfection, for the Greeks possessed no algebra in our sense. They were compelled to solve algebraic problems by geometric means. Moreover, the Pythagoreans appear to have represented numbers geometrically, by means of pebbles arranged on a flat surface. The shape of the array of pebbles representing a number was what gave rise to the concept of *polygonal numbers* (e.g., *square numbers*, *triangular numbers*, *pentagonal numbers*, etc.) In addition to his many other discoveries, Pythagoras is also said to have investigated the mathematical problem of finding right-triangles having the square on one side equal to the sum of the squares on the other two sides (in other words, a way of finding *Pythagorean Triads*.) The Pythagorean method of finding ratios of the sides of different right triangles proceeded

from odd numbers. This method takes the given odd number as the lesser of the sides enclosing the right angle. From the square of this it subtracts a unit and takes half the difference as the greater of the sides enclosing the right angle. Adding a unit to the square of the original odd number and dividing by two yields the hypotenuse. For example, starting with the odd number 3: the square of this is 9, from which a unit is subtracted, leaving 8, half of which is 4. If a unit is added to the square of 3 and we divide this sum by 2, we get 5, and in this way we have found a right-angled triangle having as its sides 3,4, and 5. Thus, Pythagoras has given us a way of finding integer-triads (Pythagorean Triads) which represent the sides of different right-triangles. If n is the given *odd integer* that we start out with (which we will call the *generatrix*) then the sides of the right triangle having this number as one of its sides will be:

$$(n) , \quad \left(\frac{n^2 - 1}{2} \right) , \quad \left(\frac{n^2 + 1}{2} \right)$$

This so because the *Pythagorean Theorem* proves that in a right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. Conversely, if this relationship holds true for a triangle we know that the given triangle must contain a right angle. That this relationship does indeed hold true for the triads generated by the above method can be ascertained by algebraically verifying that the following equation holds true:

$$(n)^2 + \left(\frac{n^2 - 1}{2} \right)^2 = \left(\frac{n^2 + 1}{2} \right)^2$$

Using this method we can find an *infinite* number of right-angled triangles having all three sides expressible as integers (although many are scalar multiples.) For example, with generatrix 3 we find the famed 3-4-5 right triangle; with generatrix 5 we find the less-known 5-12-13 right triangle; with generatrix 7 we find the still-less-known 7-24-25 right triangle, and so on *ad infinitum*. The short leg, n , of the right triangles found by this method must always be an *odd integer* because if n were not odd, then the long leg, $(n^2-1)/2$, and the hypotenuse, $(n^2+1)/2$, would not be whole numbers, as required by the problem. According to **Proclus**, it was Pythagoras who also discovered the theory of proportionals and the construction of the “cosmic figures” known to us today as the five *Platonic Polyhedra*: the tetrahedron, the hexahedron (a.k.a. the cube), the octahedron, the dodecahedron, and the icosahedron. These five regular solids (the

only five which are *possible*) were called “cosmic” because of Plato’s use of them in his *Timaeus* in the construction of the world-order (i.e., cosmos.)

Among the followers of Pythagoras it was thought that the study of theoretical geometry elevates the soul. Before Pythagoras, geometry was a practical pursuit, not a study undertaken for its own sake. Egyptian “geometry” was, as the name implies, the art of measuring land, for such surveying was necessary not only for the construction of pyramids, but it was also forced on the Nile-dwellers by the annual flooding of their river, which obliterated everyone’s boundaries. Egyptian geometry consisted mainly in knowledge of certain practical rules which had been discovered empirically, and for which it did not occur to the Egyptians to seek proof. It was in this form that geometry passed to Greece. Pythagoras freed geometry from its connection with practice. In his hands geometry became a “liberal study” — a pursuit worthy of free men. This meant not merely that mathematics was a study for freemen as opposed to slaves, but that it was a study capable of *making* a man free. When freed from practical concerns, mathematical inquiry frees the thinker from all reliance upon the uncertain testimony of the senses, for mathematical inquiry proceeds not by use of the senses but “immaterially and conceptually.” As far as the Pythagoreans were concerned, *a priori* truths arrived at through the use of pure reason were of a divine nature and constituted a higher reality than the reality of the sense perceptions (which can easily be fooled.) This was a view which, through the influence of Plato, was to affect profoundly the whole development of Western thought.

