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THE INFLUENCE OF PYTHAGOREAN
PHILOSOPHY ON THE DEVELOPMENT
OF MATHEMATICAL ASTRONOMY

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16.Ib. 189.15ff., cf. 134.36-135.7.

17. The role of κοινή αἰσθησις has been scrutinized, from a different point of view, by H. J. Blumenthal, «Some Platonist Readings of Aristotle», in *PCPhS* n.s. 27(1981), 1-16.

18. ἔνδοθεν 119.6ff., οἰκοθεν 119.10.

19. Iē.188.35.

20. The ability to perceive that we perceive is assigned also by Priscianus to the influence of *sensus communis* but in a rather different way, cf. *Metaphr.* 22.1-23. For the sake of convenience I called the author of *CAG* XI Simplicius. This attribution has been contested by F. Bossier and C. Steel, «Priscianus Lydus en de In De anima van Pseudo (?)-Simplicius», in *Tijdsch. voor Filos.* 34 (1972), 761-822, with French summary on 821-822, who assign this work to Priscianus. However, the discrepancy between their «noetic» has been noticed also by C. Steel, *The Changing Self. A Study on Soul in Later Neoplatonism: Iamblichus, Damascius and Priscianus*. Brussel 1978, and has been resolved by saying that in *Metaphr.* Priscianus follows Iamblichus while in the in *DA* he confronts his own views with those of Iamblichus (p. 149-155).

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THE INFLUENCE OF PYTHAGOREAN PHILOSOPHY ON THE DEVELOPMENT OF MATHEMATICAL ASTRONOMY

When referring to the ancient mathematical astronomy we mean the notions of spherical astronomy as well as various mathematical theories invented by Greek astronomers for the description of the motion of the planets. For this reason I do not intend to deal here with ideas concerning the physical state of the celestial bodies.

It is true that the astronomical knowledge of the Pythagoreans and their general contribution to the development of astronomy are considered as a constantly controversial subject of the history of this science because of the lack of authentic early evidence¹. But we cannot dispute the importance attributed by the Pythagoreans to the ἀδελφαὶ ἐπιστῆμαι of astronomy and harmonics, as follows from Socrate's words in Plato's Republic². For this reason a new look at the cosmological system of Philolaus of Kroton and the theories of Archytas of Taras in relation to the evolution of the astronomical knowledge of Greeks may possibly help to the deduction of some basic conclusions regarding the cosmological model of the late Pythagoreans.

In the Philolaic system, in the center of the universe there is a fire called «Hearth of the world»; the divine bodies dance around it, the following according to their order from the center: the first is the counter-earth, the second is the earth which always moves and revolves ἐξ ἐναντίας to the counter-earth, the third is the moon, the fourth is the sun, then come the five planets, and the last one is the sphere of the fixed stars³.

The general characteristics of the cosmological system of the Pythagoreans can be summarized as follows:

- 1) The spherical universe.
- 2) The circular motions of the celestial bodies around a common center.
- 3) The idea that the common center is a fire not identified with the sun.
- 4) The distinction between planets and other stars.
- 5) The circular shape of the earth.
- 6) The displacement of the earth from the center of the universe and its motion around it.⁴
- 7) The hypothesis of the counter-earth.
- 8) The existence of other celestial bodies invisible to us.
- 9) The divinity of the celestial bodies.
- 10) The harmony of the spheres.

The notions of the sphericity of the universe and the periodic circular motions of the celestial bodies already exist in Heraclitus, Parmenides, Empedocles, and Anaxagoras, who differ from the Pythagoreans by their assumption that the earth is motionless in the middle of the world.⁵ For this reason it is generally accepted that the genuinity of the Philolaic system lies mainly in the displacement of the earth from the middle of the universe and in the hypothesis of the counter-earth, which possibly aimed at raising the total number of the spheres of the celestial bodies to ten,⁶ the sacred number of the Pythagoreans.

From the remaining fragments of Philolaus's work «On Nature» we can understand the importance of the role of number in the universe: nature and all in the world are composed from «unlimited and limited» elements.⁷ All things that can be known contain number; without this, nothing could be thought or known;⁸ moreover all things are related to one another by the ratios of their numbers.⁹ Special importance is attributed to the power of the decade in the divine, the celestial, and the human life; without it all would be «infinite and invisible and unseen».¹⁰

These ideas applied to the case of astronomy lead to more substantial results apart from that of the hypothesis of the counter-earth as the tenth body of the system of the world: they introduce quantitative relations between the members of the system, which are not only static, as order and relative distances of its members between them, but also dynamic, as ratios of their periods of revolution. In this point we can distinguish between mythical or simple cosmological ideas and mathematical cosmological models. Because all efforts of later Greek astronomers pointed to the

principle of «saving the phenomena», i.e. to the introduction of a new or to the improvement of an already existing mathematical model describing the structure of the universe, the motion of the sphere of the fixed stars, and the motions of the planetary spheres, so that the arithmetical data from observations agree with those calculated on the basis of the theories.

