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THE A PRIORI WHICH IS FALLIBLE

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THE A PRIORI WHICH IS FALLIBLE

Some things not only are true, but are necessarily true. Not only are they so, but they could not have been otherwise. Current philosophies of mathematics give inadequate accounts of the necessity of mathematical truths.

Mathematics has become so sophisticated that it is easy to lose touch with the mathematical realities out of which it arises. Many nowadays think of mathematics as complicated games with forests of symbols. Yet this is an image which could not have got a grip on the ancient mathematicians, like those of the Pythagorean brotherhood, who stood so much nearer to the roots of those forests. Now we are lost in the higher branches of mathematics, and we sometimes forget that all these growing branches are nourished ultimately by roots far below in down to earth reality.

The early Pythagoreans discovered things which are thus and so, and which could not have been otherwise. Yet although these things could not have been otherwise, the Pythagoreans who discovered them could have done otherwise. Hence the truths which the Pythagoreans discovered are things which would still have been so even if no one had ever discovered them.

In the minds (and so in the brains) of the mathematicians there were perceptions or conceptions. It was contingent that these occurred in the minds they did occur in, or indeed in any minds at all. Yet in virtue of having these contingent perceptions or conceptions, mathematicians came to recognize that certain things are thus and so, and that these things could not have been otherwise. These mathematicians used words and gestures, scratched patterns in the sand, assembled arrays of pebbles, and did a wide variety of

things to convey their perceptions and conceptions to one another. The fierce formalisms of modern mathematics emerged from communicative activities of this sort. Symbols emerged, together with rules for their use, and games people played with them; yet these symbols were not mere counters in a self-contained game that had nothing to do with anything beyond itself. The symbols did refer to things beyond themselves. They expressed perceptions or conceptions in the minds of those who used them. And those perceptions or conceptions, in turn, constituted the recognition of necessary truths. Hence symbols, and the perceptions and conceptions they express, both correspond to things which would still have been so even if there had been no one to discover that these things are so, or to tell anyone about their discoveries.

Consider an illustration. Nine pebbles can be arranged in a square grid — a pattern with as many rows as columns, with the same number of pebbles in each row and in each column. Ten pebbles cannot be arranged to make a square grid like that. Sixteen pebbles can. Twenty-five pebbles can. And so on. Let us begin to list the so-called square numbers in order; the list begins:

4, 9, 16, 25,....

(I omit 0 and 1 because it is arguably anachronistic to include them as numbers, when we are trying to recapture Pythagorean ideas; and besides, I hold a theory endorses Aristotle's claim that the natural numbers begin at 2, 0 and 1 being mere notational conveniences.)

Now list the differences between each square number and the one which follows — the difference between 4 and 9, between 9 and 16, between 16 and 25, and so on. This list begins:

5, 7, 9,....

Note that each of these differences is two greater than the one before, or at least, so it seems when we survey the first few steps in the series. The question arises whether this pattern will continue indefinitely as we climb along the endless list of square numbers. Will the differences always increase by twos? I will return to this question of proof. But first I will illustrate the nature and importance of this number pattern.

The pattern we have seen instantiated by pebbles is one which can also be instantiated by things which look superficially very different from pebbles. Consider a falling object. In a unit of time, suppose it to be moving at a given average speed, so that it covers a

given distance. In the next unit of time, suppose the object preserves its previous speed, in virtue of which it falls the same distance as in the previous interval; but in addition to preserving its previous speed, it receives an additional increment of speed — gravity gives it a little tug. Due to the extra speed, it falls further than it would have fallen if it had merely preserved its previous speed. So it falls further than it fell during the previous interval. Rescale units of distance so that the distance travelled in the second interval of time is two units of distance greater than the distance travelled in the first unit of time.

Suppose then that in the third unit of time, the object retains all the speed it had in the previous unit of time, but again gravity gives it a little tug, so that it acquires another increment of speed. Suppose this results in the object falling two units of distance further in the third unit of time, than it did in the first unit of time.

If this pattern continues, then the distance fallen in each interval of time will be two distance units more than that fallen in the previous interval. The Pythagorean discovery about square arrays of pebbles showed that when you keep adding a two more than you added last time, you make bigger and bigger square numbers. This leads to the conclusion that the distance fallen by an object is proportional to the square of the time during which it falls. This is Galileo's law of free fall. The ancient Pythagoreans discovered truths not only about pebbles, but also about the patterns which those pebbles instantiated; and these truths about patterns could be reapplied to any case in which those patterns were instantiated.

