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ARCHIMEDES AND THE PSEUDO-EUCLIDEAN CATOPTRICS: EARLY STAGES IN THE ANCIENT GEOMETRIC THEORY OF MIRRORS *

WILBUR R. KNORR **

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Introduction

The study of the reflection of light, or 'catoptrics' (from Greek *katoptron*, 'mirror'), already constituted a field of major interest within ancient geometric optics in the 3rd century B.C. Among the extant mathematical writings devoted to this field are a *Catoptrics* by Hero of Alexandria (1st cent. A.D.)¹, a major portion of Ptolemy's *Optics* (2nd cent. A.D.)², and a *Catoptrics* ostensibly by

* *Dedication*. — I would like to dedicate this paper to Prof. Albert Lejeune, whose contributions have long been, and will long remain, indispensable for the study of ancient optics.

¹ *Heronis Opera*, II, ed. W. Schmidt, 301-365.

² *L'Optique de Claude Ptolémée*, ed. A. Lejeune, Books III-IV.

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Euclid, but now generally accepted to be a pseudonymous work, perhaps compiled by Theon of Alexandria (4th cent. A.D.)³. These works are elementary in character, and so do not represent the findings of more advanced research. But the tract *On Burning Mirrors* by Diocles (late in the 3rd cent. B.C.) ably demonstrates the focal property of parabolic mirrors and the phenomenon now called 'aberration' in spherical mirrors⁴. From this we can infer that the fundamental results in catoptrics date back well into that century. Indeed, the basic principle of the equality of incident and reflected angles is assumed in a geometric proof by Euclid⁵, while the same phenomenon is known to writers on natural philosophy in the 4th century B.C.⁶

The tradition in later antiquity assigns to Archimedes a prominent role in the development of this field. Three passages cite or quote from a treatise *On Catoptrics*, and a fourth appears to be lifted from the same work⁷. Archimedes' exploits in the design and mobilization of gigantic mirrors for burning the Roman fleet are described in detail by some late historians of the Punic Wars, although the earlier and more reliable witnesses transmit no such reports⁸. Their silence, confirmed by that of Diocles who knows of no contribution by

³ *Euclidis Opera*, VII, ed. J.L. Heiberg, 285-343.

⁴ *On Burning Mirrors*, ed. G.J. Toomer, props 1-3.

⁵ *Optics*, prop. 19; *Euclidis Opera*, VII, ed. J.L. Heiberg, 30.

⁶ The equal-angles principle is invoked in the pseudo-Aristotelian *Problems* (XVI, 13) to explain why objects rebound at equal angles with the ground: "just as in mirrors <the image> is seen at the end of the line where the visual ray has converged". This principle is not introduced in the accounts of haloes and rainbows in the Aristotelian *Meteorologica* (III, ch. 3 and 5), even though their cause is assigned to reflection of light. One would suppose that the comments by Plato on singly and multiply reflecting mirrors (*Timaeus* 46a-c) indicates that geometers already knew the basic principle of reflection. It is remarkable, however, that Plutarch (2nd cent. A.D.) can cite the same phenomena of multiple reflection (e.g., the images seen in double mirrors) as counterevidence to the equal-angles principle ("On the Face in the Moon", *Moralia*, 930 a-c). For the correct explanation, derived from that principle, would be well established in the technical literature, as we may infer from Hero's *Catoptrics*, and the prototypes associated with the pseudo-Euclidean and Archimedean *Catoptrics*.

⁷ The passages derive from Apuleius, Theon of Alexandria, Olympiodorus and an anonymous scholiast to the pseudo-Euclidean *Catoptrics* and will be discussed in the following sections.

⁸ Surveys of the passages on Archimedes' burning mirrors, that is, their absence from the accounts in Polybius, Livy and Plutarch, and their appearance in later historians, are provided by E.J. Dijksterhuis, *Archimedes* (Copenhagen, 1956), 28-29 and I. Schneider, "Die Entstehung der Legende um die kriegstechnische Anwendung von Brennsiegeln bei Archimedes", *Technikgeschichte*, 36 (1969), 1-11. Both are sceptical of the historical validity of the late reports of Archimedes' burning mirrors. (See also my discussion in the article cited in the next note.)

Archimedes to the study of burning mirrors, should be all we require to relegate these later tales to pure legendizing, even were we to suspend judgment on the intrinsic implausibility and technical infeasibility of the feat⁹. But that Archimedes did contribute to optics in some manner is clear from his *Sand-Reckoner*; it contains a careful discussion of the design and use of an optical instrument for measuring the angular size of the sun and reveals Archimedes' sense of mechanical detail in its elaboration of a practical procedure enabling correction for the fact that the pupil of the eye is an extended object, not a geometric point¹⁰.

This involvement in practical optics might be assumed to lend credence to the testimonia relating to his writing on catoptrics. But the decision to believe them or not must follow upon a careful examination of the passages themselves. In undertaking that project here, I will seek insight both into the character of Archimedes' contribution to optical theory and also into the provenance of the pseudo-Euclidean *Catoptrics*. Although one would hardly suppose so at first, these two questions turn out to be closely related to each other.

PART I *Archimedes' Catoptrics*

1. *The testimony of Apuleius*

The earliest extant witness to Archimedes' writing on the theory of mirrors appears in a passage from the *Apology* by the Latin writer Apuleius of Madaura (mid-2nd cent. A.D.). In the following excerpt from its ch. 16, I have inserted numbers to facilitate later references¹¹.

⁹ A detailed discussion appears in my "Geometry of Burning Mirrors in Antiquity", *Isis*, 74 (1983), 53-73.

¹⁰ *Sand-Reckoner* I, 10-16 (in *Archimedis Opera*, ed. J.L. Heiberg, II, 222-226). For an account of Archimedes' experiments with this sighting instrument, see I. Schneider, *Archimedes* (Darmstadt, 1979), 91-95.

¹¹ I translate from the Latin text of Butler and Owen; a text is reproduced by Heiberg in *Archimedis Opera*, II, 550-551. I have consulted the English translation by Butler (*Apuleius: Apology and Florida* [Oxford, 1909], 41-42), but have found it too loose, and sometimes too inaccurate, to use here. The commentary by Butler and Owen (*Apulei Apologia* [Oxford, 1914], 43-46) provides valuable references on historical and philological points, although it is inaccurate on some technical matters.

Does it not seem to you that philosophy ought to investigate and inquire into all these things [sc. matters dealing with mirrors], and that especially those [sc. philosophers] ought to look into all kinds of mirror, both moist and dry¹², – (those) for whom beyond the things which I have said also the following line of thought is necessary: [1] why in plane mirrors the beholders and the images appear practically equal¹³, [2] while in swollen and globose [sc. convex] (mirrors) all things are more diminished, [3] but on the contrary, in hollow [sc. concave] ones they are more augmented¹⁴; [4] where and why left is interchanged with right; [5] in what cases the image in one and the same mirror now removes itself inward, now thrusts itself outward¹⁵; [6] why hollow mirrors, if they are held opposite to the sun, ignite the tinder set near; [7] why it happens that arcs are seen in clouds in various ways, (or why) two suns (are seen) with rivalling appearance; [8] and beyond this many other things of the same sort, expounded

uideturne uobis debere philosophia haec omnia uestigare et inquirere et cuncta specula uel uda uel suda soli uidere? quibus praeter ista quae dixi etiam illa ratiocinatio necessaria est, [1] cur in planis quidem speculis ferme pares obtutus et imagines uideantur, [2] <in> tumidis uero et globosis omnia defectiora, [3] at contra in cauis auctiora; [4] ubi et cur laeua cum dexteris permutentur; [5] quando se imago eodem speculo tum recondat penitus, tum foras exserat; [6] cur caua specula, si exaduersum soli retineantur, appositum fomitem accendant; [7] qui fiat ut arcus in nubibus uarie, duo soles aemula similitudine uisuntur, [8] alia praeterea eiusdem modi plurima, quae tractat uolumine ingenti Archimedes Syracusanus, uir in omni quidem geometria multum ante alios admirabilis subtilitate, sed haud sciam an propter hoc uel maxime memorandus, quod inspexerat speculum saepe ac diligenter.

¹² Translating *soli* as if referring to an implied *philosophi* (instead of *philosophia*) as subject. The word has puzzled editors; for alternatives, cf. Butler and Owen, *op. cit.*, 43-44.

¹³ The equivalent of *ferme* could hardly have appeared in a technical work by Archimedes. The notion that plane mirrors produce images smaller than true size is expressed by Olympiodorus (*In Aristotelis Meteora*, ed. Stüve, 214.13-14), who also seems to follow the Archimedean catoptrical source (see sect. 2 below); this is likely to signify Olympiodorus' use of a nontechnical intermediate source.

¹⁴ The term *auctiora* would apply to the magnified virtual images seen in concave mirrors; but real images can also be produced, sometimes magnified, sometimes reduced, depending on the position of the object relative to the center of the mirror and to the observer. To be sure, the principal use of concave mirrors would be for magnification; but Apuleius must be simplifying the content of his technical source.

¹⁵ This would refer to the various positions of the image seen in a single concave mirror, as the viewer moves nearer or farther from it. Butler and Owen consider such an interpretation too complicated, and thus prefer taking Apuleius to mean the images in a simple plane mirror (*op. cit.*, 45). But as the comparison with the pseudo-Euclidean *Catoptrics* (prop. 28) reveals, the properties of concave mirrors were certainly within the technical range of Apuleius' source, so that this must surely be what he intends here.

in a great volume by the Syracusan Archimedes, a man admired much before others for subtlety in the whole of geometry, but particularly to be noted, I believe, for his having looked often and diligently into the mirror.

One cannot miss the florid language and sporting tone. These are natural features in Apuleius' "apology", the speech by which he aims to defend himself against charges submitted by his hostile in-laws that he practiced the magic arts¹⁶. Among the items raised in evidence was Apuleius' possession and use of mirrors, and the passage just quoted rounds off a lengthy section of his defense in which he affirms the philosophical and scientific merit of the study of mirrors by invoking the names of respected sages, Socrates, Demosthenes, Plato, the Pythagoreans and the Stoics (ch. 15), and as here, Archimedes. Despite Apuleius' rhetorical flourishes one can still recognize clearly the objective phenomena he alludes to, while the forensic context actually reinforces the reliability of his witness; for it is unlikely he would risk the embarrassment of exposure by his adversaries, by blatantly fabricating such a claim as that Archimedes had composed a 'great volume' of the given description on catoptrics. We may infer, then, that Apuleius is here giving an account of a specific work whose Archimedean authorship would be accepted by his audience.

It is important to recognize, however, that this does not guarantee the correctness of the attribution; for a general audience, and even Apuleius himself, so far removed from the time of Archimedes, might be mistaken in admitting such a claim to be true. The fact that Apuleius, some four centuries after Archimedes and operating within an entirely different intellectual sector, should be the oldest witness to Archimedes' catoptrical work is disturbing; for why should earlier writers in optics, Diocles and Hero, for instance, or Apuleius' contemporary Ptolemy, have omitted mentioning Archimedes' contributions? Further, false attributions were relatively common among the writers on the occult arts blossoming around Apuleius' time; appropriating the names of Democritus and Pythagoras to writings in alchemy and Hermeticism might lend prestige and authority to efforts in a relatively young tradition¹⁷. Indeed, Apuleius appeals to ancient precedent in precisely this way to justify his own

¹⁶ Butler and Owen provide an ample synopsis of this sordid episode; cf. *op. cit.*, vii-xix.

¹⁷ On the prominence of writings falsely ascribed to Democritus in the alchemical tradition around this time, see J. Lindsay, *Origins of Alchemy in Graeco-Roman Egypt* (New York, 1970), ch. 5. Pseudonymous writings are commonplace in the Hermetic and Gnostic traditions.

interests in magic. When we turn to the Archimedean corpus, however, we find no evidence of catoptrical studies. His chapter on the sighting instrument in the *Sand-Reckoner* is the only place in the extant works relating to optics at all, and this discussion has no bearing on any aspect of catoptrics.

However unsettling these considerations might be, on balance, they would probably not move one to reject this Archimedean attribution. Negative briefs, like this argument *ex silentio*, are notoriously slippery, while comparable complaints could be lodged against virtually every ancient attribution. Scholars are thus inclined perforce to accept the testimony of ostensibly reliable witnesses, save in the face of strong countervailing factors. In the present case, the 'silence' is unusually telling, for it embraces the entire ancient tradition of optics, not in the sense that other testimonia are lacking – for references to Archimedes' catoptrical studies appear in three later passages, to be discussed below –, but in the sense that no aspect of geometric technique in the extant writings gives evidence of the sophistication we should expect from an Archimedean contribution. One possible exception can be proposed in the more advanced findings exhibited by Diocles; but this renders all the more significant Diocles' failure to include Archimedes within the group of researchers engaged in the early study of burning mirrors¹⁸.

I believe the argument against authenticity here is secured through a further observation – that Apuleius' treatise on mirrors can be identified: it is the pseudo-Euclidean *Catoptrics*.

The agreement between the Archimedean work described by Apuleius and the pseudo-Euclidean treatise is quite remarkable, yet it has passed unnoticed by scholars on ancient optics and thus has not figured into any of the regularly consulted accounts of this field. Indeed, referring back to the passage from the *Apology*, we find that it reads virtually as a table of contents of the pseudo-Euclidean writing. Item (1) on plane mirrors answers to prop. 19 of the *Catoptrics*: "in plane mirrors ... the image appears equal to the object seen ..."; item (2) on convex mirrors to *Cat.* prop. 21: "in convex mirrors the image is less than the object seen"; and item (3) on concave mirrors to one case of prop. 28: "if the eyes B, G are placed above the midpoint [of the radius of the mirror] ... then the image is greater than the face ...". To be sure, Apuleius' remark on concave mirrors is oversimplified, since the pseudo-Euclidean proposition specifies other positions of the eyes where the image will be smaller, or where no image will be seen at all¹⁹. Item (4) on the reversal of orientation

¹⁸ Diocles names Pythion, Conon, Dositheus and Zenodorus as predecessors in the study of burning mirrors, but makes no reference to any aspect of Archimedes' work; cf. *On Burning Mirrors*, ed. Toomer, 34-37.

¹⁹ See note 14 above.

("left will be exchanged with right") corresponds to portions of the claim in *Cat.* props 19, 20 and 28 (that "left appears right, and right left"), although here again, cases on the concave mirror preserve true orientation (*i.e.*, "left appears left and right right")²⁰. It happens that all the cases cited by Apuleius to this point relate to the virtual images produced in mirrors. But in item (5), where he speaks of the image "now withdrawing inward, now thrusting outward", he must intend the convex mirror which produces internal virtual images when one looks into it at close distance, but suddenly produces real images above the mirror surface, right at the position of the observer when he is positioned at the center of the mirror, but then progressively closer to the mirror as the observer moves further away²¹. These cases are worked out in full detail in *Cat.* prop. 28, explicating formally the phenomena noted by Apuleius. The correspondence is almost literal in the instance of item (6) on burning mirrors and the last proposition of the *Catoptrics*:

Apol. ch. 16: why hollow mirrors, if they are held opposite to the sun, ignite the tinder (*fomes*) set near by;

Cat. prop. 30: From concave mirrors placed toward the sun, fire is ignited. ... When, then, these rays are growing warm [through their convergence], fire is gathered about the center [of the mirror's curvature], so that the flax (*stuppion*) set here will be ignited.

The specific parallel between Apuleius' "tinder" and the "flax" in the *Catoptrics* is striking. In effect, Apuleius' line reads like a paraphrase of the pseudo-Euclidean proposition.

The rest of the *Catoptrics* (props 1-18) merely provides the necessary preliminaries for these results: the equal-angles principle for reflected rays (prop. 1), general theorems on the convergence of rays (props 2-6) and on relative orientation (7-12), some applications relating to multiple configurations of mirrors (13-15), and the principle enabling the construction of the images (16-18), namely that they lie on the line drawn from the object perpendicular to the surface of the mirror. Thus, Apuleius' items (1)-(6) embrace essentially the whole of the *Catoptrics*, establishing in order the basic properties of the images in plane, convex and concave mirrors and appending the result on burning mirrors.

²⁰ Both Apuleius and the *Catoptrics* use neuter-plural adjectives to denote "right" (*dextra*; *ta dexia*, respectively) and "left" (*sinistra*; *ta aristera*).

²¹ *Cat.* prop. 28 does not recognize the possibility of real images formed behind the object, even though that is suggested from the geometric configurations (see note 78). Presumably, their theoretical possibility could not compete with the empirical difficulty of actually seeing images in this situation.

But with item (7) Apuleius touches on an area not present in the *Catoptrics*, the exposition of meteorological phenomena like rainbows and *parhelia*²². The extant form of the *Catoptrics* is incomplete, however, for the last of the postulates which preface the work formulates the principle of refraction, and this has no relevance to any of its propositions. To some critics the insertion of this postulate is a sign of the bad editing behind the *Catoptrics*²³; but it is surely more plausible that portions of the work were lost in the course of its transmission, than that it suffered such gratuitous and irrelevant interpolations. From the Aristotelian commentator Alexander of Aphrodisias we learn that in contrast to Aristotle and Posidonius, who accounted for the halo as due to the reflection (*anaklasis*) of rays, "almost all others" explain it by refraction (*diaklasis*)²⁴. Among the latter he must include the more technical writers, for Peripatetic commentators like himself and his teacher Sosigenes follow their master Aristotle in adopting the explanation through reflection. The accounts provided in the Archimedean work known to Apuleius must have used refraction to explain these phenomena; for, as we shall consider in the next section, Theon and Olympiodorus claim to derive their statements of the refraction principle from Archimedes. It is thus plausible that the refraction postulate we meet in the pseudo-Euclidean *Catoptrics* was the basis for a group of propositions, no longer extant, in which refraction was invoked for explaining these same phenomena. This further strengthens the link between the Archimedean and pseudo-Euclidean works. The fact that both works, although devoted to the study of mirrors and thus requiring only the principle of reflection, nevertheless also introduce the principle of refraction constitutes in itself an important coincidence linking them.

The passage from Apuleius thus reveals that a treatise on mirrors corresponding precisely to the subject matter expounded in the pseudo-Euclidean *Catoptrics* was familiar to scholars in the 2nd century A.D. Although one might follow the lead of J. L. Heiberg, A. Lejeune and other authorities and suppose that the *Catoptrics* was a late compilation drawing from Archimedes as one of its sources²⁵, there are difficulties with this hypothesis, as we shall discuss in

²² These phenomena are also discussed in the Aristotelian *Meteorologica*, Book III. For a modern account, see R.A. Tricker, *Introduction to Meteorological Optics*, London, 1971. A survey, with remarkable illustrations, is given by D.K. Lynch, "Atmospheric Halos", *Scientific American*, 238, no. 4 (Apr. 1978), 144-152.

²³ A. Lejeune, *Recherches sur la catoptrique grecque*, 55-56; P. Ver Eecke, *Euclide: l'Optique et la Catoptrique*, xxxi.

²⁴ In *Aristotelis Meteorologica*, ed. Hayduck, 143.7-10.

²⁵ See sect. 3 below. Lejeune holds, in fact, that the *Catoptrics* includes some pre-Archimedean works among its sources (*Recherches*, 142-145).

sect. 3. The simplest view would be to maintain that the treatise described by Apuleius was in essence the pseudo-Euclidean *Catoptrics*, but cited by him as a work of Archimedes. It thus becomes important to determine whether this view is compatible with the other testimonia to the Archimedean *Catoptrics*; we turn to this question next.

2. The testimonia of Theon and Olympiodorus

Besides the passage from Apuleius, two testimonia on Archimedes' writing on catoptrics are extant, one from the commentary by Theon of Alexandria (mid-4th cent. A.D.) on Ptolemy's Book I, the other from the commentary by Olympiodorus (early 6th cent. A.D.) on Aristotle's *Meteorologica*. Both relate to applications of the principle of refraction, rather than to properties of mirrors.

In *Syntaxis*, I, 3 Ptolemy argues that the cosmos is spherical in shape on the grounds that any other form would bring the heavenly bodies sometimes nearer, sometimes farther from us, so that we would observe corresponding increments and decrements in their sizes, "which is not seen to occur". He goes on to admit, however, that when these bodies are situated near the horizon they do indeed appear magnified, but that this results not from their being closer to us, but through an optical effect caused by low-lying moisture, "just as things tossed into water appear bigger, and by such an amount bigger the further downward they move"²⁶. Theon explicates this remark by noting that "the rays impinging from it [sc. the origin of the visual ray, that is, the eye] on the star undergo bending and make the angle of vision bigger, just as Archimedes has proved in the (writings) *On Catoptrics*"²⁷. Although there may be some room for debate as to precisely what Theon is citing from Archimedes, he seems to claim an Archimedean precedent only for the refractive enlargement of the angle of sight; the astronomical application maintained by Ptolemy and expanded by Theon later in this comment appears to have a different source²⁸.

Theon next provides a geometric proof of the claim that objects seen in water appear magnified, and the more so the further down they lie. He

²⁶ *Ptolemaei Opera*, ed. Heiberg, I, 12.19-13.9.

²⁷ *Commentaires de ... Théon ... sur l'Almageste*, ed. A. Rome, 347.3-348.1. The plural "*tois*" as here, would usually signify a writing in two or more books (sc. "in the <books> *On Catoptrics*"). This conforms with Apuleius' description of the *ingens volumen* by Archimedes.

²⁸ So Rome in his note on this passage (Theon, *op. cit.*, 350n). But the proposal developed below, by questioning the Archimedean provenance of Theon's source, will deflect Rome's principal objection, namely, that Theon's account of atmospheric magnification is wrong.

assumes two unequal bodies AB, GD which subtend the same angle at the eye E when viewed in pure air (Fig. 1); but if they are supposed to lie in water below the surface ZH, the rays from E to A, B will be bent as they impinge on that surface, and so follow the broken line E θ A, EKB, "just as Archimedes (has proved) in the (writings) *On Catoptrics*, as we said"²⁹. Since, further, "the vision is by nature to see according to straight lines", he extends E θ , EK to meet AB extended at L, M, so that AB will assume the appearance of LM, being seen under the larger angle LEM. Doing the same for the lower object GD, he introduces the broken lines ENG, EXD and extends EN, EX to O, P, so that GD will assume the enlarged appearance OP. With this, Theon concludes, "therefore AB, GD, being unequal but appearing to be equal in pure air, appear unequal in water, and the one (placed) further down (appears) bigger, since it is seen under the bigger angle"³⁰.

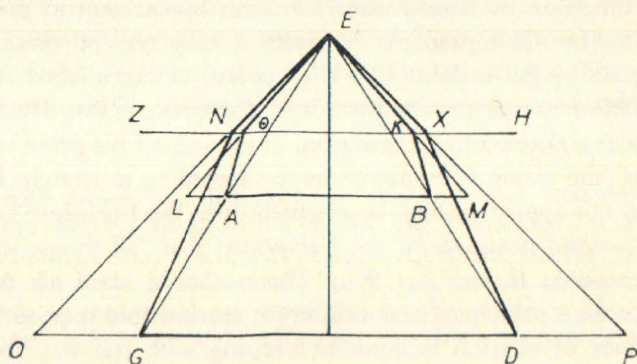


Fig. 1

Theon claims to draw from Archimedes only the enlargement of the visual angle, that is, the appropriate configuration of the rays E θ A, EKB. But the manner of its elaboration here, to conclude first the larger appearance of objects seen in water and then the greater magnification of objects lying further down, is unlikely to be original with Theon; for the results at issue are too basic to be omitted from any systematic treatment of refraction, while Ptolemy's passage indicates that they were already established in the older literature. One thus would suppose that Theon's text reproduces or paraphrases from his source. That source would surely be the same one that provided the

²⁹ Theon, *op. cit.*, 349.6-7.

³⁰ *Ibid.*, 349.16-18.

principal result on refracted rays, the result Theon assigns to Archimedes' *Catoptrics*. But Theon's extended passage has such grievous shortcomings as to prohibit its assignment to Archimedes. First, the enlargement of the visual angle is derived from Archimedes' configuration of the refracted rays, whose defining condition, to the extent that one can infer it from Theon's presentation, is the widening of the visual angle; thus the whole account becomes circular. Second, Theon has no condition to specify the relative sizes of the visual angles NEX and ΘEK . He requires the latter to be the greater; but why is it impossible that G , D be seen along the broken lines EΘG , EKD , so that the two objects AB , GD would come to have the same apparent size? Some quantitative measure of the angle of refraction as a function of the angle of incidence, for instance, the table of values presented in Ptolemy's *Optics*³¹, would serve Theon's needs, or at least some principle guaranteeing the monotonic increase of the increment of the refracted angle with increasing incident angle. This omission by Theon utterly vitiates his attempt at proof.