Perhaps equally momentous was Pythagoras’ discovery of the *musical intervals*. These were discovered by means of the *monochord*. The monochord, as the name implies, is a one-stringed instrument. By stopping the string at one point, plucking it, then stopping it at another and plucking it again, it is possible to establish a relationship between the sounds produced and the lengths of the vibrating strings. That relationship is known today as the *Law of Pythagoras*, and it states that “When a string and its tension remain unaltered, but the length is varied, the period of vibration is proportional to the length of the string.” For example, (under similar circumstances of string mass and tension) if a string of a given length is plucked it will vibrate, producing a tone of a given pitch or frequency. If that same string is then fretted at its exact mid-point and plucked again the tone produced by the string’s vibration will be exactly one *octave* higher, or in other words the *frequency* of vibration will be exactly twice as high [i.e., the frequency ratio = 1:2] when the effective length of the string is reduced by half. If that same string is then fretted at a point exactly two-thirds its length this will produce the musical interval known as the *major fifth* when the string is plucked on either side of the fret [i.e., the frequency ratio = 2:3] ; and if the string is fretted at a point exactly three-fourths its length its vibration will produce the musical interval known as the *major fourth* when the string is plucked on either side of the fret [i.e., the frequency ratio = 3:4] . These musical intervals (the octave, the major fourth, and the major fifth) were the chief

“consonances” of Greek music. But these ratios do not merely *correspond* to the consonances; *they make them what they are*, and it was in recognizing this that the genius of Pythagoras lay, for it engendered the view that *all order is in its very essence capable of being understood and expressed in terms of Number*. It was Pythagoras who established the existence of mathematical order in Nature — it was he who discovered that *the language of the Universe is Mathematics*.

Down to the time of Orpheus, Greek lyres had four strings (tuned to the notes c-f-g-c', any two of which except f-g could be played together without creating unpleasant dissonance) and these notes of the *tetrachord* formed the main notes of the music scale of ancient Greece. According to tradition, it was the lyric poet **Terpander** (who flourished circa 675 B.C.) that introduced the seven-string Lyre to Greece, however, it was Pythagoras whose mathematical analysis resulted in the standardization of the ordinary *diatonic scale* which has remained the normal scale for Western music ever since (to be improved upon only by Mersenne's *equally-tempered scale* in 1636, and later popularized by J.S. Bach.) In the *Pythagorean Scale*, the frequency of the note C' was exactly $1\frac{1}{2}$ times the frequency of the note F, the frequency of G was exactly $1\frac{1}{2}$ times the frequency of C, and so on, (although Pythagoras spoke in terms of wave-lengths, not frequencies.) This produced a scale having the ratios shown in the following table:

Pythagorean Frequency Ratio Between the Note Indicated in Boldface and the Note C in the Diatonic Scale	Pythagorean Interval Between the Boldfaced Note and the Next Note on the Scale	Frequency Ratio Between Boldfaced Note and C in the Equally-Tempered Scale
C :C = 1.0000	C →D = Tone	C :C = 1.0000
D :C = 9/8 = 1.1250	D →E = Tone	D :C = 1.1225
E :C = 81/64 = 1.2656	E →F = Hemitone	E :C = 1.2599
F :C = 4/3 = 1.3333	F →G = Tone	F :C = 1.3348
G :C = 3/2 = 1.5000	G →A = Tone	G :C = 1.4983
A :C 27/16 = 1.6875	A →B = Tone	A :C = 1.6818
B :C = 243/128 = 1.8984	B →C' = Hemitone	B :C = 1.8877
C' :C = 2.0000	C' →D' = Tone	C' :C = 2.0000

