

ARITHMETIC AND GEOMETRY IN ANCIENT ROME:
SURVEYORS, INTELLECTUALS, AND POETS

MARIO GEYMONAT

ABSTRACT

Cicero's rediscovery of Archimedes' tomb shows the interest for science in Rome when the ruling class used to be educated in Greece. Geometry, practised mainly by the Gromatici, had a more technical character in Italy. Augustine showed a good knowledge of mathematics but, as the Greek language went declining in the West, the cultural and economic exchanges became very rare across the Mediterranean. The perception of this danger induced the best scholars to translate scientific texts into Latin: e.g. Boethius translated Aristotle's logic, Nichomachus' *Arithmetica*, and Euclid's *Elements* (a few complex fragments of the latter, with authorial corrections, are found in a palimpsest in Verona). Many poets were also interested in mathematics: Catullus' 5 and 7 re-echoed the numbers of Archimedes, killed by a Roman soldier during the conquest of Syracuse; Virgil, too, found delight in arithmetic: in Constantine's times such attention is testified by Optatianus, and Ausonius offers an interesting example of combinatory geometry.

Keywords: Roman Ancient Mathematics.

The devaluation of mathematics in ancient Rome begins with a good-natured self-critical affirmation by Cicero in the *Tusculanae Disputationes* (1.2.5):

In summo apud illos [Graecos] honore geometria fuit, iaque nihil mathematicis industrius, at nos metendi ratiocinandiue utilitate huius artis terminavimus modum.¹

¹ "With the Greeks geometry was regarded with the utmost respect, and consequently none were held in greater honor than mathematicians, but we Romans have restricted this art to the practical purposes of measuring and reckoning." (*Tusculanae Disputationes*, trans. by John Edward King [Cambridge, MA: Harvard University Press, 1950 (1927)].)

It was the same Cicero who would show a clear interest in scientific subjects in his efforts to publish Lucretius' *De rerum natura*² and who boasted of having rediscovered the tomb of Archimedes when, as a quaestor in Sicily in 75 B.C., he saw on a stele hidden in some bushes the image of a sphere inscribed within a cylinder – the subject of the most important work of the Syracusan mathematician (*Tusculanae* 5.23.64). In the second book of the *Academica Priora* (*Lucullus* 116-117), he seems convinced of the logical superiority of geometry over the more nebulous arguments of the philosophers:

non quaero rationes eas quae ex coniectura pendent, quae disputationibus huc et illuc trahuntur, nullam adhibent persuadendi necessitatem; geometrae provideant, qui se profitentur non persuadere sed cogere, et qui omnia vobis quae describunt probant. non quaero ex his illa initia mathematicorum, quibus non concessis digittum progredi non possunt, punctum esse quod magnitudinem nullam habeat, extremitatem et quasi libramentum in quo nulla omnino crassitudo sit, liniamentum <longitudinem> sine ulla latitudine [...] carentem [...] Quod si geometricis rationibus non est crediturus, quae vim adferunt in docendo, vos ipsi ut dicitis, ne ille longe aberit ut argumentis credat philosophorum.³

We should not forget that in his youth Cicero prepared a poetic translation of Aratus' difficult *Phaenomena* and put an abstract discussion of astronomy in the mouths of intellectuals and political men meeting at the house of Scipio Emilianus (*De re publica* 1.13-14; 19-22), and made special reference there to Archimedes' planetaria that Marcellus had brought back as a spoil of war from Syracuse.⁴

In the same period Marcus Terentius Varro dedicated two entire sections of his encyclopedic treatise *Disciplinarum libri* to arithmetic and geometry. This treatise is probably the source of many mathematical terms later used

² There are two authoritative sources for this: Jerome, in an addition to Eusebius's *Chronicon*, refers to 94 BC (*cum aliquot libros per intervalla insaniae conscripsisset, quos postea Cicero emendavit, propria se manu interfecit anno aetatis XLIV*) and Cicero himself, when he praised Lucretius's art in a letter to his brother Quintus written in February 54 BC, QFr. 2.10 (9) (*Lucreti poemata ut scribis ita sunt, multis luminibus ingenti, multae tamen artis* ["Lucretius's poems are just as you write, born of his brilliant talent, but also of a very refined art"].)

³ "I am not asking these people about those first principles of mathematics which must be granted before they are able to advance an inch – that a point is a thing without magnitude, that a 'boundary' or surface is a thing entirely devoid of thickness, a line a thing without any breadth [...] But if he is going to refuse credence to the methods of geometry, which in their teaching exercise a compelling force, as your school itself asserts, surely he for his part will be far from believing the proofs of the philosophers." (*De Natura Deorum - Academica*, trans. by Harris Rackham [Cambridge, MA: Harvard University Press, 1933].)

⁴ On the admiration that Cicero held for Archimedes, see what I have written in *Il Grande Archimede* (Rome: Sandro Teti, 2006), pp. 95 and 105-106.

in Latin, but only very small portions of it are left to us today. Varro himself was the author of two other works dedicated to mathematics, but which have been lost: *Atticus de numeris* and *De principiis numerorum* (the latter in nine books!).

The efforts of personalities such as Cicero and Varro show the widespread interest in mathematics that developed in Rome in the 1st century B.C., when the young members of the ruling class received their most advanced education in Greek schools, in which scientific subjects had a highly formative value. Indeed, during Cicero's lifetime, Aratus' *Phaenomena* were translated by Varro Atacinus and, a little later, by Germanicus, who belonged to the *gens Iulia* and was adopted as a son by Emperor Tiberius. An additional version of Aratus was prepared in the mid-4th century by Avienus, a high-ranking member of a senatorial family. At the height of the Roman Empire, Rome itself became an important center of Greek science and culture, e.g. Galen's long stay in the court of Marcus Aurelius, a Roman emperor who even wrote of philosophy in Greek.