The account by Olympiodorus suggests a way out of these difficulties by providing additional insights into the content of the alleged Archimedean source in relation to the pseudo-Euclidean *Catoptrics*. But Theon's account already provides a noteworthy connection, in the step of his proof where he has remarked that "the vision is by nature to see according to straight lines". This postulate does not appear among those which open the Euclidean *Optics*³², but it echoes the second postulate of the *Catoptrics*: that "all things seen are seen according to straight (lines)"³³. Why Theon should leave his Archimedean source and invoke a principle from a different work would require explanation. But this manner of citation is quite in keeping with the way formal works

³¹ Introduction to Book V; see, for instance, V 7-11 (ed. Lejeune) for the values in the air-to-water case. The introduction is translated in M.R. Cohen and I.E. Drabkin, *Source Book in Greek Science*, 271-281. (See also note 100).

³² In Euclid's *Optics*, post. 1, one supposes that "straight lines drawn out from the eye are tried for a distance of great magnitudes (ed. Heiberg, 2.2-3), and in Theon's preface, this same property is assigned to light: "every light (*phōs*) is carried along straight lines", and indicated through examples and experiments (ed. Heiberg, 144.1-146.17). It is by an analogy between light and the visual ray that the same rectilinear property is applied to the latter. In the somewhat different view presented by Damianus, the visual ray is a kind of light (cf. *Optics*, ed. Schöne, ch. 1); but even here identity of the principles for visual rays and light (e.g., solar) rays is sometimes explained through analogy (cf. ch. 13). For the moments, the phenomena of vision and of light would be considered distinct, to be known through different experiences and experiments. Cf. note 86.

³³ Theon, *op. cit.*, 349.7-8; *Catoptrics*, ed. Heiberg, 286.3. Note that a similar wording occurs in Ptolemy's *Optics* in the introduction to Book III: "*cum enim visibilis radii methodo et natura sit recte procedere a principio suo in universis rebus que recte videntur*" (II, 14; ed. Lejeune, 95.13-15).

invoke their own initial assumptions within demonstrations. We thus have a link between Theon's source and the pseudo-Euclidean work.

In his commentary on Aristotle's *Meteorologica* III Olympiodorus provides extensive discussions of optical phenomena³⁴. Although the commentator is sometimes surprisingly misinformed on the views of earlier scholars, and often makes fundamental errors on technical matters, one can infer important details about his sources³⁵. In his account of the theory of haloes, rainbows and related effects, he wishes to defend Aristotle's position, that these are caused by reflection (*anaklasis*), against those who explain them through refraction (*diaklasis*). He thus presents a lengthy discussion contrasting these two optical principles: that in reflection object and image are in the same plane with the mirror disposed opposite them, while in refraction the seer and the thing seen are separated by the mirror³⁶; that rays are reflected at equal angles, but refracted at obtuse angles; and that images produced by reflection are smaller than the objects, while those produced by refraction are larger. The defects in these statements illustrate the technical weakness of Olympiodorus' account. Nevertheless, it provides some valuable information on sources, in particular, on Archimedes' catoptrical studies. The basic phenomenon of refraction is described as follows:

Archimedes proves in a different way this same fact, that the visual ray is bent, from the ring tossed into a vessel. For if you toss a ring into a vessel not containing water, it will not be apparent to you because of the interposition of the body of the vessel; but if you toss in water, it will appear displaced, as the visual ray impinges on the water in the manner of a mirror, and is bent through refraction. In this way one demonstrates the ray to be bent.³⁷

³⁴ In *Meteorologica*, ed. Stüve, 209-214.

³⁵ For instance, Olympiodorus claims that Alexander, "suffering from I don't know what", attempted to explain the halo through refraction (*ibid.*, 210.15-17); in fact, Alexander insists that reflection is the cause (cf. *ibid.*, 210n). Olympiodorus makes the false technical claim that reflected images are smaller than the objects (*ibid.*, 214.13-15). This error leads me to suspect that his information is at second hand, affected by alterations due to nontechnical writers. For no one, working directly with a technical source (e.g., *Cat.*, prop. 19) would make the claim he does.

³⁶ Olympiodorus twice uses the term *katoptron* to denote phenomena of refraction (*ibid.*, 211.22, 26-28). If this is acceptable usage, it may explain the presence of refraction in the pseudo-Euclidean and Archimedean (as attested) writings *On Catoptrics*. It may also agree with a use of *speculum* in Ptolemy's *Optics* to denote the refractive interface offered by a curved vessel (V, 51); but Lejeune, who notes the parallel with Olympiodorus, suggests that the latter is a scribal error for *superficies* (*Optique de Ptolémée*, 253n).

³⁷ Olympiodorus, In *Meteor.*, 211.18-23.

This statement may be compared with the last postulate in the pseudo-Euclidean *Catoptrics*:

If something is tossed into a vessel and assumes a distance as not to be seen, if, the distance being the same, water is poured in, the thing tossed in will be seen.³⁸

Those who have noted this coincidence of phrasing explain it through the pseudo-Euclidean editor's dependence on the Archimedean *Catoptrics* as one of his sources. But the discussion of the preceding section suggests a more direct explanation: that Olympiodorus is dependent on the pseudo-Euclidean work, which he takes to be by Archimedes. The pseudo-Euclidean wording follows a more streamlined style appropriate to a formal geometric presentation; the elaborations in Olympiodorus' statement merely recast that in a more conversational voice. One need not suppose that Olympiodorus' source differed significantly from what we read in the pseudo-Euclidean passage.

In certain respects Olympiodorus' formulation extends beyond the other. The phrase "in the manner of a mirror" is puzzling within the context of refraction. We might suppose that his sense of the word *katoptron* is broader than our "mirror", and includes any optical device. But one may observe that the passage must intend not merely to provide an example of refraction, but rather to set up a paradigmatic instance suitable as basis for a theory of refractive phenomena³⁹. This particular instance readily suggests an experimental situation for establishing the quantitative aspects of refraction: as one pours in water and perceives the displacement of the image of the ring set at G (Fig. 4), one can position a second object at Z above the water's surface so that its reflected image is superimposed over the ring's refracted image; since the path of the reflected ray ADZ is known from the positions of the eye and the second object, one can determine the path of the refracted ray ADG. This provides a rough but reliable procedure for gauging the relation between the incident and refracted rays as the water level rises. Indeed, one obtains Ptolemy's apparatus

³⁸ *Catoptrics*, ed. Heiberg, 286.17-19.

³⁹ Lejeune criticizes the pseudo-Euclidean postulate: "il n'est pas, tel quel, géométriquement utilisable" (*Recherches*, 55). But I believe he overlooks the versatility of this "thought experience", if we may so denote it, as a vehicle for the geometric theory. The propositions in Theon and Olympiodorus reveal its utility. In effect, the ring-in-the-vessel serves as a paradigm for the theory of refraction, in the most concrete sense of the term "paradigm" employed by T.S. Kuhn (cf. *Structure of Scientific Revolutions*, 1970).

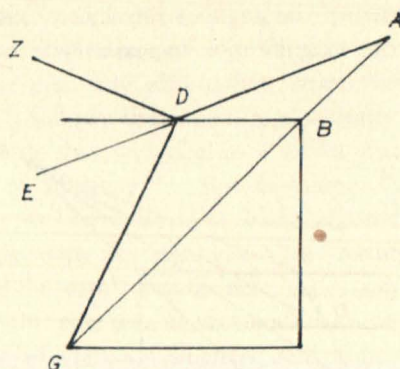


Fig. 2a

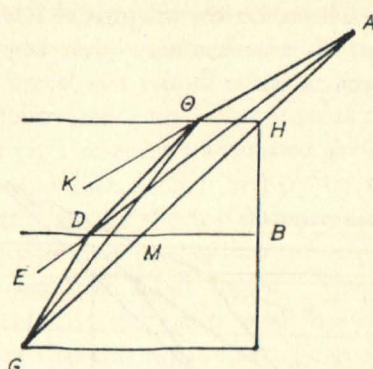


Fig. 2b

for the experimental measurement of the refracted angles merely by introducing calibrated scales into this arrangement⁴⁰.

This idealized instance of refraction provides a way to establish the two properties we noted were assumed in Theon's discussion. Initially we set the eye at A and the object at G so that the edge of the vessel just obscures the visual ray AG at B (Fig. 2a). If now water is poured in up to the level of B, the object becomes visible along the line AD; hence, the visual ray must follow the broken line ADG. This provides a straightforward basis for Theon's step, "that the rays are bent toward A, B along the lines EΘA, EKB" (cf. Fig. 1). Since he assigns this result to Archimedes, while Olympiodorus assigns the corresponding paradigm for refraction to Archimedes, it seems likely that their source adopted an approach of this kind. Further, one can establish that the amount of refractive displacement increases as the water level is higher. For, when the level is at B, the object will be seen along the line ADE, so that the apparent position of G is the same as that of an object set at D (Fig. 2b); if more water is poured in, raising the level to H, then the same principle entails that the image of D is seen displaced, say in the direction AΘK. Since the image of G must be seen along the same line, the refracted ray must follow the broken line AΘG, entailing an incremental displacement beyond the former line of ADG. In this way, one can establish the step assumed without explana-

⁴⁰ Ptolemy describes a sighting instrument (a type of 'dioptra') for studying reflection and refraction; cf. the prefaces to Books III and V; cf. also note 31 above. The passages are translated by Cohen and Drabkin, *Source Book*, 270, 274; for discussion, see O. Pedersen, *Early Physics and Astronomy*, 133-135.

In the next section Olympiodorus establishes the equality of incident and reflected angles by proving that this configuration entails a shorter distance for the ray than any alternative; since "all agree that nature effects nothing in vain", it follows that visual rays actually follow this path. This proof conforms well with that included in a Latin tract on *Catoptrics* transmitted under the name of Ptolemy⁴³. But Damianus, one of our ancient writers on optics, assigns to Hero the same basic argument: that the shortest-distance property together with the principle that "nature does not intend that our vision go around in vain" entails that reflection occurs at equal angles. This is an important part of W. Schmidt's argument that the Latin work transmits a version of Hero's *Catoptrics*, and indicates that this section of Olympiodorus' account is based on a source related to Hero⁴⁴. Since the texts of Hero and Olympiodorus are not in strict agreement, however, the commentator must have worked with an alternative version. The only source he actually names here is the Archimedean *Catoptrics*; the shortest-distance property has a natural Archimedean association, since Archimedes states the least-distance property of straight lines as one of the postulates in his *Sphere and Cylinder*, while Hero makes frequent use of Archimedean sources in his technical writings⁴⁵. This might suggest that Hero's proof was originally held in the 'Archimedean' *Catoptrics*. But we shall later discuss evidence which indicates an alternative view on the oldest treatments of the equal-angles principle.

After this proof, Olympiodorus contrasts the situation in refraction, where the ray is bent at an obtuse angle⁴⁶. His argument establishes the same configuration of the refracted rays that Theon takes from Archimedes: rays are drawn from the eye at G to points A, B on the surface of the water (Fig. 3); AD, BE are drawn perpendicular to the surface, and GA, GB are extended.

Olympiodorus then claims that the refracted rays proceed from A, B along lines intermediate between the extensions and the perpendiculars. His ar-

defective state of our evidence prevents determining whether this precise sense is intended, or a much looser sense (*i.e.*, "the more ..., the more ..."). But even in the stronger sense, it would not be an experiential claim, but rather a hypothesis suitable for the purposes of the geometric theory.

⁴³ Olympiodorus, *op. cit.*, 212.4-213.22; cf. *Liber Ptolomei de speculis*, ed. Schmidt (*Heronis Catoptrica*), prop. 4 (324-328).

⁴⁴ Damianus, *Optica*, ch. 14. Schmidt argues the Heronian identification in detail, *op. cit.*, 303-306.

⁴⁵ *Archimedis Opera*, ed. Heiberg, I, 8. On Hero's use of Archimedean sources, see my *Ancient Tradition*, ch. 5 (iv). The Archimedean provenance of the associated conception of the line (or visual ray), as the least distance between two points, is proposed by C. Mugler in "Sur l'histoire de quelques définitions ...", 343-345.

⁴⁶ *Op. cit.*, 213.23-214.13.

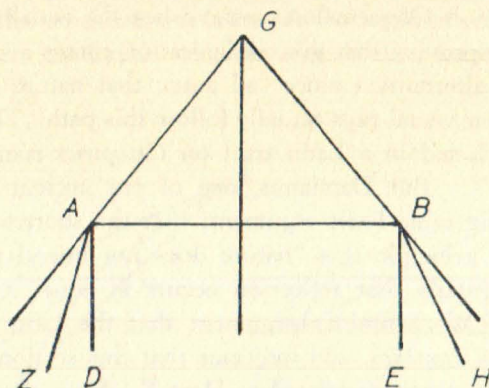


Fig. 3

gument leaves much to be desired, however; he can claim that the rays do not follow the extensions, for then there would be no refraction at all. But his reason for their not following the perpendiculars is simply, "what would compel them?". The fact that he embarks on a proof at all should indicate that some form of proof was presented in his source, even if he seems not to have the patience or ability to transmit it. A form of proof has been proposed above, and this would be adequate for the purposes of the applications in both commentators.

Olympiodorus concludes by contrasting the reduced size of reflected images and the magnified size of refracted ones. He does not attempt proofs of these claims, although they would follow easily from his previous remarks on angles. A further parallel with Theon's account appears in Olympiodorus' statement of refractive magnification:

And that the object seen under refraction appears bigger is evident; for behold, small stones seen in waters seem to be large, and by as much in depth as they are placed, by that much do they seem the bigger. But also the sun when looked at rising through mist seems to be bigger.⁴⁷

Here, the first sentence echoes the claim by Ptolemy and Theon that magnification is progressively greater as the object is placed further down. As we have seen, Theon attempts a proof of this effect; Olympiodorus merely states it. The next sentence brings us back to the claim by Ptolemy which provided the occasion for Theon's commentary: that the heavenly bodies viewed near the horizon appear magnified because of the presence of lowlying moisture. The

⁴⁷ *Ibid.*, 214.17-20.

agreement in the former case is certainly due to common dependence on the same catoptrical source. The agreement in the latter case indicates that Olympiodorus is following a source for this too; it is not likely that Theon would be that source, since the two accounts have diverged throughout. The catoptrical writing, which both attribute to Archimedes, seems the most likely source; for we know from Apuleius, as cited above, that this work moved into the explanation of meteorological phenomena, and the magnification of bodies seen near the horizon can be included among these.

In his notes to Theon's commentary, Rome considered the possibility that the section of Theon's account providing the explanation of horizon magnification derived from his catoptrical source, but ultimately rejected that view⁴⁸. Since one will be observing the heavenly bodies from within the denser refractive medium, they will actually appear diminished, not magnified; Archimedes, the alleged author of Theon's source, could hardly have committed such a gross error. But the fact that both Theon and Olympiodorus have access to the same source and transmit the same argument, supports the view that this argument was held in the source; if we doubt that Archimedes was its author, Rome's reservation loses its force. Indeed, one can propose certain notions which partially rehabilitate Theon's argument. If we assume a moist envelope about the earth, in the form of a spherical shell not reaching its surface, then the observer at A will see distant objects through a greater depth of moisture along the line ΘE than along KZ (Fig. 4). To apply Theon's basic result on refractive

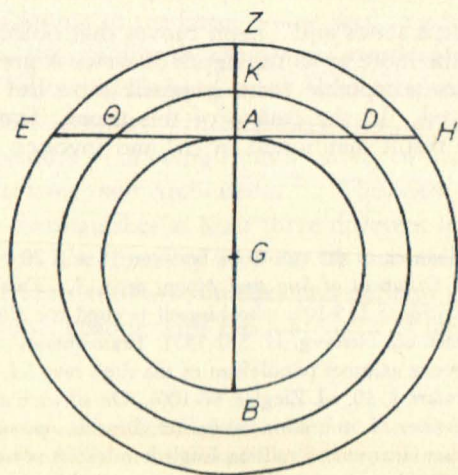


Fig. 4

⁴⁸ *Commentaires de ... Théon*, 347n, 350n.

magnification, however, we must assume that the object seen is within the moist medium, that is, set actually at E or Z, and relatively close compared to the size of the earth. These assumptions are flatly incompatible with the mature Greek cosmology adopted by astronomers from the 3rd century B.C. onward, where terrestrial dimensions are trivialized by comparison to the distances to the sun and stars⁴⁹. But the tradition from which the Aristotelian *Meteorologica* draws for its explanation of the rainbow subscribes to a different view: the sun's distance is comparable to that of clouds, and rays from the earthly observer to the sun are not all parallel⁵⁰. These assumptions are implicit in the account of burning mirrors in the pseudo-Euclidean *Catoptrics*⁵¹, and they lend a plausibility to Theon's argument. Thus, if Theon's catoptrical source can be dislodged from its Archimedean association and set in the context of the earlier meteorological and cosmological traditions, we could assign to that source the major part of Theon's discussion.

To summarize, Olympiodorus and Theon provide parallel insight into the content of their source on catoptrics:

(i) Olympiodorus paraphrases from it a statement of the principle of refraction which is in good textual agreement with the last postulate in the pseudo-Euclidean *Catoptrics*;

(ii) Olympiodorus maintains that Archimedes proved the refractive bending of rays on the basis of this principle; he provides a clumsy proof of this phenomenon (that in refraction, rays are bent at obtuse angles), while Theon assumes this result by citing Archimedes' *Catoptrics*;

(iii) Olympiodorus states and Theon proves that objects seen in water are magnified, and the more so as the depth of water is greater. Theon's proof is flawed, but an acceptable form is possible on the basis of the claims already introduced. In the course of this proof, Theon introduces from Archimedes the result mentioned in (ii) and invokes the principle of the

⁴⁹ Aristarchus set the distance of the sun to be between 18 and 20 times the distance to the moon; cf. *Sizes and Distances of Sun and Moon*, prop. 7. This dimension is cited by Archimedes (*Sand-Reckoner* I, 9-10), who himself is cited for alternative dimensions by later writers (cf. *Opera*, ed. Heiberg, II, 552-555). Eratosthenes' famous measurement of the earth's circumference assumes parallelism of the sun's rays (cf. the exposition in Cleomedes, *De motu circulari*, I, 10, ed. Ziegler, 96-100). On all such accounts, the distance to the sun runs to the order of 50 million stades (cf. Diocles, *op. cit.*, 38-39, and Toomer's commentary, 146), that is, around 5 million English miles. A review of the early distance estimates is given by O. Neugebauer, *History of Ancient Mathematical Astronomy*, 634-664.

⁵⁰ *Meteor.* III, 5. This configuration has been dubbed the "meteorological hemisphere" by C. Boyer in his account of the Aristotelian theory; cf. his *The Rainbow from Myth to Mathematics* (New York, 1959), 39-45.

⁵¹ Prop. 30; to be discussed in sect. 7 below.

rectilinearity of visual rays in terms conforming with the second postulate in the pseudo-Euclidean *Catoptrics*;

(iv) Olympiodorus states and Theon proves that these results explain why bodies like the sun appear magnified when seen through moist media; like the pseudo-Euclidean treatment of burning mirrors, Theon's account of the magnification effect is compatible with cosmological views possible in the 4th century B.C., but not with views generally accepted afterward.

These items form a deductively linked sequence of results following from the basic postulate on refraction. Their presence in the Archimedean catoptrical source can be inferred through the evidence in the commentators, and this agrees with Apuleius' account of presumably the same source, in which the general principles of catoptrics are applied toward the explanation of meteorological phenomena. But on several notable counts the commentators reveal affinities between their source and the pseudo-Euclidean *Catoptrics*, and this too agrees with the argument of the preceding section, identifying Apuleius' source with the pseudo-Euclidean work. It remains to determine whether this identification is compatible with our other evidence on ancient optics.

3. *Theon and the pseudo-Euclidean Catoptrics*

By associating the attested Archimedean work with the pseudo-Euclidean *Catoptrics*, and so assigning to the latter a date well in advance of Apuleius in the 2nd century A.D., we run directly against the standard view of the origin of the *Catoptrics*. For following Heiberg it has been accepted that the *Catoptrics* was the work of a late compiler – in Heiberg's view, of Theon of Alexandria around the mid-4th century –, drawing from a variety of sources which included writings by Hero, Ptolemy, and Archimedes⁵². This view has been elaborated by A. Lejeune, who distinguishes at least three different levels in the composition of the *Catoptrics*: the opening section, he maintains, forms a core of elementary materials antedating Archimedes and perhaps assignable to Euclid; another set of theorems, dealing with the placement of images in mirrors is of

⁵² In his earlier discussion (*Literargeschichtliche Studien über Euklid*, 150-153), although maintaining the inauthenticity of the *Catoptrics*, Heiberg did not attempt to identify the author. After completing his critical edition of the text (in *Euclidis Opera*, VII), he felt that he had the basis for a philological argument assigning the work to Theon ("Prolegomena", xlix-1). In either case, to the extent that issues of content bear on the question, Heiberg merely accepts the judgment of earlier critics on the defective technical execution of the work.

later origin, but certainly earlier than Ptolemy, and perhaps due to Hero; certain results are ascribable to the compiler himself, perhaps to be identified with Theon⁵³. Lejeune argues these claims through affinities between portions of the *Catoptrics* and items held in works by these other writers. But neither Heiberg nor Lejeune, nor those after them have taken into account the affinities noted in the preceding sections, which establish that a work conforming to the over-all structure of the *Catoptrics* was already accessible to Apuleius, fully two centuries before Theon. This at once raises questions about the validity of their dating argument.

Although the extant manuscripts name Euclid as author of the *Catoptrics*, Heiberg doubts this attribution on the grounds that the earliest notice of a Euclidean *Catoptrics* is in Proclus (latter part of the 5th cent. A.D.), and that the work suffers from errors and deficiencies in proof⁵⁴. Heiberg argues further that certain aspects of terminological usage link the *Catoptrics* (C) more closely to Theon's recension of the *Optics* (T) than to the Euclidean version (O)⁵⁵:

(1) In O the term for "visual ray" is usually *aktis* (74 times), less frequently *opsis* (20 times); in T this disparity is notably reduced (52 times vs. 20 times, respectively), while in C only *opsis* appears (70 times).

(2) In O the term *oukoun* for "therefore" is used sparingly (15 times), but in T it is frequent (50 times); in C (a work about half the length of the *Optics*) it appears 22 times, that is, with roughly the same frequency as in T.

(3) A manner of referring to angles by a single letter, e.g. as *hê A* and not as *hê pros tôi A* [lit: "the (angle) at the (point) A"] is found frequently in C, rarely in T, but not at all in O.

On these grounds Heiberg expresses his "suspicion" that the *Catoptrics* was compiled by Theon, moved by his work on Euclid's *Optics* to compose this companion piece; in Heiberg's view one has cause for doubting that Euclid ever composed a catoptrical work at all.

Heiberg's case is far from compelling. The absence of early citations of the *Catoptrics* is not altogether different from the situation of the Euclidean *Optics*: Heiberg offers citations no earlier than Theon in the 4th century, and one can find in it too errors, ambiguous or incomplete statements of principles, and

⁵³ *Recherches*, Pt. I, ch. 3 and Pt. II, chs 2 and 3.4.

⁵⁴ See the works cited by Heiberg in *Studien*, 90-91, 148, 150-151. At the same time he argues in favor of the authenticity of the Euclidean *Optics*, against the judgment of many critics, despite the presence of technical deficiencies in that work.

