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The Mathematical Passage in the Epinomis ¹⁾

A. R. LACEY

THE general theme of the *Epinomis* is set by its opening question, how can mortal man be wise? The answer is that wisdom and happiness are only for the few, and these will need an intensive intellectual training, whose objects are to be the nature of the universe, its elements and inhabitants, and the basic laws by which it is held together and governed. Nowhere are these laws better exhibited than in the complex but regular and eternal movements of the celestial phenomena, and so, absurd though this may seem to the layman, the path to wisdom lies through astronomy, studied mathematically and not merely empirically (990a2-b4). At this point we have a short but notoriously difficult passage (990c5-1b4) which starts by saying that the study of astronomy involves that of mathematics, and eventually leads up to the panegyric, which ends the dialogue, of the structural unity of the universe, which must be studied and appreciated by those who are to form the Nocturnal Council of *Laws* 12.

Various writers, notably Stenzel, Taylor, des Places, Toeplitz, and Van der Waerden, have tried to interpret this passage, but it seems to me that despite many ingenious and fruitful ideas on individual sentences none of them has succeeded in giving a satisfactory interpretation that takes account of both philosophical probability and the exact Greek text of the entire passage. In this paper I shall try to take the problem one step further by a detailed examination of the grammar and syntax of each of the five sentences into which I have divided the passage, together with an attempt (tentative enough, to be sure) to link them together into a coherent philosophical whole. It need hardly be added that any success I may have will be built on the foundations laid down by previous writers, especially those mentioned above.

I have not attempted to say anything on the vexed question of the authorship of the *Epinomis*. I have in fact assumed that the author is Plato, which the balance of evidence seems to support (see Hans Raeder: *Platons Epinomis*. Raeder disposes effectively of the view that Philip of Opus wrote it as a deliberate forgery to be passed off under Plato's name; he says regrettably little about the mathematical passage, to which he gives only two and a half pages, pp. 56-8). This question becomes most pressing in connexion with conflicting doctrines, such as the ether and the demonology; with our present passage the chief task is that of giving

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it a coherent meaning at all, not that of reconciling it with conflicting passages. It does not appear to clash with any other Platonic passages, and its very brevity and obscurity would lead us to conclude that, even if the *Epinomis* is a forgery, its author had no interest in proclaiming a new doctrine in this sphere, and so would keep as near the Platonic as possible in order to make his work the more plausible.² In fact, with the exception of *Laws* 894a and 817e-22d, our passage seems to stand in relative isolation among the dialogues. I cannot see that it has any particular connexion with the Nuptial Number (*Rep* 546ab) or the Tyrant's Number (*Rep* 587). These passages, which occur in a political context, have been recently reinterpreted by R. S. Brumbaugh in *Plato's Mathematical Imagination*. Brumbaugh does not treat the *Epinomis* passage in detail, though he connects it with *Laws* 894a and *de Anima* 404a and discusses the general significance of the three mathematical means for Plato (chapter 4).

For purposes of exposition I shall divide the passage into five sentences, and devote most of my attention to the last three, though one must bear in mind all through that the sentences form a single passage with (presumably) a consistent development from beginning to end.

Epinomis 990c 5-1 b 4

1. διὸ μαθημάτων δέον ἂν εἶη· τὸ δὲ μέγιστόν τε καὶ πρῶτον καὶ ἀριθμῶν αὐτῶν ἄλλ' οὐ σώματα ἔχόντων, ἀλλὰ ὅλης τῆς τοῦ περιττοῦ τε καὶ ἀρτίου γενέσεώς τε καὶ δυνάμεως, ὅσῃν παρέχεται πρὸς τὴν τῶν ὄντων φύσιν.
2. ταῦτα δὲ μαθόντι τούτοις ἐφεξῆς ἐστὶν ὁ καλοῦσι μὲν σφόδρα γελοῖον ὄνομα γεωμετρίαν, τῶν οὐκ ὄντων δὲ ὁμοίων ἀλλήλοις φύσει ἀριθμῶν ὁμοιώσεις πρὸς τὴν τῶν ἐπιπέδων μοῖραν γεγонуῖά ἐστιν διαφανής· ὁ δὲ θαῦμα οὐκ ἀνθρώπινον ἀλλὰ γεγονὸς θεῖον φανερόν ἂν γίγνοιτο τῷ δυναμένῳ συννοεῖν. μετὰ δὲ ταύτην τοὺς τρεῖς ἡὑξημένους καὶ τῇ στερεᾷ φύσει ὁμοίους· τοὺς δὲ ἀνομοίους αὖ γεγονότας ἑτέρα τέχνη ὁμοιοῖ, ταύτη ἣν δὲ στερεομετρίαν ἐκάλεσαν οἱ προστυχεῖς αὕτη γεγονότες·
3. ὁ δὲ θεῖόν τ' ἐστὶν καὶ θαυμαστόν τοῖς ἐγκαθορῶσί τε καὶ διανοουμένοις ὥς περὶ τὸ διπλάσιον ἀεὶ στρεφομένης τῆς δυνάμεως καὶ τῆς ἐξ ἐναντίας ταύτης καθ' ἑκάστην ἀναλογίαν εἶδος καὶ γένος ἀποτυποῦται πᾶσα ἡ φύσις.
4. ἡ μὲν δὲ πρώτη τοῦ διπλασίου κατ' ἀριθμὸν ἐν πρὸς δύο κατὰ λόγον φερομένη, διπλάσιον δὲ ἡ κατὰ δύναμιν οὔσα· ἡ δ' εἰς τὸ στερεόν τε καὶ ἄπτόν πάλιν αὖ διπλάσιον. ἀφ' ἑνὸς εἰς ὀκτώ διαπορευθεῖσα.

5. ἡ δὲ διπλασίου μὲν εἰς μέσον, ἴσως δὲ τοῦ ἐλάττονος πλεον ἔλαττον τε τοῦ μείζονος, τὸ δ' ἕτερον τῷ αὐτῷ μέρει τῶν ἄκρων αὐτῶν ὑπερέχον τε καὶ ὑπερεχόμενον - ἐν μέσῳ δὲ τοῦ ἕξ πρὸς τὰ δώδεκα συνέβη τό τε ἡμιόλιον καὶ ἐπίτριτον - τούτων αὐτῶν ἐν τῷ μέσῳ ἐπ' ἀμφοτέρα στρεφομένη τοῖς ἀνθρώποις σύμφωνον χρεῖαν καὶ σύμμετρον ἀπενείματο παιδιᾶς ῥυθμοῦ τε καὶ ἀρμονίας χάριν, εὐδαίμονι χορείᾳ Μουσῶν δεδομένη.

1. And so there will be a need of studies. Now the most important and primary study is that of numbers themselves, not of embodied numbers but of the whole coming-into-being and potentiality of increase [by squaring and cubing] of the odd and even, and of the effect which this has towards the nature of things.
2. The man who has learnt this will next study what is called by the ridiculous name of earth-measurement, but has manifestly become the assimilation to each other, by means of their share in planes, of numbers which are not naturally similar to each other, a wonder which would clearly be of divine, not human, origin, to one who could understand it. After this he will study numbers that have been increased to the third degree, and resemble solids; and those which have become dissimilar again one assimilates by another art, this art which those who first came upon it called solid-measurement indeed—
3. which is also a divine and wonderful thing to those who perceive and reflect on how, while the power and the term which consists in the correlative root to this ever revolve about the double in each proportion, the whole of Nature is stamped out, genus and species.
4. The first term is of the double, being taken numerically from 1 to 2 in ratio. The term which is a power is also a double, and so again is that which goes to the solid and tangible, having gone right through from 1 to 8.
5. Now the term which goes into the middle of a double, but is equally greater than the less and less than the greater, while the other middle position exceeds and is exceeded by the same fraction of the extremes themselves (the ratios $1\frac{1}{2}$ and $1\frac{1}{3}$ can be found in the middle of the ratio 6 : 12), in the middle of these, turning towards each of them, this term provides for men a harmonious and balanced practice for the sake of disportation, rhythm and concord, being given to the blessed choir of the Muses.

