

PYTHAGORAS AND FOUR COLOURS

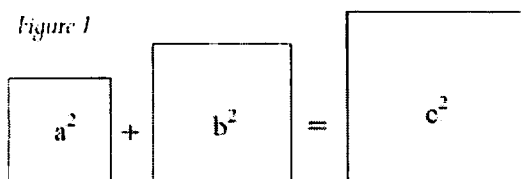
Hasan Unal describes a spatial activity based on partitioning a square into four figures and constructing two new squares from them.

One way to teach Pythagoras' Theorem is through use of puzzles. Marshall (2004:1) points out that, 'in creating their individual solutions to puzzles, students may reveal mathematical thinking on which approaches to the standard curriculum could be based.' This article describes a puzzle-like spatial structuring activity related to Pythagoras' Theorem. The tasks were tried with pre-service middle and secondary mathematics teachers in the USA and Turkey.

There are two cases:

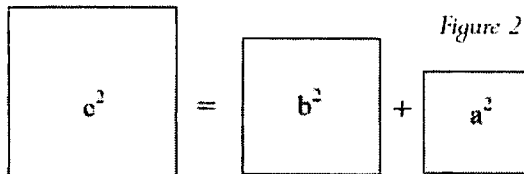
Case 1: If I give you two squares, you need to give me one square with an area equal to sum of those two squares (see *Figure 1*).

Figure 1



Case 2: If I give you one square, you need to give me two squares whose areas sum to the area of the original square (see *Figure 2*).

Figure 2



We have explored the second case in this article, but our contention is not that one way is superior to the other. The two cases provide different perspectives that might be useful for teaching Pythagoras' Theorem and developing students' visualisation skills. Furthermore it also encourages divergent thinking.

The Task

We gave the students a 5-by-5 square and asked them to construct two squares from the given square by dividing it into four regions so that these four regions can be rearranged into two squares (4-by-4 and 3-by-3).¹ This activity can be done with manipulatives or just plain paper. There are multiple answers to the problem and students' thinking skills get challenged. It also helps to improve spatial ability, since students need to rotate the images in their minds when they construct new squares. The task is fun and intrinsically motivating, and it offers just the right amount of challenge.

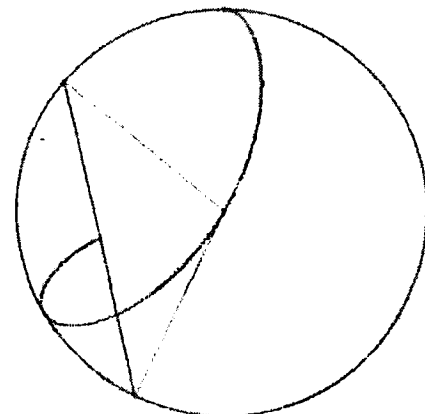
Students' solutions fall into two categories (A and B) and colour is used below to illustrate the way in which the 5-by-5 square has been dissected.

Table 1: Summary of Type A solutions

5^2	4^2	3^2	number of solutions
four colours	three colours	one colour	9
four colours	one colour	three colours	1

Table 2: Summary of Type B solutions

5^2	4^2	3^2	number of solutions
four colours	two colours	two colours	3



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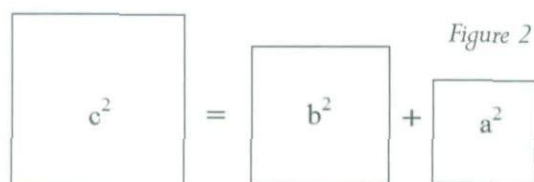
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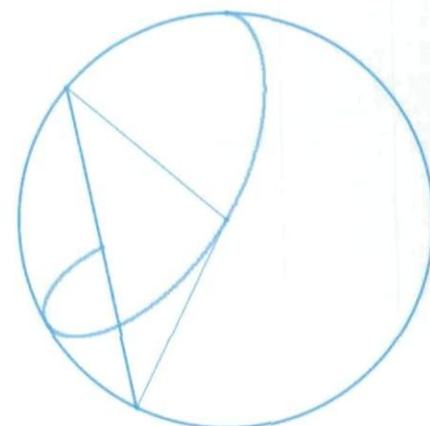
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Type A solutions

The characteristic of this type of solution is that four colours from the main square (5-by-5) are distributed as three colours in the first square (4-by-4) and one colour in the second square (3-by-3) or vice versa – see figures 3-12. The partitioning in the main square in Figure 6 was carried out using only rectangles and a square. The solution in Figure 7 was the first solution students came up with, and we find that the majority of students arrive at this solution at their initial attempt. The solutions in Figures 9, 10 and 11 took more time to discover than did the previous ones. In Figure 12, it is the 4-by-4 square that is all one colour rather than the 3-by-3 one.