⁵⁵ *Eucl. Op.*, VII, xlix-1.

surprising lapses in many of its proofs; yet the authenticity of the *Optics* is not now questioned⁵⁶. As for the stylistic claims, Heiberg is misleading in his statement that the diction *hê A* is "*inaudita*" in Euclid's *Optics*; for it appears in props 9, 38, 42alt, 48, and 53. To be sure, in each of these cases the letter stands for the vertex, so that *hê A* is equivalent to *hê pros tôi A*⁵⁷; but the nuance is far slighter than Heiberg makes out. More important, the appearance of the shorter diction, as in Theon and the *Catoptrics*, cannot be viewed as idiosyncratic of Theon; it is found in Diocles' treatment of burning mirrors (props 1-3),⁵⁸ in Hero's *Catoptrics* (props 6-8, 10), and in scholia on optical theorems⁵⁹. Thus, however rare the short form is in the geometric tradition, our evidence must lead us to view it as virtually standard in optical writing. This and other features of the terminology in *C* may be archaisms perpetuated within the optical tradition⁶⁰; but it is entirely unclear why Theon should

⁵⁶ Before Heiberg, the authenticity of the *Optics* was seriously questioned. In defending it, Heiberg insists that it is inappropriate to impose modern technical standards on work deriving from the earliest phases of a science. This is an attitude he could support by citing Kepler, who also read the work with sympathetic eyes (cf. *Studien*, 90-91). But apparently this act of grace did not extend to the *Catoptrics*, whose faults he pronounced to be more severe (*ärger*) and numerous than those of the *Optics* (*ibid.*, 150). It becomes evident, however, that a vicious circle is operating: convinced of the inauthenticity of the work, one is predisposed toward finding fault with it; see below, esp. sect. 8.

⁵⁷ The diction *hê pros tôi A* is frequent in the lemma preceding prop. 36, and in props 41, 48.

⁵⁸ *On Burning Mirrors*, props 1-3. Toomer, who suggests this might be an archaism with Diocles, notes the additional parallel with Aristotle's *Prior Analytics* I, 24, 41b5-22 (*ibid.*, 151).

⁵⁹ See the scholium to Theodosius, *Sphaerica*, III, 11, ed. Heiberg, 1927, 196.2. (It is reproduced in my "Ancient Versions of Two Trigonometric Lemmas", *Classical Quarterly*, 35 [1985], 362-391.) Of course, these usages are rife in the scholia to the *Optics*, in both the Euclidean and Theonine recensions, and to the *Catoptrics*.

⁶⁰ See note 58 above. Other ostensible archaisms may be noted: (1) The *Catoptrics* admits mixed angles, that is, the space formed at the intersection of a straight line and a circular arc; the same is found in Hero (prop. 8, 10), the Aristotelian passage (cited in note 58), the Bobbio mathematical fragment (see sect. 7), and possibly also Diocles (prop. 3; cf. Toomer, *op. cit.*, 156-157). (2) The *Catoptrics* expresses the sum of two angles by simple juxtaposition (e.g., *hê AB* for "angle A plus angle B"); it also appears in Diocles (props 1-2), Hero (prop. 8), and Theon's recension of Euclid (props 8, 44). The same notation is found for other types of magnitudes in some places: numbers (e.g., Archytas' fragment on epimoric ratios, DK 47 A19), areas (e.g., Hero's *Metrica* I, 32) or weights (e.g., Archimedes' *Plane Equilibria* I, 7); but it is hardly frequent in the standard geometric tradition. In these respects, I think we should infer that the optical tradition has perpetuated a usage present in the older geometric literature, but largely abandoned by later geometric writers following Euclid's example. These 'archaisms' are compatible with an early dating of the *Catoptrics*, but of course do not exclude a later dating; they certainly do not constitute an argument for a late dating, however.

assume such an archaic manner when he has the clear model of Euclid's usage before him.

As Heiberg acknowledges, the usage in the *Catoptrics* in these three respects does not actually *agree* with Theon's, but rather it sharply *exaggerates* Theon's in comparison with Euclid's. Thus, even if the evidence is accepted as arguing against Euclidean authorship, it by no means sustains the attribution to Theon. The above remarks on angle notations already raise doubts about Theon's authorship; the stylistic coincidences in item (2), for instance, are compatible with an alternative view that the extant *Catoptrics* represents the Theonine recension of an older work, while the other items are in better conformity with this view than with Heiberg's⁶¹.

Heiberg's view of Theonine composition faces further difficulties when one considers the sources Theon would be working with. First, if there were a

⁶¹ The Greek ms tradition strongly associates *Cat.* (C) with Theon's recension of *Opt.* (TO), rather than with the Euclidean form (EO). Of those codices which contain both works, according to Heiberg, twenty-seven pair C and TO (cf. *Eucl. Op.*, VII, xvi-xviii); only one (cod. Marc. 303) pairs C with EO, but even here, Heiberg can argue its prototype of C to be one of the Theon-related mss (Vat. gr. 192; cf. *ibid.*, xliii-xliv). The medieval tradition of C, however, reveals a strikingly different pattern. Both *Cat.* and *Opt.* were translated directly from Greek to Latin in the 12th cent., and these also frequently circulate together in the same codices (cf. *ibid.*, xv, li). Heiberg has prepared a Latin text of *Opt.*, based principally on a single ms (*ibid.*, xv), and set it in parallel with his Greek text of EO; a critical edition of the Latin *Opt.*, based principally on five mss (in the context of twenty-one cited mss), is given by W. Theisen ("Liber de Visu", *Mediaeval Studies*, 51 [1979], 44-105). From both texts it is clear that the Latin is a literal rendering of EO, rather than TO. (For a more extensive list of mss, see D. Lindberg, *A Catalogue of Medieval and Renaissance Optical Manuscripts*, Toronto, 1975, 46-55.) The Latin of C has not been edited, but Heiberg discusses its ms tradition and offers some specimens (*op. cit.*, li-liii). From my own inspection of eleven mss (or about one-fifth of those listed by Lindberg), I discovered that the text exists in two forms. One follows the Greek *Cat.* literally (e.g. Flor. I, 32; Flor. V, 30; Berlin 510; BM Harl. 13; BM Add. 17368; Oxf. Corp. Chr. 251; Oxf. Corp. Chr. 283; cf. also Heiberg's specimen of *Cat.* 30 from Torun IV^o 2, *op. cit.*, lii). The other retains the Latin wording of the enunciations in the literal version, but entirely reworks the demonstrations (e.g. BM Sloane 285; Oxf. F. Auct. 5.28; Ven. 1647 (332); Dresd. Db86; cf. also Heiberg's specimens of *Cat.* 1-3 from Dresd. Db86, *op. cit.*, li-lii). This manner of paraphrasing is reminiscent of Campanus' handling of the *Elements* (cf. J.E. Murdoch, "Euclid: Transmission of the Elements", *Dict. of Sci. Bio.*, IV, 446). One may note that the division between the two forms in the codices does not follow the same pattern that Theisen has deduced for the Latin *Optics* (cf. the stemma given in *op. cit.*, 60). A wider investigation of the Latin *Catoptrics* ought to provide insight into its provenance and use among medieval optical writers. For the present, it suffices to observe that the affiliation of C with EO in the Latin tradition conflicts with that of C with TO in the Greek. The affiliation in the Latin may be merely accidental. It nevertheless recommends caution in any use of the Greek affiliation to support the claim that Theon composed the *Catoptrics*.

genuine Euclidean *Catoptrics*, why would Theon choose to produce an entirely new work, rather than merely touch up the older one, as he had done in the case of the *Optics*⁶²? Perhaps appreciating this issue, Heiberg conjectures that Euclid never wrote on catoptrics⁶³. Lejeune, who wishes to admit such a Euclidean work, presumes that Theon found it too out-of-date to be usable as the basis for a new textbook in the field⁶⁴. It is hard to conceive, however, that Theon would have viewed the *Optics* as "state of the art"; yet he did not abandon it as a model. More to the point, the *Catoptrics*, as extant, certainly does not represent the range of the ancient tradition. The technical level evident in Diocles' writing, and others drawing from it⁶⁵, is not even hinted at in the *Catoptrics*. We would suppose, on the basis of textual correspondences, like those already noted, that Theon included among his sources catoptrical works by Archimedes, Hero and Ptolemy. But in this lies the least credible aspect of the standard view. For Theon, admittedly not a profound mathematical intellect, was nevertheless entirely competent as an editor of technical subjects, as his editions of Euclid and his commentary on Ptolemy make plain. But this level of expertise is inconsistent with the notion that he could draw from the best of the ancient optical tradition and produce a work marred by those defects which led Heiberg and others to reject its Euclidean authorship out of hand. Lejeune's proposal hardly circumvents the problem: if the more primitive parts of the *Catoptrics* derive from a source representing the level of the theory near the time of Euclid, why would Theon select such a source *in preference to* superior treatments by Archimedes and other later writers?

Doubtless, one could construct solutions to these difficulties. But the simplest response is merely to discard the hypothesis of Theonine authorship. We have established the basis of an alternative view: that the *Catoptrics* is the extant version of an older work, from whose prototype writers like Hero and Ptolemy drew in the elaboration of their own propositions; the connection with Archimedes is now explained, through Apuleius' testimony, as the result of a misattribution to him of this prototype of the pseudo-Euclidean *Catoptrics*. This radical change of view instigates the project, to be undertaken in Part II, of reexamining the evidence bearing on the relation of the extant *Catoptrics* to other ancient writings on optics.

⁶² Cf. also Theon's editions of Euclid's *Elements* and *Data*.

⁶³ *Eucl. Op.*, 1: "*tum causa est dubitandi, scripseritne omnino Catoptrica Euclides*".

⁶⁴ *Recherches*, 147.

⁶⁵ Cf. the theorems on burning mirrors in Anthemius and the Bobbio mathematical fragment, discussed in my "Geometry of Burning Mirrors".

PART II *The place of the pseudo-Euclidean Catoptrics*

Although correspondences between the pseudo-Euclidean *Catoptrics* and other ancient works on optics have long been recognized, these have invariably been explained through the hypothesis that the pseudo-Euclidean writer (in most accounts, Theon) used these others as sources. This view of the direction of dependence is never argued, at least as far as I can determine from the accounts in Heiberg, Lejeune, Schmidt, Schöne and the scholars they draw upon, but rather seems merely to follow from their acceptance of the late origin of the *Catoptrics*. In this section, then, we consider these correspondences in relation to the dating question.

4. *Damianus and the Catoptrics*

The *Optics* of Damianus of Larissa is a short tract presenting an informal, sometimes anecdotal, account of the basic principles of optics. Its twelfth chapter treats of the bending of rays due to refraction:

When we look into water, we view the surface directly and the visual ray moves unbent, but we see things swimming or lying underwater when the visual ray advances into the depth, being bent however by its resistance. Now *if into a vessel something* [is tossed] *in* [and assumes a distance so that it] *is not seen, if, the distance being the same, water is poured in, the object tossed in will be seen*, when before it was not seen.⁶⁶

The words in brackets in the second sentence are missing from the manuscripts, but can be supplied through comparison with the sixth postulate of the pseudo-Euclidean *Catoptrics*⁶⁷. For the remaining part of the sentence (shown here in emphasis) conforms *verbatim* with the reading in the *Catoptrics*.

The editors Schöne and Heiberg note well this agreement, but do not consider the difficulty it poses for their views of the dating and authorship of the

⁶⁶ I translate from the text of Schöne, 12.22-14.5. Emphasis mine.

⁶⁷ The agreement with the *Catoptrics* had already been noted by Heiberg (cf. Schöne, *ibid.*, 14n; Heiberg, *Studien*, 151n). Schöne's text would be rendered: "if into an empty (*kenon*) vessel ...", where *kenon* is his emendation for the manuscripts' *ti enon* (lit.: "something being in"). This change can be defended (cf. the parallel in Cleomedes, ed. Ziegler, 224.14, where, however, the general wording is quite different from Damianus'). But the correspondence with the *Catoptrics* suggests taking *ti enon* as a scribal attempt to emend a corrupted *ti emblêthêi* ...

two works. In Heiberg's view, Damianus' writing is based on that of a near contemporary, Heliodorus, whose dating before Theon he holds "*pro certo*"⁶⁸. Since Heiberg also proposes Theon as author of the *Catoptrics*, we must infer that the wording of its sixth postulate is taken *verbatim* from a source. In view of the nontechnical character of Damianus' work, we would hardly take it for Theon's source. Presumably, then, we would have here an instance of their parallel, literal dependence on the same source.

But without Heiberg's prior commitment to Theon's authorship of the *Catoptrics*, we would naturally have supposed Damianus merely to be copying his line from it. That conforms with the rest of his *Optics*, for it borrows from a mixed assortment of nontechnical and technical sources, including Euclid's *Optics*, Hero's *Catoptrics*, and Ptolemy's *Optics*. In this way, we might hope to use Damianus as indicating a pre-Theonine provenance for the pseudo-Euclidean *Catoptrics*.

The difficulty with this argument is that the dating of Heliodorus and Damianus is far from assured. In the event that these figures lived well after the time of Theon, which I consider far more plausible than Heiberg's opinion, the coincidence would merely signify the availability of the *Catoptrics* to writers from the 6th century on, a fact no one would doubt anyway. I will defer to the Appendix (sect. 9) of the present paper a more detailed examination of this dating issue, for it now bears only peripherally on the *Catoptrics*.

5. Ptolemy and the *Catoptrics*

In its original state the *Optics* attributed to Ptolemy was a massive compendium on physical and geometric optics. The remnants which survive from four of its books through an Arabo-Latin recension contain ample materials on the reflection and refraction of visual rays. The parallels with materials from the pseudo-Euclidean *Catoptrics* are often striking and thus raise the issue of determining the relation of the two works. Lejeune has discussed many of these parallels in detail⁶⁹. As he subscribes to Heiberg's hypothesis on Theon's responsibility for the *Catoptrics*, he takes them as signs of Theon's use of Ptolemy⁷⁰. But his analysis leads him inevitably to conclude the basic incompetence of the editor, who either did not exploit Ptolemy's work intelligently, or knew of it only indirectly, through sources representing a more primitive level

⁶⁸ *Studien*, xxxii.

⁶⁹ *Recherches*, Pt. II, ch. 2.

⁷⁰ *Ibid.*, 146-149.

of the field⁷¹. In the following examples, I will seek to show how the converse view, that Ptolemy used and extended the *Catoptrics*, provides a more credible account of these materials⁷².

The pseudo-Euclidean *Catoptrics* (*E*) and Ptolemy's *Optics* (*P*) both base their theory of reflected images on three principles: that the image appears along the rectilinear extension of the visual ray; that the visual ray is reflected at equal angles with the mirror surface; and that the image appears situated where the extended visual ray intersects the extension of the normal line drawn from the object perpendicular to the mirror surface. For *P* these principles are established empirically; Ptolemy provides a careful account of experiments done with optical instruments designed especially for the study of these phenomena⁷³. By contrast, *E* enunciates the first principle as a postulate (post. 2) and attempts to prove the second and third as geometric theorems (*E* 1 and *E* 16-18, respectively) on the basis of other postulates (post. 3 and 4-5, respectively). The procedure in *E* must be viewed as contrived, although I find Lejeune's virtual diatribe against it uncalled for⁷⁴. As I will argue later, post. 3 is likely to have resulted from an editor's change, but post. 4-5 were already in the pre-Heronian version of *E*. What is entirely unclear, however, is why any editor knowing of the approach in *P* would formulate these principles in the manner of *E*, instead, for instance, of merely stating them as postulates. In his recension of Euclid's *Optics*, for instance, Theon retains the postulates with only slight, occasional changes in wording, while in a preface, thought to be based on Theon's introductory lectures, the phenomenological aspects of the same postulates are elaborated⁷⁵. It seems to me more plausible that a similar relation links *P* and *E*: that the formal treatment of the principles in *E* (perhaps in a version somewhat different from that extant) stimulated Ptolemy to con-

⁷¹ *Ibid.*, Pt. I, ch. 3. Lejeune voices his unsympathetic view of the *Catoptrics*, often in surprisingly strong terms. For instance:

le traité pseudo-euclidien représente un stade nettement décadent de la catoptrique grecque. Son auteur, piètre logicien et encore pire physicien, tout en étant incapable de s'assimiler la pensée et les méthodes de ses prédécesseurs, prétend néanmoins faire oeuvre de novateur. [p. 67; emphasis Lejeune's.]

He insists that this view should underlie one's inspection of the propositions which comprise the work, "*surtout celles qui paraissent ne présenter aucun sens cohérent*". To be sure, the *Catoptrics* has its defects; Lejeune's attitude is guaranteed to find them all, and more.

⁷² I will throughout speak of "Ptolemy's *Optics*", although I believe the reservations about its authenticity are serious; see the Appendix, sect. 10.

⁷³ Lejeune, *Recherches*, Pt. I, ch. 1; cf. also the references in note 40.

⁷⁴ *Ibid.*, Pt. I, ch. 3, esp. 58-62; cf. note 71 above.

⁷⁵ Recension of *Optics*, ed. Heiberg, 144-154.

firm them empirically and to make his experimental findings the basis of his introduction.

Both *P* and *E* set out general preliminaries on the convergence of reflected rays before examining the explicit construction of images. In the case of the concave mirror, for instance, *P* (IV, 64-65) sets the eye at *E* and considers the object set at three different positions *T*, *H*, *Z* along the reflected ray *BZ*, such that $BH = HD$, but $BT < TD$ and $BZ > ZD$ (Fig. 5a). It follows that *EB* is parallel to *HD*, whence the two lines, however extended, do not meet. The extensions of *EB*, *DT* will intersect each other at *K* behind the mirror surface, while those of *EB*, *DZ* will intersect at *L* in front of it (Fig. 5b). This provides the background for constructions in later theorems where the images will be placed at those points of intersection.

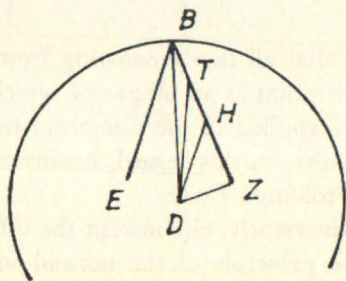


Fig. 5a

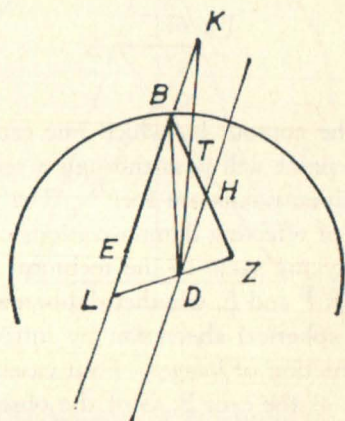


Fig. 5b

The *Catoptrics* undertakes a similar analysis in what appears to be an abortive first attempt at the construction of images. *E* 6 establishes a partial condition determining when rays drawn from the eye, set within the arc of a concave mirror, will be reflected as to intersect each other (Fig. 6a): if the chord $AB\Theta$ is equal to or greater than GBK , the reflected rays *GZ* and *AH* will meet; but if $AB\Theta$ is less than GBK , then it is asserted only that convergence sometimes occurs, sometimes does not. This result would provide a method for constructing the reflected image of the eye, were it the case that all such rays pass through the same point. That happens not to be true, however, as Diocles shows for a somewhat modified configuration⁷⁶; while Anthemius establishes

⁷⁶ *Op. cit.*, props 2-3. In Diocles' diagram (see Fig. 19), the rays enter in the direction parallel to the axis of the mirror, and are reflected through points on a segment of the axis; that point of intersection approaches more nearly the midpoint *H* of the mirror radius $A\Theta$

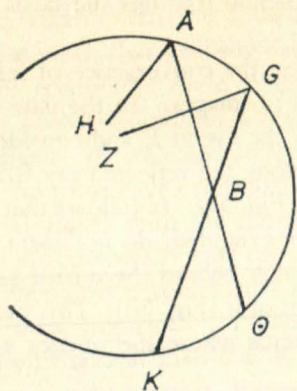


Fig. 6a

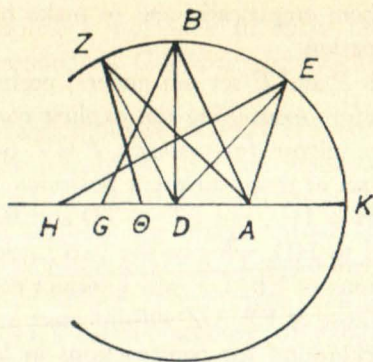


Fig. 6b

that the contour by which one can contrive that all rays emanating from one given point will pass through a second given point is an ellipse of which the two given points are foci⁷⁷. The techniques applied in the *Catoptrics* fall far short of effecting demonstrations of such results; in this regard, however, one can say the same of the technical level in Ptolemy's *Optics*.

Both *P* and *E*, whether deliberately or inadvertently, circumvent the difficulty of spherical aberration by introducing the principle of the normal for the construction of images. Four cases for the concave mirror are distinguished in *E* 28: as the eyes *B*, *G* of the observer lie between the center *D* of the mirror and the midpoint *N* of the radius *AD*, or at that midpoint *N*, or between *N* and

as the ray impinges on the mirror more closely to the axial point *A*. In this way, Diocles shows that the spherical mirror does not have a coherent focus, in contrast with the parabolic mirror (prop. 1). An elementary proof of aberration is possible on the basis of the figure in the *Catoptrics*. If it were supposed that all rays passing through *A* converged at a point *G*, then *G* could be found through the intersection of the diameter *KA* and the symmetrical ray *ABG*, where *DB* is perpendicular to *AB* (Fig. 6b); hence, $AD = DG$. But it follows that the ray reflected at any inward point *E* (between *B* and *K*) will cut the diameter at a point *H* beyond *G*. For in the triangle *AEH*, the line *ED* will bisect the angle at *E*, so that $AE:EH = AD:DH$; since $HE > EA$, *HD* must be greater than *DA*. Similarly, if the point of reflection *Z* lies outside *B*, the reflected ray *ZΘ* must cut the diameter between *G*, *D*. Ptolemy adopts an analogous procedure to demonstrate the weaker result, that rays emanating from a point on a diameter (say, *A*), upon reflection, will not converge at another point on that diameter (*Optics*, IV, 22-23; ed. Lejeune, 155-156).

⁷⁷ On *Paradoxical Mechanisms* (fragment), ed. Heiberg, 78-81. The construction is presented by T.L. Heath in *History of Greek Mathematics*, II, 541-543.

the mirror (at A), or beyond the center D (Figs 7a, 7b).⁷⁸ The cases turn on the inequalities $AG > GD$, $AG = GD$, $AG < GD$ and again $AG > GD$; in the first and last cases (Fig. 7b; corresponding to Ptolemy's point Z) the reflected ray BAG will meet normal line GD at E (although in the first case, for

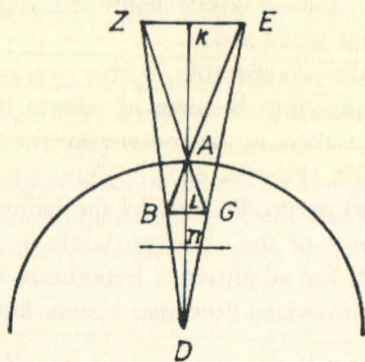


Fig. 7a

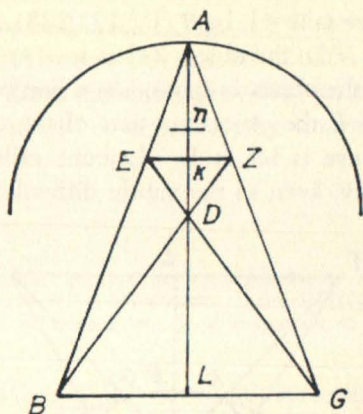


Fig. 7b

⁷⁸ The constructions in E 28 provide a straightforward derivation of the relations for object and image distance. For instance, for the case of real images (Fig. 7b), by similar triangles, $EK:BL = AK:AL$, where AK is the image distance and AL the object distance. Also by similar triangles, $EK:KD = LG:DL$, where $LG = BL$; hence, $EK:BL = KD:DL$. Thus, $KD:DL = AK:AL$. We thus have that AD is the harmonic mean of the distances AK , AL (since, according to the ancient definition of the mean, $AL-AD:AD-AK = AL:AK$). The Greeks recognized that this definition was equivalent to expressing the harmonic mean as the inverse of the arithmetic mean of the inverses of its terms; we may thus take AD as twice the focal distance (f), AL the object distance ($o.d.$) and AK the image distance ($i.d.$) to obtain: $1/f = 1/o.d. + 1/i.d.$ It is further clear that the derivation is unchanged if the object and image are interchanged; moreover, if one sets the object between A and the midpoint N of AD , the diagram for the virtual image will satisfy the same geometric relations (Fig. 7a); finally, interchanging object and image in this last case produces the configuration of the convex mirror. The Greek geometers seem not to have recognized this duality of object and image, nor to have worked out the quantitative relation for object and image distances. But in the latter instance, the arithmetic rule would be superfluous in view of the geometric construction. This simple derivation must condition somewhat Lejeune's insistence that the determination of quantitative (rather than merely qualitative) results for curved mirrors was beyond the ancient technical methods (cf. note 85 below).

eyes at E, Z and their images at G, B, since the intersection lies behind the observer, E claims that no image will appear), in the second these lines will be parallel (hence, no image will be formed), and in the third (Fig. 7a) the lines will converge behind the mirror. Embedded within these proofs in *E* 28 are precisely those "existence proofs" of the intersection of the rays which Lejeune insists are never given in the *Catoptrics*⁷⁹, following effectively the same argument as in the proposition from Ptolemy, cited above.

