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BouRODIMOS, E.
1992

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AND HARMONY OF REALITY AND BEING

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Pythagorean philosophy / ed. by Konstantinos I. Boudouris. Athens : International Center for Greek Philosophy and Culture, 1992. 257 p. ill. index). (Studies in Greek philosophy ; 7). • papers read at the third international conference on Greek philosophy, Samos, August 1991.

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E. L. BOURODIMOS

THE MATHEMATICS OF MUSIC AS "NUMBER" AND HARMONY OF REALITY AND BEING

I. SYNOPSIS AND ABSTRACT

The Pythagorean quest for truth was centered (the functional theory of numbers) as the building blocks of reality and Being.

The harmony of mathematical reasoning and knowledge was considered as certain, accurate and applicable to the real world.

The "number" — although an abstraction — was obtained by mere thinking and contemplation leaving out the need for observation (and/ or experiment) as a general rule. Numbers "simulate" reality. The numbers and their relationships — any substance of physical reality was conceived as built with numbers — were the essence of order and symmetry of the external world, the harmony of nature, all things material and immaterial. Pythagoreans for the first time in history coined the word "cosmos", as representing orderly "numerical structures" and embodying symmetry and analogies, shaping Beauty and Truth. The numbers perceived (and constructed) by Pythagoras, as shapes and independent entities, as instruments with which Mathematics (and Geometry) were founded through strict deductive logic theory and theorisis, all components of analysis and contemplative life. The mathematical abstractions — the first historical achievements in the evolution of ideas — of the Pythagorean theory, have led to the foundation of the functional analysis. They have strengthened the integrity and universality of mathematical methods and numbers representing thereby the ideal world of "noêsis" (Νόησις), more real and authentic than the

observed reality. The Platonic conception of "ideas - forms" (and idealism vs. empiricism) has its roots to Pythagorism, for which objects of thought are more "real" than those of sense perception, whereas contemplation and thought were nobler than sense perceptions.

«... I do not know of any other man who has been as influential as he was in the sphere of thought — B. Russell has suggested — the whole conception of the external world, revealed to the intellect but not to the senses, is derived from him...».

The mathematical abstraction of Pythagoras in the realm of numbers, the numerical compositions and the established observations on periodicities, tones, sounds, intonation and rhythm of musical harmony, was his greatest contribution to philosophy. The functionality and other fundamental properties of the modern "theory of functional" and mathematical analysis — a sure scientific approach for "interpretation" of physical reality originated in modern times — have their root in Pythagorean concepts.

The exponential rise of mathematical sciences of the last five centuries such as Differential and Integral Calculus (Newton, Leibniz, Laplace), the analytic geometry (Descartes) the Fourier analysis; Chemistry, Physics and Biology and all the meteoric advancements since the 16th century, are founded in the "number" analysis - synthesis, i.e. the "archetype" of the Pythagorean thought.

The Pythagorean contributions also are:

- a. in Epistemology — mainly the relationship between music and mathematics,
- b. in Cosmology — the harmony of the spheres and the rejection of "geocentric concepts" in the structure of the planetary system,
- c. in Anthropology and the structure of moral behaviour and social-political life, were all central in the evolution of philosophy of science founded on Pythagorean inquiry.

II. SAMIAN HISTORY AND POLITICS : THE IMPACT ON THE LIFE AND WORK OF PYTHAGORAS

The decision of Pythagoras at the age of about 43, to leave Samos is centered in his attempt and failure to influence the political climate and establish a free democratic society. The political developments, however, have led to the rising of the tyrannical

regime of Polycrates family and specifically, Polycrates the Second.¹ According to D. Laertios² and K. Boudouris³ Pythagoras coming from a financially strong and rich family originally, may have been in good relations with Polycrates regime originally. This has, most probably, provided him with the opportunity of traveling abroad for business, self-education and "theory" inquiring. Pythagoras has a deep drive and unyielding quest in the pursuit of knowledge and truth ("θεωρίας εἵνεκεν"). Such knowledge of the world has sharpened his ability of clear understanding of the problems of the political and social processes of that period, — a basic requirement for actual and successful involvement in the Samian history and the public affairs of the Samian Society — . Based on all these knowledge of travel, study and contemplation the genius of Pythagoras has established a very close and selective group of associates of high moral standards, courage and intelligence, who can follow a special form of life (θεωρητικὸς βίος).

III. THE THEORETICAL FORM OF LIVING: THE AXIOCRATIC CONCEPT OF PYTHAGOREAN COMMUNITY. SOCIAL AND POLITICAL LIFE PROCESS AND CONFLICT.

All members of the Pythagorean Community have promoted their mathematical and philosophical "research" and inquiry whereas they built the "theoretical form of living" (θεωρητικὸς-πυθαγόρειος βίος) outside of their contemporary society and its "style", "habits" and "format" of living.