1. 990c5-d1 διὸ... τῶν ὄντων φύσιν.

Des Places (*Revue des Etudes Grecques*, 1935, pp. 540-50; see p. 547) follows Bekker and Z in omitting καὶ before ἀριθμῶν, and Toeplitz (*Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik, Abteilung B¹ Vol. 2, p. 334ff.* ("Die mathematische Epinomisstelle"))³ favours this course. In this case ἀριθμῶν will be predicate to τὸ δὲ μέγιστόν τε καὶ πρῶτον (μάθημα). Otherwise we must take τὸ δὲ μέγιστόν τε καὶ πρῶτον as adverbial, and ἀριθμῶν as parallel to μαθημάτων, and depending on δέον. The excision seems favoured by ταῦτα δε μαθόντι in the next sentence.

It has been suggested to me that the first ἀλλ' implies there are two kinds of ἀριθμοὶ αὐτοί, the σώματα⁴ ἔχοντες and the σώματα οὐκ ἔχοντες, the former being the Forms (which "have bodies" in the sense that they appear through the medium of physical objects), and the latter the Ideal Numbers, which are superior to the Forms. It seems easier to me to follow the usual view, which takes "bodiless numbers" to mean simply numbers, considered in the abstract, and "embodied numbers" to mean sensible numbered groups (see G. Martin, *Zeitschrift für philosophische Forschung*, 1953, p. 191ff, and *Tht* 196a; cf. also des Places, *loc. cit.*), the first ἀλλ' being taken in connexion with the second; the study is not to be of sensible numbered groups, but of "the whole coming-into-being and potentiality of increase of the odd and even".

It seems best to understand δύναμιν after ὄσσην, but in a somewhat different sense from δυνάμεως, because it would be grammatically awkward to make δυνάμεως the antecedent of ὄσσην without making γενέσεως so too, which would not make sense; but we must assume that it was the proximity of δυνάμεως that made Plato omit δύναμιν after ὄσσην. γενέσεως is not easy to interpret. Aristotle tells us of the generating of numbers from the One and the Indefinite Dyad, and perhaps something of the sort is referred to here. What seems certain from the rest of the passage is that Plato is thinking mainly of the δύναμις (in whichever sense) of the numbers when already generated, rather than of their generation.

2. 990d1-e1 ταῦτα δὲ μαθόντι... γεγονότες.

This must be treated as a single unit, because there are disagreements over its punctuation. O. Becker (*QS* 4 p. 191), in an article which brings in part of our passage, suppresses the fullstop after συννοεῖν, and the δὲ between μετὰ and ταύτην, and inserts an ἔστιν after θαῦμα

(deleting the ἐστὶν before διαφανής in the previous clause). He takes δὲ δὴ θαῦμα ⁵ to have a forward reference, his general interpretation being that four studies are in question, arithmetic, geometry, stereometry and harmony, of which the first two have been known for a long time, while the last two are new and therefore of special significance. While this view no doubt has its advantages it would be better to translate the received text if we can, and to give δὲ δὴ θαῦμα a forward reference destroys the grammatical connexion of the sentence with the one before. It would also seem more natural for this interpretation that the next clause should start τοὺς γὰρ ἀνομοίους rather than τοὺς δὲ ἀνομοίους. As a second and less satisfactory course Becker suggests putting a stop after γεωμετρίαν and enclosing μετὰ δὲ ταύτην in brackets or commas; this would get rid of the δὲ δὴ θαῦμα difficulty, and avoid excising δὲ, but the bracketing is awkward, and it is hard to see why the geometrical assimilation of numbers should become any clearer to a person who knows stereometry. Keeping to Burnet's text we shall have to supply some such word as μαθητέον (Müller) with τοὺς τρις ηὐξημένους. ⁶

The more well-known textual difficulties towards the end of this passage do not seem to disturb the meaning so much. I have translated Burnet's text, and Toeplitz does so too. Des Places, who dislikes ταύτην as referring to ἑτέρα τέχνη, and the absence of a subject for ὁμοιοῖ, reads with Z τοὺς δὲ ἀνομοίους αὖτε γεγονότας ἑτέρα τέχνη ὁμοιοῖ ὁμοία ταύτη, ἣν δὲ στερεομετρίαν... ταύτη referring to geometry. Stallbaum and Müller follow A and O (against L² and Theo of Smyrna) in reading γεωμετρίαν for στερεομετρίαν, and it has been suggested to me that this is supported by δὴ; this reading will cause no trouble so long as we keep ὁμοία (or ὁμοία, according as we read ἑτέρα τέχνη or ἑτέρα τέχνη) instead of, or as well as, ὁμοιοῖ.

The general doctrine of the passage is wellknown. Two numbers are ὅμοιοι when each of them can be represented by a rectangle (if it is split into two integral factors) or a rectangular solid (if it is split into three integral factors) whose dimensions correspond to these factors, and these rectangles or rectangular solids are geometrically similar to each other (*Tht* 148a, H. Raeder: *Platons Epinomis*, pp. 56-7, des Places, p. 548, Toeplitz: *QS* 2 p. 334ff.). (Another sense of ὅμοιος, in which a number is ὅμοιος if it has symmetrical factors, i.e. if it is a perfect square or cube, is mentioned by Becker (*QS* 4 p. 185ff.), who supposes it to underlie *Rep* 546b.) But obviously only a very few numbers are ὅμοιοι in this (the first) sense (ὄντες ὅμοιοι ἀλλήλοις φύσει, as Plato puts it), and the purpose of geometry and stereometry is to assimilate

the others. Toeplitz points out that it is not at once obvious that a number which can be factorised in different ways (e.g. $30 = 6 \times 5$ or 3×10 or 2×15) will be similar to the same numbers in each case, and, drawing attention to Euclid's proof that two numbers are similar when and only when they are related to each other as two squares, he thinks the "assimilation" of two numbers means the constructing of squares (or cubes) which bear the same ratio to each other in area (or volume) as the two numbers do, and that 1 and 2 are assimilated by squares in this sense in the Meno, while the Delian problem is to assimilate them by cubes. In general this seems to be undoubtedly the right interpretation, but it is not necessary to limit it to squares. The general principle of finding the geometric mean between two lines by laying them end to end as the diameter of a circle and dropping the perpendicular from the circle to their meeting-point (see Toeplitz: *QS 1 p. 3ff.* (§ 1)) can be used equally well to enable one to construct a rectangle which will be similar to a given rectangle and which will stand to it in area as two given lines stand to each other.⁷ Thus if we have the number 6 expressed as a rectangle whose sides are 2 and 3, we can take the number 13, which cannot be expressed as a similar rectangle so long as it is factorised integrally, and express it as a similar rectangle, whose dimensions, however, are surds. Stereometry will do the same thing for numbers which when viewed as solids have become dissimilar again. Why this word "again" (αὔ)? Presumably what is meant is that if after constructing our two rectangles of areas (say) 6 and 13 we try and turn them into volumes of 6 and 13, we can in the first instance only do so by giving to each of them unit height, which will of course make them dissimilar again, because the larger one will be too flat. To remedy this we must raise its height and lessen its other two dimensions, while keeping them in the same mutual proportion. This can be done by stereometry.⁸ (That it *can* be done, whether or not this solution "counts", is shown by Archytas' solution of the Delian problem (T. L. Heath: *Greek Mathematics, Vol. 1*), so that it seems better to take ὁμοιοῖ as indicative and not, with Becker, as optative ("möge er durch eine andere Kunst ähnlich gestalten"). A further minor difficulty in translation is to know whether to take ὁμοίους after τῇ στερεῶ φύσει as technical or non-technical; the frequent use of the word in the context, and the presumably technical use of ἀνομοίους in the next clause, might suggest that it was technical here too; but the fact that the numbers mainly to be studied are those which are not similar until they are made so, coupled with the absence of ἀλλήλους and the awkwardness of the dative, τῇ στερεῶ φύσει, perhaps makes it best