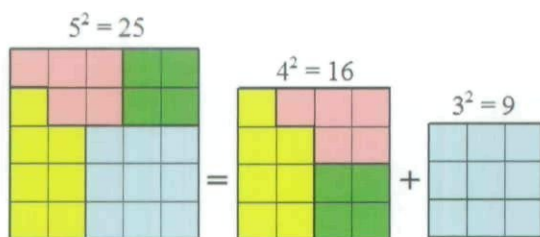


Figure 3

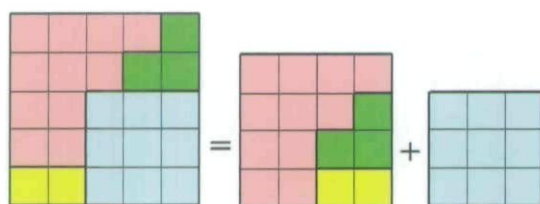


Figure 4

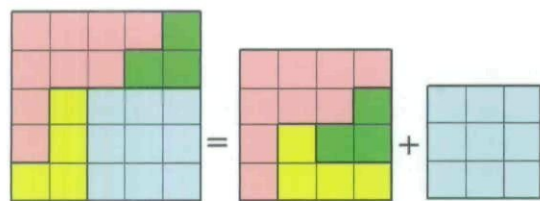


Figure 5

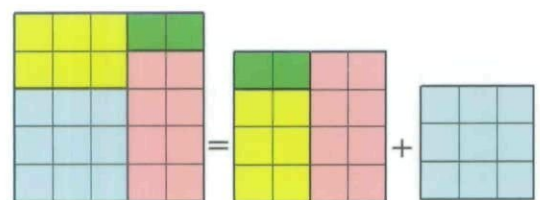


Figure 6

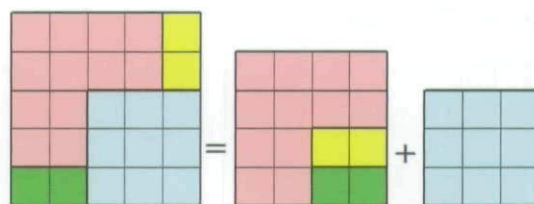


Figure 7

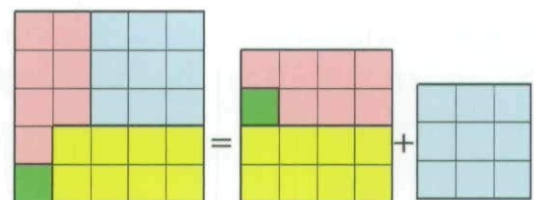


Figure 8

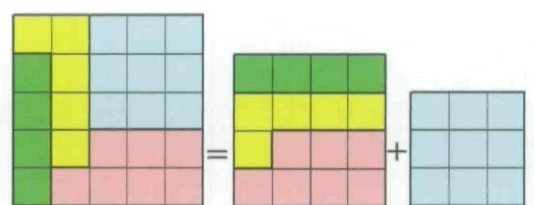


Figure 9

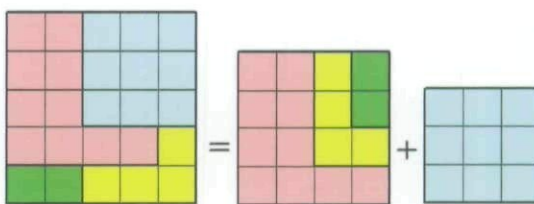


Figure 10

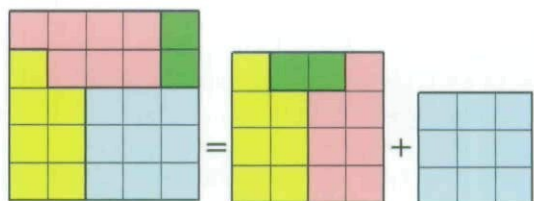


Figure 11

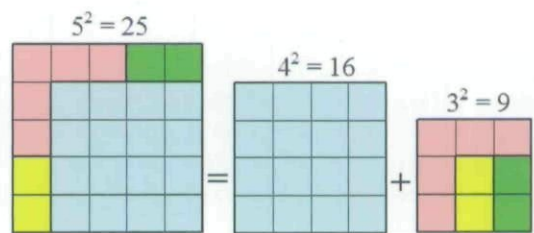


Figure 12

Type B solutions

The solutions in Figures 13 to 15 are different from the ones above. In the previous solutions, one of the two smaller squares (either the 3-by-3 or the 4-by-4) was all one colour and the other square contained three colours. In these type-B solutions, both of the two smaller squares contain exactly two colours each.