Specifically this small and closed circle of Pythagoreans — with all their mystical and ritualistic "dromena" (δρώμενα) was a true elite group discussing not only scientific problems, philosophy, music and "numbers", but also current issues of the political and social process, of the particular Samian historical period. Their discoveries and the results of their philosophical dialogue were kept "secret", restrictive only among the members of the Pythagorean Community, never being brought to the public forum: They were not for public release! Two important points should be underlined at this juncture :

- a. All these concepts of the Pythagorean inquiry were considered seriously by their contemporary thinkers — among them Heraclitus — and by Plato and Aristotle in the 5th and 4th centuries. Heraclitus, on the other hand, didn't know³ in all depth and details the Pythagorean teachings and "theories" on numbers,

harmony and music. Heraclitus, however, was entirely critical in his evaluation of Pythagoras although he was accepting the greatness of Pythagorean scholarship. Heraclitus recognized the broad and deep knowledge of Pythagoras with its consistent cosmology, ontology and metaphysics but one who missed "the Pythagorean grasp of the essence of Being", the "Substance of things".³

- b. The Pythagorean theoretical way of life (βίος) structured in a form of education — in which Mathematics and music were fundamental, pivotal elements — along with the Suppression of "bodily desires" — was for Pythagoreans — the best way in constructing and strengthening a lawful Society which nurtures ethical standards through the cultivation of the Soul. However, with all the wealth and brilliance of the Pythagorean concepts in cosmology, mathematics, music, politics and ethics being promulgated in secrecy, one may ask: When and how that this "closed group" has to confront an open society in order to teach it, convince and change it? How such an open society can accept the teachings of the secret and "closed" Pythagorean Community without an open dialogue and a legitimate criticism of its views aiming at the building a more fair, just and democratic Society?

How the principle of an open dialogue can be secured?

(«... Πάντας κοινωνεῖν τῶν ἀρχῶν τῆς Ἐκκλησίας καὶ διδόναι τὰς εὐθύνας τοὺς ἄρχοντας ἐν τοῖς ἐκ τῶν πάντων λαχοῦσι...»)

The secrecy of the Pythagorean community life may have provided authority, power and standing for all members of the group, but it has generated fear, suspicion and jealousy to the members of the society at large. Of course secrecy, secures to a great extent a strong bond and coherence among its members, whereas the "theoretical βίος supports all forms of concrete and enlightened action along with the possibility of survival in adverse socio-political conditions such as the Polycrates II tyranny. This was exactly what happened before the Pythagoras' departure from Samos and later on in Croton.

However, the efficiency of the action of that kind of a group — larger and considerable as it may become — results ultimately in psychological "Cut off" and isolation from the broader communal living and working in a free and open society. Prof. Boudouris³

correctly observes that it is an "undeclared war" between two basic concepts, ideologies and ways of life: that of the "closed secret group" and the open and free society within the frame of which the group is living and it is acting.

A hard dilemma is always surfacing in these cases: *First* to have the "closed group" — such as the Pythagorean — succeeding and prevailing as Government and transforming the open society into a *closed authoritarian society* (in accordance with its image) eliminating any element or characteristic of democracy, political freedom and social reform.

Second to permit some "openings" in the "closed group" through which some of the concepts, "happenings" and notions of the open society may permeate, influence and change the "circle" views and attitudes. None of these developments have taken place in the early Pythagorean Community life process and thereby a looming social and political conflict was apparent.

IV. "MATHEMATIZATION" OF SOCIAL AND POLITICAL PRINCIPLES: JUSTICE AS HARMONY OF NUMBER IN DEMOCRACY. NUMERICAL AXIOCRATIC ANALOGIES

The "theoretical bios" of Pythagorean Community in all its scientific-mathematical and political-social views-spectrum — being a secret religious - ethical group in the beginning — has never included (and/or considered) an initiation of either one of the two aforementioned alternatives. Accordingly the "group" actions and "secret life" was set in a collision course within the society at large.

Pythagoras, nevertheless, has thought that something could have been done to prevent collision in the social-political life. He considered that since all members of the group were men of superior moral living and religious standards, their impact and influence on social reforms, moral and ethics in public life should have been the outcome. These and not their party-politics, involvement and participation should be prevailing. The presence, the scientific teachings, the mathematical-philosophical work of the Pythagorean Community being the *elite-superstructure* — and not the political-party organizations — should be "diffused" into the open society and help and build the social change and betterment of the citizen and society at large.³

According to Philip⁴ and Buckert⁵ the Pythagorean Community in all its secret structure has a strong impact and presence although

indirect in the affairs of the Croton Society. Philip suggested that the Pythagorean community was a philosophical Society and a political organization with great power and influence in Croton. He considered the religious-mystical character of the Pythagorean circle as secondary. He presents strong arguments against Buckert's⁵ conception that Pythagorean society was mostly a religious group and that Pythagoras was a "shaman" (μάγος). Dunbabin⁶ suggested that the political influence and power of the Pythagorean group was the result of its moral and ethical standards, because "no serious scholar can avoid (and escape) to play a role concerning the public affairs of a small society". On the other side the Pythagorean circle was not only theoretical and far from the political reality and the every day affairs of the city and society: Their views and objectives were not only a calm idleness, but also a strong political set of moral and philosophical statements. This includes also realistic and unbending practical policy when the circumstances demanded such a standing according to Minar.⁷ As an example one should mention, the case of conflict and defeat of Sybaris by Croton in 510 B.C.. Although Crotonians and Pythagoreans were defending supreme moral and ethical values in that conflict with Sybaris, their magnanimity was present toward the enemy, i.e. the fugitive from Sybaris seeking protection and help.