In the Pythagorean (Diatonic) Music Scale the intervals C-D, D-E, F-G, G-A, and A-B are all *exactly* equal, with a frequency ratio of 9:8. Pythagoras described each of these intervals as a “tone.” This left with two smaller intervals, E-F and B-C, each of which is represented by the more complicated frequency ratio of 256:243 (or 1.0535). Pythagoras called such an interval a “hemitone” — an interval which is distinctly less than either the half of a Pythagorean tone or the modern semitone, as can be seen by a comparison of the frequency-ratios of these smaller intervals:

Pythagorean Hemitone	=	1.0535
Half of a Pythagorean Tone	=	1.0606
Equal Temperament Semitone	=	1.0595

Thus, the Pythagorean octave was made up of five equal tones and two equal hemitones, which were rather less than half-tones. The scale was perfect and complete — so far as it went. In addition to the concord of the octave, it contained no fewer than four fifths and

five fourths, a greater wealth of concords than can be obtained from any other selection of eight notes. The scale could be extended indefinitely in either direction by trespassing into neighboring octaves, but the severity of Greek taste restricted the melodies to a compass of an octave — and frequently even a fourth — so as to employ only the most agreeable registers of the human voice. The normal eight-stringed lyre might begin at any note of the scale, but it would end in the same note in the octave above. As there were seven choices possible for the lowest note (c, d, e, f, g, a, and b) this could be done in seven ways, which were referred to as *modes*. The choice for a lyre's lowest note — and hence its *mode* — also varied the positions of the hemitones in the Pythagorean (or diatonic) octave. The resulting modes were as follows:

Ancient Greek Name of Mode	Range of Mode	Scale (Beginning with c)
Lydian	c - c'	c, d, e, f, g, a, b, c'
Ionian	g - g'	c, d, e, f, g, a, b $\flat$, c'
Phrygian	d - d'	c, d, e $\flat$, f, g, a, b $\flat$, c'
Aeolian	a - a'	c, d, e $\flat$, f, g, a $\flat$, b $\flat$, c'
Dorian	e - e'	c, d $\flat$, e $\flat$, f, g, a $\flat$, b $\flat$, c'
Myxolydian	b - b'	c, d $\flat$, e $\flat$, f, g $\flat$, a $\flat$, b $\flat$, c'
Syntolydian	f - f'	c, d, e, f $\sharp$, g, a, b, c'

Of the three most popular modes, the Dorian was considered “virile”, the Phrygian “emotional”, and the Lydian “plaintive” (although it should be recalled that the original ancient Greek names of the modes were later jumbled-up by Medieval church musicians.)

As we have seen, Pythagoras' mathematical analysis of music and harmony led to the standardization of the Pythagorean (or diatonic) music scale, but this was a practical development which had philosophical ramifications as well. Once it was seen that the properties and ratios of musical consonances were expressible in numbers, this way of looking at things, when applied to other subjects, made it seem that *all things* are wholly modeled in their nature upon numbers. As Aristotle put it, "... [The Pythagoreans] took numbers to be the whole of reality, the elements of numbers to be the elements of all existing things, and the whole heaven to be a musical scale and a number." It is hardly necessary to point out the importance of this generalization for the history of science. Within two centuries it was to give rise, in the hands of **Archimedes**, to the science of *mechanics*; and **Galileo**, at the beginning of the Modern Period, took it as the starting point for his own work:

"[Natural] Philosophy is written in the great book which is ever before our eyes — I mean the Universe — but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. This book is written in the mathematical language, and the symbols are triangles, circles, and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth."

— Galileo Galilei, (1564-1642)

This was the world-view of Pythagoras and his followers: that *mathematics is both the basis and the language of the Universe*. And it was upon this assumption that the foundations of classical physics were securely laid. But this is not all of Pythagoras' legacy. Clearly a genius of the first magnitude, Pythagoras was not content to restrict himself merely to seminal developments in music theory, and in to establishing the underlying tenets of Physics, but he also ventured boldly into the realm of *metaphysics*. According to Aetius, "Pythagoras was the first to call what surrounds us a *cosmos*, because of the order in it." According to Pythagoras, the cosmos ("world-order") is a number, and the way in which a number is generated is the same as the way in which the cosmos itself could have been created. As we have already said, the ancient Greeks viewed numbers geometrically — as arrays of pebbles, not as symbols. If we take a pebble (say, a small ball of moist clay) and split it into two pebbles, laying them side by side, we have created the number two out of the number one. We tend to think of the number 2 as being made up of the pebbles only, but this is not the case; the number two is the whole of the figure — not only the pebbles, but the space between them as well. This is because the space *between* the pebbles is as much a part of the number as the pebbles themselves; for if the space were not there to hold them apart they would merge again into a single pebble. This metaphysical model suggests an elegant answer to how the Universe might have come into being by itself — a cosmogony which does not depend upon a God or gods: Space and Number are both generated simultaneously when