to take it as non-technical. With the technical meaning the phrase would mean: “study numbers with respect to their similarity (sc. or lack of it) in the third dimension”).

3. 990e1-4 δ δὲ θεῖον... πᾶσα ἡ φύσις.

It is not easy to know whether to give δ δὲ θεῖον a forward or a backward reference. Becker gives it a forward one, as he does to δ δὴ θαῦμα, and so does Toeplitz. This makes the sentence rather awkward grammatically, since the ὥς clause has to serve as the antecedent of δ, although one would naturally take it with τοῖς ἐγκαθορῶσί τε καὶ διανοουμένοις, and I have preferred to give it a backward reference, as this helps to unify the passage, and relates the study of stereometry to what is coming.⁹

The next difficulty concerns δυνάμεως. Reference to Liddell and Scott, and Ast, shows that the mathematical meaning of this is normally “root”¹⁰ or “power”, though Philo uses it for “product”; it seems to mean “that which is potentially something” or “that which something potentially is” (cf. Becker: *QS* 4 p. 185ff., who quotes Proclus: *in Remp.* 51.9ff. in interpreting δυνάμενα and δυναστευόμενα of *Rep* 546b). The very fact that it covered both these meanings suggests that the use of it can hardly have been very strict¹¹, and it would seem difficult if not impossible to take it strictly in either of these senses in our present passage. We only seem to have one instance of a square ($4 = 2^2$) and one of a cube ($8 = 2^3$), but we have mention of the number one (ἐν πρὸς δύο, 991a2), and of the step from 4 to 8 (or else from 1 to 8; ἡ δ’ εἰς τὸ στερεόν, 991a3). But the geometrical use of δύναμις in connexion with the process of squaring seems to depend on taking a line as being potentially a square, and as becoming a square when drawn out into the second dimension (cf. *Leg* 894a), and it seems to be part of this view that in just the same sense the point is potentially a line, and the plane potentially a solid; otherwise the symmetry which gives the view its attractiveness disappears. (Cf. Stenzel: *ZG* 94¹²: “Das, Können’ besteht gerade in dem Produzieren der nächsten Dimension”. Stenzel refers to *Tht* 148b). It seems reasonable enough to add that a point is potentially (though perhaps only indirectly so) a solid, and so with the other cases. This view would enable us to say that any one of the numbers 1, 2, 4, 8 is a δύναμις with respect to any of the others, though primarily only with respect to the terms standing next to it in this series.

The natural interpretation of τῆς ἐξ ἐναντίας ταύτης will on this view be the number with respect to which the δύναμις is a δύναμις, or the correlative of the δύναμις. Des Places seems to incline tentatively to this

view ("Ce pourrait être la racine", p. 549), and Müller takes the tempting step of deleting ἐξ. (The rather radical alteration of Theiler (*Gnomon* 1931 p. 345n3), who reads τὸ ἐξ ἐναντίας for τῆς ἐξ ἐναντίας, seems to be without authority, and it will be better to interpret a safer reading if we can.) Van der Waerden (*Hermes*, 1943, pp. 185-7) translates: "die Kraft und die aus der Gegensatzung zu ihr... resultierende Kraft", which he does not comment on in detail, but which if taken literally would suggest a noun ἐναντία, like ἐναντιότης, which does not seem to exist. If we are to keep ἐξ the only alternatives seem to be to supply some feminine noun (like χώρα) meaning "region" or "direction", or else to understand δυνάμεως again after ἐναντίας and to take ἐξ in a vague sort of way as meaning "formed out of" or "consisting in" ("that δύναμις which consists in (or is) the opposite δύναμις to this"). Taking τῆς δυνάμεως and τῆς ἐξ ἐναντίας as correlatives in this way makes it unnecessary to consider which is root and which power (or which is lower term and which higher term).¹³

Toeplitz takes δύναμις in quite a different sense, as "Wirkungsweise (Art zu functionieren)", and this is followed by Van der Waerden, who takes the δύναμις to be the power (in the ordinary, non-mathematical sense) of the double in creating the διπλάσιον series, and the converse to it as the power of inserting means; in other words both these writers take δύναμις in the non-mathematical sense.¹⁴ In interpreting διπλάσιον Van der Waerden recalls the Euclidean use of διπλασίῳ λόγος to signify the squaring of a ratio (*Mathematische Annalen*, 118, (1942), p. 287; *Euclid* 8.8, 18), and says that sentence 4 of our passage has to do with squaring, and sentence 5 with the converse process of inserting means, and in particular the arithmetic and harmonic means, the geometric mean underlying sentence 4 (*Hermes*, 1943, p. 186, n. 1). But there are difficulties in this. Euclid uses διπλασίῳ to signify the square of a ratio, but he uses τριπλασίῳ to signify the cube (*Euclid* 8.9, 19), and there is no reference to a τριπλάσιον here, though there is a reference to a process "from 1 to 8". The theory seems to be that δύναμις refers to the multiplication of ratios, of which the particular case where they are multiplied by themselves (διπλάσιον) is being used here. In this case we should expect the converse process to be the division of ratios; but this is not altogether satisfactory as a term for mean-building, for though the arithmetic mean is obtained by dividing the sum of the extremes by two, and the harmonic mean by dividing the product of the extremes by the arithmetic mean, the geometric mean is not obtained by dividing at all, but by taking the square root of the product of the extremes, and taking

a square root does not seem to be a special case of division in the sense that squaring is a special case of multiplication. In any case is it not very vague to talk about the faculty of multiplication and the faculty of division “revolving about the double” when what is meant is that the particular multiplication in question is multiplication by itself, or by two (according to which view one takes)? (It is true that in the first sentence of our passage δυνάμεως is used in a sense related to, but not identical with, the technical sense. This merely seems to support Souilhé’s view of the vagueness of the term.)