But in Rome, as Cicero suggested in the passage cited at the beginning, the separation of science and technology was much less pronounced than in the Greek world, and the specific interest in arithmetic and geometry presented a characteristically technical/practical character. An example from the Augustan age is the funny interrogation of a boy on the division of a coin that we read in Horace, *Ars Poetica* 325-330 and, at a superior level, Vitruvius's *De Architectura*. From the 1st century A.D. we could think to the *De aquaeductu urbis Romae* of Frontinus (*geometricae artis inspector providissimus*), who was consul various times during the reigns of Vespasian and Trajan. In late antiquity, such a technical/practical approach is evidenced in particular by the *Gromatici*, whose technique of measurement and delimitation of agricultural properties is best illustrated in a splendid late-antique manuscript kept in Wolfenbüttel.⁵ The *mensores* were charged with military, civil and bureaucratic surveying, and, among other things, they took care of the calculation of agrarian taxes, the measurement of communal property, the construction of camps, the division of colonies, the construction of bridges and viaducts, and the inspection of borders. A number of professional schools were founded

⁵ For the writings and doctrines of the *Gromatici* see the still fundamental studies *Die Schriften der römischen Feldmesser*, herausgegeben und erläutert von Friedrich Blume, Karl Lachmann und Adolf August Friedrich Rudorff (Berlin: Reimer, 1848-52, reprint Hildesheim, 1967) and MORITZ CANTOR, *Die römischen Agrimensoren und ihre Stellung in der Geschichte der Feldmesskunst; eine historisch-mathematische Untersuchung* (Leipzig: Teubner, 1875, reprint Wiesbaden: Sändig, 1968).

across the Empire for their training and in the instructional texts they used a few elementary explanations of geometry: the definition of point, line, plane figure, the calculation of the area of a square, of a rhombus, of a parallelogram and of a trapezoid, the calculation of the perpendicular bisector of a triangle, given the sides, and that (though not always exact) of the areas of regular polygons, and even the very approximate relationship between the diameter and the circumference. The presence of their extracts helped to maintain a specific connection between geometry and land surveying for the entire Middle Ages.

Of much more significance for the relationship with practical disciplines, however, were the scientific interests that developed in the learned circles of Rome along with the broad influence of neo-Pythagoreanism and that exerted by the captivating personality of one Nigidius Figulus (praetor in 58 B.C.). Even beyond Cicero and Varro the interest of the best Latin intellectuals in science had revealing examples. One need only think of Pliny the Elder, the author of the *Naturalis Historia*, who died heroically in an effort to help people in distress and to better understand the tragic eruption of Vesuvius in 79 A.D., or Quintilian, who suggests complex problems of geometry as being fit for the formation of the young orator (1.10.34-49). In the 2nd century we remember Apuleius, to whom we owe the first Latin version of the *Introduction to Arithmetic* of Nichomachus of Gerasa, now lost but recorded together with that of Boethius by Cassiodorus.⁶ Still at the end of antiquity a grammarian of great success such as Priscian wrote a brief treatise dedicated to a specific mathematical subject, the *De figuris numerorum quos antiquissimi habent codices* (*Grammatici Latini* II, 406-417). One must recognize, in any case, that the Latin authors, who cited almost only Euclid and Archimedes (even in works of philosophy, literature, and rhetoric), were never able to get close to the questions discussed by contemporary Greek scientists. This explains why no Latin author is ever mentioned in any analogous Greek work; moreover, not even the Latin grammarians, with famous names such as Varro and Priscian, were ever cited by their Greek counterparts.

⁶ *Institutiones* 2.4.7: *reliquae vero quae sequuntur, sicut eius iam qualitas ostendit, ut sint atque subsistant, indigent arithmetica disciplina. quam apud Graecos Nicomachus diligenter exposuit. hunc prius Madaurensis Apuleius, deinde magnificus vir Boethius Latino sermone translatus Romanis contulit lectitandum; quibus, ut aiunt, si quis saepius utitur, quantum hominibus fas est, lucidissima procul dubio ratione perfunditur* ["indeed the remainders that follow, just as its quality has already shown, as they are and stand, require mathematical discipline, as among the Greeks Nichomachus diligently explained. First Apuleius of Madaura, then the magnificent man Boethius completed a Latin translation of it and it must be read often. As they say, if someone makes use of something as often as it is right for such men, doubt is washed away by brilliant reason"].

One significant development in mathematical interests is revealed in some Latin authors of the 4th and 5th centuries, when the influence of neo-Platonism became more tangible. It should be of no surprise that the greatest intellectual of the age, Augustine, makes explicit use of geometry in his intimate conversations with his Reason (*Soliloquia* 1.4.10-11):

RATIO. Ergo lineam in duas lineas per longum scindi, manifestum tibi est nullo modo posse?

AUGUSTINUS. Manifestum.

R. Quid, transversim?

A. Quid, nisi infinite secari posse?

R. Quid, sphaeram ex una qualibet parte a medio, ne duos quidam pares circulos habere posse pariter lucet?

A. Pariter omnino.

R. Quid linea et sphaera? unumne aliquid tibi videntur esse, an quidquam inter se differunt?

A. Quis non videat differre plurimum?

R. [...] Sic enim nosti lineam et nosti sphaeram, cum se non sic habeat linea ut se habet sphaera. Quamobrem responde utrum tibi satis sit sic Deum nosse, ut pilam illam geometricam nosti; hoc est, ita de Deo nihil, ut de illa, dubitare.⁷

Augustine's interest in arithmetic and geometry is specifically connected to that in astronomy and music in the second book of the *De ordine*, in chapters 14.39-15-43,⁸ and it is not by chance that Sandro Botticelli

⁷ "REASON: So it is clear to you that a line cannot be divided in two along its length?

AUGUSTINE: It is clear.

R: What about transversally?

A: Can it not be divided infinitely?

R: And is it not equally clear that of all the circles that pass through a part more or less distant from the center of a sphere, not even two of them can be equal to one another?

A: It is equally clear.

R: What about the line and the sphere? Do they seem the same to you, or is there a different between them?

A: Who doesn't see that they differ greatly?

R: [...] So you know the line as you know the sphere, although the line isn't the same thing as the sphere. Answer me now whether it is enough for you to know God as you know the sphere in geometry; that is, having no doubt about God, as you have none about it?"