Imagine a square number of pebbles, say nine:

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o o o
o o o
o o o

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To make the next square number, we need to add a row and a column:

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o o o o
      o
      o
      o

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This added row-and-column makes an L-shape (or ° shape) which the Greeks called a *gnomon*. Every time you add a *gnomon* to a square, you get the next larger square. And each added *gnomon* has to have two more pebbles in it than the one before: one extra pebble

to make the row one pebble longer, and the other to make the column one pebble longer. This is why the differences between square numbers must always increase by twos.

When you think about square patterns made with pebbles, necessary truths leap out at you from time to time, and catch you by surprise. Yet what is it that leaps out at you? Not language games you could play with symbols. Not introspective insights into the workings of our own mind. When you think about arrays of pebbles, you are not thinking about your own mind, or about anyone else's. You are finding out things about pebbles; and beyond the pebbles, you are finding things about the patterns which the pebbles instantiate.

Thus, what the ancient mathematicians discovered were truths about patterns, not truths about words or thoughts. I propose a theory about just what these discoveries involve. Necessary truths, I conjecture, are truths about patterns. More specifically, they are truths about how one complex pattern contains another simpler pattern as a constituent. When a large pattern contains a constituent pattern, then this is not an accidental, contingent matter. The larger pattern would not have been the pattern that it is, had it contained a different constituent pattern. The constituent pattern is an essential part of the pattern which contains it.

Consider for instance a square array of pebbles. This pattern contains gnomons; indeed it is constituted entirely of a series of gnomons added onto a single initial pebble; and each of these gnomons contains exactly the pattern of the gnomon before it but with one extra pebble added to each end. The distinctive necessity of the Pythagoreans' discovery arises from the containment of one pattern in another.

I conjecture that this containment of one pattern in another is the source of all necessities in mathematics. I conjecture also that this theory recaptures some of the spirit of Pythagorean doctrines about number and nature. Pythagoreans were realists about mathematics; but unlike later Platonists, they were not transcendent realists. They saw mathematical truths in the physical world, not in some other world. It is difficult to justify such sweeping statements about what the early Pythagoreans thought, and there is a risk of distortion when we describe their thoughts using words which are currently fashionable, like the word «realism». Yet I am willing to take this

risk. The immanent mathematical realism implicit in Pythagoreanism lies at the heart of the scientific advances of such great modern minds as those of Galileo, Kepler and Newton.

Mathematics has a subject matter: namely, patterns. And it discovers truths, indeed necessary truths, about this subject matter. When mathematicians discover these truths about patterns, there is a distinctive way in which they set about justifying their opinions about these patterns. Mathematics not only has a distinctive subject matter, it also has a distinctive methodology. The characteristic form of justification found in mathematics is one which has traditionally been called *a priori*. A mathematical opinion is supported, not by assembling sensory observations, but by proofs which do not make any appeal to any contingent observations that anyone happens to have made. Such proofs are said to be *a priori* because the order of justification begins without (prior to) any assumptions about any experiences of any agents. Experiences will be required to help someone understand a proof; but no assertions about those experiences feature as premisses in the proof. Different people have different experiences which lead them to understand the proof; but it is the very same proof that they come to understand.

One of the key ideas in Pythagoreanism is that of a harmony between the patterns in the world around us, and the patterns in the mind of the enquirer who seeks to understand this world. Music is the model; beauty arises from a match between mathematical patterns in the world, and mathematical patterns in the mind. It is this harmony between the microcosm and the macrocosm that holds the key to the distinctive *a priori* status of mathematical justifications.

When a person perceives or imagines or in any way thinks of a pattern, there will be some pattern in the person's mind or brain. For instance, when people represent to themselves a square grid of pebbles, there will be something structured inside them which serves to represent this square grid. And when they represent to themselves a gnomon of pebbles, there will be something structured inside them which represents this gnomon. One thing which can sometimes happen, then, is that the pattern which represents the gnomon is contained within the pattern which represents the square grid. It may happen that the part-whole relation which holds between the square and the gnomon in the world is mirrored in the mind of the enquirer, and the representations stand in the same part-whole relation as the things they represent.