There is a problem: what do we know about the application of these mathematical relations in the case of Pythagorean astronomy? In the remaining fragments of Archytas's work «Harmonic», there is a passage referring to the mathematicians who know well the *ἀδελφεὰ μαθήματα* of geometry, arithmetic, spherics, and harmonics. The reason why these sciences are so called is evident, as they treat numbers or magnitudes and their relations to one another, i.e. their ratios and proportions. But I would like to comment further on the same passage referring to «the mathematicians who gave us exact means of distinguishing the speed of stars and their heliacal risings and settings.»¹¹

It is noteworthy that the passage mentions only one speed of the stars in contrast to the plural used for their heliacal risings and settings. The terms *ἐπιτολή* (=heliacal rising) and *δύσις* (=setting) for the stars are used in astronomy to denote the various phenomena of the first appearance (or disappearance) of a fixed star a little before sunrise (or after sunset), according to the season, while the corresponding phenomena for the planets are their conjunctions or oppositions to the sun. Consequently, the reference to the «one» speed of stars implies that the passage speaks about the fixed stars, which participate in the revolution of the celestial sphere without changing their relative angular distances.

Such knowledge relating the aspects of the celestial sphere to the seasons is already found in Hesiod,¹² and in 5th cent. B.C. the observations of Meton and Euctemon are registered on the famous «paraepgmata»,¹³ the material of which was enriched by more accurate observations of later astronomers until Ptolemy (included).¹⁴ The problem is, what are the speeds of the planets according to the Pythagoreans, as no one source mentions their order accurately; it is simply said that after the Sun come the five planets. I think the reason for the silence about the exact order of the five planets is perhaps that it was not needed, and I will immediately explain why.

In antiquity the speeds of the planets refer to the time needed in order that they describe a whole circle in the zodiac, i.e. to their sidereal periods. In the case of the Moon, Mars, Jupiter and Saturn

the differences between these periods are so great that no problem has ever appeared in the estimation of their order and distances from the earth. The problem appears in the case of Mercury, Venus and the Sun, because the first two do not seem moving far from the sun and they describe a whole circle in about a year. Consequently, when sources mention that the five planets come after the sun, even without giving their sidereal periods, the order of the planetary sequence is unclear only as far as Mercury and Venus are concerned, i.e.: central fire, Counter-earth, Earth, Moon, Sun, Mercury - Venus (or inversely), Mars, Jupiter, Saturn, sphere of fixed stars. Generally, this vagueness regarding the order of the Sun, Mercury and Venus also continues in later Greek astronomy.¹⁵

Another question is, whether the Philolaic system took into account the obliquity of the ecliptic, i.e. that the zodiac belt, where the Sun, the Moon and the planets seem moving, is inclined in respect to the circle of the revolution of the celestial sphere, i.e. the celestial equator. Aëtius referring to the motion of the earth says: «Some say that the earth is at rest; but Philolaus the Pythagorean says that it is carried in a circle round the fire on a slanting circle in a similar fashion to the sun and moon». ¹⁶ As Aristotle and Simplicius do not mention inclined orbits,¹⁷ the general opinion is that all about the «inclined circle», namely the ecliptic, is Aëtius's interpolation. But the discovery of the obliquity of the ecliptic is attributed to Oinopides of Chios,¹⁸ who flourished c. 450 B. C., while Philolaus flourished c. 430 B.C. and he could have known about Oinopides's discovery. In the literary sources there also is the opposite opinion, that Oinopides appropriated this Pythagorean discovery, or that he learned it from the Egyptians.¹⁹

To two other Pythagoreans, Hicetas²⁰ and Ecphantus of Suracuse, is attributed the idea of the rotation of the earth around an axis passing through its center, which explains the apparent diurnal motion of the celestial sphere: We do not know when they exactly lived, but they are usually considered as Philolaus's contemporaries (middle of 5th cent. B.C.). Although according to Ecphantus the earth is in the middle of the world and moves without involving change of place but by revolution,²¹ it is very important that one of these two kinds of true motion is attributed to the earth by Philolaus and the other kind of motion by Ecphantus and Hicetas.