The analyses of specular images are also comparable in the two works. Consider Ptolemy's construction for investigating the sizes of images in the concave mirror. In *P* (IV 121-128), for instance, in the concave mirror ABG, center at D, the object ZH is seen as if at TK (Figs 8a, 8b).⁸⁰ Here, just as in *E* 28, the object is figured as a line bisected perpendicularly by the radius DB. But in *E* the object consists of the two eyes of the observer, while in *P* the (one) eye is set at E. Lejeune rails at *E* for admitting a hypothesis which Ptolemy, keen to the subtle difficulties in analyzing binocular vision, knew to

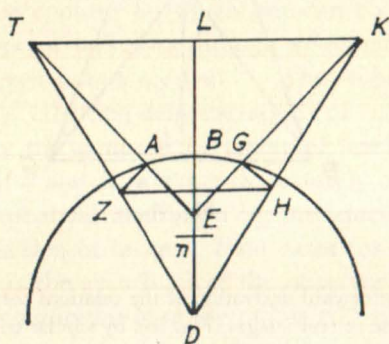


Fig. 8a

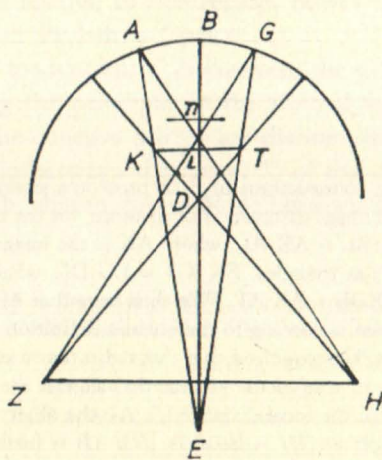


Fig. 8b

⁷⁹ *Recherches*, 126.

⁸⁰ As Ptolemy shows, the location and size of the image TK is dependent on the position of the observer E (props 65-67; cf. note 84 below). In the companion case for real images (Fig. 8b), the diagram produced by Lejeune shows K lying on ZG and T on AH. This condition yields a much simpler figure than the one I have drawn; but it is not assumed in the proof and, in general, is not valid.

avoid⁸¹. But this ploy in *E* seems straightforward and permits a geometric demonstration along effectively the same lines as that in *P*. In separating the eye from the object, *P* may have avoided the difficulties arising from double vision; but it thereby introduces complications which eventually overreach its technical abilities. We may overlook the difficulties of actually constructing the ray paths EAZ, EGH, for given points E, Z, H; for this issue is not even raised in *P*.⁸² The symmetrical disposition of the object ZH relative to the diameter BD (*P* denotes this special configuration '*secundum oppositionem*') is a feature *P* here shares with *E*, and in both it is a convenient, perhaps necessary, device for effecting the analysis⁸³. For the real images (Fig. 8b), *P* shows that the relative sizes of object ZH and image KT follow the ratio of ZD:DT.⁸⁴ The simpler configuration in *E* 28 (Fig. 7b) effects the same, providing an implicit quantitative relation of sizes to image and object distances; by contrast, the

⁸¹ *Ibid.*, 133:

jamais Ptolémée, ou quelqu'un qui tiendrait compte de ses résultats, ne se serait hasardé à une démonstration qui fit intervenir les deux yeux. Il saurait trop bien à quels phénomènes de fusionnement ou de dédoublement il risque de se heurter irrémédiablement.

Lejeune is correct in claiming that the virtual image actually seen under the conditions of *E* 28 (Fig. 7a) will be a one-eyed cyclops; for each eye will see the image of the other superimposed along the line through the axial point A. But *E* 28 need not be read as making a claim about the *appearance* of the image; the same cyclopean aspect would obtain if the eyes gazed in similar manner on a real face of the same size and distance as the image-face at EZ. The author of *E* 28 might assume that the location of the image does not change as our eyes shift their direction toward the mirror; indeed, if one traces the reflected rays joining B to L and G to L, the image is not likely to fall far off from K on EZ. Further, this manner of constructing the images cannot be invalidated by the criticism raised by Lejeune, since it is the same as that adopted by Ptolemy. *E* may be understood as effecting a geometric explanation of a familiar experience: that we do perceive our magnified image when we look into concave mirrors in this manner. The value of Lejeune's critique here is that it underscores some important procedural differences between *E* and *P*. The fact that *E* overlooks the subtleties of binocular vision in *P* is just what one would expect on the thesis of *E*'s earlier composition.

⁸² Lejeune notes the constructive difficulties (*Recherches*, 72, 74). On the techniques adopted by Alhazen for determining the reflected ray between two given points and a spherical mirror, see A.I. Sabra, "Ibn al-Haytham's Lemmas for Solving 'Alhazen's Problem'", *Archive for History of Exact Sciences*, 26 (1982), 299-324.

⁸³ III, 77-78; ed. Lejeune, 87. *P* introduces this restriction in order to circumvent the distortions of images in spherical mirrors, but adopts it even in the plane case, where it is unnecessary; contrast *E* 19.

⁸⁴ The construction must be completed by referring to the results on the distance of images in prop. 65 (IV, 114-117); Lejeune presents the combined analysis (*Recherches*, 88-90), and so gives it a far more comprehensive guise than it has in *P*.

more complicated figure in *P*, with the eye separate from the object, obscures this relation⁸⁵. Indeed, the image now loses its objective status, becoming dependent on the position of the eye; the analysis in *P* thus moves closer to the complexity of actual experience, but must settle for a more limited theoretical range⁸⁶. The resulting configurations in *P*, while constituting a richer field for study than that set up in *E*, nevertheless entails technical complications which *P* is often not equipped to resolve⁸⁷.

The thesis that *E* represents the early catoptrical theory provides a ready explanation for these aspects of *P*. The more ambitious project of *P*, to provide geometric demonstrations of detailed phenomena seen in mirrors, is not likely to have been undertaken without precedents of a humbler sort. In comparison, the simplifications assumed in *E* are effective, surprisingly so, for establishing the basic phenomena of mirror images. But in some respects, particularly with regard to the concave mirror, they are too gross to accommodate the results of observation. The identification of observer and object in *E* 28, for instance, would prove incompatible with *P*'s findings on binocular vision. One might consider modifying the diagram in *E* 28 (Fig. 7b), by setting the eye at *L* and constructing the image of *BG* (it would lie closer to *D* than *EZ* does); from this, one would recognize at once the more general configuration, for arbitrary position of *E* on the diameter, as in *P* (Fig. 8b). In this way, the limitations of *E* could motivate the experimental efforts of *P*, as in the study of binocular vision and the formation of images in concave mirrors. Lacking geometric techniques beyond those already applied in the proofs of *E*, however, *P* ultimately fails to establish the geometric theory corresponding to these phenomena. This view accounts for the general conformity in geometric approach in the two

⁸⁵ In his account of Ptolemy's theory of mirrors, Lejeune notes repeatedly that the assumptions of the theory prohibit a quantitative determination of the relation of objects and images in spherical mirrors (*Recherches*, Pt II, ch. 1; cf. 77-78, 83-84, 88, 90, 110-111). Strictly speaking, this is true; for spherical mirrors do not have a well-defined focus. But Lejeune apparently misses the possibility of deriving the familiar object-image rule in the context of the simplified configuration adopted in the *Catoptrics* (cf. note 78 above).

⁸⁶ Lejeune emphasizes that *P*'s adoption of visual rays, rather than luminous rays, as the model for optical theory, necessarily limits the range of the theory (*ibid.*, 73-74, 108-111). *P* thus measures the image distances along the line of the visual ray, rather than along the normal to the mirror (*ibid.*, 82, 86), and the image sizes in terms of the visual angle subtended at the eye (*ibid.*, 86-87). In the latter instance, *P* adheres to the conception of apparent size adopted in Euclid's *Optics*. But *P*'s approach, in both respects, diverges from that in the *Catoptrics*. The distinction between visual and luminous rays has been noted above (note 32), and will be a factor in our later discussion of burning mirrors (sect. 7).

⁸⁷ Lejeune cites from G. Loria the adverse judgment that if the *Optics* alone had survived of Ptolemy's treatises, we should have considered him a mediocre geometer (*ibid.*, 99). Lejeune is throughout Ptolemy's apologist in deflecting such criticisms (cf. *ibid.*, 110-111).

works, where *P* emerges as an effort to improve upon *E*. By contrast, the Heiberg-Lejeune hypothesis must portray these parts of *E* as an ill-managed paraphrase of *P*, in ignorance of its principal findings. But even this notion fails Lejeune, who ultimately proposes that *E* is based not directly on *P* but on pre-Ptolemaic sources. Lejeune's view thus becomes effectively equivalent to that now proposed, save that he requires the hypothesis of a lost treatise serving as common source for *P* and *E*. An important advantage of the new proposal, then, is that by taking *E* itself as Ptolemy's source, one has a working hypothesis for marking off Ptolemy's innovations from the content of the earlier theory.

E 13 solves the problem of "seeing the same object by means of several mirrors". It is here proved that for the object at *A* and eye at *B*, if three mirrors are set at right angles along the broken line *GDEZ*, then *A* will be apparent to *B* in the line of *BΦ* (Fig. 9). The proof introduces the points Θ , *K*, *L* in order to construct the triply reflected ray *BΦMPA* so that equal angles are made where it impinges on each of the mirrors; the normal principle is not introduced, so that the apparent location of the image observed by *B* is not constructed. Ptolemy takes up a remarkably similar configuration in IV 175.⁸⁸ Here one proposes to position mirrors so that a given object at *B* can be seen from a given point *A* (Fig. 10). The broken line *ADEB* is drawn⁸⁹, the angles at *D*, *E* are bisected by *ZD*, *EH*, respectively, and lines *TD*, *LEM* are drawn perpendicular to these bisectors. The equality of angles entails that mirrors set along *TD* and *LM* will reflect the ray from *A* to *B* through the points *D*, *E*, as

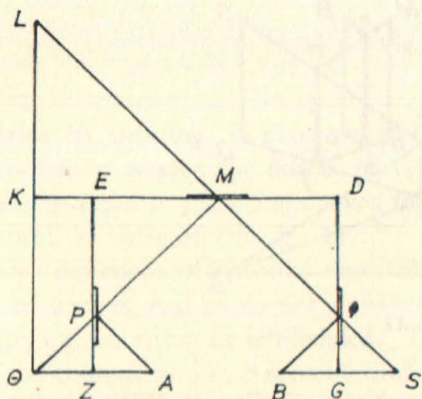


Fig. 9

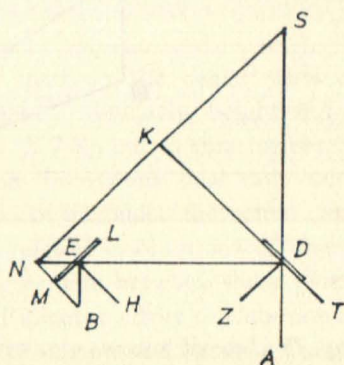


Fig. 10

⁸⁸ IV, 175-177; ed. Lejeune, 217-219.

⁸⁹ The broken line *ADEB* may be arbitrarily drawn; but the ms. figures (as shown by Lejeune) draw *DE* at right angles to the parallel rays *AD*, *EB*.

required. Ptolemy introduces the extensions to N, K and S to effect the construction of the images in accordance with the normal principle; it thus follows that A sees B positioned as if at S and that the image distance AS equals the object distance ADEB.

Ptolemy's construction, although straightforward, goes well beyond *E*, so that the view of *P*'s extension and elaboration of *E* seems possible. But *P* has expressed more clearly the practical situation to which the configuration applies, and this suggests a connection with the "theatral" mirror described by Hero in his *Catoptrics*⁹⁰. Hero does not give a full account of the geometric construction, nor does he discuss the question of image distance or provide a proof. But even if Hero's account is unlikely to be a source for *P*, it suggests the topicality of the construction. The geometric procedure that *P* adopts for orienting the mirrors, by drawing angle bisectors and perpendiculars to them, is precisely that used by Anthemius in his practical configurations of burning mirrors⁹¹. Since Anthemius is arguably drawing from sources predating Ptolemy⁹², I consider that the similarities noted here in *P* and *E* might follow from their parallel dependence on related sources, rather than *P*'s direct use of *E*.

In the related construction which immediately follows in *P* (IV, 178-182) one examines the orientation of multiply reflected images. Much as in the preceding, Ptolemy constructs the image CQ of the line BG seen from A through reflections by mirrors at KL, HT and EZ (Fig. 11). Lejeune notes a parallel to Hero's configurations (*H* 12 and 15).⁹³ But there is a fundamental

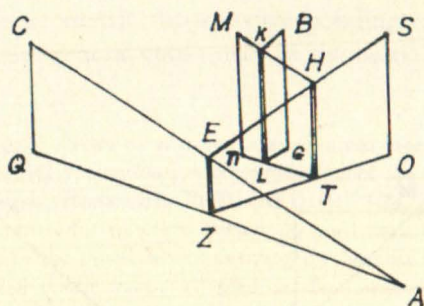


Fig. 11

⁹⁰ Prop. 15. Lejeune suggests a connection with prop. 17 (*Optique*, 219n); this may be a misprint, however, for the two configurations seem unrelated.

⁹¹ *Op. cit.*, 82-83. The same procedure for orienting the reflecting surface is applied continually in his practical construction of elliptical and parabolic mirrors.

⁹² In my "Geometry of Burning-Mirrors", I argue that Anthemius' practical constructions derive ultimately from a pre-Dioclean source, that is, at or before the mid-3rd cent. B.C.

⁹³ *Optique*, 221n.

difference between Hero and Ptolemy: in *H* a two-element (doubly reflecting) mirror is termed *dextrum*, or "right-handed", because the right hand of the object corresponds to the right hand of the image; in *P* it is the odd-numbered arrangements which yield *positio equalis*, while the even-numbered ones "make right appear left and left right". Thus, the conception of orientation in *H* is the opposite of that in *P*. Elsewhere, in the general discussion of the formation and properties of mirror images⁹⁴, Ptolemy is reluctant to speak of direct and reversed images in the manner of *H*.

The analogous sections of *E* on the orientation of images agree with *H* in this (*E* 19-20, 28). But earlier the manner of *P* is adopted: in *E* 9-10, 12 one looks at "lengths set to the side" (*plagia mêkê*); those seen in plane or convex mirrors appear "such as they truly are", while in concave mirrors they appear sometimes as they are, sometimes "turned about" (*antestrammena*). This terminology reverses the conventional manner used later in *E* and in *H*. But *E*

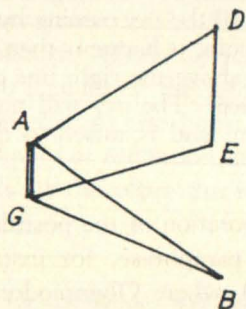


Fig. 12

explains its meaning: in plane mirrors, for instance, the nearer parts of the object appear nearer, the farther parts farther⁹⁵. Similarly, height and depth appear reversed in plane and convex mirrors (*E* 7-8), but in concave sometimes reversed, sometimes true (*E* 11).⁹⁶ These discussions deal only with the relative directions of reflected rays; they do not introduce the actual construction of images, and so do not consider the relative orientations of object and image, e.g., as right- or left-handed. The agreement between these parts of *E* and the construction in *P* seems to indicate Ptolemy's effort to elaborate on the plane case in *E*, by generalizing the number of mirrors and locating the images.

⁹⁴ *Recherches*, 100-103.

⁹⁵ *E* 9; ed. Heiberg, 302.5-8. Cf. Fig. 12.

⁹⁶ In these propositions one must assume that the mirror is set horizontally and viewed from above or below.

The converse relation is not likely, since *E*'s adoption of the alternative proof technique, by omitting the construction of the images, and its consideration of the convex and concave mirrors, in addition to the plane, have no analogue in the treatment in *P*.

Relating the treatments of refraction in *P* and *E* is more difficult, since only the postulate on refraction survives from *E*. But on the view set out in sect. 2, that the oldest version of *E* included the materials on refraction assigned to Archimedes by Theon and Olympiodorus, we obtain a larger basis for comparison.

P cites the familiar paradigm of refraction in the introductory remarks to the fifth book:

[That rays are refracted] is possible for us to know from the coin (*nummus*) in the vessel (*vas*)⁹⁷ ... For when the eye (*visus*) stands fixed in a place where the ray passing by the edge of the vessel is made higher than the coin, and then, its position remaining in this condition, water is poured into the vessel gently, until the ray passing by the edge of the vessel is bent within and falls on the coin, it happens then that the thing which was not seen before is now seen above the right line drawn from the eye to a place higher than the true place. The ray will not be thought to be bent, but that the coin itself swims and is raised to the ray.⁹⁸

This reads well as an elaboration of the postulate in *E*. But certain specific parallels to Olympiodorus' paraphrase, for instance, the designation of the object as a "coin" (*nummus*), where Olympiodorus reads "ring" (*daktylios*)⁹⁹, suggest that the extant text of *E* preserves a somewhat altered text. But if, as proposed above, Olympiodorus is working from a secondary source, it is possi-

⁹⁷ The text inserts here "*quod vocatur baptistir*" (*Opt.*, ed. Lejeune, 225.8-9). In Eugene's allusion to this very passage in his preface, the analogous phrase is "*quod vocatur fostir*" (*ibid.*, 8.14-15). Lejeune notes that the former corresponds to Greek *baptistērion*, while the latter is either a corruption of the same, or more likely, a cognate of *phôtistērion* (*ibid.*, 8n). In medieval Greek both terms refer to baptizing vessels, as is appropriate to the present context; but the manner of ancient usage is less clear. It seems to me most likely that the phrase derives from Eugene; this accounts for the use of different terms in the two passages, the first of which must be due to him. I suppose we should have expected *baptisterium* in this case; and the spelling *-ir* suggests an Arabic provenance. But Eugene might have transliterated *baptistēr* (an alternative form) in this manner. These difficulties seem less troubling to me than explaining why the original Greek author, or his Arabic translator, would have found this a helpful illustration of his procedure.

⁹⁸ *Optique*, V, 5; ed. Lejeune, 225.

⁹⁹ In Cleomedes' version of the phenomenon, the same term *daktylios* appears; cf. *op. cit.*, II, 6, ed. Ziegler, 224.12-23.

ble that his witness to the account he assigns to Archimedes might have been modified by, or even confused with, the account in Ptolemy.

The distinctive feature of Ptolemy's treatment is its application of detailed experimental procedures not only to establish the general character of refraction in various media (air, water and glass), but also to provide the basis for a quantitative relation between incident and refracted angles¹⁰⁰. In the accounts of the phenomena of refraction in the rest of the book, however, these empirical results are not applied. Instead, a purely geometric exposition, in the style of the Archimedean passages in Theon and Olympiodorus, is adopted. In its remarkable account of the displacement of the apparent positions of stars when viewed near the horizon, for instance, no attempt is made to quantify the displacement¹⁰¹. This omission would be surprising in the work of a scientist like Ptolemy, for whom geometric theory in astronomy is aimed toward practical implementation in computation, where discovery of the relevant numerical parameters depends on observation¹⁰². This, coupled with Ptolemy's silence on the need for refractive corrections in the *Syntaxis*, fortifies doubts which have been raised on the authenticity of the *Optics*¹⁰³. The disparity between the empirical and quantitative manner of the introduction and the theoretical and qualitative manner of the rest of the book can be taken, I believe, to signify that the body of the theory of refraction in the *Optics*, like the remarks in the commentators, depends on the older theory of refraction.

¹⁰⁰ *Optics*, ed. Lejeune, V, 5-21; for translations and discussions, see note 31. Lejeune examines Ptolemy's experiments and data on refraction in *Recherches*, Pt. III, ch. 1, where theoretical values computed by G. Govi (editor of Ptolemy's *Optics*, Turin, 1885) are reproduced for comparison. Ptolemy's listed values conform to a principle of constant second differences (that is, as the incident angles change by increments of 10 degrees, as measured from the normal, the refracted angles (in the air-to-water table) change by increments of $7\frac{1}{2}$, 7, $6\frac{1}{2}$, 6, etc. Although the values are computed, they must be based on actual observation, however, for when Govi's data are rounded to the nearest half-degree, Ptolemy's deviation is only rarely more than a half degree (*ibid.*, 155, 159). Lejeune notes that Ptolemy had access to astronomical instruments calibrated to $1/6$ degree; his use of a cruder scale for the refraction experiments may follow from the small size of the instruments and the inutility of more precise measurements (*ibid.*, 156-157). The second-difference method is a fixture of tabular work in Mesopotamian astronomy and appears in some Greek astronomical texts; cf. the accounts of "System B" methods in Neugebauer, *History*, Books II and IVD.1.

¹⁰¹ *Optics*, ed. Lejeune, V, 23-30; translated in Cohen and Drabkin, *op. cit.*, 281-283. See the discussion of Fig. 22 in sect. 10 below.

¹⁰² The current debate over Ptolemy's 'crime', of presenting computed data as if observed, would not, I suppose, lead one to maintain that actual observation was not a serious part of his activity as an astronomer.

¹⁰³ This issue is taken up in the Appendix, sect. 10.

Ptolemy, like Theon and Olympiodorus, provides the geometric explanation for the magnified appearance of objects seen in water. He explains also the converse phenomenon: that objects appear smaller when the eye is set in the denser medium and the object in the rarer¹⁰⁴. As noted in sect. 2, this would contradict Theon's account of the magnified appearance of the sun when seen near the horizon; Rome assigns the discrepancy to Theon's error¹⁰⁵. But even if the *Optics* insists that this same magnifying effect owes to causes other than refraction¹⁰⁶, it need not follow that the account of refractive diminution was a novel section of the *Optics*. For this is a natural complement to the account of magnification; its absence from the commentators would follow merely from its being irrelevant to their purposes. Ptolemy's account goes beyond that in the commentators in that it applies the normal-principle to refraction in order to locate the refracted images. That is, the refracted image of Z seen by D lies on the intersection of the visual ray DB and the perpendicular ZN (Fig. 13a). No

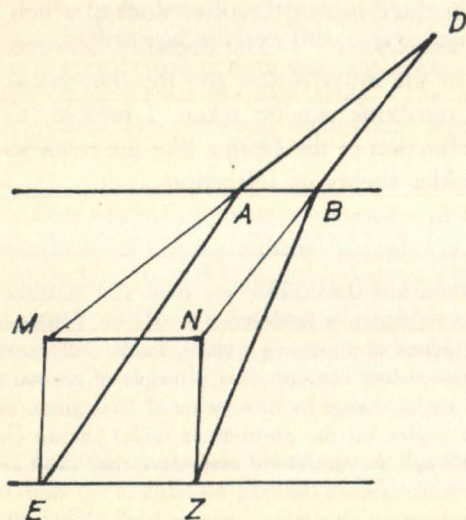


Fig. 13a

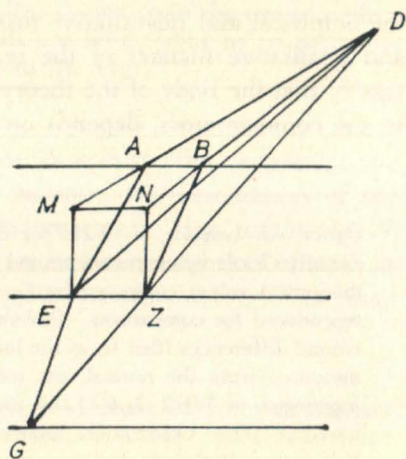


Fig. 13b

¹⁰⁴ *Optics*, V, 78.

¹⁰⁵ *Commentaires de ... Théon*, 347n, 350n.