⁸ See, for example, 2.15.42: *Hinc est profecta in oculorum opes et terram coelumque collustrans, sensit nihil aliud quam pulchritudinem sibi placere, et in pulchritudine figuras, in figuris dimensiones, in dimensionibus numeros; quaesivitque ipsa secum utrum ibi talis linea talisque rotunditas vel quaelibet alia forma et figura esset, qualem intellegentia contineret. Longe deteriorem invenit et nulla ex parte quod viderent oculi cum eo quod mens cerneret comparandum. Haec quoque distincta et disposita in disciplinam redegit appellavitque geometriam. Motus eam caeli multum movebat et ad se diligenter considerandum invitabat. Etiam ibi per constantissimas temporum vices, per astrorum ratos difinitosque cursus, per intervallorum spatia moderata, intellexit nihil aliud quam illam dimensionem numerosque dominari. Quae similiter definiendo ac secernendo in ordinem nectens, astrologiam genuit, magnum religiosus argumentum tormentumque curiosi* ("Thus she

painted *Saint Augustine* in his *Study* with geometric designs and figures in the background (Florence, Uffizi, c. 1490). A specific interest in geometry can be seen in chapters 6.10-13.22 of *De quantitate animae*, a catechistic dialogue strangely ignored by historians of mathematics, from which Pseudo-Boethius deduced the principal arguments of the *Altercatio duorum geometricorum*, a brief treatise quite popular in the Middle Ages, published 25 years ago by Menso Folkerts.⁹ A certain theological Platonism is evident in the *De quantitate animae*, but it is also full of literal references to definitions and postulates of the first book of Euclid's *Elements*, as one can see, for example, in this passage from chapter 8.13:

AUGUSTINUS. Nam si et quid sit linea, et quid sit figura iam tibi notum est, dic quod abs te quaero, id est utrum putes ullam figuram fieri posse, si linea ex utraque parte, aut ex altera per infinitum ducatur?

EVODIUS. Nullo modo id fieri posse confirmo.

A. Quid igitur agendum est ut figuram faciamus?

E. Quid, nisi ut illa linea infinita non sit, et ducatur in circulum, ut ex alia parte se contingat? non enim video, quomodo aliter possit ex una linea concludi aliquod spatium; quod nisi fiat, secundum tuam descriptionem, figura non erit.

A. Quid, si rectis lineis figuram facere velim? potest fieri ut de una linea fiat, an non potest?

E. Nullo modo.

A. Quid, duabus?

E. Ne hoc quidem.

A. Quid, tribus?

E. Video posse.

A. Bene igitur nosti ac tenes, cum figura lineis rectis facienda est, minus quam tribus non posse. An si ulla tibi adversetur ratio, de hac te sententia devocabit?

E. Plane si quis mihi hoc falsum esse monstraverit, nihil erit quod me scire posse confidam.

crossed into realm of the eyes and went over the earth and heaven. She sensed that what pleased was nothing other than the harmony, and in the harmony figures, in the figures the measurements, and in the measurements numbers. And she reflected in herself whether this line or this circle or any other form or figure is similar to that which belongs to the intelligence. She found that they are much more imperfect and that what the eyes sees can absolutely not be compared to what the mind sees. And so she analyzed and systematized these notions, placing them into a discipline that she called geometry. The movement of the heavens attracted her very much and encouraged her to study it attentively. She understood that also here, through the constant successions of times, the fixed and defined course of the stars though exactly established distances, was the realm of none other than the measurement and numbers. She systematized these notions as well with definitions and divisions, and thus inaugurated astronomy, which is a great subject of study to the religious and a torment to the superstitious.")

⁹ MENSO FOLKERTS, *Die Altercatio in der Geometrie I des Pseudo-Boethius. Ein Beitrag zu Geometrie im mittelalterlichen Quadrivium*, in *Fachprosa-Studien*, edited by Gundolf Keil et al. (Berlin: E. Schmidt, 1982), pp. 84-114.

A. Nunc ergo illud responde, quomodo tribus lineis figuram feceris?

E. Cum se finibus iungunt.

A. Quid? ubi se iungunt, nonne videtur tibi angulus fieri?

E. Ita est.

A. Quot ergo angulis haec figura constat?

E. Totidem quot lineis.

A. Quid? ipsas lineas pares constitui, an impares?

E. Pares.

A. Quid? anguli tantumdem omnes patent, an est alius alio contractior vel apertior?

E. Etiam ipsos pares esse video.



A. Potestne fieri, ut in figura, quae tribus rectis paribus lineis facta sit, impares anguli sint; an non potest?

E. Nullo prorsus modo.

A. Quid? si rectis lineis tribus, sed imparibus figura constet, possunt etiam in ista pares esse anguli, an aliud intellegis?

E. Omnino non possunt.

A. Recte dicis: sed dic, quaeso, quaenam tibi figura melior videatur et pulchrior? eane quae paribus, an quae imparibus lineis constat?

E. Quis dubitet eam esse meliorem in qua aequalitas praevalet?¹⁰

¹⁰ AUGUSTINE: If you know what a line is and what a figure is, answer my question. Do you think that a figure can exist if a line extends infinitely from both parts or even from just one?

EVODIUS: I affirm that it cannot happen in any way.

A: So what must be done in order to make a figure?

E: What, if not that the line be made finite and extended into a circle so that it reconnects to itself from the other side? For I do not see how a space can be marked out with a single line; if this is not done, according to your explanation, it will not be a figure.

A: And if I should want to make a figure out of straight lines? Is it possible that one be made from a single line, or not?

E: Impossible.

A: With two, then?

E: Not even with two.

A: With three?

E: I see that it is possible.

A: So you have learnt well, and you remember that when a figure is to be made from straight lines, it cannot be formed with less than three. If an argument seemed wrong to you, would it make you withdraw from this conviction?

E: Clearly, if someone showed me that it is false, there would be nothing that I would trust myself to be able to know.

A: So answer me now: how do you make a figure from three lines?

E: The lines are connected at their ends.

A: And where they are connected, does it seem to you that angles are formed?

Towards the end of antiquity, science gradually acquired a weighty importance in Latin schools, especially arithmetic and geometry, both part of the *quadrivium*, were the subject of synthetic treatments in the encyclopedias of Martianus Capella, Cassiodorus, and Isidore. Particularly meaningful is the interest in numbers – considered Platonically as the basis of geometry – that one can see in Macrobius, a politician (*vir clarissimus et illustris*), the highest level of the senatorial class) and scholar from the first half of the 5th century, who had a huge influence on the whole medieval culture of magic and wisdom. Let us read, as an example, §§ 7-12 and 17-18 of chapter five of the first book of the *Commentarii in somnium Scipionis*, which he dedicated to the final chapters of the *De re publica*, Cicero's most important work of political philosophy conserved for us thanks to Macrobius. Note how in this text some Greek terms are widely used in a very natural manner; evidently, it was the language used in that period for scientific discourse, even in the West.