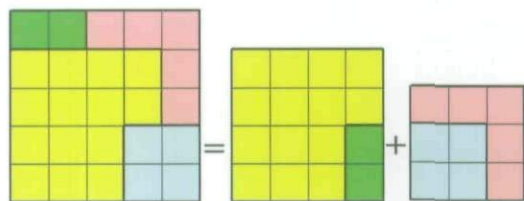


Figure 13

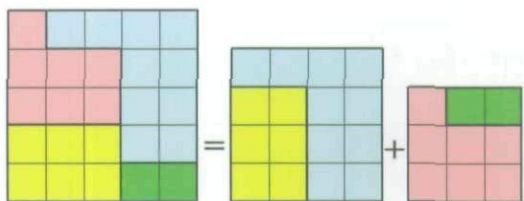


Figure 14

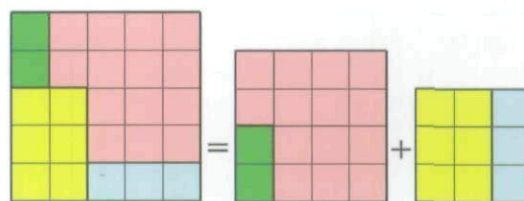


Figure 15

An interesting feature of some of the students' solutions (both A- and B-type) were ones where pieces from the 5-by-5 square needed to be turned over – as opposed to merely rotated: see figures 16 (type A) and 17 (type B), for instance. In other words, both sides of the paper would need to be coloured. These solutions were not valued by the students; they felt that allowing these made the task insufficiently challenging. It will be up to the teacher to decide at the beginning of the activity if flipping the figures is going to be allowed or not.

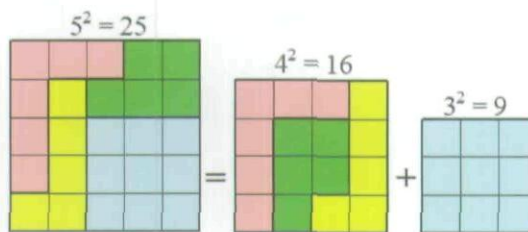


Figure 16

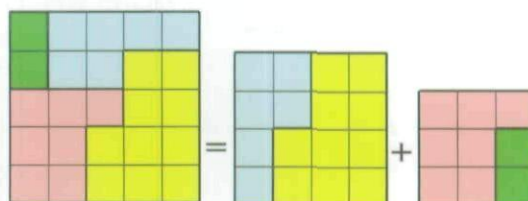


Figure 17

Conclusion

We believe that this approach to Pythagoras' Theorem makes the lesson more active and encourages students to consider why it works. Presenting geometry in ways that stimulate curiosity and encourage exploration can enhance students' learning and improve their attitudes towards mathematics.

Hasan Unal is an assistant professor in the mathematics department at Yildiz Technical University, Istanbul, Turkey.

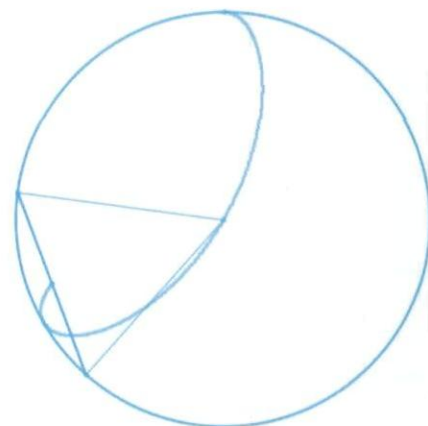
Note

1 The problem was originally investigated by Bolt (2001).

For a summary of the solutions and an activity sheet to use with students, go to www.atm.org.uk/mt206

References

- Bolt, B. (2001) Two squares from one, *Mathematics in School*, November, 30-31
- Marshall, A. J. (2004) Construction of meaning: urban elementary students' interpretation of geometric puzzles, *The Journal of Mathematical Behavior*, 23 (2), 169-182



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