It is also a fact that the character of the Pythagorean brotherhood being secret, selective and elitist was by nature "aristocratic" (mostly in the Spiritual sense) and thereby "undemocratic" by the common standards. That was the main reason of looming conflict between *democratic* and aristocratic parties in Croton.^{9 10} Even the word number (ἀριθμός) was of that *supreme* aristocratic structure and quality.

«An important — Buckert has stated — aspect of the word number (ἀριθμός) is the aristocratic color and feature of the word. It is significant whatever can be counted. *In order to know and seek to learn about numbers is to seek the essence of things, the inner substance of reality...*»⁵

Philolaus in a classical piece and statement relates the presence and power of numbers — as indicators of another reality³:

«θεωρεῖν δεῖ τὰ ἔργα καὶ τὴν οὐσίαν τῷ ἀριθμῷ κατὰ τὴν δύναμιν ἅτις ἐστὶν ἐν ταῖς δεκάδι: μεγάλα γὰρ καὶ παντελὴς καὶ παντοεργὸς καὶ θείω καὶ οὐρανίῳ βίῳ καὶ ἀνθρωπίνῳ ἀρχὰ καὶ ἀγεμῶν κοινωνοῦσα *** δύναμις καὶ τᾶς δεκάδος. ἄνευ δὲ

τούτας πάντ' ἄπειρα καὶ ἄδηλα καὶ ἀφανῆ.

γνωμικὰ γὰρ ἡ φύσις ἂ τῷ ἀριθμῷ καὶ ἡγεμονικὰ καὶ διδασκαλικὰ τῷ ἀπορουμένῳ παντός καὶ ἀγνωομένου παντί. οὐ γὰρ ἡ δῆλον οὐδενὶ οὐδὲν τῶν πραγμάτων οὔτε αὐτῶν ποθ' αὐτὰ οὔτε ἄλλω πρὸς ἄλλο, εἰ μὴ ἡ ἀριθμὸς καὶ ἡ τούτων οὐσία. νῦν δὲ οὗτος κατὰ ψυχὰν ἀρμόζων αἰσθήσει πάντα γνωστὰ καὶ ποτάγορα ἀλλάλοις κατὰ γνώμονος φύσιν ἀπεργάζεται σωματῶν καὶ σχίζων τοὺς λόγους χωρὶς ἐκάστους τῶν πραγμάτων τῶν τε ἀπείρων καὶ τῶν περαινόντων.

ἴδοις δὲ καὶ οὐ μόνον ἐν τοῖς δαιμονίοις καὶ θείοις πράγμασι τὰν τῷ ἀριθμῷ φύσιν καὶ τὰν δύναμιν ἰσχύουσιν, ἀλλὰ καὶ ἐν τοῖς ἀνθρωπικοῖς ἔργοις καὶ λόγοις πᾶσι παντὰ καὶ κατὰ τὰς δημιουργίας τὰς τεχνικὰς πάσας καὶ κατὰ τὰ μουσικὰν.

ψεῦδος δὲ οὐδὲν δέχεται ἂ τῷ ἀριθμῷ φύσις οὐδὲ ἀρμονία· οὐ γὰρ οἰκεῖον αὐτοῖς ἐστὶ. τὰς τῷ ἀπείρῳ καὶ ἀνοήτῳ καὶ ἀλόγῳ φύσιος τὸ ψεῦδος καὶ ὁ φθόνος ἐστὶ.

ψεῦδος δὲ οὐδαμῶς ἐς ἀριθμὸν ἐπιπνεῖ· πολέμιον γὰρ καὶ ἐχθρὸν ταῖ φύσει τὸ ψεῦδος, ἂ δ' ἀλήθεια οἰκεῖον καὶ σύμφυτον ταῖ τῷ ἀριθμῷ γενεᾷ.

Central point and target, the key moving spiritual element was the scientific and sociopolitical inquiring in seeking the perfection of the communal life in order and place — similar to the order and harmony of numbers. The members of the Pythagorean inner circle were first the mathematicians — who were subjected to hard trials and education for years in order to acquire equal status as associates of Pythagoras. Other members of the external Pythagorean Circle were “listeners” (οἱ ἀκουσματικοὶ) who were working and thinking somehow in the periphery of Pythagorean teachings. The key member of the inner circle have had a community of all material things, (κοινότητα ἀγαθῶν) living together in common houses. They have assumed also the responsibility in regard with the interests of the community affairs (economic, financial advisers, law specialists and statesmen). Plato has analyzed the Pythagorean way of life in the “Republic” (600 a9 and b5) in a brilliant statement:

«Ἀλλὰ δὴ εἰ μὴ δημοσίᾳ, ἰδίᾳ τισὶν ἡγεμῶν παιδείας αὐτὸς ζῶν λέγεται Ὀμηρος γενέσθαι, οἱ ἐκείνον ἡγάπων ἐπὶ συνουσίᾳ καὶ τοῖς ὑστέροις ὁδὸν τινα περέδοσαν δῖον ὁμηρικὴν, ὥσπερ Πυθαγόρας αὐτὸς τε διαφερόντως ἐπὶ τούτῳ ἡγαπήθη, καὶ οἱ ὑστεροὶ ἔτι καὶ νῦν Πυθαγόρειον τρόπον ἐπονομάζο-

ντες τοῦ δῖου διαφανεῖς πῃ δοκοῦσιν εἶναι ἐν τοῖς ἄλλοις;» (Πολ. 600a₉ - b₅).

It is apparent — Prof. Boudouris observes — that the Pythagorean way of life — which can only be compared with that one established by Homer — the prince of Hellenic Paedeia — combines knowledge of mathematics and wisdom, rationality and calm and apollonian spiritual disposition and the capacity of an abstraction as in the pose of numbers and within a frame of mystic, metaphysical, irrational and emotional elements. All these contribute to the Pythagorean life scientific superiority and purity. All these are building blocks, numbers in a hierarchy securing social peace and justice as the foundation of democracy resembling the harmony and perfection of the realm of numbers. The mathematization of the social process and political life was introduced by Pythagoras in that model context.

V. NUMBERS AS ENTITIES AND FIRST PRINCIPLE AND CAUSE OF REALITY — THE HARMONY OF MATHEMATICAL REASONING — DEMOCRATIC AND AXIOCRATIC EQUALITY AS ARITHMETIC AND GEOMETRIC ANALOGIES

Pythagoreans work hard and care for the cultivation of character and soul. The key tools of this pythagorean convictions and self discipline were music and mathematics. Pythagoras believed that the uncontrolled way of life leads ultimately to social conflict and destruction of the State, because of human inefficiencies and material desires and glutony. The structure of a society should be — according to the Pythagorean views — founded on a frame of just (and rational) laws the main ingredients of social justice, harmony and peace for all. The law should be:

«...κοινωφελὴς καὶ διὰ πάντων διατείνων...».

Only in that society harmony and democracy — founded on justice and freedom — can flourish. And such a harmony is always a balance of adverse and contrary forces and tendencies, a unique synthesis of opposing points of view. In the Pythagorean mathematical context and analogy to numbers, it is the synthesis of the infinite (ἄπειρον) and the limited (πεπερασμένον) (συναρμογή τοῦ πέρατος καὶ τοῦ ἀπείρου) in the application of the power and perfection of numbers. This mathematical scheme is a

reminder —mutatis mutandis — of the Hegelian approach to the historical and social evolution of society and ideologies always passing through "thesis, antithesis" and "synthesis". Hegel of course, presented his scheme in the socio-economic and historical field in the struggle and conflict along with that of the diverging ideologies and the scientific-technological status of each historical period.

Human reasoning was considered as mathematical reasoning (and calculation) which in the proper time moment (καιρός - kairos) may be applied to resolve and harmonize social conflict with the help of the State law. Of course this attitude of the few leaders, follows a social hierarchy, since all members of the Society are not equal (in the sense that their spiritual endowments by nature are different). The meaning of this hierarchy in essence is associated with the Pythagorean mathematical thinking, an approach not likable by most of the people who cannot conceive it.

Tetrachtys (Τετρακτύς), the perfect pythagorean number, strengthens the right arrangement and symbolizes an ideal and harmonious hierarchy. Accordingly, the fundamental concepts of the pythagorean politics (political "tactics" and "strategy") were: harmony, order, hierarchy and the exclusion of disorder (ἀμετρία) of the extreme and undisciplined behavior, leading ultimately to lower denomination of equalization (ἰσοπέδωσις). The social harmony can be established only with the prevailing of Logos and Justice which is associated with the harmony of numbers in proper order. First and foremost leaders in a society — according to Pythagoras — are those of higher intellect, courage and virtue. This is democracy at its best: The axiological order of society and not the indiscriminate "equality" of all which becomes a chaos of social irresponsibility and actual inequality at the end.

In that respect Pythagoreans have employed several numerical analogues to which some "social analogies" can be construed. The *arithmetic equality* made up from the numbers 6, 4, 2 such as $6 - 4 = 4 - 2$ corresponds to the *democratic equality* whereas the relationship between 8, 4, 2 $8/4 = 4/2$ is a *geometric equality* corresponds to the *axiocratic equality* form of democracy. The *democratic equality* makes all citizen equals regardless of their intellect, courage and virtue, resulting ultimately into social *inequality*, whereas the geometric equality corresponds to the real meaning of social justice and value, the very foundation of democracy. There is also the harmonic relationship among 6, 4, 3 such as $6 - 4 = 2$ and $4 - 3 = 1$. In both cases: $2 = 1/3^6$ and $1 =$

$1/3^3$ Pythagoreans have also associated social justice with numbers. Justice is a number equally - equal (ἰσάκως ἴσος) whereas justice is considered as equality among citizens, based on the citizen's merits, intellect and moral courage. For the first time the use of mathematics is employed to systematize exactly the axiocratic tenets and structure of the political relations and virtues of social life. In an ethical sense this resembles the Kantian Categorical imperatives of the citizen's moral life and behaviour. Pythagoras experience — Boudouris observes — from the Samian politics and the tyrannical regime as a deviation from a democratic society has led him in Croton to ask for a strong and solid foundation of political "bios" (βίος) composed of freedom, justice, human dignity, the essence of democracy.