[7] haec superficies, sicut est corporum terminus, ita lineis terminatur, quas suo nomine γρομμιάς Graecia nominavit: punctis lineae finiuntur: et haec sunt corpora quae mathematica vocantur, de quibus solerti industria geometricae disputatur. [8] ergo haec superficies cum ex aliqua parte corporis cogitur, pro forma subiecti corporis accipit numerum linearum: nam seu trium ut trigonum, seu quatuor ut quadratum, seu plurimum sit angulorum, totidem lineis sese ad extrema tangentibus planities eius includitur. [9] hoc loco admonendi sumus quod omne corpus

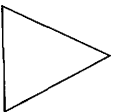
E: Yes.
A: How many angles make up this figure?

E: There are as many angles as lines.

A: And are the lines equal in length, or unequal?

E: Equal.

A: And do the angles have the same degree, or is one more closed or more open than the others?
E: These too I see as equal.



A: Is it possible or impossible to make a figure made of three equal straight lines, of which the angles are unequal?
E: Completely impossible.

A: And if the figure is formed by three unequal straight lines, can the angles in it be equal? Or do you see things differently?

E: It is absolutely impossible.

A: You are correct; but tell me, please, which figure seems to you to be better and more beautiful? The one made of equal lines or the one made of unequal lines?

E: Who would doubt the one in which equality prevails is better?

longitudinis, latitudinis et altitudinis dimensionibus constat: ex his tribus in lineae ductu una dimensio est – longitudo est enim sine latitudine – planities vero quam Greci ἐπιγώνεια vocant, longo latoque distenditur, alto caret, et haec planities quantis lineis contineatur expressimus, soliditas autem corporum constat cum his duabus additur altitudo: fit enim tribus dimensionibus impletis corpus solidum quod στερεόν vocant, qualis est tessera quae κύβος vocatur. [10] si vero non unius partis sed totius velis corporis superficiem cogitare, quod proponamus esse quadratum, ut de uno quod exemplo sufficere disputeremus, iam non quattuor sed octo anguli colliguntur: quod animadvertis si super unum quadratum quale prius diximus alterum tale aliud impositum mente conspicias ut altitudo quae illi plano deerat addidatur fiatque tribus dimensionibus impletis corpus solidum quod στερεόν vocant ad imitationem tesserae quae κύβος vocatur. [11] ex his apparet octonarium numerum solidum corpus et esse et haberi, si quidem unum apud geometras puncti locum obinet, duo lineae ductum faciunt quae duobus punctis ut supra diximus coercentur, quattuor vero puncta adversum se in duobus ordinibus bina per ordinem posita exprimentur quadri speciem, a singulis punctis in adversum punctum eiecta linea, haec quattuor ut diximus, duplicata et octo facta, duo quadrata similia describunt, quae sibi superposita additque altitudine formam cybi quod est solidum corpus efficiunt. [12] ex his apparet antiquiorem esse numerum superficiei et lineis ex quibus illam constare memoravimus formisque omnibus, a lineis enim ascenditur ad numerum tamquam ad priorem, ut intellegatur ex diversis numeris linearum, quae formae geometricae describantur [...] [17] Pythagorici vero hunc numerum iustitiam vocaverunt, quia primus omnium ita solvitur in numeros pariter partes, hoc est in bis quaternam, ut nihil minus in numeros aequae pariter partes divisio quoque ipsa solvatur, id est in bis bina. [18] cum ergo et contextio ipsius pari aequalitate procedat, et resolutio aequaliter redeat usque ad monadem, quae divisionem arithmeticam ratione non recipit, merito propter aequalem divisionem iustitiae nomen accepit, et quia ex supra dictis omnibus apparet quanta et partium suarum et seorsum sua plenitudine nitatur, iure plenus vocatur.¹¹

¹¹ [7] As the terminus of a body is the surface, so the termini of the surface are the lines, *grammata* in Greek; and lines terminate in points. Now these are what are known as mathematical bodies, about which geometers dispute with skill and zeal. [8] When we consider the surface of one side of a body, we find that the number of its lines depends upon the form of the underlying body: whether it be triangular, quadrangular, or polygonal, its surface is enclosed by the same number of lines as it has angles. [9] At this point it would be well to keep in mind that all bodies have three dimensions: longitude, latitude, and altitude. Of the three we obtain one dimension by drawing a line (in longitude we do not have latitude); a surface, called by the Greeks *epigoneia*, is marked by length and breadth and lacks thickness (we have indicated how many lines confine this surface); bodies acquire solidity by adding altitude to the other two dimensions (a solid body is produced by filling the space enclosed by three dimensions, and it is called *stereon*, an example being the cubical die). [10] If you should wish to consider not the surface of one side, but all the surfaces of a solid figure, which we may assume for the purpose of illustration to be rectangular and equilateral, there are now not four but eight angles to be reckoned with. This you will recognize if you imagine that above one quadrat (a surface such as

A little later, however, in the years that follow the formal end of the Western Empire, knowledge of Greek becomes ever more uncertain, and these cultural and commercial exchanges across the shores of the Mediterranean become rare. It was a sense of the danger this represented for the entire Latin culture that pushed the most responsible of the learned men of Italy to endeavor not to lose the knowledge accumulated in previous centuries by translating some of the most important texts of Greek science and philosophy into Latin. Such was the generous attempt of the aristocrat Serverinus Boethius, who translated Aristotle's major works on logic (the *Categories*, the *De interpretatione*, and the *Analytica priora* which we have along with his version of Porphyry's *Isagoge*). Boethius also published in Latin the *De institutione arithmetica*, which was actually a re-elaboration of the *Introduction to Arithmetic* of Nichomachus of Gerasa (1st-2nd c. A.D.), a version of which had already been prepared by Apuleius.¹² Up until his last, most difficult years, when he wrote the *De consolazione Philosophiae* in prison, Boethius remained convinced of the fundamentally numeric structure of the world (*Tu numeris elementa ligas*, "You bind the elements with numbers": book 3, metr. 9, v. 10). This great intellectual also wrote a *De institutione musicae* to complete the

was described above) you have placed another exactly like it, so that altitude, which was lacking in the plane, is now added: with the three dimensions filled up a solid body is produced which geometricians call a die or cube. [11] Hence it is apparent that the number eight both is and is considered a solid body, if indeed one is represented by a point, two by the drawing of a line (which, as we said above, is limited by two points), and four by points arranged at right angles to each other, with lines extending between the points to form a square. When these four are duplicated and made eight, forming two equal squares, and one is superimposed upon the other, giving the figure altitude, the result is a cubical figure, which is a solid body. [12] Thus it becomes clear that numbers precede surfaces and lines (of which surfaces consist), and in fact come before all physical objects. From lines we progress to numbers, to something more essential, as it were, so that from the various numbers of lines we understand what geometrical figures are being represented [...] [17] The Pythagoreans, indeed, called the number eight Justice because it is the first number that may be divided into two equal even numbers and divided again into two more equal even numbers. It is also the product of equals: two times two times two. [18] Since it is the product of equal even numbers and may be divided equally, even down to the unit, which does not admit of division in mathematical computation, it deserves to receive the name Justice. And since it is clear, from what has previously been said, to what extent it depends both upon the fullness of its parts and upon its own fullness, it deserves to be called full." (*Commentary on the Dream of Scipio*, trans. by William Harris Stahl [New York: Columbia University Press, 1952], pp. 96-99.)