¹⁰⁶ Cf. *Optics*, III, 59, ed. Lejeune, 115-116; and Lejeune, *Recherches*, 20. The view in the *Syntaxis*, elaborated by Theon, conforms with testimonia from Strabo and Posidonius; cf. Rome, *op. cit.*, 348n, 350n. A résumé of views is given by Cleomedes, *De motu circulari*, II, 1. P's explanation is not altogether clear. Rome speaks of "fatigue" thus suggesting a physiological factor. But P's *debilitas sensuum* and *difficultas actionis* must surely refer here to the "weakness of the optical effect", that is, our inability to detect parallax on distant objects (cf. III, 21, discussing our difficulty to discriminate distant objects); for P's

such explicit placement of the image is given in Theon's account, for instance; but it would provide the basis for a simple proof of the property he assumes, as having been proved in his source, on the relative increments in displacement for deeply submerged bodies¹⁰⁷.

Yet the application of the normal principle in the *Optics* is defective: the claim in P (V, 79-82) that the refracted image of a rectilinear magnitude will appear rectilinear; this requires that the points E, Z in the horizontal line will appear displaced to M, N, also in a horizontal line. Since N is determined by the horizontal through M and the vertical through Z, the visual ray DBN is also determined; one must thus introduce an additional condition establishing that the functional relation between incident and refracted rays is in fact compatible with this alignment. In fact, it is not. The refracted image of EZ will be a curve (in some regions nearly linear) oblique to the horizontal (Fig. 14a);¹⁰⁸

explanation concludes his account of stereoscopic vision. Lejeune is probably correct in describing P's explanation as psychological, but cannot be right in supposing that the apparent distances of overhead objects appear reduced in comparison to those at the horizon (*Opt.*, 116n). P maintains that we see objects on the horizon *secundum consuetudinem*, but objects overhead *extra consuetudinem*; thus the latter appear small, even though they might actually subtend visual angles equal to those of the former. The explanation, however loose, accommodates the phenomenon; but it seems to reverse the straightforward account. The familiar objects (like houses and trees) to which we compare the moon, for instance, when it is near the horizon, are here seen under an unwontedly small visual angle; but we interpret the disparity not as a diminution of their angle, but as a magnification of the moon's. Presumably, if we could see the culminating moon alongside a distant city skyline, the effect would be the same. Perhaps this is what P intends by the phrase *extra consuetudinem*: we do not habitually see familiar objects in positions elevated high above us.

¹⁰⁷ Suppose that a lower point G, initially along the line of DZ when the vessel is empty, has its image lying along the line DE when the level of water is at Z (Fig. 13b). If the water level is now raised to AB, the image of EZ will be lifted to MN, so that the refracted ray DAE must lie beyond the refracted ray DBZ. It thus follows, as Theon requires, that the refracted image of G (seen along the line DA) must lie beyond that of Z (seen along the line DB).

¹⁰⁸ I have used Ptolemy's data for refraction from air to water; the values for the ratio $i:r$ for the incident and refracted rays, in degrees measured from the normal are 60:40.5, 50:35, 40:29, 30:22.5, 20:15.5 for the respective positions A_1, A_2, A_3, A_4, A_5 . In each case, the refracted ray meets the base at E, where the perpendicular is drawn to meet DA extended at M. The image points M here happen to be approximately collinear, although markedly oblique to the base line. The extensions for i of 80 and 10 degrees (not shown) produce image points respectively below and above this line. That the image of the E line cannot itself be straight is evident through symmetry: if the figure is continued to the right, the image points M' will be set at distances and altitudes equal to those of the corresponding points on the left. Since the variation of i with r is continuous, the curve defined by the M's will be smooth; it is symmetrical with respect to the vertical axis through D, attaining a minimum at a point on the axis determined by the index of refraction.

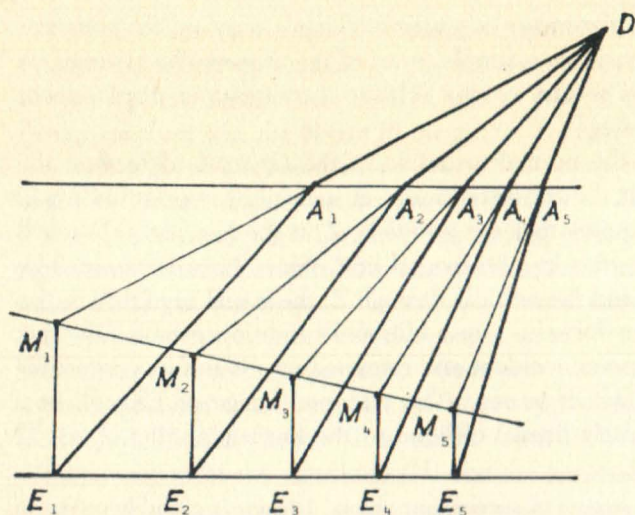


Fig. 14a

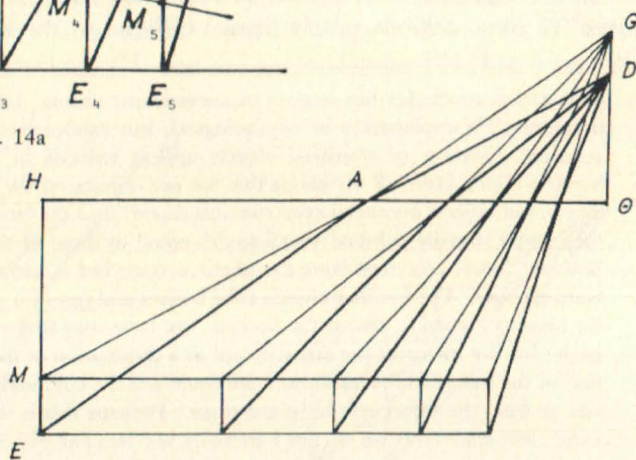


Fig. 14b

conversely, the relation which secures the disposition claimed by *P* is inconsistent with Snell's law, to which the data in *P* are a good approximation¹⁰⁹.

¹⁰⁹ For comparison, the values $i:r$ which yield a parallel line (at the level of the lowest of the above values) for the locus of the images *M*, in accordance with Ptolemy's claim would be, approximately, 20:16, 30:24.5, 40:33, 50:43, 60:53.5; these values have been adopted in Fig. 14b, and in each case agree to within one-half degree with the theoretical values, satisfying the relation $\tan i : \tan r = HE:HM$. Since *HE*, *EM* are constant, this relation is not consistent with Snell's law (where $\sin i : \sin r$ is constant), save for very small values of *i*. Thus, it could not conform to the values supposed by Ptolemy. The tangent relation entails that the refracted rays converge at a common point *G* above *D*, such that $G\Theta:\Theta D = EH:HM$; thus, when the eye is set at *G* in the denser medium, the refracted image will undergo a corresponding translation from *M* to *E*. The fact that the author of the *Optics* could be guilty of such major technical lapses must strengthen the case against its attribution to Ptolemy; see the Appendix, sect. 10, for further discussion.

One finds it difficult to accept that either Ptolemy or a genuine Archimedean source could have committed such a slip. Although I have taken pains to defend the basic competence of the pseudo-Euclidean *Catoptrics*, it is clear that an oversight of this kind is not inconsistent with the range of its technical proficiency.

On this view, *P* could find a precedent in *E* for its account of the displacement and size of the refracted images viewed over a plane surface; similarly, *P*'s next section, on the images seen in cylindrical vessels (V, 83-87) could also draw from a version in *E*. Because of the defective state of our sources, we lack evidence to argue a more specific proposal. But it should be clear that *P* has made substantial use of sources for its exposition of refraction, so that we should find it difficult to discriminate between the older materials and those due to Ptolemy. Lejeune, in effect, must hold a similar position, for he cannot deny *P*'s debt in some manner to the Archimedean *Catoptrics*¹¹⁰. The principal difference of the present view is that it does not accept the accounts by Theon and Olympiodorus as witness to the complete range of the older theory of refraction, so that more of Ptolemy's treatment is maintained to have been present in the source than Lejeune supposes.

By identifying the Archimedean and pseudo-Euclidean *Catoptrics* we obtain a subtly different impression of the relation of Ptolemy's *Optics* to its sources. For *P* would be drawing from *E* not only a preliminary version of the theory of reflection, but also the elements of its theory of refraction. The older catoptrical theory comes to appear more unified and comprehensive than one has usually presumed, while Ptolemy's responsibility for new discoveries appears somewhat less. Further, in being able to derive most of the background to his geometric theory from a single work, instead of from several, *P* no longer stands as the first attempt to synthesize these portions of the ancient field of optics; that honor must go to the anonymous author of the *Catoptrics*.

6. Hero and the *Catoptrics*

In his edition of Hero's *Catoptrics*, W. Schmidt calls attention to several passages linking Hero's work to the pseudo-Euclidean *Catoptrics*, and these are noted by Heiberg in his synopsis of ancient optics¹¹¹. These observations have

¹¹⁰ *Recherches*, 176-179. Lejeune emphasizes how Ptolemy has advanced the refraction theory initiated by Archimedes; but he expresses their relation in terms of generalized "stages" of development, rather than of Ptolemy's explicit use of Archimedes as a source.

¹¹¹ *Heronis Opera*, II, ed. Schmidt, 330-336, 342-343, 394-399; Heiberg, *Geschichte der Mathematik ... im Altertum*, 78.

been expanded by Lejeune in his effort to recover the various stages of development of the ancient field¹¹². As Schmidt notes, props 7, 9 and 10 in Hero (*H*) are in almost *verbatim* agreement with props 4, 24 and 5, respectively, of the pseudo-Euclidean work (*E*). But the comparison runs much deeper: both works present theorems on the sizes of images (*H* 6; *E* 19), on polygonal configurations of mirrors (*H* 17; *E* 14), on doubly reversing arrangements (*H* 11; *E* 28-29), and on composite mirrors producing multiple images (*H* 12; *E* 29). Since the Latin version which furnishes us our text for the lost Greek is defective, Lejeune supposes that Hero included some of the related additional materials which are held in *E*, but are absent from *H* as extant. In this way he makes tentative claims about the state of catoptrics at the time of Hero, about a century before the substantial advancement of the field by Ptolemy. Presumably, Archimedes' catoptrical studies would serve as primary source, but Lejeune is not clear on how he wishes to relate the Archimedean and Heronian strata in his analysis of the *Catoptrics*.

According to the view propounded by Heiberg and Lejeune, these correspondences follow from the use of Hero's work by the later compiler (Theon) of the pseudo-Euclidean *Catoptrics*. But even if we admit that our text of Hero is only an abridgment, this view of the dependence of the two works is not convincing. The pseudo-Euclidean work is a formal geometric effort, articulating certain principles as postulates and from these deriving proofs of the basic phenomena of mirrors. One may raise complaints about the order and accuracy of its proofs, but its ambition to provide a systematic exposition of the field is clear. What we have of Hero's *Catoptrics*, however fragmentary, accords with the form of his other extant writings, such as the *Pneumatics*, *Mechanics*, *Belopoeica* and others¹¹³. The style is eclectic, not systematic; Hero's major objective is to provide a descriptive survey of the designs of specific mechanisms. In these works he attempts to explain, as well as describe, so that he typically includes discussions of the appropriate technical principles, as these are relevant for understanding why the devices operate as they do. Sometimes, but only rarely, does he present proofs for the geometric notions introduced. In the case of his *Catoptrics*, only props 4-10 offer proofs; the remaining propositions (11-18) take the form of geometric problems¹¹⁴, but in each case only the geometric construction without proof is given. Arguably, the absence of proofs might follow from omissions by an abbreviator; but their absence is fully

¹¹² *Recherches*, 137-142.

¹¹³ On stylistic connections between the *Catoptrics* and other Heronian writings, see Schmidt, *op. cit.*, 303-306.

¹¹⁴ E.g., prop. 11: "to construct a right-handed mirror", that is, a mirror which produces images with true orientation.

consistent with Hero's procedure in his other technical writings.

The standard view would maintain that where they overlap, *E* has effected a formalization of *H*. But *H* would provide a model for only a small portion of *E*. For instance, although both works are concerned with the manner of images in mirrors, only *E* takes up the questions about the location and size of images. In props 16-18 *E* establishes that the image lies at the intersection of the visual ray extended from the eye and the line drawn from the object perpendicularly to the mirror; from these *E* deduces the properties of size, location and orientation of images in plane, convex and concave mirrors. By contrast, *H* nowhere refers to this normal line and its role in the formation of images. Lejeune must entirely ignore this gap, when he proposes that the accounts of images in *E* 19-20 and 22, for instance, were derived from *H*; ¹¹⁵ if we take his dissection of the pseudo-Euclidean *Catoptrics* seriously, however, we would have to set *H* at the earliest stage in the development of the field. Lejeune admits that the dependence of *E* on *H* might not be direct, but rather that their agreement could follow from parallel dependence on a common source. Presumably, this source would have the formal style of *E* and would include suitably extended treatments of the materials found in both *E* and *H*. In a word, *it would be E*; the hypothesis is otiose.

The geometric parts of *H* read well as an informal abridgment of the corresponding sections of *E*. In displacing *E* 24 forward, setting it in the context of *E* 4-5 (as *H* 7-10), *H* may be seen to repair what Lejeune and others take to be a notable flaw in the formal organization of *E*; ¹¹⁶ the converse hypothesis, that *E* has disrupted a perfectly natural ordering in *H*, would leave us unable to grasp the editor's motives.

Another discrepancy also suggests Hero's effort to improve on his source. The fourth and fifth postulates of *E* set out a principle on which the propositions on image location (*E* 16-18) will be based; for instance,

(*E*, post. 4): in plane mirrors, if the place is occupied (*katalêphtheis*) at which the perpendicular falls from the seen object, the seen object is no longer seen. ¹¹⁷

¹¹⁵ *Recherches*, 113, 127-142; cf. 142:

Il n'est pas invraisemblable que la seconde partie – la plus récente – de la Catoptrique pseudo-euclidienne reproduise plus ou moins la théorie des images spéculaires formées par les miroirs simples dont l'abrégé de Héron ne nous a conservé que les théorèmes préliminaires. [Emphasis Lejeune's.]

¹¹⁶ *Ibid.*, 116-117.

¹¹⁷ *Catoptrics*, ed. Heiberg, 286.10-12. Post. 5 enunciates the analogue for convex and concave mirrors.

The claim being made is not quite clear, and on the simplest reading is in fact false. For the perpendicular from the object need not actually touch the mirror (that is, the mirror need not extend that far, or be clear at that point) in order for the image to be seen¹¹⁸. I would suppose that the postulate intends the eye to be occupying the specified position, or to be set somewhere on the perpendicular; the failure to see the image from this position would thus indicate that the eye is blocking its formation, so that the image lies along that perpendicular. One of Hero's propositions is quite similar:

(H 6): in plane mirrors, there is a place such that when it is occupied (*apprehensus*) the image is no longer seen.¹¹⁹

The wording is almost identical, but a different situation is intended: one finds the point on the mirror where the broken line from the eye to the object makes equal angles with its surface; then if that point is covered with wax or the like, the image will not be seen. This is of course a correct claim; but it is superfluous in view of the results in H 4 (establishing the equal-angles property for reflected rays) and H 2 (an account of the rectilinear path of visual rays). The convergence of wording in these statements from *E* and *H* can hardly be accidental; the radical difference in sense must thus indicate a deliberate alteration of one into the other. The view that Hero has taken an unclearly worded, yet very important principle from *E* and recast it as a clear, yet unnecessary proposition in *H* is, I believe, more plausible than the hypothesis of a converse dependence of *E* on *H*.

Both *H* and *E* provide demonstrations of the equal-angles principle for reflection (*i.e.*, *E* 1 and *H* 4-5). Hero adopts the ingenious strategy of establishing that the broken line making equal angles at the mirror is the least of all lines proceeding from the eye to the mirror and thence to the object; in his argument, this must be the path of the visual ray, because the principle of the infinite (*sc.* immeasurable) speed of visual rays entails their passage along minimal paths¹²⁰. An entirely different approach is adopted in *E*; in its second postulate the following claim is made:

¹¹⁸ Heiberg notes this false claim, *Studien*, 153.

¹¹⁹ *Heron. Op.*, II, ed. Schmidt, 330. As noted by Heiberg (*Studien*, 153), this correspondence of *E* and *H* was recognized by Kepler.

¹²⁰ *Heron. Op.*, II, 320-322. In the alternative formulation transmitted by Olympiodorus, the underlying principle is that "nature does not act in vain" (*In Meteor.*, 212.5-10). Damianus, who cites Hero, conceives the argument in the manner of Olympiodorus (*Optics*, ch. 14). This discrepancy between Hero's account and the two others could result from the later writers' use of secondary sources.

if a mirror is set in a plane and one looks at a certain height which is at right angles to the plane, then the lines become proportional – as the line between the mirror and the on-looker is to the line between the mirror and the height at right angles, so is the height of the on-looker to the height at right angles to the plane.¹²¹

In *E* 1 this configuration is used for establishing the similarity of the two triangles formed by the eye and the object and their perpendiculars relative to the plane of the mirror, whence follows the equality of the corresponding angles. The procedure seems contrived, for the postulate is hardly a self-evident claim on which to base the proof of equal angles; by contrast, Hero's approach does attempt to work toward the result through a manifest principle, namely, the maximal speed of the motion of visual rays.

The correspondences already noted, linking *E* and *H*, suggest the possibility that Hero might have drawn from the same catoptrical source for his treatment of the reflection principle, so that in the case of this result, the extant text of *E* could result from changes by a later editor. The motives to make such a change can be understood: in the context of a formal effort like *E*, one might be uneasy at the introduction of a physical postulate (*i.e.*, the infinite speed of rays), or worse, a metaphysical one (*i.e.*, the economy of nature, if Olympiodorus' testimony on Hero is correct). The origin of the geometric substitute is also clear: in Euclid's *Optics*, prop. 19, the use of mirrors for measuring the heights of distant objects is explained; the unknown height has to a known distance BH a known ratio, that of $G\Theta$ to ΘH (Fig. 15). This proportionality follows from

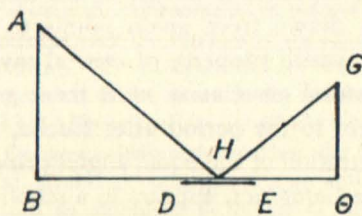


Fig. 15

the similarity of triangles $G\Theta H$, HBA , and this in turn follows from the equality of the angles $GH\Theta$, AHB , formed at the mirror by the reflected ray, "as is said in the *Catoptrics*"¹²². Even if the line is an interpolation, the fact that Euclid can assume this result here indicates not only that the principle of

¹²¹ *Cat.*, ed. Heiberg, 286.4-9.

¹²² *Opt.*, ed. Heiberg, 30.3.

equal angles was familiar at his time, but also that it had received some form of geometric exposition. What we find in the pseudo-Euclidean *Catoptrics* is the converse of Euclid's procedure, where the proportionality of lines $DA:AK = BG:GK$ is now postulated, and from this the equality of the angles E and Z is derived through the similarity of the triangles BGK , DAK (Fig. 16).

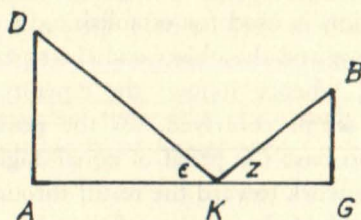


Fig. 16

One would suppose, then, that the extant text of *E* presents an alternative version of the proof of equal angles, following Euclid's theorem as a model for revising an older form. The version in *H* might represent that older form; but its use of the minimal-distance property is so strongly allied to geometric work in the later part of the 3rd century B.C. that its presence in a work that Euclid could have assumed is unlikely. In particular, the postulate that the straight line is the least distance between two given points receives its first formal statement by Archimedes, in the opening section of *Sphere and Cylinder*, Book I.¹²³ Further, the study of isoperimetric figures is advanced by Archimedes and Zenodorus¹²⁴, while Apollonius adopts as his definition of normals their being the minimal lines drawn from given points to given curves¹²⁵. One cannot deny that the least-path property of optical rays might have been proposed earlier; but its natural association with these geometric studies recommends assigning its origin to the period after Euclid, rather than before.

The form of a demonstration of the equal-angles principle, appropriate to the level and purposes of the *Catoptrics*, appears in a scholium to *E* 1 (cf. Fig. 16):

¹²³ *Arch. Op.*, ed. Heiberg, I, 8.

¹²⁴ Archimedes proves the maximal property of the hemisphere among all isoperimetric spherical segments in *Sph. Cyl.* II, 9; Pappus transmits the analogous result for semicircles and circular segments in his *Collection*, Book V; Zenodorus proves the maximal property of circle and sphere among isoperimetric plane and solid figures, respectively. For synopses, see Heath, *History*, II, 390-393 and 206-213, respectively. I have proposed that Archimedes himself anticipated some of Zenodorus' results; see my "Archimedes and the *Elements*", *Archive for History of Exact Sciences*, 19 (1978), 238-239, 283-284.

¹²⁵ See Heath, *History*, II, 158-167 for a résumé of Apollonius' theory of normals.

Archimedes says thus, that the angle Z is either equal to E or less or greater. Let Z first be greater than E ; therefore E is less. Now let the eye next be supposed (to be) D , and from the eye let (the visual ray) be reflected toward the seen object B . Thus the angle E will be greater than Z ; but it was also less, which is absurd.¹²⁶

The Archimedean attribution has spurred debate, one scholar considering this argument unworthy of Archimedes, another considering it not unworthy¹²⁷. Clearly, the subjective issue will not be settled to everyone's satis-

¹²⁶ Schol. no. 7 to *Cat.*, ed. Heiberg, 348.17-22. The lettering is that of prop. 1 (*ibid.*, 289). A paraphrase of the argument appears in Olympiodorus:

That reflection is at equal angles is manifest, since when, there being three points, one where the object is, another where the viewer is, and yet another where the mirror is, if they will transfer the object to where the viewer was, the same angle again will arise as was before when they were in their own places, and neither more nor less – clearly on the provision of the mirror's being kept in the opposing place [*In Meteor.*, 211.30-212.4].

This is followed immediately by the "geometric proof" in the Heronian manner; Olympiodorus concludes that the principle has thus been proved, "according to the two enterprises (*epicheirēmata*), the physical and the mathematical" (*ibid.*, 213.21-22). Lejeune reconstructs from this passage an empirical procedure, to be assigned to Archimedes (*Recherches*, 50-53). Although Olympiodorus describes this procedure as "physical", both his account and that in the scholia are phrased in standard geometric terminology. Even the ostensibly kinetic "transfer" (*ameibein*) has analogues in the geometric literature (cf. *metapiptein* in *Data*, props 25-30). I think that Olympiodorus could have introduced the term "physical" through suggestion by a source which had labelled it a *phenomenon*. This is a term used in the *Catoptrics* to denote its own postulates; specifically, props 16-18, refer thus to posts 4-5 (although, alternatively, in prop. 1 the term *boros*, "definition", refers to post. 3). The relation between the "phenomena" in posts 4-5 and the theorems in props 16-18 is comparable to that between the versions of the equal-angle principle in Olympiodorus and the scholium; that is, Olympiodorus has not offered the proof of the principle, but only the postulate assumed in the proof in the scholium. We have here another suggestion of Olympiodorus' dependence on a secondary source; for it is clear that no technical writing, say, a *Catoptrics* attributed to Archimedes, would hold both this and the Heronian versions of the equal-angle principle, but an anthology of proofs would. An important example of such an anthology is the set of cube duplications compiled by Olympiodorus' contemporary, Eutocius, in his commentary on Archimedes' *Sphere and Cylinder* (cf. *Arch. Op.*, ed. Heiberg, III, 56-106). Use of such a source could account for the discrepancies between Olympiodorus and Hero (see note 120).

¹²⁷ Lejeune defends the Archimedean attribution in "Archimède et la loi de réflexion" and *Recherches*, 50-53. But Rome demurs: "tel que la scolie le rapporte, ce raisonnement n'est pas concluant, et nous avons peine à croire que ce soit de l'Archimède authentique" ("Notes sur les passages des Catoptriques d'Archimède", 34). Pedersen also criticizes the proof, for its assumption that the reflected rays will meet the mirror at the same point K in both conditions (*Early Physics*, 130-131). In the passage discussed in the preceding note, however, it appears that the effort to articulate some such assumption was part of the ancient version.

faction. But the very fact that there could arise such a disagreement over the intrinsic merit of the proof attests to its primitive quality, suiting it to an early stage of technical expertise more readily than to what we would expect from Archimedes. The key notion is the assumption of symmetry, that one and the same broken line marks the path of the visual ray from the eye to the mirror to the seen object, regardless of which of the two extremities is the eye and which the seen object. This would certainly qualify as a postulate, for inclusion among the other principles prefacing the work; or it might be considered sufficiently evident that the need for its formal articulation could escape notice.