ἀναλογία means primarily mathematical proportion (*Tim* 31c, 32c). It is also used of the three progressions, and of proportion generally and analogy, and later was used for relation, correspondence, or resemblance (Liddell and Scott; cf. also Stenzel: *QS* 1 p. 34ff.). It seems to be generally agreed that in our present passage καθ’ ἐκάστην ἀναλογίαν refers to the three types of progression, arithmetic, harmonic and geometric, though it is worth noting that it probably could be used to refer to the three instances of the διπλάσιον ratio mentioned in sentence 4, in which case the subject of ἡ in sentence 4 (and presumably therefore in sentence 5) would be ἀναλογία rather than (as I shall suggest) δύνამεις – though in this case we should perhaps rather expect καθ’ ἑκάστον λόγον instead of καθ’ ἐκάστην ἀναλογίαν. What is not so generally agreed is whether καθ’ ἐκάστην ἀναλογίαν is to be taken with τῆς ἐξ ἐναντίας ταύτης (Toeplitz, Van der Waerden), or with στρεφομένης (des Places), or with ἀποτυποῦται πᾶσα ἡ φύσις (Müller, Boulliau (apud Stallbaum)).

ἀποτυποῦται is taken as passive by Boulliau and Toeplitz (though Toeplitz thinks it might have an active sense, with εἶδος καὶ γένος as object, the sense being the same), and as middle by Van der Waerden, des Places, Müller, Taylor (apud Harward), and Stenzel. Stenzel (*ZG* 100) argues that Plato uses the term in the middle on the other occasions when he uses it (*Leg* 681b, *Tht* 191d, *Tim* 39e), and that the last of these is especially significant. But because Plato uses a verb in the middle on three occasions he need not necessarily be using it in the middle on the fourth, and a striking difference between *Tim* 39e and the present passage is that there the subject is the Demiurge, and here πᾶσα ἡ φύσις; it is one thing for the Demiurge to do the stamping, but another thing for Nature herself to do it. Also why πᾶσα ἡ φύσις? Surely there is no reason to emphasise that *the whole* of Nature does the stamping – but it would be much more relevant to emphasise that the whole of Nature was stamped, and not just one part of it (such as numbers, as Plato’s readers might have expected from the talk about δύνამεις and

διπλάσιον). If ἀποτυποῦται is then to be taken as passive we are left with the questions: Is the subject πᾶσα ἡ φύσις, or εἶδος καὶ γένος, πᾶσα ἡ φύσις being an explanatory apposition (or even spurious; Reuther, quoted by Stenzel: ZG 99n)? If the former, is εἶδος καὶ γένος an apposition, or an internal accusative ("stamped into species and genus")? Stenzel, as is well-known, lays considerable stress on this passage for his triple diaeresis theory, and des Places says one can go further, and that in each number there are two constitutive elements, the genus (the Great and Small) and the specific difference, or species (the One). Whether these or similar interpretations can be accepted cannot be decided on the basis of this passage alone, and all we can do at the moment is to point out that, whereas it is not necessary to take ἀποτυποῦται as middle in order to support Stenzel's view, it is also possible that the phrase εἶδος καὶ γένος is an innocuous apposition (possibly underling πᾶσα: "all Nature, both species and genera"). A minor point is that on the technical interpretation of Stenzel, and others, one might perhaps have expected γένος καὶ εἶδος instead of εἶδος καὶ γένος, the things which are being stamped being written in the order in which they are being stamped.

Toeplitz however takes ἀποτυποῦται to mean not "stamped out" but "mirrored" or "typified" (abgespiegelt), and considers that the double is used as an example in the various spheres that are mentioned in our passage, since the "assimilation" of 1 and 2 in the Meno gives an example of plane assimilation, and the Delian problem of doubling a cube gives an example of solid assimilation. But to this there are two objections. It is surely not in accordance with the usual meaning of ἀποτυποῦσθαι, which at its other Platonic occurrences (*Leg* 681b, *Th* 191d, *Tim* 39e) seems to mean "stamp out" or "impress" rather than "typify" or "give an example of". Secondly, in the final sentence of our passage, which, whatever its detailed meaning, obviously refers in some way to music and harmony, the double has a significance in its own right, and not merely as an instance of a wider set of ratios, for the octave, when one listens to it, at any rate seems to be a fundamental datum of harmony, and not merely something imposed on it for the convenience of theorists.

We still have not considered the meaning of στρεφομένης, and this brings us to the general interpretation of the whole sentence. What can it possibly mean to say that the root and the power, or the lower and the higher term in a series, "turn about the double"?¹⁵ One thing that suggests itself (especially in view of ἀεί. This need not have anything to do with infinite series, but certainly implies that whatever is happening happens more than once. It may be no more than a reinforcement of καθ'

ἐκάστην ἀναλογίαν, if this is taken with στρεφομένης) is that the situation is not just a static one, with two δυνάμεις and a διπλάσιον between them, but that there are either several δυνάμεις and διπλάσια, or anyway a δύναμις and its correlate and a διπλάσιον appearing in several positions. This can best be accounted for if we are considering a series. καθ' ἐκάστην (rather than ἐκατέραν) ἀναλογίαν suggests that not less than three types of proportion are being referred to. If we construct a scale where each term is half its successor (i.e. its successor is διπλάσιον), and then put in the three means, we can write down three scales as follows:

Arithmetic:

1 $1\frac{1}{2}$ 2 3 4 6 8

Geometric:

1 $\sqrt{2}$ 2 $2\sqrt{2}$ 4 $4\sqrt{2}$ 8

Harmonic:

1 $1\frac{1}{3}$ 2 $2\frac{2}{3}$ 4 $5\frac{1}{3}$ 8

Now it is obvious here that not only are the basic terms in each of these series (i.e. 1, 2, 4, 8,) in a διπλάσιον relation, but the mean terms in each series are so too (3 is twice $1\frac{1}{2}$, $2\sqrt{2}$ is twice 2, $2\frac{2}{3}$ is twice $1\frac{1}{3}$), and so if the δύναμις and its correlate can be treated as general terms for any terms in the basic series (1, 2, 4, 8) then it might be said that the δύναμις which occupies each position in this series in turn, "revolves round" the intervening means, which also form pairs in a διπλάσιον relation. The same could be said of course about the correlate to the δύναμις, and similarly the means can be regarded as the δύναμις and its correlate, revolving round the terms of the original διπλάσιον series. This is not a very exact account, and it would only apply to the series looked at generally, and not to the first step ($1\frac{1}{2}$, $\sqrt{2}$, and $1\frac{1}{3}$ are not doubles, though they are, in a looser sense, terms in the διπλάσιον relation, in that they are halves), but it does pay some attention to the meaning of the terms involved, and seems no more metaphorical than such a passage as *Leg 894a*. It will now be seen that it is best to take καθ' ἐκάστην ἀναλογίαν with στρεφομένης. To have taken it with the succeeding phrase would have been awkward because it seems most obviously to refer to something mathematical rather than to the physical or metaphysical world that is somehow being derived from or compared with the mathematical one, while to take it with τῆς ἐξ ἐναντίας ταύτης would suggest that the δύναμις and its correlate were the basic term (x) and the mean next following ($\frac{3x}{2}$, $\sqrt{2}x$, or $\frac{4x}{3}$), but though the pro-

portions ἡμιόλιον and ἐπίτριτον are shortly to be mentioned there seems no way in which the geometric mean ($\sqrt{2}x$) could be referred to without mentioning both its extremes (x and $2x$), and not merely the lower one (i.e. to mention the geometric mean one must mention three terms, the mean itself and its two extremes, but (in the case of the double) the other means can be named by reference to the lower term only – one-and-a-half, one-and-a-third; $\sqrt{2}$ is not one-and-an-anything). Also this interpretation would weaken the force of δύναμις (if my interpretation of that is right), since there is no particular sense in calling a mean the correlate of the lower extreme; and anyway the Greeks had a perfectly good word for “mean” (μέσον), but did not have a fixed terminology for distinguishing between roots and powers, or lower and higher terms.