¹² As Jean-Yves Guillaumin affirms in his introduction to his edition of the *De institutione arithmetica* (Paris: Les Belles Lettres, 1995), p. xxxix: "Telle est la substance de l'enseignement pythagoricien que Boèce, armé de son excellente connaissance du grec, entreprend de transmettre au monde latin dans son *Institution Arithmétique*. Titre justifié: le caractère méthodique et progressif de l'ouvrage fait bien du traité de Boèce une *institutio* au sens exact du terme, 'exposé méthodique', à la fois 'manuel scolaire' et ouvrage de référence, qui se propose de mettre la science arithmétique à portée des débutants, et en même temps de fournir au professeur un aide-mémoire et un exposé systématique".

quadrivium, along with a treatise on astronomy, of which the former has been preserved to us whereas the latter is now lost.

What the ancients admired him for most, however, was his version of the entirety of Euclid's *Elements*, a translation that confronted very complex logic and semantic values, that never had been tackled in the language of the West. Let us read what Cassiodorus writes of this great innovation in his *Institutiones* (2.6.3):

*Cuius disciplinae [i.e. geometriae] apud Graecos Euclides, Apollonius, Archimedes necnon et alii scriptores probabiles extiterunt; ex quibus Euclidem translatum Romanae linguae idem vir magnificus Boethius edidit. Qui si diligenti cura relegatur, hoc quod praedictis divisionibus apertum est manifestae intelligentiae claritate cognoscitur.*¹³

For the definitions, the postulates, and the propositions of the first theorems, Boethius's translation was preserved to some degree during the Middle Ages, and two small works that synthesize and resume it were attributed to him (the so-called *Geometria I* and *Geometria II*, in five and two short books respectively). In some of the manuscripts that conserve these works one can find the first western notations of Arabic numerals, which Boethius obviously did not know.

Boethius's translation of Euclid was dedicated to Symmachus (according to a 9th-century gloss in the ms. Vaticanus Lat. 3123, f^o 54v), who held the title of *patricius*.¹⁴ In my opinion, when treating the subject of Latin geometry, it is not very fruitful to stop on the *Geometriae* of Pseudo-Boethius in medieval translation (the *Geometria I* was produced in Corbie at the end of the 8th century and the *Geometria II* was probably made in Lorraine at the beginning of the 11th century). They display scant interest in proofs, which are the true fulcrum of Euclid's work, and they preserve only the very elementary ones in theorems 1-3 of the first book of the *Elements*. In order to study the good level that Latin scientists reached in the field of geometry in late antiquity, it would be better to reflect on a few fragments of Latin translations of Euclid, which contain much more complex proofs and which were the last product in that field from the moment when

¹³ "Of this discipline [i.e. geometry], among the Greeks, there were Euclid, Apollonius, Archimedes and probably other writers as well. Of these writers, the magnificent man Boethius published a translation of Euclid into the Roman language. Whereby, if it is reread with diligent care, that which is opened up by the aforementioned divisions becomes known by the brilliance of clear intelligence."

¹⁴ This form is echoed by the *Patricius* to whom the *Geometria* is dedicated, published under Boethius's name by Gottfried Friedlein in 1867 (Leipzig: Teubner), p. 373, 21-24.

Fol. 9 (341r)

XIII

1. ADEUMIGITURQVIZHΘ
2. SITCIRCULUMCONUE
3. (NITQUODFUTE)XPEDI
4. RERATIONES

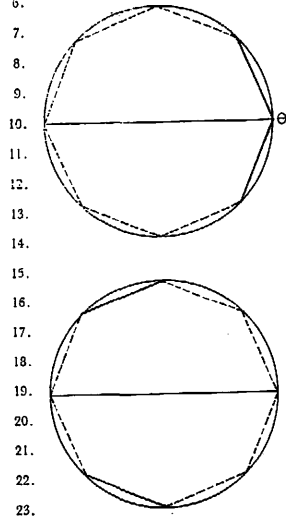


Fig. 3

In particolare alle righe 10-11 il traduttore si trova davanti a una parola, $\pi\rho\acute{\iota}\sigma\mu\alpha\tau\alpha$, che non sa come rendere; anche alla riga 8 era in dubbio: aveva scritto *sectiones* poi corretto in *segmenta*; andando oltre ha trovato la parola che gli pareva più fedele al testo [...] La differenza tra *segmentum* e *sectio* sembra essere quella tra $\tau\eta\mu\mu\alpha$ e $\kappa\alpha\tau\alpha\tau\omicron\mu\eta$. Il $\pi\rho\acute{\iota}\sigma\mu\alpha$ era qualcosa di diverso; era effetto di una risezione, ma era poi anche un solido con caratteristiche ben definite, quantunque tutto ciò non si possa affermare senz'altro.²⁰

²⁰ "Especially at lines 10-11 the translator finds himself faced by a word, $\pi\rho\acute{\iota}\sigma\mu\alpha\tau\alpha$, that he does not know how to render. He was also in doubt at line 8: he had written *sectiones*, then corrected it with *segmenta*. Proceeding, he had later found the word that he thought more faithful to the original text [...] The difference between *segmentum* and *sectio* seems to be that between $\tau\eta\mu\mu\alpha$ and $\kappa\alpha\tau\alpha\tau\omicron\mu\eta$. The $\pi\rho\acute{\iota}\sigma\mu\alpha$ was something different; it was the effect of a resection, but was also a solid with well-defined characteristics, although all of this cannot be confirmed for certain."