the One is divided into *the Many*, and in the process, *Time* is also generated. Moreover, not only does Number (greater than 1) generate Space and Time, but, conversely, Space and Time generate Number as well! In other words, the elements of physical existence *create each other!* [For a more detailed exposition of how *Time* is created in this cosmogonic theory, see pages 526-528 of Khan Amore's novel, *Hypatia*.]

According to tradition, Pythagoras was the first to recognize the *sphericity of the Earth*, he was the first to recognize that the "Evening Star" and the "Morning star" were one and the same, and he was the first to recognize the *Obliquity of the Ecliptic*, and the movement of the planets therein. As if these were not honors enough for one mortal, Pythagoras is also said to be the discoverer of the *golden section*, or *divine proportion*, a rule of harmonious proportionality which underlay not only the best of Greek art and architecture, but which is built into Nature itself (for example, the divine proportion is to be found within the logarithmic spiral of Chambered Nautilus shells, or within the spiral seed patterns in sunflowers, or within the pattern of florets of the common daisy, etc. Amazingly, a plurality of divine proportions are even to be found in the harmonious proportions of the most beautiful human bodies!) [For more information on the mathematical basis of beauty, check out H.E. Huntley's *The Divine Proportion*, A Study in Mathematical Beauty; Dover]

To the Pythagoreans the Cosmos was considered both animate and intelligent — intelligent because it is capable of imposing order on all within its sphere. The orderliness of Nature was central to Greek thought from the start; but the word "cosmos" brings into sharper focus the implications which "order" had for the Greek mind. For the corresponding verb means not merely "to set in order," but "to set in an order which is fitting" and which is therefore *beautiful* and *good*. Indeed, in Greek there is no sharp distinction between "beautiful" and "good," and the word "cosmos" suggests them both. It signifies not merely the regularity which is to be found in the world-order, but also the harmonious ordering of things in due measure, which is the hallmark of beauty and divinity. This conception was part of the Ionian tradition from the first, but was first made explicit in Pythagoreanism. In the Pythagorean view, not only does the existence of a cosmos depend upon the observance of due proportion, but the *soul*, too, is ordered by the observance of due proportion. For the soul, like the cosmos, is a "harmony" of things blended proportionately. Because they held this view, the Pythagoreans strove always to live in harmony with each other and with Nature. Pythagoras himself was noted for his serenity, equanimity, *ataraxia*, and *sophrosyne*:

"Pythagoras would chastise neither slave nor freeman in anger. He called admonition a 're-tuning.' "

— Diogenes Laertius

Thus, Pythagoras thought it best to avoid anger, but in the event of a dispute, choosing instead to reason with a man to bring his soul into harmony with the world-order, for it is