4. 991a1-4 ἡ μὲν δὴ πρώτη... διαπορευθεῖσα.

The first difficulty here is to decide whether the antecedent of ἡ (which presumably has the same antecedent in its three occurrences here, and also, unless there is an extreme stylistic barbarity, in sentence 5) is δύναμις (Toeplitz, Van der Waerden) or ἀναλογία (Müller, Stallbaum, des Places, Taylor (apud Harward; cf. also Harward’s note ad loc.) Stenzel translates the subject of διαπορευθεῖσα as “Kraft” at ZG 99, but writes ἀναλογία for it at the top of p. 93). It presumably is not φύσις. If it is ἀναλογία, and the sentence is meant to carry on from the last one, it would seem that we should have to assume that πρώτη was answered by ἡ δὲ διπλασίου in sentence 5, in order to keep to our decision to refer ἐκάστην to the three types of proportion, for arithmetic and harmonic proportion are obviously mentioned only in sentence 5. This would involve saying the διπλάσιον δὲ... and ἡ δ’ εἰς τὸ στερεόν ... clauses were parenthetical – but it seems much more natural to take them as answering πρώτη, and this in turn would suggest it is these three clauses that are explaining ἐκάστην; but we have already seen that, though this is not perhaps impossible, we should really expect καθ’ ἑκάστον λόγον rather than καθ’ ἐκάστην ἀναλογίαν. Toeplitz and Van der Waerden, who take the antecedent to be δύναμις, both take δύναμις in the non-mathematical sense of a “Wirkung” (Toeplitz) or “Kraft der Verdopplung” (Van der Waerden) an interpretation whose difficulties I have already discussed.

On any interpretation this sentence is probably the most difficult to interpret in detail, but perhaps taking δύναμις and its converse to mean higher and lower term in a series (and in fact, in the present context,

just “term”)¹⁶ we can, despite the awkwardness of the phrase ἡ κατὰ δύναμιν οὕσα δύναμις, find an interpretation that is open to less objections than those so far considered. Both Toeplitz and Van der Waerden take τοῦ διπλασίου as forming part of the subject with ἡ; grammatically this is perfectly possible, but the appearance of διπλάσιον as predicate in the two following clauses suggests that it should be taken as predicate here too.¹⁷ Van der Waerden explains the general sense of the sentence as being that if one doubles a line one has a geometrical image of the numerical ratio 1 : 2, while if one doubles the side of a square one has the surface-ratio 1 : 4, and if one doubles a solid in each of its dimensions one has the ratio 1 : 8. This seems to be very near the truth, but how does it work out in detail, on the present interpretation of δύναμις?

The words φερομένη and διαπορευθεῖσα (and also στρεφομένης in sentence 3) suggest that this δύναμις is something which moves. Now we know that the notion of a point “moving” so as to generate a line, and then this line moving so as to generate a plane, was a common one among the Pythagoreans, and a similar conception applies at *Leg 894a*, where the initial ἀρχή remains the subject throughout the whole process. There are really only the static terms, which are considered successively in thought, but this thought is as it were hypostatised and considered as though it were itself a term moving among the others and coming to rest on each of them successively. In this case the occurrences of δύναμις or its equivalent in the text will refer primarily to the term in the διπλάσιον series which has been reached, but also secondarily to this hypostatised moving term which has reached it. The sentence then means that the first term is of the series of the double, being moved numerically from 1 to 2 in ratio, while the term which is a power¹⁸ ($4 = 2^2$) is a double, and so again is the term which takes us to the solid and tangible, having gone right through from 1 to 8. At first sight it would seem that what was being referred to was just this generation of the dimensions that is described at *Leg 894a*. But there are two reasons why such an interpretation would by itself be inadequate. Firstly, if καθ’ ἐκάστην ἀναλογίαν does in fact refer to the three types of proportion, of which sentence 5 seems to mention the arithmetic and harmonic, then we are left with the task of fitting in somewhere some reference to geometric proportion; and secondly, the earlier part of our passage dealt with geometry and stereometry, with high praise of their usefulness, and whether ὁ δὲ θεῖον is given a forward or a backward reference, it is desirable to find some connexion between the earlier part of our passage

and this later part. It is also necessary to suppose that in writing this sentence Plato was doing something more useful than merely writing down part of the two-times table. The words *κατὰ λόγον* suggest that the first change has something to do with a ratio, and *κατ' ἀριθμόν* suggests that this first clause refers to ordinary numbers (the *πρῶτον μάθημα* of sentence 1, leaving the other two clauses to refer to geometry and stereometry respectively, all of which leads us to Van der Waerden's conception of a line which is doubled, only perhaps it will work out better if we say there are two lines, one of unit length and one of length 2, which are to be compared. If we make squares on both these lines, these squares will bear to each other the ratio 4 : 1, which is double the previous ratio of 2 : 1, and similarly if we make cubes on these squares we get the ratio 8 : 1, which is again double 4 : 1 (this seems a slightly better way of putting it than Van der Waerden's, who says that if one doubles a cube in each dimension one gets the 8 : 1 ratio, because if the last clause meant this it would hardly be a step *εἰς*¹⁹ τὸ στερεόν, since the whole process would be *ἐν τῷ στερεῷ* the whole time; therefore it seems better to make the third clause refer to a process which arrives at, but does not start from, a cube – though of course the underlying point is the same). This last process one might strictly expect to be described as *ἀπὸ τετραγῶν εἰς ὀκτώ* rather than *ἀφ' ἑνός*, but here we must notice the difference in tense between *διαπορευθεῖσα* and the preceding *φερομένη*; our hypothetical moving term arrives at this last term “having gone (during the whole process, and not only this last stage of it) right through (*δια-*) from 1 to 8”²⁰; the phrase is a sort of summing-up of the whole process. The significant point is that a numerical ratio which is a double will become doubled again when squares are formed on the lines which are its terms, and yet again when cubes are formed; this implies that in order to find squares and cubes which are double other squares and cubes in area or volume one must seek the geometric mean or means, and it is just that that geometry and stereometry help us to do, geometry by the process referred to by Toeplitz: *QS* 1 p. 3ff (§ 1), and stereometry by Archytas' solution of the Delian problem. Hence geometry and stereometry are *θεῖόν τε καὶ θαυμαστόν* to those who understand this later part of the passage. In so far as these geometrical and stereometrical constructions enable one to construct squares and cubes in any ratio, and not merely in that of double, the selection of *ἐν πρὸς δύο* for the first clause is arbitrary, and to that extent Toeplitz's “Abspiegeln” theory is right. The selection of the double for this purpose no doubt depends partly on the symbolising of the dimensions

by 1, 2, 4, 8, and partly on the real part which the double plays when we come to harmony in the next sentence.

5. 991a5-b4 ἡ δὲ διπλασίου ... δεδομένη.