OMNISPY(RA)MISQUAE
 HABETTRIANGULAM
 SE(D(EMDI)UIDITURTA(M)
 [(INDUASP(AM)IDAS
 AEQUALESTR[(L^{CONAS})]
 SEDESHABENTESACSI
 MILES(SI)BIADQUEINDU(AS)
 SECTIO(ME)AEQUALES
 SCILICETEXARTEDISCOE
 D((E)TA(C)DUO(R)ECISAMEN
 (TATOT)IUSPY(RA)MIDIS
 QUAEFORTIETCONTIGERIN
 MAI(O)IRASUNTABEOQUOD
 DIMIDIUMESSSEGERNI
 [(TORSITPYRAMIS)UNA
 CUM(IN)PRO(PRI)UMEX
 ARTEPRODUCTACUIUS
 (B)AS(IS)EADEMQUAE
 (AB)T-SITTRIANGULUM
 SITQUEAPEXEIFUSEX
 DICOUADIUIDITUR
 TAMINDUASP(AM)IDAS
 DASAQUALESTRIA^{CONAS}

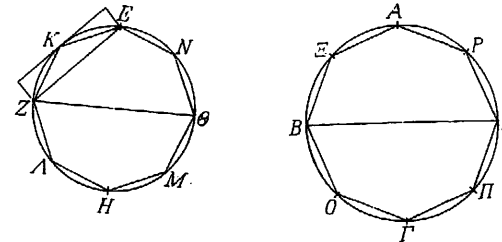


Fig. 4

Fabio Troncarelli offers confirmation that the author of these corrections was Boethius himself in a recent lecture given in Los Angeles²¹ With convincing evidence, he builds on Bernhard Bischoff's opinion that the fragments of XL (38), rewritten in Luxeuil script, likely came to Verona from Bobbio ("several monks of Luxeuil used to live in the sister abbey of Bobbio during the 7th century – four of them became Bobbio's abbots!"). These manuscripts must have belonged to "a great scholar, acquainted with very rare commentaries to Virgil and a very rare version of Livy. He was the only one at the end of the 5th century to have a Latin translation of Euclid, revised by the author himself... The name of Boethius seems so obvious...". Troncarelli's original conclusions about the provenance of ms. Verona XL (38) are that: "We must remember that Boethius was kept prisoner and executed in Pavia or nearby. During his imprisonment he certainly had some books, because he quotes different authors literally in his *Consolatio*. Two years after his death, Boethius was rehabilitated by queen Amalasunta and his books were probably preserved in Pavia, whose bishop had been till 521 Ennodius, a relative of Boethius, a town very important and later the capital of the Lombard's kingdom. Pavia is only 30 miles from Bobbio, and after its foundation in 612 the Monastery of St Columban obtained a lot of ancient manuscripts from Pavia and the Lombard's kings: it could have obtained also the remains of the books that Boethius owned during his imprisonment. Not by chance, Bobbio's library owned the only extant manuscript of Boethius of the 6th century: a fragment of the *De Institutione Arithmetica*, now in Turin (CLA IV, 450), a simple and professional work copied in uncial, in two columns, similar to the other manuscripts, possibly related to Boethius".

²¹ FABIO TRONCARELLI, *Thrice-born Boethius: the Last of the Romans from Late Antiquity to Renaissance*, summer 2007, now in print in BRIAN P. COPENHAVER, *Thrice-born Latinity* (Los Angeles: University of California).

Troncarelli's paleographic observations are revealing: "It is noteworthy that the manuscript [of Euclid], in two columns to save space, although not a calligraphic masterpiece, is written in *capitalis rustica*, the noblest ancient script, typical of the oldest copies of Virgil (the use of the *capitalis* for translation of Euclid means that Euclid was considered a classic, such as the old Latin classics). It is remarkable also that the textual corrections in an elegant, calligraphic small capital, with some uncial letters, are by no means the author's autograph corrections. My opinion is based on palaeographical evidence: the corrector's hand is very fluent and elegant, as only the hand of an upper class, learned writer could have been".

A separate chapter in the knowledge of arithmetic and geometry in Rome is represented by the writings of poets. In the volume cited at, note 4, *Il grande Archimede*, I raised the problem of whether and to what extent, in the attention they gave to numbers, Catullus and Virgil hid in their poetry an amiable memory of Archimedes, whose death at the hands of a Roman soldier provoked frustration and shame in the most sensitive Latin intellectuals.

In the *Arenarius* (Ψαμμίτης) Archimedes had constructed a series of huge numbers. It was the most well-known of his works in Rome, still cited by Hyginus Gromaticus at the end of the 1st century AD:

Nam et Archimedem, virum preclari ingenii et magnarum rerum inventorem, ferunt scripsisse, quantum arenarum capere posset mundus, si repleretur.²²

In the realm of poetry the *Arenarius* had been openly exalted in the mid-1st century by Silius Italicus, whose readers had heard of it (*Punica* 14.350-351):

Non illum mundi numerasse capacis harenas
vana fides.²³

So it seems very probable to me that Catullus is winks at this highly original work on numbers in lines 7-13 of poem 5:

Da mi basia mille, deinde centum,
dein mille altera, dein secunda centum,
deinde usque altera mille, deinde centum;
dein, cum milia multa fecerimus,

²² CARL OLOF THULIN, *Corpus Agrimensorum Romanorum* (Leipzig: Teubner, 1913), p. 148, 4-7 Carl Olof Thulin ("Indeed they say that Archimedes, a man of brilliant genius and inventor of great things, wrote how much sand the world can contain, if it should be filled up.").

²³ "Not without reason men believed that Archimedes had counted the sands of this great globe" (*Punica*, trans. by James D. Duff [Cambridge, MA: Harvard University Press, 1934]).

conturbabimus illa, ne sciamus,
aut ne quis malus invidere possit,
cum tantum sciat esse basiorum.²⁴

Perhaps there is also an allusion here to the columns of the *abacus*, the instrument for mathematical calculations commonly used in the market place. Certainly, however, the references to mathematics are substantiated as learned ones in poem 7:

Quaeris quot mihi basiationes
tuaе, Lesbia, sint satis superque.
Quam magnus numerus Libyssae arenae
lasarpiciferis iacet Cyrenis,
oraclum Iovis inter aestuosi
et Batti veteris sacrum sepulcrum,
aut quam sidera multa, cum tacet nox,
furtivos hominum vident amores,
tam te basia multa basiare
vesano satis et super Catullo est;
quae nec pernumerare curiosi
possint nec mala fascinare lingua.²⁵

Here, in line 5, the oracle of Jove is the temple of Ammon in the African oasis of Siwa, in line 4 *Cyrenis* is the region of birth of the admired poet Callimachus, and Battus, in line 6, is the mythical founder of Cyrene, whereas *magnus numerus* in line 3 and *pernumerare* in line 11 are terms characteristic of mathematics. If the number of kisses requested of Lesbia is compared to the enormous number of grains of sand on the African coast, it seems that the Latin poet truly had in mind the beginning of the Ψαμμίτης, where Archimedes says that he intends on working out the number of grains of sand "not only of those around Syracuse and in the rest of Sicily, but also of those in every region, both inhabited and uninhabited."