In summary the Pythagorean theory of numbers applied to physical and social reality can be put as follows:

Numbers are not an abstract entity of the human mind nor as typical *model* of the prototype substances but autonomous real entities, independent, authentic substances. This does really imply that a) each real substance is a number and its composing parts are also numbers. b) The harmony and symmetry of the relations of those real substances up to the "Cosmos" — the sun and the totality of all existing substances — are those of the symmetry, beauty and harmony of numbers.

The Pythagorean definition of "Cosmos", does indicate its harmonious analogies and symmetrical order as the living representation and hierarchical embodiment of numerical relationships.

No existing substance and/or entity per se, nor in its relationship with other substances (or entities) can be coherent and concrete in its essence without the particular number and its *numerical functioning* and *geometric shapeness*.

Mathematical entities such as numbers and shapes were the *archetype substances*, the *ultimate stuff*, the forms (The Platonic Ideas) out of which the real substances and entities of the external world experienced and constructed by our senses and perception.

Pythagoras has work out the adjustment to our mind (and noësis - νόησις) everything our senses are observing by giving shape and a bodily form and *determining numerically* their "limited" and infinite relations. Accordingly Pythagoras' theory strengthens our "ability of knowing" after this scrutiny and evaluation of reality in terms of *quantity and quality evaluation* with the certain tool of numbers.

The nature and power of numbers is being seen by Pythagoras as prevailing not only in the *divine and demonic realm* but in all real things of the physical world (φύσις - nature) and in the human artifacts and in music. Nature (φύσις) and Number's harmony cannot afford a "lie" nor a falsification: it will be contradiction in terms!

It is important to add at this juncture that *the Pythagorean theory of numbers, their beauty and harmony was mostly associated with the magic of music. The chaos of the multitude of sounds and tones, their random "chaotic intonation" can be transformed into music only when it is subjected to a numerical arrangement in an orderly discipline. The chaos of sounds — with the numbers as regulators — becomes harmonious analogy and rhythm melody of music and music of medody.*

Pythagoras has reached the conclusion that the harmony and the beautiful analogies of music and numbers is running throughout the external world. The remarkable creation of orderly numbers and numerical relationships — independently of perceptual experience, is a step of the mind (and pure noêsis) was viewed by Pythagoras as divine revelation, a spiritual enlightenment and inspiration of another realm, the sense cannot grasp.

VI. PYTHAGOREAN CONTRIBUTION TO THE EVOLUTION OF MODERN MATHEMATICS AND SCIENCE

6.1 A. N. WHITEHEAD'S ANALYSIS

Alfred North Whitehead¹¹ in his classical contribution and analysis of mathematical reasoning and logic "*Science and the Modern World*" (1925) has stated the following:

«...The harmony of the logical reason which divines the complete pattern as involved in the postulates, is the most general aesthetic property arising from the mere fact of concurrent existence with unity of one occasion. This aesthetic relationship is that which is divined in the exercise of rationality. Whatever falls within that relationship is thereby exemplified in that occasion, whatever falls without that relationship is thereby excluded from exemplification in that occasion. The complete pattern of general conditions, thus exemplified, is determined by any one of many select sets of these conditions. These key sets are sets of equivalent postulates. This reasonable harmony of being, which is required for the unity of a

complex occasion, together with the completeness of the realization (in that occasion) of all that is involved in its logical harmony, is the primary article of metaphysical doctrine. It means that for things to be together involves that they are reasonably together. This means that thought can penetrate into every occasion of fact, so that by comprehending its key conditions, the whole complex of its pattern of conditions lies open before it. It comes to this: — provided we know something which is perfectly general about the elements in any occasion, we can then know an indefinite number of other equally general concepts which must also be exemplified in that same occasion. The logical harmony involved in the unity of an occasion is both exclusive and inclusive. The occasion must exclude the inharmonious, and it must include the harmonious».

The significance of the great pythagorean notion of mathematical abstraction and recreation of the physical entities and substances runs throughout the classical world. Plato has elevated the Pythagorean doctrine that "number lies at the base of the real world" to the superb and divine conception of "forms", the Platonic world of ideas. On the other hand Aristotle puts emphasis on classification (taxonomy) of things instead of measurement and its transcendence into mathematical reasoning and mathematical logic.

Whatever significance this Aristotelian Logic and classification can be attributed to the orderly classification of sciences it is a fact that the advancement of physical sciences retarded throughout the Middle Ages up to the end of sixteenth century. It is *critical* to underline that Aristotle was not a mathematician like Pythagoras and Plato — even he was not ignorant of mathematics.