Although the *Catoptrics* does not employ arguments from symmetry, the principal theorems on reflections in concave mirrors (*E* 27-28) adopt a comparable configuration: visual rays from each of the two eyes are drawn out toward the other eye as the seen object¹²⁸. Further, the results established in *E* 2-3 adopt the same configuration and follow the same simple line of argument that one finds in the scholium¹²⁹. These parallels suggest that an older version of the *Catoptrics* followed the line of proof in the scholium, but that a later editor, somehow dissatisfied with it, substituted the artificial mode now extant, based on the mensurational application in Euclid's *Optics*, prop. 19.

Since the scholiast names his authority, he is evidently following a source. His account is a paraphrase, however, for he remains faithful to the lettering of the figure in *E* 1 as well as its terminology. What one must explain, under the present view, is how he came to attribute this proof to Archimedes. From the discussion of the passage from Apuleius' *Apology* we have seen that a work conforming to the description of *E* circulated in the 2nd century A.D. under the name of Archimedes. Now, any technical work contains a number of intrinsically interesting results, suitable for excerpting in general compendia; in the present case, this would certainly apply to the statement and proof of the equal-angles principle. Olympiodorus provides an example of a commentator lifting a proof of the equal-angles principle out of its technical context (namely, Hero's *Catoptrics*) for insertion into a broader discussion of optical principles¹³⁰; he and Theon do the same for the refraction principle (extracting from their Archimedean source), as does Damianus (although without proofs). We cannot be certain that such secondary writers prepared their comments through direct consultation of primary sources; Theon may have, but the others

¹²⁸ Cf. Figs 7a, 7b and note 78.

¹²⁹ In *E* 2 it is claimed that the ray falling perpendicularly on the mirror is reflected back on itself, for otherwise unequal angles would be equal. *E* 3 shows that an oblique ray will not be reflected back on itself or into the part of the plane by the lesser angle, for here too the lesser and greater angles would become equal.

¹³⁰ See sect. 2 above.

probably did not. Several compendia of mathematical materials were compiled at different times in later antiquity, among which the multi-volume encyclopedia of Geminus (1st century A.D.) was an especially important source for Proclus and other commentators; Geminus also produced a commentary on the meteorological work of Posidonius¹³¹. Alexander cites Geminus, doubtless from the latter work, for an argument about the cause of the rainbow¹³². It is thus clear that compendia and commentaries were among the sources consulted by the scholars in later antiquity for their technical information.

Since this type of text could have diverse lines of transmission – through the primary treatises, through derivative works like encyclopedias and miscellanies, through commentaries on other works, and so on, – the separate transmissions could effect the result that one source for a particular datum came to present a different account from another. I think this can explain why our testimonia to the *Catoptrics* differ on their evidence about the equal-angles proof. Under my view, the scholiast's form of the proof appeared in the oldest version of the *Catoptrics*, a work which had come to be accepted as Archimedean by the time of Apuleius, and doubtless also at the somewhat earlier time of Geminus. Then, any anthologist, like Geminus, in excerpting from the *Catoptrics*, would attach Archimedes' name to these extracts. The Archimedean attribution is still recognized by Theon, if we accept that he refers to his catoptrical source directly; but the extant *Catoptrics* has reached us under the name of Euclid. Thus, a stage of editorial revision must be supposed, responsible not only for the changed attribution, but also for a new proof of the equal-angles principle, doubtless also a variety of minor stylistic changes, and perhaps the loss of the sections on refraction and meteorology. If Proclus' allusion to Euclid's *Catoptrics* refers to our work¹³³, this editing would have occurred between the

¹³¹ A description of Geminus' work is given by Heath, *History*, II, 222-234; his commentary on Posidonius is cited there, p. 231.

¹³² That is, that it is due to reflection (*In Meteor.*, 152). Alexander cites from Philip, Plato's associate, the phenomenon that rainbows move to the side along with the observer, just as reflected images do. From Geminus and Aelius he cites another phenomenon, that they appear to recede from or approach toward the observer, with the respective recession or approach of the observer, also in the manner of reflections. Alexander admits that the latter phenomenon does indeed obtain for mirror images; but with justified caution he proposes that one ought to investigate its validity for the rainbow. Since the rainbow subtends a constant arc, it will of course recede from the approaching observer; this is quite the opposite of Geminus' claim.

¹³³ According to Proclus (*In Euclidis Elementorum Librum I*, ed. G. Friedlein, 69), Euclid produced many scientific treatises (*synggrammata*) of remarkable precision: "such are both the Optics and the Catoptrics, and such also are the Elements of Music, and further the Book on Divisions". His pairing of the two optical works might be taken to suggest their circulation together. This conforms with the pattern that most of the Greek mss of the

mid-4th and mid-5th centuries. That Olympiodorus still associates the refraction principle with Archimedes could follow from his relative closeness in time to this transitional phase, for consultation of the older version would then still be possible; but I consider it just as likely that he worked with secondary sources based on that older version. Heiberg assigns the scholia to the 9th century¹³⁴; by that time access to the primary Archimedean sources would be limited¹³⁵, so that our scholiast would most likely have resorted to a secondary source for the 'Archimedean' proof of the equal-angles principle he has attached to *E* 1.

The present account lends an ironic touch to this reunion of the *Catoptrics* and its original proof of the equal-angles principle. The route is perhaps more twisted than some might like. But such convolutions of transmission are within the range attested frequently in the extant record of ancient technical writing, and no feasible account of the evidence bearing on this case could be simple.

7. *Diocles and the Catoptrics*

Certain issues bearing on the *Catoptrics* have already suggested links to the technical activity near the time of Euclid. For instance, that Euclid can assume the equal-angles property in the *Optics*; that Hero's alternative proof (based on the least-distance property) has associations with geometrical researches in the 3rd century B.C.; that Theon's wrong account of horizon magnification is compatible with cosmological assumptions current in the late part of the 4th century B.C., but not thereafter – these items point to an early dating for the *Catoptrics*. The same association is more clearly indicated in the pseudo-Euclidean treatment of the burning mirror.

The last theorem of the *Catoptrics* (*E* 30) demonstrates in two parts the caustic property of the spherical concave mirror. In the first part, it is proved that an oblique ray from the sun will be reflected to intersect the mirror's diameter at a point between the mirror and its center (Fig. 17). The proof is

Catoptrics and *Optics* (in Theon's recension) are preserved in the same codices (see Heiberg's survey in *Eucl. Op.*, VII, xvi-xviii. In Heiberg's view, Theon composed the *Catoptrics* as a companion work to his recension of the *Optics* (*ibid.*, 1). But the evidence agrees no less with the view that Theon produced two works, each the recension of an older work.

¹³⁴ *Eucl. Op.*, VII, xlviii.

¹³⁵ Note that no such catoptrical work is included in the extant Archimedean corpus, whose prototype manuscript, Heiberg maintains, was compiled at the order of Leon of Byzantium around this same time (*Arch. Op.*, III, xc).

correct, although it suffers from minor omissions, clarified by the scholia¹³⁶. In the second part one considers solar rays coming in along the mirror's diameter; since these rays have a common intersection at the center, it is claimed that burning will take place there (Fig. 18). The second part does not use the result from the first, and in fact is inconsistent with its finding that reflected rays do not converge at the center.

Diocles treats of spherical reflectors in props 2-3 of his tract *On Burning Mirrors*. He effectively trivializes the result in *E* 30, by showing that solar rays converge along the line between the mirror at *A* and the midpoint *H* of the radius $A\Theta$ (Fig. 19). Where *E* 30 is not entirely clear as to whether the convergent point *K* is supposed to be the same for all pairs of rays, Diocles establishes in detail that it is not: the spherical mirror does not have a well-defined focal point¹³⁷. With this he effectively refutes the claim of the second part of *E* 30. Diocles has already shown in his first proposition that the

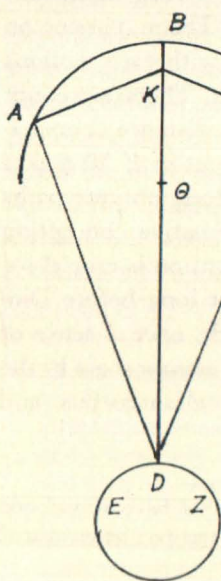


Fig. 17

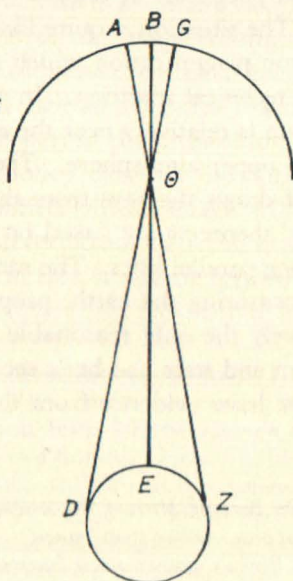


Fig. 18

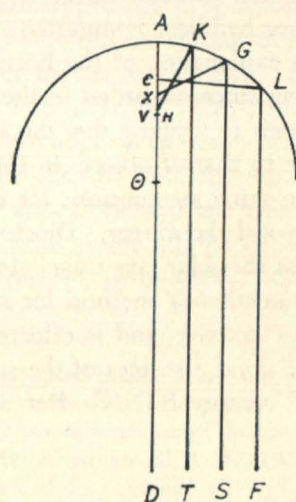


Fig. 19

¹³⁶ The proof assumes as obvious certain inequalities of angles which are elaborated in scholia nos. 56, 57 (ed. Heiberg, 360-361). Ptolemy (IV, 81-82) employs the same diagram for showing the manner of convergence of rays in concave mirrors.

¹³⁷ In Fig. 19 parallel rays from *F*, *S*, *T* are reflected at *L*, *G*, *K*, respectively, to intersect the diameter at *E*, *X*, *V*. In the limit, the intersections approach *H*, the midpoint of the radius $A\Theta$, as the point of reflection at the mirror surface nears *A*.

parabolic mirror reflects incoming rays to a point, so that the contour of the ideal burning mirror is paraboloid, not spherical. In his preface, he refers to this finding thus:

The intensity of the burning in this case [sc. of the parabolic mirror] is greater than that generated from a spherical surface, for from a spherical surface the rays are reflected to a straight line, not to a point, *although people used to suppose [zanna] that they are reflected to the center.*¹³⁸

As the line set in emphasis indicates, Diocles lodges against his predecessors precisely that error ostensibly proved in *E* 30. Under the circumstances, it is understandable that Diocles refuses to grace that claimed result with the term "proof", but speaks of it merely as a "supposition".

The hypothesis of a late dating for the *Catoptrics* must assign to its composer a phenomenal ignorance of the optical literature. The same would apply, but to a much reduced degree, to any redactor or copyist of an older work where this error had been committed. The situation is quite like that of Theon, passing on an explanation of the horizon magnification which accords with early notions long since discarded in the technical tradition. In particular, Theon's account seems to suppose that the sun is relatively near the earth, its distance comparable to that of objects in the upper atmosphere. The argument in *E* 30 makes the same assumption: for it draws the rays from the sun along oblique paths toward the mirror. Diocles' theorems are based on the alternative conception that the solar rays travel along parallel lines. The same conception is crucial for Eratosthenes' method for measuring the earth, proposed not long before Diocles' activity, and is effectively the only reasonable approach, once a sense of the great distances of the sun and stars had been secured by astronomers in the 3rd century B.C.¹³⁹ But we have evidence from the Aristotelian corpus, and

¹³⁸ Toomer's translation from *On Burning Mirrors*, 36; emphasis mine. I have altered one word, rendering *zanna* as "suppose" rather than "guess". This passage may be compared with the following from the Bobbio mathematical fragment:

With reference to the arc of the circle one may next establish toward how great an arc and where it will effect burning; *but the ancients used to maintain [dialambanein] that burning was effected about the center of the mirror.* (Ed. Heiberg, 88; emphasis mine.)

Toomer argues that this fragment is based, perhaps indirectly, on Diocles (cf. Diocles, 20, 142-143) and I have elaborated this view in my "Geometry of Burning Mirrors", sect. 3. It seems clear that the words in emphasis correspond to Diocles' preface, and that the term *dialambanein* (lit.: mark off distinctly) corresponds to the term which Diocles' Arabic translator has rendered by *zanna*.

¹³⁹ See note 49.

perhaps also from the Epicureans, that a consensus on this matter had not yet been effected in the 4th and early 3rd centuries¹⁴⁰. Indeed, even Diocles feels compelled to discuss at length this very principle, the "astronomers' assumption" that the earth is, as it were, a point in relation to celestial distances"¹⁴¹. Of course, Ptolemy does the same in the *Syntaxis* (I 6), as do other writers on astronomy even in late antiquity¹⁴². But Diocles' book is a technical treatise addressed (one would suppose) to a professional colleague, rather than to the more general audience of learners assumed by Ptolemy and the others. I thus submit that around the turn of the 3rd century the assumptions implicit in *E* 30 would not be the manifestations of ignorance or dogmatic anti-scientism they would become toward the close of that century.

It is important to realize that the failings in the argument in *E* 30 are conceptual rather than technical. The author has in his findings on concave reflectors all that would be needed for anticipating Diocles' findings on spherical mirrors.

Specifically, in the diagram of *E* 28 where the observer stands at a distance beyond the center of the mirror, the image is formed between the surface of the mirror and the observer. Moreover, as the observer moves further away, the image becomes reduced in size and recedes toward the mirror. The critical position is not stated in this part of the proposition, but appears earlier in *E* 28: when the observer is positioned between the mirror and the midpoint of the line to its center, a magnified virtual image will appear behind the mirror surface; but when the observer stands between this same point and the center, no image will appear (for, in fact, it will be formed behind the observer). One

¹⁴⁰ Cf. the Aristotelian account of the rainbow in *Meteor.* III, 5, cited in sect. 2 above and in my *Ancient Tradition*, ch. 4(i). Specific dimensions of the cosmos do not appear in Aristotle's accounts in *De caelo*, Books I-II; but his view of the immense size of the sphere of the fixed stars can be inferred from his assigning it the fastest motion (II, 4; 287a23-30) and from his asserting that the 400,000 stade circumference of the earth makes it "of no great size" (II, 14; 297b30-32; 8a15-20). But testimonia on the cosmological views of Presocratics like Democritus and Anaxagoras betray notions of a smaller cosmos, while Epicurus and his followers in the time immediately after Aristotle maintained ideas of the relative nearness and smallness of the heavenly bodies; for a survey, see G.E.R. Lloyd, *Early Greek Science* (New York, 1970), ch. 4; and *Greek Science after Aristotle* (New York, 1973), ch. 3. The thesis of the anti-scientific attitude of the Epicurean school, leading them dogmatically to deny the basic principles of astronomy, is developed by D. Sedley in "Epicurus and the Mathematicians of Cyzicus", *Cronache Ercolanesi*, 6 (1976), 23-54.

¹⁴¹ *Burning Mirrors*, 38-43.

¹⁴² See the commentary by Theon of Alexandria on *Syntaxis* I, 6 (ed. Rome, 417-421); Cleomedes, *De motu circulari*, I, 11; and Theon of Smyrna, *Expositio rerum mathematicarum*, ed. Hiller, 1878, 120-127.

thus easily recognizes that the image of distant objects will be formed in front of this midpoint, becoming closer to that point as the objects are more distant¹⁴³. From this, the basic result for burning mirrors follows immediately: that the solar image will be formed at, or imperceptibly close to, the midpoint of the mirror's radius (Fig. 20).

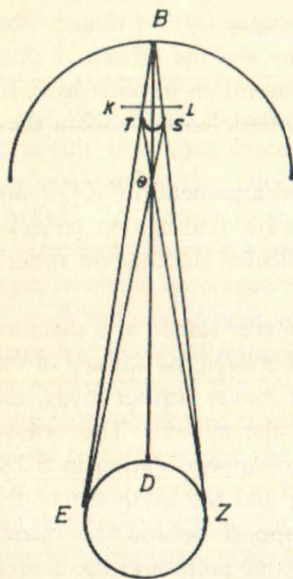


Fig. 20

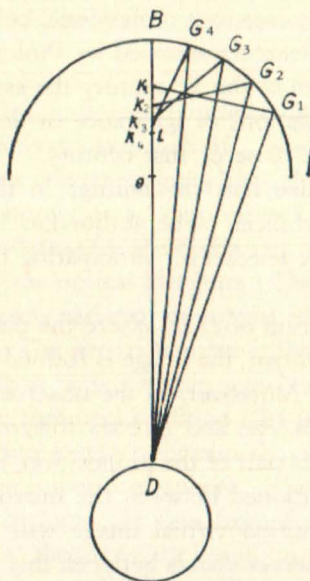


Fig. 21a

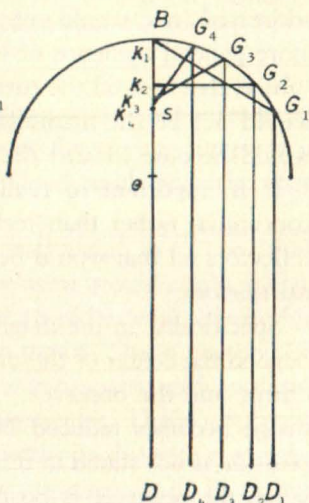


Fig. 21b

What has kept the author of *E* from securing this result is not a limitation of geometric technique, but rather a failure to comprehend clearly the nature of the real images formed by concave mirrors in this configuration. In effect, one needs only to take literally the analogy between the behavior of visual rays and luminous rays; then, since the sun (or an observer placed at the sun) will see its own image concentrated at the midpoint of the mirror's radius, the solar rays will converge at the same point, and this is where the mirror will cause ignition. Thus, aware of the error in the latter part of *E* 30, one could see how its former part, combined with *E* 28, entails the correct result¹⁴⁴. Rays from a

¹⁴³ A proof of the critical position follows immediately from the relation derived in note 78.

¹⁴⁴ Namely, that cited in the preceding note. The conceptual distinction of visual and luminous rays seems characteristic of ancient geometric optics, although physical theorists sometimes attempted to relate them as corporeal phenomena; cf. note 86 and notes 214-217.

distant object at D will be reflected to a series of points K along the diameter of the mirror converging toward L (Fig. 21a), the theoretical image distance implicit from the construction in *E* 28 (cf. Fig. 20). Introducing the assumption of parallel rays, one can adapt the technique in *E* 30 to specify the points of intersection K corresponding to the reflection of solar rays, and show that their limit is S, the midpoint of BΘ (Fig. 21b).

Diocles thus provides valuable insight into the problem of placing the pseudo-Euclidean *Catoptrics*. Not only does he identify the erroneous result in *E* 30 as a view maintained by geometers in the generation preceding him; but also his manner of working out the description of the reflective properties of spherical mirrors can be explicated as an adaptation and correction of the faulty analysis in *E*.

8. *The Unity of the Catoptrics*

The comparisons of the treatments of concave mirrors in the *Catoptrics* and in Diocles have reinforced other indications setting the *Catoptrics* within the context of technical studies near the turn of the 3rd century B.C. An examination of pre-Euclidean passages on optics, such as those compiled by Heiberg, Mugler and Schmidt, confirms that this work represents well the level of geometric expertise and optical knowledge attained around the time of Euclid¹⁴⁵. I will not here attempt to scrutinize the case arguing for a pre-Euclidean provenance for the *Catoptrics*, but wish instead to consider how the present findings on its strong associations with early researches alter the usual view, based on the hypothesis of its late composition.

The role of any hypothesis in this matter ought to be to account for the correspondences linking the *Catoptrics* to other ancient writings, in particular, the *Catoptrics* of Hero, the *Optics* of Ptolemy and the alleged Archimedean *Catoptrics*. In adopting a late date for the pseudo-Euclidean *Catoptrics*, Heiberg and Lejeune must suppose that its author used these others as his sources. The determination of the direction of dependence in cases of textual coincidence can often be slippery, and the present situation is complicated by the woeful condition of the manuscript evidence for Hero and Ptolemy and by the absence of any manuscript tradition of the Archimedean work. Since the pseudo-Euclidean work generally provides a more satisfactory treatment of those items it shares with the others, one is thus forced to appeal to recon-

¹⁴⁵ Heiberg, *Geschichte*, 73-74, 77-79; Mugler, "Sur l'histoire de quelques définitions"; Schmidt, *Her. Op.*, II, 311-315.

structed versions of its presumed sources. The standard hypothesis multiplies its difficulties by proposing Theon as the compiler of the *Catoptrics*. A competent mathematical commentator like Theon, taking advantage of sources like these, could be expected to produce a more satisfactory account of the field than the *Catoptrics* does. The overlap with Ptolemy, while it leaves out much that would surely have interested Theon keenly (e.g., the instrumental verification of the principles of reflection and refraction), is nevertheless sufficient to establish, on this view, that Theon had access to Ptolemy's *Optics*. Yet in his commentary on Ptolemy's *Syntaxis* Theon neglects to follow a correct account of refraction from the *Optics*, and instead fashions an incorrect argument based on the Archimedean *Catoptrics*¹⁴⁶. The inconsistency is remedied only through an unconvincing appeal to *ad hoc* hypotheses about Theon's access to and selective use of sources. Further, we search in vain for signs of geometric sophistication which the purported dependence on Archimedes should have instilled into Theon's treatment of catoptrics, or indeed into the efforts by any other ancient writer in this field.

The Heiberg-Lejeune hypothesis discourages a sympathetic appraisal of the *Catoptrics*, by drawing attention to features presented as defects in its composition. In particular, their view of its dependence on multiple sources leads them to emphasize any aspect which might be interpreted as a sign of its disunity. Lejeune dissects the work into three or more levels, where the principal break occurs at *E* 16.¹⁴⁷ The postulates (4 and 5) on the location of images (i.e., that images appear along the normals to the mirror surfaces) enter into none of the propositions before *E* 16, but are present either explicitly or implicitly in almost all propositions thereafter. In Lejeune's critique, the work suffers from redundancy, in that, for instance, several of the results on the orientation of images (*E* 7-12) are subsumed under later propositions (*E* 19-20, 28) with superior proofs¹⁴⁸. He thus proposes that the earlier theorems, where the principle of the normal does not enter, derive from the earliest (Euclidean) stage of development of the field, while later theorems, like *E* 19-22, 27-29, which apply the principle for determining the size, orientation and distance of images, represent a later stage. Pushing this criterion, Lejeune notes that as *E* 22 (on the reduced size of images in convex mirrors) does not actually locate

¹⁴⁶ This anomaly is cited by Rome as casting doubt on the authenticity of the *Optics*; see *Comment. de Théon*, 348n; and "Notes sur les passages". This issue is discussed in the Appendix, sect. 10 below.

¹⁴⁷ *Recherches*, Pt. II, ch. 2, esp. 113.

¹⁴⁸ *Ibid.*, 114-122. This alleged defect is included among several formal criticisms already cited by scholars in the 17th century to question the authenticity of the *Catoptrics*; cf. Heiberg, *Studien*, 152-153.

the image, it must derive from the earlier theory, even though all the propositions in its vicinity do locate the images, and thus derive from the later theory¹⁴⁹.

I believe Lejeune has adopted a questionable critical procedure here. We have no clear *historical* evidence for maintaining that the normal-principle was as yet unknown at the time of Euclid¹⁵⁰. Lejeune's view, then, depends strictly on the *logical* status of that principle within the formal structure of the *Catoptrics*. But logical distinctions do not necessarily have chronological significance. One may consider the famous case of Euclid's theory of parallels: the "parallel-postulate" (*Elements*, post. 5) appears for the first time in I, 29, and is implicated in all the propositions thereafter. But one would not wish to suppose that the first 28 propositions constitute an earlier stage of the theory of plane geometry and the remaining propositions a later stage. Indeed, we have testimonia indicating the importance of materials from the latter part of the book (e.g., I, 32 on the sum of the angles in a triangle, and I, 47, the "Pythagorean theorem", on the hypotenuse of a right triangle) within geometric studies well over a century before Euclid¹⁵¹. By contrast, the first enunciation of the postulate in Euclid's manner seems to be an outgrowth of formal studies only a few decades before Euclid¹⁵². Thus, when Euclid demonstrates an inequality related to the angles of a triangle in I, 16 without reference to the postulate, the fact that the same result is a trivial corollary to I, 32, where the postulate is used, does not connote an earlier dating for I, 16; to the contrary, it suggests the rather sophisticated objective of exhibiting the extent of the field of results which can be demonstrated without recourse to the postulate.