Part-whole relations among representations recall the Kantian idea of *analytic* truths. Kant defined analytic truths in the special case of simple subject-predicate judgements. An analytic truth, Kant said, is a proposition in which the concept corresponding to the predicate is already contained in the concept of the subject. Representations are, in fact, complex structures; and one representation can contain another quite literally as a part. The Kantian idea is worth giving a run for its money. It is a point in its favour, that it offers some promise of capturing the distinctive «Aha!», the «Eurika!» of recognition which accompanies the proofs of simple mathematical truths like those discovered by the early Pythagoreans. When you get clear and distinct representations of a complex pattern, and of one of its constituent patterns, then you can just *see* that one necessarily contains the other: in fact, you can just *see* one pattern *as* containing the other.

Caution is called for, however. History has taught us some hard lessons. Again and again people have taken truths to be self evident which turned out, in the course of time, to be far from self evident. Some things people thought to be self evident turned out in the end not to be truths at all. The *a priori* status of mathematical justifications should not be conflated with anything like certainty. People can be quite certain about things which are not true. It can be rational for a person to be uncertain about *whether* something is a necessary truth. Hence *a priori* justifications should not be misconstrued as providing infallible proofs in any epistemic sense. They cannot prove anything beyond rational doubt.

History has taught us to be wary of any pretensions to infallibility. Yet this should not mislead us into denying the existence of *a priori* justifications in mathematics. It is right and proper to reserve a degree of doubt about any claims we might make about *which* justifications really are *a priori*. Something may seem to be *a priori* and yet may not be. Nevertheless, the fallibility of judgements about what is *a priori* does not establish the nonexistence of anything *a priori*. Mathematical justifications often do have a distinctive status, even if they do not establish their conclusions beyond all reasonable doubt. We need a name for the distinctive kinds of justifications which we find in mathematics; and «*a priori*» is the name which best fits. However epistemically fallible mathematics may be, its justifications are nevertheless *a priori*.

What makes a mathematical justification *a priori*, I propose, is the way in which the part-whole relations which constitute the

necessities in nature are reflected in part-whole relations among the representations in the mind or in language. This proposal gains credibility from the imaginative exercise of recreating the early discoveries of the simplest mathematical necessities in nature. Most rival, contemporary philosophies of mathematics do not score very well at all, by comparison, when they are held up against simple mathematical discoveries of the earliest mathematicians.

The ancient Greeks were filled with a touching optimism. Their *a priori* proofs were often accompanied by a degree of confidence which it is not rational for us to share. History has dealt us too many lessons in humility over the many years between ancient times and the twentieth century. Yet it is a mistake to conflate fallibility with contingency. Mathematical truths are necessary, not contingent; and mathematical justifications are, characteristically, *a priori* not experimental. We must not allow the overwhelming sophistication of modern mathematics to fog our vision. We must not lose sight of the harmonies between representations in our mind, and the patterns in the world which they represent. These harmonies are the source of the necessary truths and the *a priori* justifications which lie at the heart of mathematics.

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THE PYTHAGOREAN COMMUNITY CREATION, DEVELOPMENT AND DOWNFALL

The examination of the problem concerning the creation, development, nature and downfall of the Pythagorean Community is not a matter that concerns only the philologist, the historian, the political scientist and the sociologist, but it is without doubt connected with the philosophical examination and understanding of the Pythagorean *doctrines* and unquestionably involves the accepted activities of the specialist in political philosophy since testimonies of whatever nature need to be related to the hermeneutical approach and the philosophical evaluation of the Pythagorean doctrines.

Moreover, in connection with the above, and given the nature of the existing evidence concerning the Pythagorean Community, the method judged to be the most suitable for the examination of the problem in question consists of putting forward and adopting (and, in consequence, rejecting) certain *hermeneutical hypotheses* by a process of *reductio ad absurdum*.

Without doubt, a clarification of the problem concerning the Pythagorean Community might be accomplished by answering the following main questions, which, to some extent, provide an outline of all those axes and parameters that constitute the essence of the problem: 1. What were the reasons that led to the creation of the Pythagorean Community and where did this begin to take shape for the first time? 2. Were there various stages in the development of the Pythagorean Community and how can these be determined? 3. How was the Pythagorean Community organised and internally structured?