A. N. Whitehead has stated:

«The importance of an individual thinker owes something to chance. For it depends upon the fate of his ideas in the minds of his successors. In this respect Pythagoras was fortunate. His philosophical speculations reach us through the mind of Plato. The Platonic world of ideas is the refined, revised form of the Pythagorean doctrine that number lies at the base of the real world. Owing to the Greek mode of representing numbers by patterns of dots, the notions of number and of geometrical configuration are less separated than with us. Also Pythagoras, without doubt, included the shape-iness of shape, which is an impure mathematical entity. So to-

day, when Einstein and his followers proclaim that physical facts, such as gravitation, are to be construed as exhibitions of local peculiarities of spatio-temporal properties, they are following the pure Pythagorean tradition. In a sense, Plato and Pythagoras stand nearer to modern physical science than does Aristotle. The two former were mathematicians, whereas Aristotle was the son of a doctor, though of course he was not thereby ignorant of mathematics. The practical counsel to be derived from Pythagoras, is to measure, and thus to express quality in terms of numerically determined quantity. But the biological sciences, then and till our own time, have been overwhelmingly classificatory. Accordingly, Aristotle by his Logic throws the emphasis on classification. The popularity of Aristotelian Logic retarded the advance of physical science throughout the Middle Ages. If only the schoolmen had measured instead of classifying, how much they might have learnt!

Classification is a halfway house between the immediate concreteness of the individual thing and the complete abstraction of mathematical notions. The species take account of the specific character, and the genera of the generic character. But in the procedure of relating mathematical notions to the facts of nature, by counting, by measurement, and by geometrical relations, and by types of order, the rational contemplation is lifted from the incomplete abstractions involved in definite species and genera, to the complete abstractions of mathematics. Classification is necessary. But unless you can progress from classification to mathematics, your reasoning will not take you very far».

The Concept of Continuity of a function (and the limitation of continuity) along with the invention of infinitesimal changes and variations — and its reverse of integral summation — leading to the foundation of "Calculus of Variation and Integration" is the greatest step ahead in the meteoric rise of modern Physics, Cosmology and Computer Sciences.

These concepts were not conceived by Pythagoras. A. N. Whitehead brilliantly analysed this topic.¹¹ It is important to present this exact analysis in its entirety:

«Finally the particular functions, such as the trigonometrical functions, and the logarithmic functions, and the algebraic functions, are generalised into the idea of 'any function'.

Too large a generalisation leads to mere barrenness. It is the large

generalisation, limited by a happy particularity, which is the fruitful conception. For instance the idea of any *continuous* function, whereby the limitation of continuity is introduced, is the fruitful idea which has led to most of the important applications. This rise of algebraic analysis was 'concurrent with Descartes' discovery of analytical geometry, and then with the invention of the infinitesimal calculus by Newton and Leibniz. Truly, Pythagoras, if he could have foreseen the issue of the train of thought which he had set going would have felt himself fully justified in his brotherhood with its excitement of mysterious rites.

The point which I now want to make is that this dominance of the idea of functionality in the abstract sphere of mathematics found itself reflected in the order of nature under the guise of mathematically expressed laws of nature. Apart from this progress of mathematics, the seventeenth century developments of science would have been impossible. Mathematics supplied the background of imaginative thought with which the men of science approached the observation of nature. Galileo produced formulae, Descartes produced formulae, Huyghens produced formulae, Newton produced formulae.

As a particular example of the effect of the abstract development of mathematics upon the science of those times, consider the notion of periodicity. The general recurrences of things are very obvious in our ordinary experience. Days recur, lunar phases recur, the seasons of the year recur, rotating bodies recur to their old positions, beats of the heart recur, breathing recurs. On every side, we are met by recurrence. Apart from recurrence, knowledge would be impossible; for nothing could be referred to our past experience. Also, apart from some regularity of recurrence, measurement would be impossible. In our experience, as we gain the idea of exactness, recurrence is fundamental.

In the sixteenth and seventeenth centuries, the theory of periodicity took a fundamental place in science. Kepler divined a law connecting the major axes of the planetary orbits with the periods in which the planets respectively described their orbits: Galileo observed the periodic vibrations of pendulums: Newton explained sound as being due to the disturbance of air by the passage through it of periodic waves of condensation and rarefaction: Huyghens explained light as being due to the transverse waves of vibration of a subtle ether: Mersenne connected the period of the vibration of a violin string with its density, tension, and length. The birth of modern physics depended upon the application of the

abstract idea of periodicity to a variety of concrete instances. But this would have been impossible, unless mathematicians had already worked out in the abstract the various abstract ideas which cluster round the notions of periodicity. The science of trigonometry arose from that of the relations of the angles of a right-angled triangle, to the ratios between the sides and hypotenuse of the triangle. Then, under the influence of the newly discovered mathematical science of the analysis of functions, it broadened out into the study of the simple abstract periodic functions which these ratios exemplify. Thus trigonometry became completely abstract; and in thus becoming abstract, it became useful. It illuminated the underlying analogy between sets of utterly diverse physical phenomena; and at the same time it supplied the weapons by which any one such set could have its various features analysed and related to each other. For a more detailed consideration of the nature and function of pure mathematics cf. my *Introduction to Mathematics*, Home University Library, Williams and Norgate, London.