¹⁴⁹ *Recherches*, 123, 128-129; on similar grounds, Lejeune suggests a Heronian provenance for E 22 (*ibid.*, 141).

¹⁵⁰ The passage from the Aristotelian *Problems* (XVI, 13; cited in note 6), in speaking of the image as being located "at the end of the line where the visual ray has converged", may indicate efforts to discover the condition for localizing images. The normal-principle would follow at once from the assumption that any object will have its image appearing in the same place to all observers (cf. the discussion of E post. 4-5 in sect. 6 above).

¹⁵¹ Proclus derives from Eudemos a "Pythagorean" proof for I 32 (*In Eucl.*, 379). The Babylonians already knew the relation for right triangles a millennium before the Greeks; see B.L. Van der Waerden, *Science Awakening* (New York, 1963), 76-80. An extensive survey of the ancient knowledge of Pythagorean number triples is included in his *Geometry and Algebra in Ancient Civilizations*, Berlin, 1984.

¹⁵² I. Tóth has argued that geometers were exploring some consequences of non-Euclidean (or "anti-Euclidean") hypotheses around the middle and latter parts of the 4th cent. B.C., and that the formulation of Euclid's postulate emerged from these researches; cf. his "Das Parallelenproblem im Corpus Aristotelicum", *Archive for History of Exact Sciences*, 3 (1967), 249-422. The view develops around several passages where the theorem on the sum of the angles of the triangle is introduced as a dialectic premise.

Thus, Lejeune's insight on the role of the normal-postulate in the *Catoptrics*, when taken as a chronological indicator, encourages one to interpret as marks of primitive technique or inept editing what could be viewed alternatively as marks of formal sophistication. The *Catoptrics*, in fact, follows a reasonably coherent formal plan. It first provides a proof of the equal-angles principle (E 1), and then applies this to establish the paths of rays and the manner of their convergence or divergence for all cases of mirror (E 2-6). In Ptolemy's *Optics*, the same results justify statements of the conditions under which mirror images are unique¹⁵³; for instance, only one ray can pass from eye to object via a plane or a convex mirror (cf. E 2-4), but there are configurations in concave mirrors where multiple images of the same object appear (cf. E 6). Although the *Catoptrics* does not make explicit assertions on the uniqueness of images, we may infer that motive in these propositions. The next set of results (E 7-12) deals with the orientations of images in each case of mirror: where height and depth appear reversed, and where "such as they are"; where oblique lengths appear "as in truth" (i.e., near appears near and far far), and where reversed. In the following set of problems (E 13-15) one contrives to make a given object appear through composite arrangements of mirrors. One can detect here an interest in explaining the operation of optical devices already familiar in the 4th century B.C.¹⁵⁴

None of these results requires the actual construction of the image. The manner of the construction, at the intersection of the visual ray and the normal, is presented in E 16-18 for each of the three kinds of mirror¹⁵⁵. There follow

¹⁵³ For the concave cases, see IV, 64; cf. Lejeune, *Recherches*, Pt. II, chs 1-2.

¹⁵⁴ The configuration has a practical aspect in that it bears on the understanding of folding mirrors of two or more elements. Such devices are familiar artefacts from ancient times, as revealed in literary and archeological witnesses. See the article "speculum" in *Dictionnaire des antiquités grecques et romaines*, ed. C. Daremberg and E. Saglio. Composite mirrors are suggested in the passage from Plato's *Timaeus* 46a-c (cf. note 6) and appear in Hero's *Catoptrics*, Lucretius' *De rerum natura*, and other sources (see the surveys cited in note 145 above).

¹⁵⁵ Lejeune objects to these proofs on the grounds that they do not establish the existence of the intersections of the critical lines (*Recherches*, 126). But apparently, their convergence is obvious enough to be assumed in the plane case (E 16, 19) and convex case (E 17, 20); in the concave cases (E 28), however, the convergence is proved. One could trivially adapt these proofs to fill the gaps in the other cases. Lejeune describes the project of the equivalent constructions in Ptolemy as a form of existence proof: "il restait à démontrer que les trois lois de la réflexion ne comportent, géométriquement, aucune contradiction" (*Rech.*, 74). But in the concave cases, at least, Ptolemy's procedure is hardly different from that in E 28 (cf. the discussion in sect. 5 above). Further, since Ptolemy has instituted an empirical method for establishing the basic principles of reflection, he hardly requires a proof of consistency; for the existence of the images is a matter of empirical fact. Le-

results on the size, orientation, distance and shape of images in plane mirrors (E 19), convex mirrors (E 20-23), and concave mirrors (E 24-28). The main proposition on concave mirrors (E 28) makes the simplifying assumption that the observer is looking at his own image, and considers four separate cases, as the observer stands between the mirror and the midpoint of the radius (to see a magnified virtual image), or at the midpoint (to see no image), or between the midpoint and the center of the mirror (to see no image – because it is formed behind the observer), or beyond the center (to see a reduced real image between himself and the mirror). One regrets that certain opportunities are missed, for instance, to articulate the relation between object and image distance, or appreciate the symmetrical relation between object and image¹⁵⁶; or that the equivalence of real visual images and projected luminous images is not grasped. But these are shortcomings the *Catoptrics* shares with the whole later tradition of the ancient field, while it provides an adequate basis for the somewhat more refined treatments we find in Ptolemy. E 29 merely summarizes these findings in the form of a problem: to contrive a composite mirror in which all manner of images (*e.g.*, reversed and true, magnified and reduced) will be seen. This sense of the practical potential of its propositions extends to the last theorem (E 30), which attaches the incorrect claim that convex spherical mirrors induce burning at their centers, to the correct result that rays impinging on such mirrors are reflected to a point between the center and the mirror. Although this analysis of the burning mirror is wrong, for the reasons discussed above, it is compatible with general notions still current early in the 3rd century B.C., and provides a start for the correct analysis worked out by Diocles at the close of that century.

This completes the *Catoptrics*, as extant, but for its last postulate, providing a paradigm instance of the phenomenon of refraction. The formal character of the work entails that this postulate served as the basis for theorems on refraction, and the materials in Apuleius, Theon and Olympiodorus relating to the Archimedean *Catoptrics* provide insight into what the oldest theory of refraction would have contained: results on the manner of bending of rays, the enlargement and distance of refracted images seen in water, and their applications to explain certain meteorological and astronomical phenomena. We can only surmise the extent of this theory, but can presume that the line separating

jeune's attitude exemplifies a characteristic tendency in modern accounts to interpret ancient techniques in an existential mode; for a criticism of this attitude, see my "Construction as Existence Proof in Ancient Geometry", *Ancient Philosophy*, 3 (1983), 125-148. As for Ptolemy, the combination of empirical and geometric approaches can best be understood, I propose, through his adoption and extension of the results presented in E.

¹⁵⁶ See note 78.

the early results from the later ones, as extant in Ptolemy's *Optics*, would be very difficult to determine.

The *Catoptrics* is neither primitive nor inept. The unity of its formal organization is impressive. Despite claims by Heiberg, Lejeune and others, the geometric technique in the proofs is generally sound, and the number of its outright errors is few. On all these counts the *Catoptrics* equals or surpasses the *Optics* of Euclid¹⁵⁷; the chief argument for pseudonymity thus falls: the *Catoptrics* cannot in principle be ruled unworthy of the master. This, of course, does not establish Euclidean authorship. But the manuscripts agree in assigning the work to Euclid, and I see merit in taking their witness seriously and allowing that the extant *Catoptrics* might represent a recension of a legitimate Euclidean writing.

This alternative proposal views the prototype of the *Catoptrics* as an attempt to effect a formal synthesis of what was known in the field around the time of Euclid. Those aspects of technique and terminology which some have cited in favor of a late dating are in fact fully consonant with documentation relating to research from early in the 3rd century B.C.¹⁵⁸. On only one item, the treatment of the equal-angles principle in *E* 1, is a post-Euclidean date clearly indicated. But this item must surely have been the work's most frequently cited result, so that the assumption of its alteration by a later editor is natural and need not affect one's view of the rest of the work. To be sure, the technical level of optics was soon notably advanced through the activities of Diocles and the colleagues he cites, and later in the work reported by Ptolemy. But the older writing would retain its value as an elementary introduction into the field of catoptrics, and thus be recommended as the essential nucleus for new recensions. The correspondences with Hero and Ptolemy thus point to the continued use of this writing as the basis of more advanced study; moreover, our testimonia on Archimedes' *Catoptrics* portray a work no different from the pseudo-Euclidean writing, and thus indicate a pattern of misattribution. On the present view, then, the pseudo-Euclidean *Catoptrics* — notwithstanding certain editorial changes of only minimal significance — represents to us the introductory account underlying the entire ancient tradition of geometric catoptrics.

Ideally, a thesis on such issues should be true, or at least arguably so on the basis of a straightforward reading of the full range of evidence. The Heiberg-Lejeune hypothesis is not; to the extent it mounts any case at all on behalf of its principal suppositions, it requires such contortions as should long since have effected its abandonment. Beyond this, when the documentary

¹⁵⁷ See note 56 above for references to the formal criticisms of both the *Optics* and *Catoptrics* on which were based the early arguments against their authenticity.

¹⁵⁸ See sect. 3 above.

record does not permit a definitive determination on the validity of theses, they may still have value for stimulating a fresh examination of the primary sources and bringing forth new insights into their concepts, techniques and interrelations. The Heiberg-Lejeune hypothesis can only stifle interest in the *Catoptrics* by assigning its composition to an inept compiler, poorly informed on the work of his predecessors, and placed late in antiquity, when the pursuit of science had degenerated into an arid scholasticism. This negative argument, however, is not the primary objective of the present inquiry. The association of the ancient testimonia on Archimedes' optical studies with the extant *Catoptrics* does not merely convert a pseudo-Euclidean writing into a pseudo-Archimedean one. It opens up the prospect of a markedly new conception of the organization and development of the field of catoptrics in antiquity, and it suggests that the ancient writings, the *Catoptrics* in particular, may have virtues one has long managed to overlook.

APPENDIX

Two Problems of Authorship

9. *The Optics of Damianus of Larissa*

Damianus' tract on optics consists of fourteen short chapters providing a general account of fundamental principles¹⁵⁹. The style is discursive, unsystematic and nontechnical. The author weaves in anecdotal details, like the emperor Tiberius' reputed ability to see in the dark (ch. 2). His lack of system is manifest in his explaining the phenomena of refraction (ch. 12) before the equal-angles principle of reflection (ch. 14), and his mixing of physical principles (like the infinite speed of visual rays) with geometric (their rectilinearity and conic forms; cf. ch. 3). Geometric constructions and proofs are entirely absent; and the author occasionally betrays lapses in understanding (e.g., his claim that refraction, like reflection, occurs at equal angles; ch. 14), such that our confidence in his technical acumen wanes.

The principal virtue of the work lies in its eclectic nature, for its allusions to various sources provide us valuable information on the ancient optical literature. In particular, its reference to Ptolemy's use of instruments for tracing rays

¹⁵⁹ *Damianos: Schrift über Optik*, ed. R. Schöne, Berlin 1897; for earlier editions and discussions, see *ibid.*, v-vi.

(ch. 3) is one of only three ancient testimonia to Ptolemy's *Optics* (to be discussed in the next section); and its reference to Hero's use of the least-path principle for proving that rays are reflected at equal angles (ch. 14) provides a vital clue for identifying the Latin tract *Ptolomei de speculis* as a fragment of the *Catoptrics* not of Ptolemy, but of Hero¹⁶⁰.

Since Damianus and his *Optics* are not attested outside the manuscript tradition of the work itself, dating him is difficult. As Tannery and Schöne do not take up the issue, the firm statements made by Heiberg, Hultsch and Lejeune ultimately trace back to the arguments of the learned editor of Damianus, Erasmus Bartholinus (1657).¹⁶¹ Their view, now generally accepted, if with reservations¹⁶², is that Damianus was son, or perhaps disciple, of Heliodorus of Larissa; the latter is claimed to have lived sometime before Theon of Alexandria, for Theon draws from him in remarks in the preface to his recension of Euclid's *Optics*. As remarked in sect. 4 above, a pre-Theonine dating for Damianus' work would have implications for dating the pseudo-Euclidean *Catoptrics*. It is thus pertinent to reconsider the dating argument here.

The relation of Damianus to Heliodorus is indicated in the title. The manuscripts employ a double genitive *Damianou tou Hêliodôrou Larissaïou kephalaia tôn optikôn hypotheseôn*, literally: "of Damianus of Heliodorus the Larissan Chapters on the Optical Hypotheses". According to Bartholinus, Voss had proposed that the second genitive should be taken as "son of Heliodorus", in accordance with a common pattern of names¹⁶³. But Bartholinus himself, noting a parallel with *Hêrôn tou Ktêsibiou*, recommended the view of a master-disciple relationship; that is, Damianus, as auditor, disciple and perhaps foster-son of his teacher Heliodorus, compiled and published his master's papers to ensure their survival. Bartholinus' view seems better than that of Voss, for patronymics are quite uncommon in the transmission of the titles of ancient

¹⁶⁰ See W. Schmidt, *Her. Op.*, II, 303-304.

¹⁶¹ *Damiani De optica*, ed. E. Bartholinus, Paris, 1657; for commentary, see his "Animadversiones", 94ff. Tannery discusses the manuscripts of Damianus, but no issues of dating or content; cf. "Rapport sur une mission en Italie" (1888), *Mémoires scientifiques*, II (Paris, 1912), 319-324. Heiberg draws on the early editions by Dasypodius, Bartholinus and others; see *Studien*, 90, 148, 150-151; "Prolegomena", *Eucl. Op.*, VII, xxxi-xxxii; *Geschichte*, 76-77. Hultsch's account derives uncritically from Heiberg (cf. "Damianos (3)", in *Pauly Wissowa Real-Encyclopädie* (1901), 4, cols 2054-55), as does that of Lejeune (cf. *Recherches*, 4, 13, 20, 27).

¹⁶² Cf. G. Sarton, *Introduction to the History of Science*, I (Baltimore, 1927), 354-355, who states the accepted view on Damianus, but remarks that "the whole question is very obscure".

¹⁶³ *Dam. Opt.*, 97.

works¹⁶⁴. But the very example cited by Bartholinus works against his own view: for we know that Ctesibius and Hero were separated by at least three centuries. The double genitive in the title *Hêrônōs Ktêsibiou Belopoiika*, for instance, must signify Hero's responsibility for a work based on an earlier work by Ctesibius¹⁶⁵, that is, the second genitive is part of the *title* of the work, not a part of its author's name.

In the case of Damianus, we would thus render: "Damianus' (edition of) Heliodorus' *Optics*". The parallel with Hero shows that the double genitive entails nothing more specific about relative date than that Damianus lived after Heliodorus. As for the character of the two indicated works, Damianus' version could be an up-dating, in the manner of Hero's treatment of Ctesibius¹⁶⁶, or merely a light retouching, in the manner of Theon's editions of Euclid. But the concise, informal nature of Damianus' book suggests that it abridged and simplified an earlier treatment, perhaps serving as a general preface toward the study of a more substantial work. The distance of time between Heliodorus and Damianus is more likely to be long than short, since a work would have to gain a certain standing before a new edition was called for.

Explicit citations in the work itself are of little help for dating: we can claim only that it must be later than Euclid, Tiberius, Hero and Ptolemy. But two passages have been construed as indicating a date before Theon. In ch. 1 the shape of the eye is related to the physical nature of the process of vision:

That, upon the projection of something from us, we engage the things seen makes clear also the figure of the eyes, not concave nor made for the reception of anything, like the other sense organs, but being spheroid.¹⁶⁷

A passage from the introduction to Theon's recension of the *Optics*, makes the same observation¹⁶⁸. Further, in Damianus' ch. 9 an example is given in

¹⁶⁴ Who could name the father of Euclid, for instance, or Apollonius, or Ptolemy? That the name of Archimedes' father is known owes to a chance reference in the *Sand-Reckoner* (I, 9), not to a patronymic in any titles.

¹⁶⁵ On the title, see E.W. Marsden, *Greek and Roman Artillery: Technical Treatises* (Oxford, 1971), 18, 42. Hero's date has been set by means of a lunar eclipse in 62 A.D.; cf. O. Neugebauer, *History*, 846. A.G. Drachmann assigns Ctesibius a date early in the 3rd cent. B.C.; cf. "Ctesibius", in *Dictionary of Scientific Biography* (New York, 1971), vol. 3, 491-492.

¹⁶⁶ Hero's writing actually introduces very little material representing the advances in artillery construction between Ctesibius' time and his own. Marsden notes and attempts to explain this surprising fact (*op. cit.*, 2).

¹⁶⁷ *Dam. Opt.*, ed. Schöne, 4.2-5.

¹⁶⁸ *Eucl. Op.*, VII, ed. Heiberg, 150.9-27.

order to illustrate that vision is clearest along the line of the axis of the visual cone from the eye:

Because of this also if there chances to be a needle (*raphis*) lying close by, when we project the visual ray toward the place where it happens to lie, we don't see it until we engage it with the rays along the axis or those round about it.¹⁶⁹

The same illustration appears in the Theonine preface¹⁷⁰. On the basis of these parallels, Heiberg maintains that both accounts "without doubt" follow the same author, "for that Heliodorus was older than Theon I hold for certain"¹⁷¹.

It should be clear *a priori* that Theon did not base his accounts here on a source resembling Damianus' *Optics*. Theon was professor of mathematical sciences at Alexandria, and the leading scholar in the field at his time. His lectures on optics might well draw from many sources, just as his commentaries on Ptolemy do. But it is hard to imagine he would find anything new or useful in the kind of low-level, carelessly organized and inaccurately executed compilation of materials represented by Damianus' writing. Conversely, the preface to Theon's recension of Euclid, providing insights into the physical principles and phenomena corresponding to Euclid's geometric propositions, would be precisely suited to Damianus' needs.

But beyond this, it is clear that the Theonine preface provides a fuller and more accurate account in these parallel passages than does Damianus. In the case of the first, what Damianus merely asserts, Theon elaborates in an argument of twenty lines: that the sense organs of hearing, smell and taste are concave in order to take in moving bodies from outside and retain them for a period of time, so that

if in the case of the visual organ, moving bodies had to impinge on it from without, rather than that it emit something from itself, then its figure would also have to be concave and adapted for receiving bodies impinging on it; but now one sees that this is not the case, but rather the visual organ is seen to be spheroid.¹⁷²

The argument is counterfactual in form, intended to refute the theory that vision involves the stimulation of the visual organ through its reception of an

¹⁶⁹ *Dam. Opt.*, 10.20-24.

¹⁷⁰ *Eucl. Op.*, VII, 146.26-148.4.

¹⁷¹ *Ibid.*, xxxi-xxxii.

¹⁷² *Eucl. Op.*, VII, 150.21-27.

image from the object ¹⁷³. The reasoning is valid, if one accepts the teleological notions standard among all but the atomists in antiquity; indeed, its view of the adaptation of the form and function of bodily organs is a commonplace in Hippocratic medical writings and Aristotle's zoology ¹⁷⁴. Theon hardly needed the sentence from Damianus to suggest this argument, and would have found with Damianus nothing more a brief statement of its conclusion, anyway. By contrast, Damianus' version could well be viewed as a summary of the extended account in Theon, and the verbal agreements indicate that this is precisely where he got it.

The resemblance between Theon and Damianus relative to the second passage, on the needle, is at first sight merely superficial; but closer examination reveals again the priority of Theon. As with the other passage, Theon adopts a counterfactual mode, here to elucidate Euclid's first proposition, that one does not see the whole of a body all at once:

for often, when a pin (*belonê*) or other such small object has fallen to the ground, people would get down diligently in search and explore the same place over and over again, nothing standing in the way of the object they sought after. Later, however, casting the visual ray at the place where the object was, they saw the pin. It is clear, then, that formerly the fallen object was not seen, nor was the place where it was. Thus, not all the parts lying under the visual ray of the one seeking are seen. For if they were seen, then the sought object would also have been seen; but it wasn't. ¹⁷⁵

Theon provides a second illustration supporting the same conclusion: that we can visually take in a whole column of writing without registering individual letters; thus, again, we cannot have seen all the parts of the visual field simultaneously ¹⁷⁶. In the next passage, Theon casually refers back to these examples: they call into question the intramission theory, that vision is a matter of taking in images of things; for the pin or the letters are perceived only when we concentrate on them, not merely when they happen to lie in the general visual field ¹⁷⁷. But in this context he develops at greater length the argument,

¹⁷³ The proposition is stated at *ibid.*, 148.20-22 and then challenged through three phenomena: the occasional failure to see pins or letters in books (recalled from Theon's earlier discussion); the greater efficiency of searching when the mind is concentrated; and the spherical shape of the eye.

¹⁷⁴ See Hippocrates, *On Ancient Medicine*, chs 22-23 and Aristotle, *Parts of Animals*, I, chs 1, 5.

¹⁷⁵ *Eucl. Op.*, VII, 146.24-148.7.

¹⁷⁶ *Ibid.*, 148.7-15.

¹⁷⁷ *Ibid.*, 148.20-150.8.

which we have already cited, dealing with the shape of the eye¹⁷⁸.

For Damianus, the example of the needle in ch. 9 demonstrates the greater concentration of light at the axis of the visual cone. This physical notion is developed further in ch. 10, illustrating how the "visual power" (*optikê dynamis*) has a greater effect in the forward direction than to the side. A closely related phenomenon appears earlier (in ch. 5): that the range of "exact" (*akribês*) vision is much narrower than that of "comprehensive" (*holoscherês*) vision¹⁷⁹. The distinction is introduced to resolve an ostensible contradiction with Euclid:

But against the author of the Elements [sc. Euclid], who says "none of the things seen are seen simultaneously" [*Opt.*, prop. 1], we seem to be saying the contrary, that the quadrant of the heaven is seen simultaneously.¹⁸⁰

Thus, Damianus can admit Euclid's proposition with reference to "exact" vision, yet accommodate a more general notion of the visual cone through his notion of "comprehensive" vision.

It is remarkable that Theon, who uses the same term *holoscherês*, but in a different sense¹⁸¹, makes no such distinction about kinds of vision and perceives no need for nuance in handling Euclid's claim about simultaneity. He thus seems unaware of the issues raised by Damianus, despite his elaboration of the pin example in two contexts. By contrast, Damianus treats the needle example quite casually (ch. 9), to illustrate his physical notion of the visual cone; but he is conscious of the potential discrepancy with Euclid on simultaneity (ch. 5). One can view his remarks on "comprehensive" vision and his form of the needle example as adaptations of a source like Theon, where in the one case Damianus has introduced new features, while in the other he has applied a borrowed example for a different purpose. The converse view, of Theon's dependence on Damianus, would explain nothing about Theon's treatment of these items, but merely raise the puzzle of why he is silent on the issues which concern his alleged source.

Damianus subscribes to the conception that vision is an actual form of light (e.g., "the light projected from us" in ch. 2), and on this basis accounts for its basic geometric properties, like its rectilinear passage and rapid speed (ch. 3). His examples, such as those here cited, attempt to provide direct support for

¹⁷⁸ *Ibid.*, 150.9-27.

¹⁷⁹ *Dam. Opt.*, 8.12-24.

¹⁸⁰ *Ibid.*, 8.12-16.

¹⁸¹ *Eucl. Op.*, 150.4: "when thinking about it [sc. the lost pin] they seek and utterly fail to find it".

claims about the physical nature of this light. Theon makes no specific claims about the essential nature of the visual emanation, but adopts the analogy with light rays to explicate Euclid's postulates about visual rays. In his accounts the examples either merely illustrate Euclid's meaning, or are mobilized within counterfactual arguments to refute alternative notions. In these ways, one perceives the greater coherence of Theon's presentation, the appropriateness of his examples, and its careful adaptation to Euclid's text. We derive little from supposing that Theon included Damianus' *Optics* among his sources, other than the puzzle as to what value Theon could have seen in this popularizing miscellany. But Damianus' work can easily be understood as the uneven product resulting from excerpts of Theon and other sources for the purposes of expounding a theory of vision different from Theon's.