Comparison with *Tim* 36a 2-5 and Archytas: *Fr* 2 (*Diels*) seems to make it clear that the words from ἴσως to ὑπερεχόμενον are a description of the arithmetic and harmonic means (Toeplitz, *QS* 2 p. 286ff; Müller's rejection of τὸ δ' ἕτερον ... ὑπερεχόμενον as a gloss, repeating what has gone before, in which he follows Ast, seems quite unnecessary, but it has been suggested to me that ἴσως δὲ ... τοῦ μείζονος refers to the geometric rather than the arithmetic mean. This might seem to be supported by the definition of the three μέσαι in Archytas: *Fr* 2 (*Diels*), to which Reuther refers in interpreting the passage (see Stenzel: *ZG* 99). Archytas says that in arithmetic proportion the interval between the greater terms is less than that between the lesser terms, and vice-versa in harmonic proportion, while in geometric proportion the intervals are equal. But his basic definitions of arithmetic and geometric means are respectively as follows: ὃ πρῶτος δευτέρου ὑπερέχει, τούτῳ δεύτερος τρίτου ὑπερέχει and οἷος ὁ πρῶτος ποτὶ τὸν δεύτερον, καὶ ὁ δεύτερος ποτὶ τὸν τρίτον. Now it is possible that Plato was bent on describing the ways in which each of these means was equally removed from its extremes, the geometric mean being so in the sense described in Archytas' riders, and the harmonic in the sense described in our text; but on the other hand the use of ὑπερέχειν (which occurs in the definitions but not the riders of Archytas) in the description of the harmonic mean at least suggests that the same concept is to be understood in the description of the other mean, and this suggestion is strongly supported by the use of ἡμιόλιον in the following parenthesis, if that parenthesis is to have any relevance at all).

Plato now turns explicitly to dealing with the means, and with the arithmetic and harmonic ones because he has already implicitly discussed the geometric mean (cf. Van der Waerden, p. 186 n. 1). We have already seen that the series of the means, like the series of the extremes, is itself a διπλάσιον series, so that it would be possible to take διπλασίου with ἡ as part of the subject, as is usually done, but it seems much easier nevertheless to take διπλασίου with μέσον, since in the harmony that we are about to discuss the arithmetic and harmonic means are in fact between extremes which are of the form $x : 2x$. Moreover if we take διπλασίου with ἡ it is difficult to see what answers the μέν – one would expect a δὲ clause with some contrasted subject –, while if we take it

with μέσον then the μέν is answered quite naturally by the δὲ after ἴσως. Theiler (*Gnomon*, 1931, p. 345) sees the difficulty when he says that what we should expect is: ἡ δὲ διπλασίου [μέν] εἰς μέσον, ἴσως [δὲ] <τὸ μὲν (sc. τῶν μέσων)> τοῦ ἐλάττονος πλέον . . . On the present interpretation, however, there is no need to alter the text. Possibly my interpretation is also favoured by the lack of an article before διπλασίου, since it is easier to talk of a term going into the middle of “a” double rather than to talk of the power of “a” double. We should understand some word like βαίνουσα (cf. συνέβη below) after μέσον; the aorist, συνέβη, presumably signifies merely that a particular example is being given.

Van der Waerden’s interpretation of this sentence follows a suggestion by Tannery, which is that, after dividing the octave at the harmonic and arithmetic means to get the fourth and fifth (Mese and Paramese), one should then take a fifth down and a fourth up from the Mese, and divide these, which will give the Enharmonic Lichanos and the Tritē respectively, for all the ἐπιμόριον intervals (i.e. those where the numerator exceeds the denominator by 1) up to the tone (9 : 8) are found in the three scales (Enharmonic, Chromatic, Diatonic) mentioned by Archytas (apud Ptolemaeus), 5 : 4 and 8 : 7 between neighbouring terms in the Enharmonic and Diatonic scales respectively, 7 : 6 from the Mese to the Tritē common to all the scales, and 6 : 5 from the Mese to the Enharmonic Paranete. Van der Waerden’s article is on Pythagorean music in general, and his purpose in bringing in the *Epinomis* at all is just to give textual evidence for this otherwise textually unsupported hypothesis of Tannery. Our sentence means, he says, that “dieselbe mittelbildende Kraft, die bereits die Verhältnisse 3 : 2 und 4 : 3 hervor-gebracht hat, nun noch einmal von der Mitte aus auf diese beiden angewandt wird” (p. 187). Unfortunately Van der Waerden’s commentary, in the bare two pages which he gives to the whole of sentences 3, 4 and 5, including text and translation (pp. 185-7), is not at all clear in detail. It is hard to see how he can translate διπλασίου μὲν εἰς μέσον as “Was nun endlich die (Kraft) der Verdopplung anlangt, die sich nach der Mitte wendet”. If διπλασίου is to be taken with ἡ the phrase must surely for Van der Waerden refer to the squaring of ratios, which hardly seems to come into this process at all, and it would seem much easier for him to take διπλασίου with μέσον (in fact it was his interpretation that suggested to me that this should be done). It is also not clear how his interpretation quoted above fits with his interpretation of τούτων αὐτῶν ἐν τῷ μέσῳ: “von eben diesen (Verhältnissen) in der Mitte (stehend)”; he explains that the “Verhältnissen” are the ἡμιόλιον and ἐπίτριτον just

mentioned, and not, as Toeplitz thought, 8 and 9, and he presumably means that the power, which has just created the ἡμιόλιον and ἐπίτριτον, now stands in the middle of each of them and faces both ways. But the ἡμιόλιον and ἐπίτριτον in which the power stands are not the ἡμιόλιον and ἐπίτριτον just created, but a new fifth and fourth created by going a fifth down and a fourth up from the Mese. To divide in this fashion the ἡμιόλιον and ἐπίτριτον just created will not give any of Archytas' scales at all. The sense in which the ἐπιμόριον intervals up to 9 : 8 are called the fundamental intervals of Archytas' three scales also seems to be rather a vague sense; it is true that they all occur somewhere in the scales, and that each of the scales contains some of them, but a lot of other intervals occur too, and they do not seem to occur in a very systematic fashion. The three scales of Archytas which Van der Waerden quotes are built with tetrachords of the following intervals (going from Mese to Hypate²¹):

$$\begin{array}{lll} \text{E:} & 5 : 4, & 36 : 35, \quad 28 : 27 \\ \text{C:} & 32 : 27, & 243 : 224, \quad 28 : 27 \\ \text{D}_1: & 9 : 8, & 8 : 7, \quad 28 : 27 \end{array}$$

Later, after he has finished his discussion of the Epinomis, he mentions three other types of Diatonic scale:

$$\begin{array}{lll} \text{D}_2: & 9 : 8, & 9 : 8, \quad 256 : 243 \\ \text{Dm:} & 8 : 7, & 10 : 9, \quad 21 : 20 \\ \text{Ds:} & 10 : 9, & 9 : 8, \quad 16 : 15 \end{array}$$

Of Archytas' three scales²², as quoted by Van der Waerden, only E contains a harmonic progression (M, Pn, N), except of course for the progression (H, M, N) common to all the scales; both E and D₁ contain arithmetic progressions (L, M, Pn in E; Ph, L, M and M, T, Pn, and T, Pn, N in D₁), and if one takes the συνημμένον scale (i.e. the one where the tetrachords have a note in common, as opposed to the διεzeugμένον scale, where they are separated by a tone) from Pm to M of the octave above, whose H is the original N, D₁ contains a harmonic progression (T, Pn, Ph) and another arithmetic progression (Pn, Ph, L), while E contains another harmonic progression (Pn, N, M) and another arithmetic progression (Pn, L, M). The C scale however contains no arithmetic or harmonic progressions at all (except those common to all the scales), in either its διεzeugμένον or its συνημμένον form, though, like D₁ and the συνημμένον form of all the scales, it does contain geometric progressions. We can if we like consider D₁ as defined by the continued arithmetic progression (M, T, Pn, N), E as a variant which includes the pure major third, but keeps the T of D₁, and C as defined by the geometric progressions (M, Pm, Pn) and (L, Pm, N). It is true that we can get

EPn by dividing M-N harmonically at Pn, but this does not give us the other member of the tetrachord, T (and hence Ph), and does not itself account for the words ἐπ' ἀμφοτέρω. It is interesting, however, to note that the scale which comes nearest to being generated by a harmonic or arithmetic division of the ἡμιόλιον and ἐπίτριτον is Ds, which Van der Waerden says (p. 189) does not appear until Didymus, writing about 50 A.D. In this scale L is the harmonic mean between H and Pm, and T is the harmonic mean between M and N.