Nothing is more impressive than the fact that as mathematics withdrew increasingly into the upper regions of ever greater extremes of abstract thought, it returned back to earth with a corresponding growth of importance for the analysis of concrete fact. The history of the seventeenth century science reads as though it were some vivid dream of Plato or Pythagoras. In this characteristic the seventeenth century was only the forerunner of its successors.

The paradox is now fully established that the utmost abstractions are the true weapons with which to control our thought of concrete fact. As the result of the prominence of mathematicians in the seventeenth century, the eighteenth century was mathematically minded, more especially where French influence predominated. An exception must be made of the English empiricism derived from Locke. Outside France, Newton's direct influence on philosophy is best seen in Kant, and not in Hume».

6. 2. MUSICAL RHYTHM AS SOURCE OF PHILOSOPHICAL KNOWLEDGE : BECKING'S EVALUATION

It is relevant to add at the juncture the theoretical search and systemic classification of musical rhythm and music in relationship to philosophy by G. Becking.

In his deep and systematic study of the subject in his treatise entitled "*The Musical Rhythm as Source of knowledge*"¹² Becking is

considering a rigorous classification of the great composers — mostly Germans — in groups based on "Rhythm" as the key element of their musical creations excluding any relationship with existing ideologies and/or personal philosophical ideas and/or disposition towards their contemporary ideas. His attempt was clear: to classify the great composers (and compare them with the great philosophers of the period) along the line of psychological forms -types— originated by W. Dilthey¹² and established by K. Jaspers.¹³ Along three fundamental standings and views of the human soul: idealism, pantheism and naturalism. Becking's three categories based on the synthesis of these philosophical triptych are:

Category I: Monism - Spiritualism - Idealism

II: Dualism - Materialism - Idealism

III: Dualism - Spiritualism - Naturalism.

Without going in the depth of this classification attempted by Becking — which touches religious doctrines and views along with specific philosophical schools of the period of the great composers — the verdict is that Mozart and Bach are spiritualists and Beethoven a materialist!

«...The composers of the Beethoven group — Becking has stated — in their creations fight with "dead material". All circumstantial elements resemble obstacles without soul and life, obstacles blocking man's effort which must be overcome. There is no close relation of Man with all these. Man should go ahead and leave them behind in the best case. On the contrary Mozart and Bach live within that world with all these elements and obstacles...».

«...Rhythm, according to Becking, is resembling the human blood rhythm and order or the watch orderly movement. Rhythm is ringing as it is by nature, it is not a human creation but life itself full of intensity and natural purpose for which the composer is not responsible...».

Becking is dividing the history of German music in three great chapters: (a) the pre-classical period in which the divine power is present everywhere, (b) The classical period of German music when the personal responsibility of the individual is present and (c) The romantic rhythm in Germany is identified with search inquiries in the irrational...»

Prof. E. T. Rakintzis in his excellent treatise entitled "*Music and*

Philosophy — The classification of Gustav Becking (Greek Philosophical Review 1985 (pp. 162-168)) describes thoroughly these views and states:

«...With all reservations and scepticism, one may have — regarding the objectivity of Becking music classification, it is apparent that his analysis is unique. We are confronting the problem of musical rhythm as key parameter of the struggle and agony of the human soul in confronting creativity the animosity of the external world, and in good standing toward Cosmos and God...».

VII. CONCLUDING REMARKS

1. The Pythagorean *quest for truth* and harmony was centered on *the number as the building block* of physical and social reality.

2. The harmony of mathematical reasoning being an abstraction — was considered as *authentic knowledge*, accurate certain and applicable to the real world.

3. Pythagorean notion of numerical abstraction and formulation as the basis of science and mathematics, in *recreating* the theoretical structure *representing physical reality*, its entities and substances. *The numbers and the numerical relationships, the orderly blocks of harmony in nature permeate (and regulate) all functions and processes of ecosystems: Any disturbance or destruction of this order result in ecological unbalance and disequilibrium with a fatal result on the biological "structure-roof" of life, nature and mankind.*

4. The Pythagorean *abstractions in the realm of numbers*, the "numerical compositions", the observations on periodicities, tones, intonation and rhythm of musical harmony was his monumental contribution to mathematics and philosophy.

5. The concepts of mathematical continuity and functionality along with other fundamental properties of the modern "theory of functions", the "Calculus of variations" and mathematical analysis, the "sure" and elegant approach for interpretation of physical reality today (originated and "intergrated" in the 16th century) have their roots to Pythagorean thought. The exponential rise of mathematical science of the last five centuries (Newton, Leibniz, Laplace, Euler; Lagrange, Cauchy, Gauss, Poincaré), the "Analytic Geometry" (Descartes), the "Fourier Analysis", the Physics, Chemistry, Biology and Cosmology, meteoric advancements, has been a gigantic movement of human mind with Pythagorean contribution on numbers, as their point of departure.