²⁴ "Give me a thousand kisses, then a hundred, then another thousand, then a second hundred, then yet another thousand, then a hundred. Then, when we have made up many thousands, we will confuse our counting, that we may not know the reckoning, nor any malicious person blight them with an evil eye, when he knows that our kisses are so many" (*Poems*, in *Catullus, Tibullus, Pervigilium Veneris*, 2nd edition, trans. Francis Warre Cornish, rev. George Patrick Goold [Cambridge, MA: Harvard University Press, 1988 (1962)]).

²⁵ "You ask how many kissings of you, Lesbia, are enough for me and more than enough. As great as the number of Libyan sand that lies on sulphium-bearing Cyrenaica, between the oracle of sultry Jove and the sacred tomb of old Battus; or as many as are the stars, when night is silent, that see the stolen loves of men, – to kiss you with so many kisses, Lesbia, is enough and more than enough for your mad Catullus; kisses, which neither curious eyes shall count up nor an evil tongue bewitch" (*Poems*, cit. note 24).

The reference to the grains of sand in Africa returns in epithalamium 61 (206-210), where the quantity of love games that he wishes on the newlyweds is a palpable poetic exaggeration, even if we consider the teachings of the *Kamasutra*:

Ille pulveris Africi
siderumque micantium
subducat numerum prius,
qui vestri numerare vult
multa milia ludi.²⁶

In the generation that followed Catullus, Virgil found delight in numbers as well. He – according to the *vita Donatiana* (§ 15):

inter cetera studia medicinae quoque ac maxime mathematicae operam dedit.²⁷

Lines 73-75 of *Eclogue VIII* should be read from this perspective, where the conclusive affirmation *deus ... gaudet* indicates how certain numbers please divinities:

Terna tibi haec primum triplici diversa colore
licia circumdo, terque haec altaria circum
effigiem duco; numero deus impare gaudet.²⁸

In the *Georgics* (2, 103-106) as well, Virgil refers to the numbers of grains of sand in Africa in relation to the qualities of wine known at the time (today they would be even more!):

Sed neque quam multae species nec nomina quae sint
est numerus, neque enim numero comprehendere refert;
quem qui scire velit, Libyci velit aequoris idem
discere quam multae Zephyro trurbentur harenae.²⁹

²⁶ "Let him first count up the number of the dust of Africa and of the glittering stars, who would number the many thousands of your joys." (*Poems*, cit. note 24).

²⁷ "Among other studies he also interested himself in medicine and especially mathematics."

²⁸ "Three threads here I first tie round you, marked with three different hues, and three times this altar I draw your image. In an uneven number heaven delights." (*Eclogues - Georgics - Aeneid I-VI*, trans. by Henry Rushton Fairclough, revised by Goid [Cambridge MA: Harvard University Press, 1999 (1935)]).

²⁹ "But for the many kinds, or the names they bear, there is no numbering – nor, indeed, is the numbering worth the pains. He who would have knowledge of this would likewise want to learn how many grains of sand on the Libyan plain are stirred by the West Wind" (*Eclogues*, cit. note 28).

Proceeding chronologically, it is known that in the first part of the Tiberian age the poet Manilius was interested in astronomy and astrology, both subjects a combination of mythology and science, but it is striking that in his poetry there are even a few specific references to geometry, as in lines 545-547 of the first book of the *Astronomica*, where he gives a measurement (albeit a generic one) of the relationship between the circumference and diameter of a circle, the famous π that Archimedes had measured with precision in his treatise on the *Measurement of the Circle*:

Quaecumque inciditur orbis
per medium, pars efficitur tum tertia gyri
exiguo dirimens solidam discrimine summam.³⁰

Continuing almost three centuries later, a cryptic attention for numbers and geometric figures shows up Publilius Optatianus Porfyrius, a sort of Apollinaire of the age of Constantine. Take as an example his poem VI, an hermetic composition (Fig. 5), the geometrical system of which is explained in §§ 7-9 of the *scholium* that accompanies it:

[7] Idem versus et per amfractus varios sursum ac deorsum discurrunt. [8] In hac eadem pagina quattuor quadrata in angulis sunt senarum litterarum, duo trigona, et duo octogona maiora dimidiata, et unum hexagonum in medio, et scalena per angulos quattuor, omnia pari numero litterarum et pari ordine crescentia vel decrecentia; [9] id est omnia similia, trigona trigonis, octogona octogonis, quadrata quadratis, scalena scelenis.³¹

To conclude this list, and to be certain of the real interest in geometry and arithmetic on the part of the best Latin intellectuals, a final example may be found in the small lesson of combinatory geometry from the second half of the 4th century offered by Decimus Magnus Ausonius in the introduction to his *Cento Nuptialis* (XVIII, lines 37-48), where he explains with the help of Greek terms the many possible combinations of the 14 tiles into which a square had been proportionally divided – evidently Archimedes's *stomachion*. Its single parts had the form of isosceles and scalene triangles,

³⁰ "Where a circle is cut through the middle, the lined formed amounts to a third of the circumference, a line so dividing the whole as to leave a small difference" (*Astronomica*, trans. Goid [Cambridge, MA: Harvard University Press, 1977]).

³¹ "[7] The same verses also run along various broken lines upwards and downwards. [8] In this page alone in the corners there are four squares of six letters each, two triangles, and two larger halved octagons, and an hexagon in the middle and four trapezoids pointing toward the corners, all with the same number of letters and with the same increasing and decreasing functions; [9] that is, everything is equal, the triangles to the triangles, octagons to octagons, squares to squares, trapezoids to trapezoids."