What then shall we say that the Epinomis sentence does mean? Van der Waerden takes τούτων αὐτῶν to refer to the proportions $1\frac{1}{2}$ and $1\frac{1}{3}$, because these have just been mentioned. But they have been mentioned in a parenthetical clause giving an example, and it seems just as permissible to refer τούτων αὐτῶν to τῶν ἄκρων αὐτῶν²³, also called τὸ ἔλαττον and τὸ μεῖζον, in which case τούτων αὐτῶν ἐν τῷ μέσῳ will perform the natural function of taking up again what was said at the beginning of this rather long and complex sentence, that the δύναμις goes into the middle of the double (i.e. the octave). If now we want to interpret the sentence as referring to the generation of a whole scale, would it not be better to take that scale as being D₂, which appears in the Timaeus, and which Van der Waerden himself says Plato favoured (p. 190), adding that this depends on a conservatism which is especially apparent in the Laws (to which the Epinomis is after all closely connected)? D₂ is constructed by going a fourth up from M and then a fifth down, and then another fourth up and fifth down (Van der Waerden, p. 189). This interpretation would fit better with ἀναλογία rather than δύναμις (in either sense) as the subject of the sentence, though it would not be impossible with δύναμις as subject. But need the sentence in fact refer to the whole scale at all? Need it refer to any more than the four basic notes, H, M, Pm, N? Granting that a theory which can bring in all the notes has much to be said for it, yet all we really need is something that will account for the final words of the sentence, from τοῖς ἀνθρώποις σύμφωνον χρεῖαν το Μουσῶν δεδομένην²⁴ – in fact the key words are σύμφωνον καὶ σύμμετρον χρεῖαν²⁵, since these express what is actually said to be provided (ἀπενείματο); and since the octave, fifth and fourth are undoubtedly the most fundamental intervals and harmonies of Greek musical theory, will not these be sufficient to provide a “symphonious (= harmonious) and balanced practice”? They in any case provide the basis for the rest of the scale, but Plato need not be referring explicitly to the whole scale here. If we are looking for a safe interpretation, that does not go beyond the evidence, it seems quite plausible to take στρεφομένη ἐπὶ, “turning

towards”, as meaning “considered with respect to”, since it is when considered with respect to one or other of the extremes that the means create the harmonies (fifth and fourth) for which they are responsible.

That the point of the whole passage is to emphasize the mathematical unity behind widely diverse phenomena is made clear by 991*e*:

ὁ δὲ τρόπος ὁδε - ἀνάγκη γὰρ τό γε τοσοῦτον φράζειν· πᾶν διάγραμμα ἀριθμοῦ τε σύστημα καὶ ἁρμονίας σύστασιν ἅπασαν τῆς τε τῶν ἄστρον περιφορᾶς τὴν ὁμολογίαν οὖσαν μίαν ἀπάντων ἀναφανῆναι δεῖ τῷ κατὰ τρόπον μανθάνοντι, φανήσεται δέ, ἅν, ὃ λέγομεν, ὀρθῶς τις εἰς ἓν βλέπων μανθάνῃ - δεσμός γὰρ πεφυκώς πάντων τούτων εἰς ἀναφανήσεται διανοου- μένοις - εἰ δ' ἄλλως πως ταῦτα μεταχειριεῖται τις, τύχην δεῖ καλεῖν, ὥσπερ καὶ λέγομεν.

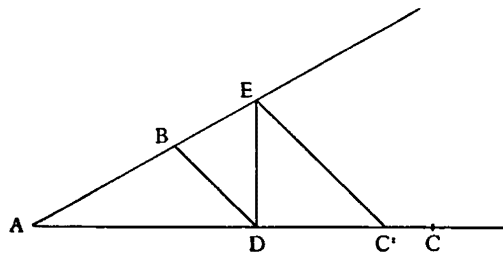
“And the method is this (for this much we must state): Every geometrical figure, and scale of numbers, and the whole system of harmony and of the revolution of the stars – the oneness of the agreement of all these must become clear to him who learns by the method, and it will do so if, as we say, one learns rightly by looking to unity; for to those who consider the matter there will appear a single bond naturally linking all these things – and if anyone practises these things in any other way one should attribute (his results) to chance, as we maintain.”

What our original passage says, in brief, is that we must study the workings of the universe as exhibited in astronomy; this will involve arithmetic, geometry and stereometry, which will appear wonderful sciences to those who understand the theory of proportions, which is so important in music.

Three connexions are asserted here, between astronomy and mathematics (arithmetic, geometry and stereometry), between mathematics in this sense and the theory of proportions, and between the theory of proportions and music. The first and the last of these connexions are tolerably obvious. Harmony depends on the theory of proportions, and astronomy involves mathematics, and especially arithmetic, which is τὸ μέγιστον as well as τὸ πρῶτον (990*c* 5); geometry and stereometry are brought in primarily because of what is coming. It is the connexion between these latter and the theory of proportions that is the most difficult to see the full point of. We have seen that probably the best solution is to follow up Van der Waerden's suggestion that sentence 4 deals with the geometric mean and sentence 5 with the arithmetic and

harmonic means, and to suppose that sentence 2 supplies the method by which the geometric mean serves the purpose it does serve, that namely of providing a mathematical link between the dimensions. Thus the whole of Nature, in all its parts, is built upon the structural laws of mathematics, where connexions are made by the geometric mean or means (for the importance of means cf. *Tim* 31b-2c), and the formal laws of harmony, where the connexions are made by the other two means.²⁶

Various methods for connecting the third dimension with the others were thought of, but the earliest achievement was that of Hippocrates of Chios, which was in effect as follows (see T. L. Heath: *Greek Mathematics*, Vol. 1):



Given a solid of side AB, to construct a similar solid the ratio of whose volume to that of the first solid will be as AC is to AB.

Draw BD perpendicular to AB to meet AC in D.

Draw DE perpendicular to AC to meet AB produced in E.

Draw EC' perpendicular to AE to meet AC (or AC produced) in C'.

By choosing a suitable angle at A, C' can be made to coincide with C, whereupon AB, AD, AE, AC will be in continuous proportion (by similar triangles), and AD will be the side of the required solid. (A method for getting the correct angle at A was first discovered by Archytas, who used a complicated construction, involving the definition of a point by the intersection of three surfaces, which does not concern us here.)

It is tempting to try and relate this to the triangles in the *Timaeus*, whose grades of size increase in the same way (see Cornford: *Plato's Cosmology*). Unfortunately however when $AC = 2AB$ in our diagram the angle at A is not 30° , as it is in Plato's triangles, and the ratio of the volumes of pyramids built out of his triangles of successive grades of size would be not $2 : 1$ but $\frac{8}{3\sqrt{3}}$ to 1.