These observations set Damianus in the period after Theon. Another indication of a date no earlier than the 5th century A.D. may be noted in Damianus' expression of the isoperimetric property of the circle (ch. 3): that it is the "most spacious" (*polychôrêtotatos*) of the plane figures isoperimetric to it¹⁸². This is reminiscent of the wording adopted in an anonymous writing entitled: "that the circle is more spacious (*polychôrêtotos*) than the isoperimetric figures"¹⁸³. Hultsch considers this term to be an innovation by the anonymous editor, for it differs from the standard expression (*i.e.*, that the circle is "greater" than the associated figures) used by writers like Pappus and Theon, and even from that adopted throughout the treatise itself¹⁸⁴. The anonymous writer's term is found with Simplicius in the 6th century¹⁸⁵, and this dating is consistent with Mogenet's ambitious effort to assign the anonymous writing to Eutocius, the 6th-century commentator on Archimedes and Apollonius¹⁸⁶.

Dating our optical work to the 5th or 6th century still leaves unclear whether this date applies to Damianus or to his predecessor Heliodorus. But it suggests an identification of the author with the Heliodorus who was a scholar prominent within the circle of Aristotelian studies at Alexandria early in the 6th century¹⁸⁷. This Heliodorus included reports of astronomical observations among his contributions, and is argued to have figured significantly in the manuscript tradition of Ptolemy's *Syntaxis*. As we have seen, the connection

¹⁸² *Dam. Opt.*, 6.1-2.

¹⁸³ See the text edited by F. Hultsch in *Pappi Collectio*, III (Berlin, 1878), 1138.1-2.

¹⁸⁴ *Ibid.*, 1139n; cf. 1138.3-5, 1156.26-27.

¹⁸⁵ *In De caelo*, ed. Heiberg, 412.16, 414.15.

¹⁸⁶ *L'Introduction à l'Almageste*, Brussels, 1956.

¹⁸⁷ On Heliodorus, see Neugebauer, *History*, 1038-1041; and F. Boll, "Heliodorus (13)", *Pauly Wissowa Real-Encyclopädie*, 15 (1912), cols 18-19. See also Ae. Boer, *Heliodori, ut dicitur, In Paulum Alexandrinum* (Leipzig, 1962), vii-viii.

between astronomy and optics is firmly established in the ancient traditions of these fields; it thus becomes plausible that a scholar like this Heliodorus should also have written on optics. While this identification can be proposed only as a possibility, it accords well with the passages here discussed, and with the didactic character of the writing over all. It has the additional advantage of associating the work with a figure of some significance in late antiquity. This is as we would expect from the citation of his name in the title, calling attention to his role, either as master of its author Damianus or as author of the writing Damianus has reedited.

The dating of Damianus remains obscure: he would be either a 6th-century disciple of Heliodorus, or more likely, a Byzantine scholar writing considerably later. But either dating appears to exclude an older view, that Damianus can be identified with Domninus of Larissa, the 5th-century arithmetic writer and philosophic competitor of Proclus¹⁸⁸.

10. *The Optics of Ptolemy*

The transmission history of Ptolemy's *Optics* is vexed. Its most recent editor, Lejeune, worked with a dozen manuscripts dating from the 14th to the 16th centuries, these all stemming from the Latin translation made by Eugene of Sicily around the middle of the 12th century¹⁸⁹. Eugene worked with an Arabic translation, not a Greek manuscript, and this was already defective, lacking at the least its first book and the concluding portions of the fifth. For a work as compendious as this, the ancient testimonia are surprisingly sparse. Theon does not consult it in the preparation of his comment on *Syntaxis* I, 3, on the phenomenon of horizon magnification¹⁹⁰. Damianus (ch. 3) reports Ptolemy's use of instruments for tracing the rectilinear path of visual rays¹⁹¹; although instrumental procedures appear in the extant *Optics*, it does not transmit this one specifically. Damianus' witness retains its interest, even if his date is transferred from the 4th century (or earlier) to the 6th century (or later). Two

¹⁸⁸ See P. Tannery, "Domninos de Larissa" (1884), *Mémoires scientifiques*, II (Paris, 1912), 105-117.

¹⁸⁹ See the introduction by Lejeune to his edition, *L'Optique de Claude Ptolémée* (1956); a synopsis of the textual issues appears in the introduction to his *Recherches* (1957). The first six sections of the former (*Opt.*, 9*-30*) reappear almost *verbatim* in the latter (*Rech.*, 9-30); the repeated portion includes Lejeune's extensive argument defending Ptolemy's authorship of the *Optics*.

¹⁹⁰ *Commentaires de ... Théon*, ed. Rome, 346-352. For discussion see sect. 2 above.

¹⁹¹ *Dam. Opt.*, ed. Schöne, 4.17-20, citing the "Optical Treatise (*pragmateia*)".

citations from the 6th century (Simplicius and Olympiodorus) and another from the 11th (Simeon Seth), all on items unrelated to the contents of the extant *Optics*, complete the round of ancient testimonia compiled by Lejeune¹⁹². This is a meager record for a treatise which we should have expected to become the definitive textbook in its field, just as his *Syntaxis*, *Tetrabiblos*, *Harmonics* and *Geography* did in theirs.

The question of authenticity must thus be considered especially serious in the case of this work. The possibilities of textual disorder, resulting from this fitful transmission, include misattribution; but they also provide some assistance for defenders of its legitimacy. For physical damage and scribal errors may account for the disruptions in expository order, the frequent unclarity of enunciations, and the questionable soundness of many of the proofs. On the other hand, a strong inducement for accepting its authenticity lies in the work's empiricism; it includes sections describing instruments and their application for establishing empirically the equal-angles principle of reflection, the properties of concave mirrors, the relation of incident and refracted rays in refraction, and the basic phenomena of binocular vision. Such experimentalism is consistent with Ptolemy's procedures in the *Syntaxis*¹⁹³ and fits so comfortably with modern notions of scientific method, that most scholars have been understandably reluctant to question it.

The empirical manner, however, does not extend to the major portion of the work, devoted to the exposition of the geometric properties of visual rays in reflection and refraction. We have noted an interesting case in sect. 6: the work gives a theoretical explanation of how refraction is responsible for displacing the images of stars viewed near the horizon (V, 24-30),¹⁹⁴ yet makes no attempt to quantify this displacement or to elaborate the theory through observations. The relevance of the phenomenon of displacement for astronomical work is evident; but the failure to present observations does not well conform with Ptolemy's approach in the *Syntaxis*.

Lejeune attempts to explain this oversight by calling attention to the author's statement (in V, 30) that the quantitative amount of the displacement has been

¹⁹² Greek texts of the four "Fragmenta" are reproduced by Lejeune in his edition of the *Optics*, 271; for their use in his reconstruction of the lost first book, see *Euclide et Ptolémée*, Pt. I.

¹⁹³ For reconstructions of Ptolemy's instruments, see A. Rome, "L'astrolabe et le météoroscope d'après le commentaire de Pappus ...", *Annales de la Société Scientifique de Bruxelles*, 47 (1927), 77-102; and "L'instrument parallactique", *ibid.*, 129-140. A survey is given by D. de S. Price, "Precision Instruments: to 1500", in C. Singer *et al.*, *A History of Technology*, III (Oxford, 1957), 586-594.

¹⁹⁴ Cf. sect. 2 above.

impossible to determine, owing to the unavailability of certain critical data¹⁹⁵. Lejeune thus makes the approach in the *Optics* appear to be consistent with an empirically oriented procedure. But this illusion vanishes when we consider the matter more closely. First, the elusive datum named by the author is the altitude of the interface between the ethereal medium and the atmosphere, that is, the height above the earth where the bending of the optical ray occurs. This is an ingredient not of an empirical determination of the amount of bending, however, but of a theoretical one. Second, even with this datum, a far more critical parameter would be lacking: the equivalent of the index of refraction for light passing from ether to air. By the very nature and location of the ancients' ether, this would in principle be unattainable through experiment.

Third, one might nevertheless mount a hypothetical determination of the refraction effect: on the assumption of possible values for the unknown parameters, one could at least obtain a general sense of the order of magnitude of the effect. For instance, referring to the diagram used by *P* for explaining the refraction displacement (Fig. 22), we may assume that *D* is placed not far below the sphere of the moon (thus varying between around 30 and 60 earth radii) and so obtain an estimate of between a half and a quarter of a degree¹⁹⁶.

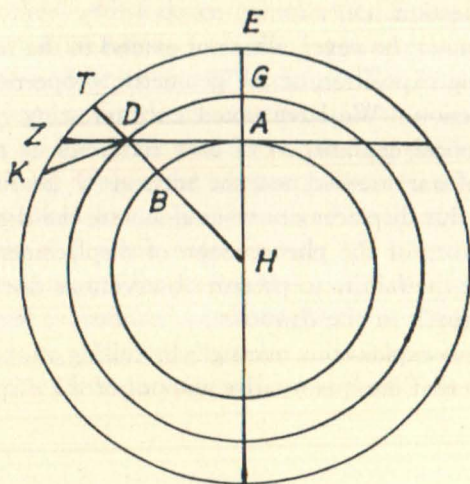


Fig. 22

¹⁹⁵ *Rech.*, 20.

¹⁹⁶ With reference to Fig. 22, we obtain the angle of refraction r (that is, angle ADH) from the relation $\sin r = AH/HD$. If we assume that D is set near the distance of the moon, about 60 earth radii, then $\sin r = 1/60$, or $r = 1$ degree, approximately. If, further, we assume that ether: air refraction is about the same as that for air: water, $i = 10$ would correspond to $r = 8$. If, finally, we assume a proportionality for small variations of i , we

The computational technique can be effected with consultation of the trigonometrical table assembled by Ptolemy (*Syntaxis* I, 10), for instance, and yields a figure remarkably consistent with modern measurements¹⁹⁷. Presumably, a practicing scientist like Ptolemy would find in such results a stimulus for seeking their empirical confirmation. Fourth, a fully observational determination would be straightforward to conceive and within Ptolemy's instrumental range to execute. For instance, one might note which stars on the equator happen to be rising and setting simultaneously with the culmination of other stars; the difference from 90 degrees will be a measure of the effect of refraction. Similar comparisons could be worked out for stars along the parallel circles¹⁹⁸. The effect would maximally amount to around half a degree, a magnitude accessible to Ptolemy's observational methods¹⁹⁹.

These considerations suggest that the author of this section of the *Optics* had a slender grasp of the relevant empirical issues. Moreover, the treatment of refraction in the *Syntaxis* does not extend to this aspect, a potentially significant factor in astronomical observations; Ptolemy alludes to refraction only to offer a loose explanation for the apparent magnification of bodies observed near the horizon, a phenomenon introduced incidentally in his account of the shape of the cosmos (I, 3).²⁰⁰ But in his commentary on this remark, Theon draws from the Archimedean *Catoptrics*, not the *Optics* of Ptolemy. This discrepancy

obtain $i = 5/4$ degrees. Thus, the displacement $i - r$ would be about $1/4$ degree. If, alternatively, we assumed D at only 30 earth radii, then $r = 2$ degrees and $i = 2\ 1/2$, so that the displacement would be $1/2$ degree. Such results, however tentative, could be a reasonable guide for empirical research.

¹⁹⁷ The agreement with modern data on refractive displacement is, of course, accidental. The estimates made above, both for the height of the atmosphere and for the ether: air refraction, are far too large, but their net effect is to cancel the discrepancy. For comparison, taking the refractive index of air to be 1.00029 (the value for water is about 1.33) and the height of the atmosphere to be about 2.5 miles, we obtain a displacement of about 32.5' for celestial objects viewed near the horizon. Under standard conditions, the displacement is set at about 34'; thus, the idealized situation is in this result roughly equivalent to the actual atmosphere. Assuming a height of 5 miles reduces the displacement to 21'; a height of 10 miles to 14'; and a height of 20 miles to 10'. Although the atmosphere extends far higher than these limits, the attenuation of the air correspondingly reduces the refractive index. Thus, the simplified model of a uniform atmosphere only yields a general estimate.

¹⁹⁸ An observational procedure of this type is described by Tricker, *Meteorological Optics*, 14-15.

¹⁹⁹ Lejeune notes that Ptolemy's astronomical instruments were graduated to sixths of a degree; the experiments on refraction list results to half of a degree (*Recherches*, 157).

²⁰⁰ Toomer suggests that another passage (*Syn.* IX, 2) might possibly relate to refractive displacement; cf. *Ptolemy's Almagest* (New York, 1984), 421n.

induced Rome to express doubt on the authenticity of the *Optics*²⁰¹. Lejeune offers further explanations: that Theon found the Archimedean account better suited to his specific purposes; that Ptolemy first appreciated the significance of refraction not at the time he wrote the *Syntaxis*, but only in the later *Optics*; that Ptolemy's view in the *Optics* was no longer consistent with the remark in the *Syntaxis*; and so on²⁰². But one's ability to contrive possible rationalizations for these difficulties does not dismiss them as challenges to one's position. They recommend that Lejeune's case be submitted to a careful scrutiny.

Lejeune's general argument for the authenticity of the *Optics* may be sketched as follows:

I. Ptolemy did compose an *Optics*: for (1) three ancient passages attest it; (2) we would naturally expect him to write a work on this field, to complement his other treatises on the mathematical sciences.²⁰³

II. The extant *Optics* represents (in an incomplete and corrupted state) that work by Ptolemy: for (3) the silence of Theon can be explained (as above); (4) terminological discrepancies can be ascribed to its translators; (5) the discrepancies with the *Syntaxis* can be explained (as above); (6) no other such discrepancies have been detected; (7) the intellectual style of the extant *Optics* is in harmony with that of the recognized treatises by Ptolemy.²⁰⁴

This must be considered an extremely weak presentation for such an important issue. Lejeune places more faith in (1) than he ought; for by doubting the 4th-century dating of Damianus we are left with the 6th-century writers Simplicius and Olympiodorus as our earliest witnesses, nearly four centuries after Ptolemy. The possibility of misattribution, which Lejeune dismisses perfunctorily²⁰⁵, thus increases, especially since these writers are likely to have depended on secondary sources for the isolated remarks they transmit²⁰⁶. Items (3)-(5) are negative, explaining away legitimate difficulties which may be raised against authenticity; in particular, (4) effectively discounts the hope of applying

²⁰¹ *Comm. de Théon*, 348n; a more detailed discussion appears in "Notes sur les passages", 35-36, 39-40.

²⁰² *Recherches*, 19-24.

²⁰³ *Ibid.*, 13-15.

²⁰⁴ *Ibid.*, 19-25.

²⁰⁵ *Ibid.*, 21.

²⁰⁶ Olympiodorus, for instance, cites Ptolemy on the colors in rainbows (*In Meteor.*, 242.26), but earlier refers only to Archimedes for his account of refraction (*ibid.*, 211.18-19). This could follow from his use of a secondary source on general physical principles, where passages from Ptolemy were included, and another source on geometric principles, which mentioned Archimedes, but not Ptolemy.

the regular philological methods. Item (6) conceals a further difficulty: the absence of overt contradictions results in large part from the disjunct nature of the subject matter: the physical and geometric theories of optics have but limited bearing on the content of Ptolemy's other scientific treatises. It thus becomes all the more striking that on one important issue where overlap exists, the relevance of refraction both in astronomy and optics, the treatments in the *Syntaxis* and the *Optics* are so utterly unrelated to each other.

As for (2) and (7), these are largely Lejeune's subjective impressions. He overlooks at least one prominent aspect of style which sets the *Optics* apart from the others: its manner of citing earlier work. Ptolemy's treatises in astronomy, harmonics and geography are our principal witness not only to the maturest technical level of these ancient fields, but also to their history. Ptolemy regularly cites his predecessors, often providing substantial information on their contributions. By contrast, the *Optics* does not make a single allusion to prior efforts, and this despite its manifest dependence on Euclid's *Optics* and the pseudo-Euclidean *Catoptrics*, and perhaps also Hero²⁰⁷; had the author given specific references for his background sources on mirrors and refraction, for instance, answers sought in the present inquiry on the relation of the Archimedean and pseudo-Euclidean catoptrical writings would simply have been a matter of record. Further, one may reasonably suggest that the author of the *Syntaxis* is less concerned over the complete formal geometric demonstration of his subject, yet more masterful in the execution of those results which are proved, than is the author of the *Optics*²⁰⁸.

This matter, I believe, deserves a systematic examination, to be settled, if possible, on the basis of objective considerations. Although Lejeune's richly detailed discussions will be indispensable for any further research, his inclination to subjective impressions in defense of a preconceived position can only cloud the issue. For the present, I recommend merely that the decision on authenticity be viewed as still undecided. The following remarks are not intended to argue on behalf of an alternative view, but to forestall the objection: who could the author of the *Optics* have been, if not Ptolemy?

From the disorder of the extant Latin manuscripts, one can surmise that the tradition of the *Optics* had already suffered corruption before entering the

²⁰⁷ A possible tie with the catoptrical sources exploited by Hero lies in Ptolemy's treatment of image distances in plane mirrors. Unlike the pseudo-Euclidean *Catoptrics* (prop. 19), where the object and image are shown to be set at equal distances from the mirror surface, as measured along the normal line, *P* measures along the visual ray. Thus, in Fig. 2a, for instance, *P* would claim that the image distance ADE equals the object distance ADZ. This recalls Hero's proof of the equal-angles principle, where it is shown that ADZ is the minimal path of all broken lines between A and Z.

²⁰⁸ Cf. Loria's opinion of the geometric technique of *P*, cited in note 87.

hands of the Arabic translators. Thus, although we now possess the major portion of four books, we cannot with certainty assume that only a single book was lost from its beginning, or that no further books followed its fifth²⁰⁹. Now, the scanty and late testimonia to Ptolemy's *Optics* have raised the possibility of misattribution, calling into question that Ptolemy even wrote a treatise on optics at all. But no such doubts can be raised against the eight-book treatise *On Vision* (*Peri Opseôs*) written by the Peripatetic teacher Sosigenes, for it is attested by his own disciple, Alexander of Aphrodisias²¹⁰. Sosigenes' works do not survive, but testimonia to them indicate a range of substantial efforts in logic, astronomy and other technical fields, and they were an important authority for the later Aristotelian commentators²¹¹.

The connection between Sosigenes' work and the subject matter of the Ptolemaic *Optics* can be inferred through Alexander's testimony on the former:

that such opinions about the halo [sc. that it is caused by the bending of light rays] are false has been adequately indicated by our teacher Sosigenes in the eighth (book) *On Vision*.²¹²

In the discussion of Apuleius' testimony on Archimedes we have seen that ancient catoptrical studies embraced the field of meteorological applications²¹³. That Ptolemy's *Optics* did the same is evident through Olympiodorus' testimony that, unlike Aristotle who spoke of three colors in the rainbow, Ptolemy discerned seven²¹⁴. In Lejeune's reconstruction of the lost parts of the *Optics*, this passage is taken to relate to the first book, a synthesis of general physical principles²¹⁵. But it seems quite plausible that a section relating to alleged

²⁰⁹ The Arabic recension of Diophantus offers a related example: it purports to present Books IV-VII which, if correct, indicates that the six books extant in Greek must be Books I-III and VIII-X (cf. the discussion by J. Sesiano, *Books IV to VII of Diophantus' Arithmetica* [New York, 1982], 4-8). In this instance, the Greek tradition has somehow lost track of the original numeration of the books.

²¹⁰ *In Meteorologica*, ed. Hayduck, 143 (see note 212 below).

²¹¹ For a survey of testimonia, see Rehm, "Sosigenes (7)", *Pauly-Wissowa Realencyclopädie* (1927), 5 (ser. 2), cols 1157-59. Simplicius makes several references to Sosigenes for planetary theory in his commentary on Aristotle's *De caelo*. Neugebauer notes his association with an annular eclipse of 164 A.D. (*History*, 104n4).

²¹² *In Meteor.*, 143.12-14. Since Olympiodorus, citing only Archimedes by name, provides just such a refutation of the refraction explanation (*In Meteor.*, 210.38-214.28), we may have an indication that these phenomena were explained by reflection in the 'Archimedean' treatment (that is, the prototype of the pseudo-Euclidean *Catoptrics*).

²¹³ See sect. 1 above.

²¹⁴ *In Meteor.*, 242.26.

²¹⁵ *Euclide et Ptolémée*, 27.

explanations of such phenomena through refraction should include speculations on their colors. In this way, one of the books following the extant "fifth" would be identified with the eighth book of Sosigenes cited by Alexander.

The other testimonia to the *Optics* can also be related to Sosigenes' works. Themistius (4th cent.) cites Sosigenes' third book for an explanation of the fluorescence of animals, namely, that their nature is like that of "the fifth body and fire"²¹⁶. Parallel to this, Simeon Seth (11th cent.) cites Ptolemy's *Optics* for the view that the "optical spirit" (*optikon pneuma*) is "ether-like in form, and of the fifth substance"²¹⁷. Simplicius indicates that the same work held an account of the natural motion of the elements, where the mention of "those elements which move in a circle when in their natural places" indicates that Ptolemy included there a discussion of the "fifth substance"²¹⁸. The question of the nature of light, e.g., whether it was corporeal like fire or ether, is at the heart of the Peripatetic study of the soul, from Aristotle's *De anima* onward²¹⁹. These references to Ptolemy's comments on the elements, the "fifth substance" and the "optical spirit" thus tie him securely to the same field of natural philosophy marked out by Sosigenes and the other Aristotelian commentators.

If we conflate these testimonia on the treatises of Sosigenes and Ptolemy, we obtain the description of a work consisting of three books on general physical principles, four (those extant in the Latin translation of Ptolemy's *Optics*) on geometrical principles, and at least one, the eighth, on meteorological applications. The notion that commentators over four centuries later could come to misattribute to Ptolemy a treatise by Sosigenes is not unthinkable, given that the later writers are likely to have used secondary sources for much of their information. As Sosigenes lived in the latter half of the 2nd century A.D. and included astronomical work among his scholarly interests, it is even possible that he studied with the aged Ptolemy and incorporated findings due to Ptolemy in his own optical writing. This could explain the lack of coordination between the *Syntaxis* and the *Optics* on the matter of refraction and its astrono-

²¹⁶ *In de anima*, ed. R. Heinze (*Comm. in Arist. Gr. V. pt. 3*) (Berlin, 1899), 61.22-25.

²¹⁷ See "Fragmenta" in Lejeune, *Optique*, 271; discussed in *Recherches*, 65-66. Reminiscent of this *optikon pneuma* is the *horatikon pneuma* mentioned by Cleomedes in his version of the refraction paradigm (*De mot. circ.*, II, 6, 224.16, 20); the latter term appears in testimonia on the vision theory of the 3rd-cent. B.C. Stoic, Chrysippus (cf. passages cited by Butler and Owen, *Apulei Apologia*, 42).

²¹⁸ *In De caelo*, ed. Heiberg, 20.10.

²¹⁹ Cf. Themistius, *In de anima*, 60, who argues against a corporealist conception of light. The parallel cited in note 216 above suggests that Sosigenes may have attempted to subsume the Stoics' *pneuma* under the Peripatetics' fifth element.

mical implications; and it would be conducive to the false attributions bearing on the *Optics*²²⁰.

As long as the decision on authenticity depends heavily on subjective considerations, the balance of evidence leans as clearly or more so on the side of authorship by an eclectic Peripatetic like Sosigenes as on that of Ptolemy. The disorder of the geometric proofs, the mix of physical, philosophical and technical topics, the various discrepancies with Ptolemy's expository style in his other treatises, find their straightforward explanation. But I do not presume to have established this identification, but merely to have shown that its possibility indicates the need for a more thorough and objective examination of the question than Lejeune has given it. In sum, I advocate the reversal of Lejeune's assessment, and recommend that, for now, one treat the *inauthenticity* of the *Optics* as the working hypothesis of greater probability and advantage²²¹.

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²²⁰ For instance, if the experimental studies in the *Optics* were indeed initiated by Ptolemy, and if Sosigenes included them in his treatise, a later excerptor from Sosigenes might well have identified these with Ptolemy, and by implication, assigned the entire treatise to him.

²²¹ Cf. Lejeune, *Optique*, 26*:

Ceux qui conserveraient des doutes sur ce point [on the authenticity of the *Optics*] admettront néanmoins que, dans l'état actuel de la science, la présomption d'authenticité est l'hypothèse de travail la plus probable et la plus féconde.

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