According to another tradition, the cosmological system as exposed in Timaeus is Plato's plagiarism, who bought and studied Philolaus's works.²² Let us have a closer look at this subject. Firstly,

both Platonic cosmological systems place the earth in the middle of the world. In the system exposed by Plato in the Republic, the outermost whorl of Ananke's (=Necessity's) spindle is the sphere of fixed stars, while the inner whorls, which correspond to the planets, the sun and the moon, are concentric and also have the same axis with the outermost whorl; moreover they move with different speeds on the same plane with it, but in opposite direction in relation to it.²³ On the contrary, the cosmological system in Timaeus takes into account the circle of the Same, which corresponds to the celestial equator and refers to the diurnal rotation of the celestial sphere from east to west, as well as the circle of the Different, i.e. the ecliptic, where the planets seem moving from west to east when they are in direct motion, and from east to west when in retrograde motion. Moreover it is clearly said that the circles of the Same and the Different do not concide, but they intersect and are like the letter X.²⁴

We do not know the chronological difference between the two Platonic models; but we can at least ask, where Plato was influenced from. Although both models, the one in the Republic and the one in Timaeus, display great differences from the Philolaic model, they also share some common elements with it. The first is the principle of the circular motion as the only one which can be attributed to the divine celestial bodies (fixed stars and planets), and which takes place around a common center. The second is the order of the planets in the two Platonic models. According to the descriptions contained in the texts this order is as follows: Earth, Moon, Sun, Venus, Mercury, Mars, Jupiter, Saturn, sphere of the fixed stars. Namely, the sun comes immediately after the moon²⁵, after it Venus and Mercury —although the latter two have the same speed as the sun,²⁶ while the remaining planets display different speeds in relation to each other as well as to the former three planets.²⁷ Plato does not give any numerical value for these speeds, as he does not write a mathematical treatise, but he says that only a few men know them, evidently the mathematicians - astronomers.²⁸ In the Philolaic model we also have the five planets after the sun. It is true that Aëtius does not mention any word on planetary speeds in the Philolaic model; in spite of it, I very much doubt that there were not some basic correspondences —even inaccurate— between planets and speeds, because of the Pythagorean «Harmony of spheres», i.e. their Harmonics applied to the spheres of the celestial bodies,²⁹ and which Plato took from them.

Consequently, although Plato did not plagiarize Philolaus's system, nevertheless he adopted some of its basic principles regarding the motion of the celestial bodies, which actually seem to describe circles on heaven. This is well understood by us, because we are familiar with basic geometrical knowledge. But in antiquity it was the Pythagorean philosophy and especially its principles regarding numbers and geometrical figures, and their relation to the universe, which initiated the use of mathematics for the description of the structure of the universe and of the motions of its parts.

From Plato on, great mathematicians - astronomers improve the two models of the geocentric system, i.e. the eccentric and the epicyclic models, aiming to an explanation of the apparently irregular motions of the planets by introducing one or more cyclic paths.³⁰ Their persistence in circular orbits leads to a more and more sophisticated structure of the models by the continuous addition of new cycles and epicycles.³¹

Aristarchus of Samos, another mathematician - astronomer, is the main introducer of the heliocentric system. According to Archimedes, «the hypotheses of Aristarchus of Samos are that the fixed stars and the sun remain motionless, and that the earth revolves in the circumference of a circle about the sun, which lies in the middle of the orbit».³² Although the passage refers to the earth, we may suggest that also the planets revolve around the sun, as the fixed stars and the sun are mentioned as motionless. Aristarchus's system adopts the Pythagorean circular planetary orbits around a common center which is the sun. The circular orbits are in the zodiac and the path of the earth coincides with the ecliptic. If Aristarchus ascribed one circular orbit around the sun for every planet, then the rejection of its system by the astronomers may not be due to religious prejudices but to scientific reasons, because it could not «save the phenomena». Namely, it is sure that in his system there was a difference between observed and calculated planetary positions, as we now know that the planets move in elliptical orbits.

This view is further supported by a case of silent deviation from the circular orbits in the geocentric Ptolemaic system. From some erroneous observations Ptolemy concluded that Mercury's orbit had two points at which the planet was nearest to the earth, i.e. that it had two perigees.³³ This implied that the locus of the center of the epicycle was not a circle, but an ovoid curve with the earth situated not in the middle of its main axis, but in the side of the two perigees, namely in the narrow part of the ovoid.³⁴ As Ptolemy was a

famous geometer, it is sure he knew that he violated the tradition regarding circular orbits; nevertheless he did it, but only in Mercury's case, in order to «save the phenomena» according to his observations.

For the same reason Copernicus, although he plagiarized Aristarchus's heliocentric idea, he applied it with respect to the geocentric system, by using a great number of cycles and epicycles,³⁵ in order to minimize the differences between observed and calculated planetary positions. The principle of circular orbits dominated more than a millennium. Only in the beginning of the 17th century Johannes Kepler formulated the three laws of planetary orbits, one of which says that the planets move around the sun in elliptical orbits, one focus of which is occupied by the sun.