VI

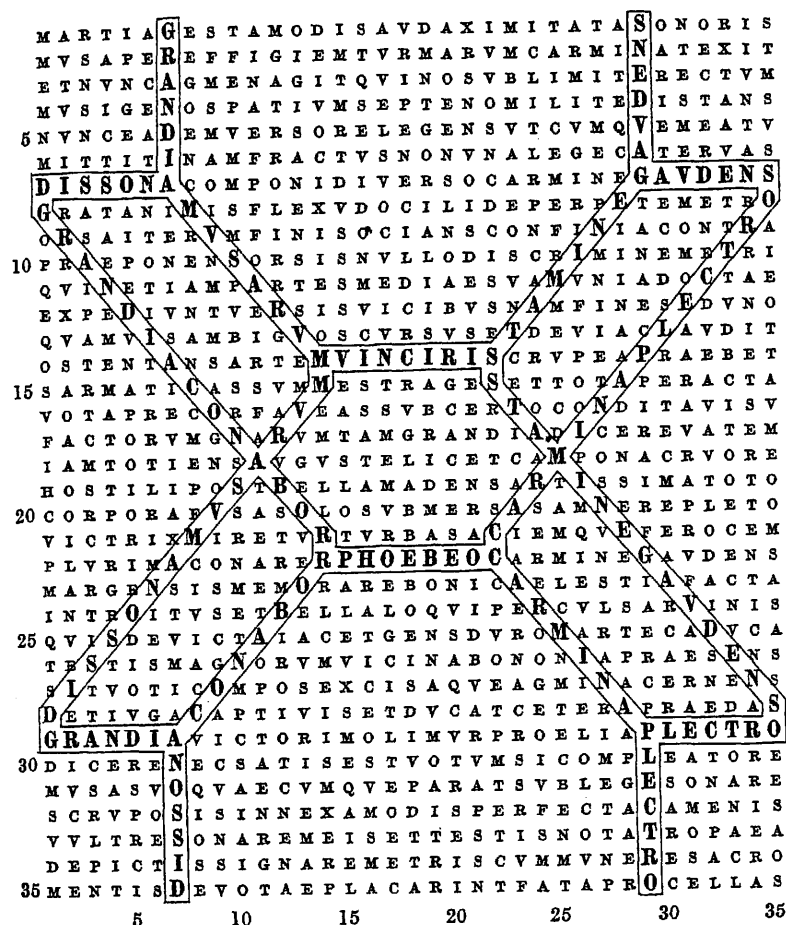


Fig. 5

quadrangles, and other polygons, from which one could compose various figures with enough fantasy and perhaps by repeating a few pieces:

simile ut dicas ludicro, quod Graeci ὁστομάχιον vocavere. Ossicula ea sunt: ad summam quattuordecim figuras geometricas habent. Sunt enim aequaliter triquetra vel extentis lineis vel <eiusdem> frontis, <vel rectis> angulis vel obliquis: ἰσοσκελῆ ipsi vel ἰσοπλευρά vocant, ὀρθογώνια quoque et σκαληνὰ. Harum verticula-

rum variis coagmentis simulantur species mille formarum: helephantus belua aut aper bestia, anser volans et mirmillo in armis, subsidens venator et latrans canis, quin et turris et cantharus et alia huiusmodi innumerabilium figurarum, quae alius alio scientius variegant. Sed peritorum concinnatio miraculum est, imperitorum iunctura ridiculum.³²

No less interesting from our point of view is Ausonius' short poem *Griphus ternarii numeri* ("The enigma of the number three"), dedicated to describing all of the functions of the number *three*. The first six lines read:

Ter bibe vel totiens ternos: sic mystica lex est,
vel tria potanti vel ter tria multiplicanti,
inparibus novies ternis contexere coebum.
Iuris idem tribus est, quod ter tribus, omnia in istis:
forma hominis coepti plenique exactio partus
quique novem novies fati tenet ultima finis.³³

Let us continue with lines 50-60:

Per trinas species trigonorum regula currit,
aequilatus vel crure pari vel in omnibus inpar.
Tris coit in partes numerus perfectus, ut idem
congrege ter terno per ter tria dissolvatur.
Tris primis par, impar habet mediumque: sed ipse
ut tris, sic quinque et septem quoque, dividit unus,
et numero in toto positus sub acumine centri
distinguit solidos coebo pergente trientes,
aequipares dirimens partes ex inpare terno.
Et paribus triplex medium, cum quattuor et sex
bisque quaternorum secernitur omphalos idem.³⁴

³² "So that you may say it is like the puzzle which the Greeks call *stomachia*. There you have little pieces of bone, fourteen in number and representing geometrical figures. For some are equilateral triangles, some with sides of various lengths, some symmetrical, some with right angles, some with oblique: the same people call them isosceles or equal-sided triangles, and also right-angled and scalene. By fitting these pieces together in various ways, pictures of countless objects are produced: a monstrous elephant, a brutal boar, a goose in flight, and a gladiator in armor, a huntsman crouching down, and a dog barking – even a tower and a tankard and numberless other things of this sort, whose variety depends upon the skill of the player. But while the harmonious arrangement of the skilful is marvelous, the jumble made by the unskilled is grotesque." (*Ausonius*, trans. by Hugh G. Evelyn White [Cambridge, MA: Harvard University Press, 1919]).

³³ "Thrice drink or else as many times three cups: thus stands the mystic law – whether three draughts thou drinkest or three thrice multiplieth, with nine times three uneven form the cube. The same virtue is in three as in thrice three: all things are in terms of these; the first forming of the human shape, the due completion of the act of birth, and the limit which marks man's extreme span, years nine times nine." (*Ibid.*)

³⁴ "Over three kinds ranges the figure of the triangle, equilateral, isosceles, and scalene. Three

Finally, let us observe the last three lines of the short poem (88-90), where, in order to exalt the number *three*, the poet from Bordeaux does not hesitate to hint at the Christian mystery of the Trinity:

Ter bibe. Tris numerus super omnia, tris deus unus.
Hic quoque ne ludus numero transcurrat inerti,
ter decies temos habeat deciesque novenos.³⁵

In conclusion, we have good reason to believe that in antiquity the best Latin intellectuals did have an interest in arguments and problems of arithmetic and geometry that had been introduced into scientific knowledge by scholars writing in Greek.

IN NUNC USQUE XXIV

parts combined make up the perfect number, in such wise that if a group thrice three be formed, by three times three the same may be resolved. Three is the first number which has an odd, an even, and a medial unit; but, as the unit itself divides three, so does it five and seven; and when it is placed under the central point of the full number, it parts in two a series of thirds forming a continuous cube, by separating even and equal groups from the uneven threes; and even numbers thrice find a center, when the same midmost point of four, six, and twice four, is bracketed." (*Ibid.*).

³⁵ "Thrice drink! The number three is above all, Three Persons and One God! And that this conceit may not run its course without significance of number, let it have verses thrice ten times three, or nine times ten!" (*Ibid.*).