only by reasoning with a man that his soul can be brought into the only state in which it is stable, and will not be subject to violent perturbations. Now, things which are similar or identical have no need for harmony — that is *unison*, not harmony — harmony comes about as the result of the linking together *dissimilar* elements (such as *different* notes), so that through well-tuned diversity the whole may be better and richer than the sum of its parts. The Pythagoreans meant this doctrine of harmony to be applied to the social order as well. Indeed, the Pythagoreans were actively political, and the Pythagorean order amounted to not a democracy, but a virtual aristocracy [literally, a “rule of the best.”] Two of Pythagoras’ most famous sayings were, “friends have all things in common,” and “friendship is equality,” and so, in the best spirit of communism, his disciples actually put their possessions into one common stock. In the Pythagorean view, *the principle of order is justice*. But justice is founded upon *reciprocity*. They thought it just that what ill a man had done he should suffer in return, and what acts of kindness he had done should return to him as well. This view tempered their system of aristocratic communism, for the Pythagoreans held that the soundest basis for the distribution of wealth in a community was by merit or virtue. But how could the distinction between the better and the worse elements in the city be maintained without sacrificing the principle of equality? Leave it to Pythagoras to come up with a mathematical answer even to this question, and, indeed, the answer was found in his theory of proportions. Now, Pythagoras gave to the world three different kinds of means: the *arithmetic mean*, the *geometric mean*, and the *harmonic mean* (which Pythagoras called the *subcontrary mean*). The *arithmetic mean* is the middle term in a series of three numbers that are in *arithmetical progression*, and an arithmetical progression is a number series in which each term is obtained from the preceding one by adding a constant, called the *common difference*. Thus, three terms which are related by the arithmetic mean are proportional by virtue of a difference between them such that the first exceeds the second by the same amount by which the second exceeds the third. It turns out, however, that in this proportion the *ratio* between the greater of the terms is less than the ratio between the lesser. This did not appeal to Pythagoras’ mathematical sense of “justice,” so he did not adopt this mean as the mathematical basis for the distribution of wealth in his communistic system. The *geometric mean* is the middle term in a series of three numbers that are in *geometrical progression*, and a geometric progression is a number series in which each term is obtained from the preceding one by multiplying the preceding term by a fixed number, called the *ratio*. Thus, three terms which are related by the geometric mean are proportional by virtue of the fact that the first term is to the second as the second is to the third. In this case, the greater of the terms are in the same ratio as the lesser. This appealed to Pythagoras’ mathematical sense of “justice” so this was the mean which Pythagoras adopted for the distribution of wealth in his communistic system. [Note: Pythagoras’ third mean, which he called the *subcontrary mean*, but which was renamed the *harmonic mean* by the Circle of Hippasus, is that in which, by whatever part of itself the first term exceeds the second, the middle exceeds the third by the same part of the third. In this proportion the ratio between the greater of the terms is greater than

the ratio between the lesser. This also did not appeal Pythagoras' mathematical sense of "justice," so it was clear to him that the *geometric mean* was indeed the most logical choice as the mathematical basis for the distribution of wealth in his communistic system.] To Pythagoras, the *geometric mean* represented justice in the world order, and it was also the mathematical basis of the Greek avoidance of excess and deficiency which was summarized by the ancient Greek commandment, "Everything in moderation." The use of the geometric mean in the distribution of wealth was thought to reconcile the disparate principles of unequal merit and equality under the law. This formula preserves the differences recognized by the aristocratic conception of justice while at the same time admitting the democratic claim of equality; for proportion, as Aristotle puts it, is "equality of ratios." In other words, citizens should receive "equal treatment" — but "equal" relative to their merit, not equal in an absolute sense. For example, the physician should not be paid the same wages as the stable-sweeper, even though in a democracy they are considered "equal," for their votes carry the same weight. In this way, the Pythagoreans reduced the question of social justice to a mathematical problem, for it is in essence a problem of placing things into the proper order, and the key to all order is number. What was peculiar to Pythagoreanism was not the preoccupation with order — even mathematical order; in these respects Pythagoreanism merely offers explicit expression of what was implicit in the Ionian tradition. What was new in Pythagoreanism was its preoccupation with the *soul*. This too, was an outgrowth of Ionian thought, but the contemplation of the order of Nature had now a new purpose unknown to the Milesians: *the freeing of the soul from the bondage of the body*. This conception was to leave its impress upon the whole body of classical Greek philosophy, and, through **Plato**, upon the whole of Western thought. Pythagoras held the physical world of the body to be inferior to the conceptual world of the mind, for in number — and in the associated science of geometry — there is a truth which is incontrovertible, changeless, and transcendental. Mathematical entities such as circles and lines are conceptual, not empirically observable fact. Every line, no matter how finely it is drawn, has dimension, and is therefore not a line at all, strictly speaking, only the representation of the *concept* of a line. This simple observation has profound philosophical implications:

"Mathematics is ... the chief source of the belief in the eternal and exact truth, as well as in a super-sensible intelligible world. Geometry deals with exact circles, but no sensible object is exactly circular; how[ever] carefully we may use our compasses, there will be some imperfections and irregularities. This suggests the view that all exact reasoning applies to ideal as opposed to sensible objects; it is natural to go further, and argue that thought is nobler than sense, and the objects of thought are more real than those of sense perception. Mystical doctrines as to the relation of Time to eternity are also reinforced by pure mathematics, for mathematical objects, if real at all, are eternal and not [transient] ... Such eternal objects can be conceived as *God's thoughts*. Hence Plato's doctrine that *God is a geometer*."

— Bertrand Russell

Pythagoras was worshipped as a semi-deity — in fact, the reincarnation of Apollo — in part because he had undergone a severe ritual initiation at the sacred Orphic temple on Crete. Orphism — the cult associated with Orpheus — postulated, in a way very similar to the concept of *karma*, the pre-existence and indestructibility of souls: *If the human being in whom the soul temporarily resided led a sufficiently worthy life, it would return to dwell among the gods with whom it was originally created, but if the human being failed to achieve a worthy and enlightened death, then the soul would be forced into a further reincarnation.* (Readers of Khan Amore's *HYPATIA* may recognize strong eschatological elements of Orphism and Pythagoreanism in this historical science-fiction novel, although these elements are given a trans-temporal twist which offers a *technological* answer to how such reincarnation might occur, rather than a mystical one. The science-based alternative to religion offered in this novel may thus be seen as a modern-day reincarnation of Orphism.)

Not only have the mathematical developments and the music theory of the Pythagoreans profoundly influenced all of Western thought, but so also has their moral philosophy. By the laws of *karma*, man is not driven inexorably by fates over which he has no control: he has a choice between good and evil, a responsibility for determining his own fate, with the consequences of his choice being reflected in subsequent reincarnations. Under this doctrine, a deer hunter, for example, might come back in the next life as a hunted deer, in order to learn what it's like to be the *recipient* of his own actions. Thus, *karma* might be seen as an automatic form of *cosmic justice* — a divine justice which does not depend upon the existence of a vindictive God. Even if specific memories of past lives do not make the transition when one is reincarnated, perhaps the lessons learned in past lives are imprinted indelibly — if subconsciously — on the soul.

In the hands of Pythagoras even music became mystical — numinous. His discovery that musical harmony was based upon numbers had profound philosophical ramifications. The significance of the discovery lay in the fact that this relationship is not created by man, or by the action of performing the experiments which led to the discovery on the monochord, but it is mathematically *inherent in the Universe*. The *cosmic harmony*, insofar as it could be discovered and enjoyed by mankind, depended on the imposition of

limit (“*peras*”) on the unlimited (“*apeiron*”). All the notes possible, sounded together without limit, produce nothing but noise — the Pythagorean exemplar of evil. Imposing limits, recognizing the natural order and structure inherent in the Universe and sounding only well-chosen notes produces harmony, which Pythagoreans would call divine — the exemplar of good. Owing to Pythagoras’ discovery, music provided for his followers the best instance of this principle at work. The beauty of music only served to illustrate the importance of order and harmony in life, for the Greeks were especially sensitive to the beauty of music, and the word *cosmos* carried to a Greek the suggestion of beauty as well as order. Music was thus further evidence for the equivalence of limit and goodness, for the imposition of limit on the field of sound brought beauty out of disharmony. This limit was represented by the numerical system of ratios between concordant notes which has the power to transform a cacophony into a symphony. This mathematical relationship which underlies harmony was seen to be part of an intelligent plan. The plan, however, is not invented by man, but has been there all the time awaiting his discovery.