The ideas of the Pythagoreans about the harmony of the spheres of the planets producing sounds as they move, as well as those regarding³⁶ the five regular solids and their correspondence to the four primary elements and the world, influenced very much Plato's cosmology in Timaeus, and other later cosmological systems. Kepler also was deeply influenced by them, as it is shown by two of his works.³⁷ The influence continues up to now, as famous scientists find again the regular solids in atomic and molecular structure of matter and they refer to the Pythagorean philosophy, which strongly linked the numbers with the universe both in micro and macro scales.³⁸

NOTES

1. D. R. Dicks, *Early Greek Astronomy to Aristotle*, Cornell U. Pr. 1970, 62.
2. Plat. *Repub.* 530d, in: *Platonis opera*, i-v, ed. I. Burnet, Oxford U. Pr., here iv (1902 / repr. 1978).
3. Aëtius II 7, 7 & III 11, 3 = Philol. Frg. A 16 & A 17. Stob. *Ecl.* I 21, 8 = Philol. Frg. B 7, in: H. Diels - W. Kranz, *Die Fragmente der Vorsokratiker* (=DK), Weidmann, Berlin 1952, 101961.
4. Aristot. *De caelo*, B 13, 293a18, ed. D. J. Allan, Loeb, Oxford U. Pr. 1936 / repr. 1973.
5. D. R. Dicks, *op. cit.*, 44-45, 48-49, 51-55, 57.

6. Aristot., *op. cit.*, B 13, 293a23.
7. Diog. VIII 85 = Philol. Frg. B 1. Stob. *Ecl.*, I 21, 7a = Philol. Frg. B 2, in: *DK*.
8. Stob. *Ecl.* I 21, 7b = Philol. Frg. B 4, in: *DK*.
9. Stob. *Ecl.* I prooem. cor. 3 = Philol. Frg. B 11, in: *DK*.
10. See note 9.
11. Iambl. *In Nicom.* 9, 1 = Archyt. Frg. B 1, in: *DK*.
12. Hesiod, *Works and days*, 383-7, 414-9, 479-80, 564-7, 571-2, 597-8, 609-11, 615-21, 663-5.
13. D. R. Dicks, *op. cit.*, 107.
14. Claude Ptolémée, *Apparitions des fixes et annonces*, éd. Halma, Paris 1819-20.
15. O. Neugebauer, *A History of Ancient Mathematical Astronomy*, i-iii, Springer, 1975, here i 148.
16. Aëtius III 13. 1. 2 = Philol. Frg. A 21, in: *DK*.
17. Aristot. *op. cit.*, B 13, 293a18. Simplic. *In Aristot. De caelo* 511, 26, in: G. S. Kirk - J. E. Raven, *The Presocratic Philosophers*, Cambridge U. Pr. 1957 / repr. 1973, 260.
18. Theon. Smyrn., p. 198, 14. Macrobian. *Sat.* I 17, 31 = Oinop. Frg. 7, in: *DK*.
19. Aëtius II 12, 2. Diod. I 98, 2 = Oinop. Frg. 7, in: *DK*.
20. Cic. *Akad. Pr.* II 39, 123 = Hiketas Frg. 1, in: *DK*.
21. Hippol. *Refut.* I 15 = Ekphant. Frg. 1. Aëtius III 13, 3 = Ekphant. Frg. 5, in: *DK*.
22. Diog. VIII 84. 85 = Philol. Frg. A 1, in: *DK*.

23. Plat. *Repub.* X 616c - 617b.
24. Plat. *Tim.* 36b-d.
25. *Ibid.* 38d, 39b.
26. Plat. *Repub.* 617a, *Tim.* 39d.
27. Plat. *Repub.* 617c, *Tim.* 36d.
28. Plat. *Tim.* 39c.
29. Aristot., *op. cit.*, B 9, 290b12.
30. Olaf Pedersen & Mogens Pahl, *Early Physics and Astronomy*, MacDonald & Janes, London 1974, 74-87.
31. O. Pedersen & M. Pahl, *op. cit.*, 90-99.
32. Archimed. *Arenar.*, I 4, in: Greek mathematics, i-ii, trans. by Ivor Thomas, Loeb, Harvard U. Pr. 1941 / repr. 1980, here ii 2.
33. Claude Ptolémée, *Composition mathématique*, IX 8, éd. Halma, Paris i (1813), ii (1816), here: ii 172.
34. O. Neugebauer, *op. cit.*, here: i 163-4, iii 1253 fig. 148. O. Pedersen & M. Pahl, *op. cit.*, 96 fig. 7. 19.
35. O. Pedersen & M. Pahl, *op. cit.*, 308-314.
36. J. Kepler, *Astronomia Nova* (1609). It is noteworthy that ellipse is one of the conic sections, which had been fully studied by Apollonius of Perga in his famous work on them, written about the middle of third century B.C.
37. *Mysterium cosmographicum* (1596), *Harmonice mundi* (1619).
38. Lecture of Prof. Roald Hoffmann (Nobel Prize for Chemistry 1981) at the University of Athens (20 May 1991).

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