Somehow or other the double has come to play a large part in Plato's

It can hardly be claimed that these notes solve all the problems of the passage satisfactorily, or that everything I have said will meet with general acceptance. What I do hope to have done is to have brought out

the main problems into the light of day by a detailed examination of the grammar and syntax of the passage, and to have made explicit some of the assumptions that are implicit in many of the commentaries on it.

London.

¹ I wish to acknowledge considerable help in discussions, references, etc., from my former supervisor Mr. G. B. Kerferd of Manchester University, who also read the paper at one stage. I have also had the benefit of several very useful comments and suggestions from Professor J. B. Skemp and Mr. D. J. Allan. Finally I am grateful to my friend Mr. I. P. V. Carter of Ferranti's, Manchester, for mathematical help.

² It is, I suppose, just possible that the *Epinomis* is a forgery and the mathematical passage a deliberate piece of meaningless mystification. If so, the unity and coherence which I think the passage contains would seem to show that the forger was more inspired than he thought!

³ Hereafter called QS.

⁴ σώματα L, Theo Smyrnaeus, marginal scribe on O: σώματος A O Stephanus.

⁵ The minor point whether θαῦμα is to be included with the subject or the predicate does not seem to have any material effect.

⁶ I accept Bekker's τρις for the τρεῖς of the mss.

⁷ Cf. P. H. Michel: *De Pythagore à Euclide*, pp. 505-8.

⁸ Cf. θεὸς ἀεὶ γεωμετρεῖ (Plut.: *Symp.* Bk 8 Qun 2). Plutarch ends the Question by using the above theorem (which he attributes to Pythagoras) for a symbolic interpretation of the Timaeus.

⁹ The Loeb translation gives it a backward reference. Professor Skemp suggests δ δὴ θεῖον, which would certainly give an easier sense.

¹⁰ "Root" occurs only at Tht 147c-8d, but it occurs 6 times there.

¹¹ J. Souilhé: *Etude sur le Terme δύναμις dans les Dialogues de Platon*, pp. 105-6 calls its usage in Plato's time "un peu flottante", though Tannery (quoted *ibid.*) would emend the Theaetetus text. Souilhé does not mention our passage.

¹² References to *Zahl und Gestalt* are to the first edition (1924).

¹³ Stenzel (ZG 101) follows Reuther in referring ἐξ ἐναντίας to ὑπεναντία, Archytas' word for harmonic proportion (since if x, y, z are in arithmetic proportion, $1/x$, $1/y$, $1/z$ will be in (descending) harmonic proportion, and vice-versa). But Stenzel then seems to assume that ὑπεναντία can apply to the reciprocals of a geometric progression (i.e. to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$); but these are themselves in geometric progression, and in *fr.* 2 (Diels) Archytas defines ὑπεναντία in terms of harmonic proportion. This, Stenzel says, is because, though no doubt ἐξ ἐναντίας has reference to ὑπεναντία, yet "zugleich soll hier auch die eigentliche Bedeutung, die Entstehung dieses Terminus angedeutet werden".

¹⁴ Stenzel (ZG 101) refuses to identify δύναμις exclusively with either the mathematical or the non-mathematical senses.

¹⁵ In common with I think all commentators I prefer ὥς περὶ (O²) for ὥσπερ (AO). With ὥσπερ we seem to have three alternatives: (1) τὸ διπλάσιον is the object of ἐγκαθορῶσί τε καὶ διανοοῦμένοις, ὥσπερ meaning "as it were" (in which case δ δὲ θεῖον must have a forward reference, as there is no connective after τὸ διπλάσιον). (2) τὸ διπλάσιον is the object of ἀποτυπῶνται, ἐγκαθορῶσί τε καὶ διανοοῦμένοις being absolute ("a wonderful thing to those who reflect on it, as though nature were

stamping the double, while..."). (3) τὸ διπλάσιον is the subject of ἀποτυποῦται, πᾶσα ἡ φύσις being in apposition, the sentence otherwise being as in (2). None of these seems to give a satisfactory sense. Perhaps ὥσπερ ἐλ arose through an error from dictation, if it and ὡς περὶ were pronounced the same; cf. τρις : τρεῖς above and δις : δ' εἰς below.

¹⁶ This does not imply a general usage of δύναμις for "term"; the meaning is partly shown by the context here.

¹⁷ Ficinus reads διπλάσιον κατὰ δύνάμιν οὔσα (i.e. omitting δὲ ἡ), and translates: "duplum potentiam possidens". This is not the usual text, and the fact that the first clause has something to do with 2, and the last clause with 8, itself suggests a middle clause having something to do with 4. The words πάλιν αὖ in the last clause might suggest too that something was being said for at least the third time, though Robinson (*Plato's Earlier Dialectic*, p. 172) denies any emphasis to this phrase, and Plato's later style is in any case well known to be prone to hyperbole.

¹⁸ At *Tim* 54b and *Pol* 266b κατὰ δύνάμιν means "potentially", being followed in each case by a predicate ("potentially something"). Here, unless we adopt Ficinus' reading, it has no predicate. At *Rep* 587d it might be taken as "potentially" with ὅσῃν ἀπόστασιν as predicate, but it is perhaps better to take it absolutely, meaning "by squaring". Perhaps the present usage is derived from this: "That which exists (comes into existence) by squaring", or "which is a power". Stylistically this would be hardly more barbaric than many things which are to be found in the *Laws* and *Epinomis*.

¹⁹ δ' εἰς margin of A²: δις ALO.

²⁰ Cf. the Loeb translation.

²¹ Abbreviations: E = Enharmonic. C = Chromatic. D¹ = Diatonic tonaion (Archytas' Diatonic). D² = Diatonic ditonaion. Dm = Diatonic malakon. Ds = Diatonic syntonon. H = Hypate. Ph = Parhypate. L = Lichanos. M = Mese. Pm = Paramese. T = Tritone. Pn = Paranete. N = Nete.

²² The following and similar facts will become clear from tables giving the intervals between each pair of notes in each of the various scales. I give here, as an example, the Synhemmenon form of the Enharmonic scale; the rest (excluded for want of space) can easily be constructed.

	Pm	T	Pn	N=H	Ph	L	M
E:	1	$\frac{28}{27}$	$\frac{16}{15}$	$\frac{4}{3}$	$\frac{112}{81}$	$\frac{64}{45}$	$\frac{16}{9}$
		1	$\frac{36}{35}$	$\frac{9}{7}$	$\frac{4}{3}$	$\frac{48}{35}$	$\frac{12}{7}$
			1	$\frac{5}{4}$	$\frac{35}{27}$	$\frac{4}{3}$	$\frac{5}{3}$

Each fraction here represents the ratio between the note in whose column it occurs and the note in whose column the figure 1 in the same row occurs. E.g. the ratio of L to T is $48/35$.

²³ Meibom (apud Stallbaum) and Müller take the antecedent to be 6 and 12.

²⁴ Van der Waerden incidentally makes a good point against the Taylor-Toeplitz interpretation, of an infinite series approaching $6/\sqrt{2}$ from above and below alternately,

when he says (p. 187): "Es ist aber auch sachlich nicht einzusehen, wie man durch eine solche fortschreitende Approximation jemals zu anmutigem Spiel, Rhythmus und Melodie kommen sollte".

²⁵ Toeplitz and Van der Waerden take χάρις as a noun governing the preceding genitives and qualified by σύμμετρον. I do not think this would make very much difference to the point at issue.

²⁶ This passage therefore links together the three studies (arithmetic, the study of length, breadth, and depth, and astronomy) which are simply mentioned seriatim as necessary parts of education at *Leg* 817e-22d.