Although it is not known with certainty whether or not all five *Platonic Solids* were known in Pythagoras’ time (as Proclus claimed they were), the Pythagoreans were clearly acquainted the penta-dodecahedron, whose construction in turn requires knowledge of the construction of the regular pentagon. The Pythagoreans were particularly enthralled by the regular pentagon because if all of this figure’s sides are extended outward until they intersect they produce a five-pointed star called a *pentagram*. The pentagram was associated with the division of line in extreme and mean proportion, with the *golden section* or *divine proportion*, and also with the fourth of the regular solids, the *dodecahedron*. The dodecahedron was taken as a symbol of the Universe because its twelve regular pentagonal facets were thought to correspond with the twelve months and twelve signs of the zodiac — not to mention the fact that three mutually-perpendicular golden rectangles (whose length-to-width ratio are in divine proportion) could be centered in a dodecahedron with their twelve corners touching the centers of all of the pentagonal faces of the solid. The pentagonal faces of the dodecahedron themselves held great interest for the Pythagoreans, for the point of intersection of two diagonals of a pentagon divides each in the divine proportion both internally and externally. Not only that, but the ratio of the edge-lengths of the triangular “points” of a pentagram to the edge-lengths of the pentagon residing at its center are *also* in divine proportion. This so-called *Mystic Pentagram* was in fact so revered that it became a symbol of recognition between the members of the secret Pythagorean order, and was taken to be a symbol of health. The Greek writer, Iamblichus, transmitted to us the tale of how once a member of the Pythagorean fellowship, while traveling far from home, stayed one night at a wayside inn. He fell ill, and despite the care of a sympathetic landlord, who tried at considerable expense to restore him to health, he died. Before his death, recognizing that his situation was dire and being unable to compensate his host, he had a board brought to him and he constructed on it a pentagram. Giving this to the landlord he requested that the board be

affixed where all passers-by would see it. In due course a traveler passing by saw the symbol, made inquiries and, on hearing the story of the landlord, generously recompensed him for the care given to his deceased “brother”.

Because Pythagoreanism had three major elements in common with Druidism, it has been postulated that there must have been some intercourse between these two sects. The first of these connections is the belief in reincarnation and of *karma*, which was common to both Pythagoreanism and Celtic Druidism. The second connection was in their code of *silence* — five years of “Pythagorean silence,” to be exact. Pythagorean disciples were bound to secrecy, and they were required to learn their lore by heart, as was the case with the Druids. No written accounts were allowed. Several ancient religions were secret and preserved their mysteries by select initiation, but the specific ban on written records and the requirement to learn great tracts of lore by heart seem to be exclusive to Pythagoreanism and Druidism. The third connection between Pythagoreanism and Druidism was that both adhered to a principle of social organization which was quite unusual and remarkable in the ancient world: *they treated women as equals*. It is said that the Pythagorean women taught mathematics (which means that they taught *divinity*) and were accorded absolutely equal status with men — although, of course, **Hypatia** is the first female mathematician known to us by name. Other than in ancient Egyptian society and in the Pythagorean cult, there is only one other ancient society in which any similar equality of opportunity and status for women is observable — and that is early Celtic tribal society.

The basic tenets of Pythagoreanism are so deeply embedded in Western thought that it is not difficult to see connections with other cultures. Through **Plato**, **Galileo**, **Copernicus**, and **Leibniz** there flows a continuous stream of ideas which is essentially Pythagorean. Even **Sir Isaac Newton**, founder of our present conception of the mathematical and physical realities of the Universe might be called a Pythagorean. Pythagoras was a great sage, a genius of the first magnitude. He was the first to declare the Earth to be spherical, the first to call the world-order “cosmos,” the first to see the order, harmony, and beauty of the Universe as it is revealed through mathematics, and the first Greek to afford women a status equal to men in his social order. His discoveries were legion and seminal, and his philosophy compassionate to all living things. He changed the course of human thought for all time to come, setting the foundation for the way scientists and mathematicians still think today. In the words of Bertrand Russell:

“I do not know of any other man who has been as influential as [Pythagoras] was in the sphere of thought; I say this because what appears as Platonism is, when analysed, found to be in essence Pythagoreanism. The whole conception of an external world, revealed to the intellect but not to the sense, is derived from him.”