6. Pythagoreans coined for the first time in history the word "cosmos" to indicate the beauty and harmony coming out of the chaotic disturbance and conflict of competing and diametrically opposing elements in nature and life.

7. The greatest contribution of the Pythagorean thought, however, concerning the essence of the relation music and harmony vs. the numbers as their inner structure is this: *For the first time in history and the evolution of ideas and human mind* this undisputed fact was conceived and shaped as follows: The infinite number of tones, *the chaotic and disorderly presence of several sounds and intonations — a noise indeed without any aesthetic result — become music and harmony upon the use of numbers: the mathematical arrangement based on numerical order. Music and harmony is founded on numbers and the power and elegance of numerical discipline.*

8. The "mathematization" of social and political processes is another Pythagorean contribution. It is the concept that justice, social harmony and freedom are the attributes of "numbers" in proper order, that the numerical categories, the accurate arithmetic or geometric equalities being applied to social categories and classes, secure the true hierarchy of the several levels of social equality reflecting the value, the intellect, the courage i.e. the status of each citizen.

9. The Pythagorean Logos and the "numbers" conceived as entities and innate ideas of another universe, beyond their mathematical impact and contribution in the structural formation of modern mathematical science, were "building" blocks to the foundation of the School of Idealism in history. These "Pythagorean numbers" became a century later (in the 5th century) the "Platonic forms", the "Ideas" (ἰδέαι) of Plato's philosophy, the authentic and unchanged reality behind "the appearances" (δόξαι).

These "Pythagorean numbers" became Berkeley's (1685-1753) "Idea" in the 17th and 18th century, that the world is nothing but "our knowledge of it", a brilliant "integration" of "Mind and the World" in a *unity*, as response to Locke's (1632-1704) "Mind - World" dualism (and the corresponding "theory of Truth", the basis of Empiricism vs. the Subjective Idealism of Berkeley).

The Pythagorean numbers became the foundation of the 18th - 19th century Kantian Idealism, the "Copernician Revolution" and the "transcendental idealism" that : *it is not the mind that conforms to the World, but rather the World that conforms to the mind.* (Within that context and in this respect Kant broadens the narrow kind of *subjective idealism* — which reduces the World to the level of

sensation and there is still *no way* of knowing another by sensation. Kant has shown — in a parallel to Pythagorean way, the binding and necessity of mathematics — a necessity that is binding on the way we think as well as on the world of experience.

10. A. N. Whitehead — has stated brilliantly — the following concerning Pythagoras mathematical inquiries: «...Pythagoras, is said, to have taught that the mathematical entities, such as numbers and shapes, were the ultimate stuff out of which the *real* entities of a perceptual experience are constructed. As thus, badly stated, the idea seems silly indeed. But undoubtedly, he had hit upon a philosophical notion of considerable importance; a notion which has a long history and which has moved the minds of men and has even entered the Christian theology...».

«Also — A. N. Whitehead continues — Pythagoras, without doubt, included the “*shape-iness*” of shape, which is an impure mathematical entity. So today when Einstein and his followers proclaim that physical facts, such as gravitation are to be construed as exhibitions of local peculiarities of spatio-temporal properties, they are following the pure Pythagorean tradition. Pythagoras stands nearer to modern physical science than does Aristotle...».

NOTES

1. Baron J. P., “*The Sixth Century tyranny at Samos*”, Classical Quarterly 1964 (p. 210-229).

2. Diog. Laertios 8. 1, Heraclitus B. 129 and Herodotus 4.96.

3. K. Boudouris: (a) *Presocratic Political Philosophy*, Ionia Edition, Athens 1988 and (b) *Presocratic Philosophy*, Ionia Edition, Athens 1988.

4. Philip, J. A. *Pythagoras and Early Pythagoreanism*, University of Toronto Press 1966 (p. 179-180).

5. Burkert, W. *Love and Science in Ancient Pythagoreanism*, Cambridge, Mass. 1972.

6. Dunbabin, T. J., *The Western Greeks*, Ares. Publishers, Chicago and Oxford 1948.

7. Minar, E. L. *Early Pythagorean Politics*, Waverly Press, Baltimore, 1942.

8. Burnet, J. *Early Greek Philosophy*, 4th Edition, London 1930.

9. Guthrie W. K. C., *A History of Greek Philosophy*, Cambridge University Press, 1962.

10. Fritz, Kurt von *Pythagorean Politics in Southern Italy*, Octagon Press 1977.

11. G. Becking, *Der Musikalische Rhythmus als Erkenntnisquelle*, B. Fisher, Augsburg (1928).

12. W. Dilthey, *Dilthey's Philosophy of Existence: Introduction to Weltanschauungslehre*, (Transl. by W. Klubach and M. Wehnbaum), Greenwood Press, Westport, Connecticut (1967).

13. K. Jaspers, *Psychologie der Weltanschauungen*, Springer Verlag, Heidelberg (1919).

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