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THE « TIMAEUS » SCALE

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NOTE: For the sake of simplicity the tone B natural is indicated by *h*, B flat by *b*, F sharp by *fs* and E flat by *es*.

I.

INTRODUCTION

Timaeus is one of the latest dialogues by Plato (429-347 B. C.). I think the figure of Plato, as reflected by this dialogue, does not exactly correspond to that which is current. One is accustomed to look upon him as a thinker whose idealism takes a hostile view of the world of phenomena, despising beauty as perceived by the senses. But this touch of pathetic pessimism is absent in Plato's last dialogues. His attitude here may be described as one of contemplative objectivity which leads, as it necessarily must, to the conception of a world order established by a Divine power. In fact, this order is revealed by so many perceptible signs that, even where to us it seems as disorder, we must assume that it is order in a higher sense, not yet disclosed to us; also Plato in *Timaeus*, 56 C, 68 B-D, 87 C, is confident that planning and order,

symbolized by numerical ratios, must exist even where we are not able to verify them. This conviction has always been the property of thinkers representing the so-called « objective idealism », as for example, Johannes Erigena and Leibniz; but it is found — as it can not be otherwise — also with thinkers to whom we should not especially ascribe that variety of thought, because the problem of correlation of the Objective and the Subjective is of first importance in thinking and it can not be answered without assigning a rôle to a mind transcending ours.

I think it was the merit of J. Stenzel to have done justice to that stage of evolution in Plato's thought presented by his last dialogues (and by his famous lecture about the Good, which is unfortunately lost). Starting from the Socratic idea of moral freedom of the individual, which establishes its law by himself (yet in conformity with a superior and therefore objective idea), Plato first sought the « Good » in the human soul and believed that demonstrating the Idea to man would suffice to induce him to realize it. Hereupon (in his *Republic*) he turned to the problem of human community, at first believing that also in this case the cognition of the Idea (represented by the Constitution) would suffice for the realization of the Good, afterwards recognizing that a constitution is relative in value and that a ruler would be required who by his insight could be qualified to alter the law. At last (in *Timaeus* itself) Plato advanced to the idea of a world ruled by a Divine creator, he being the prototype of the ruler of the state. In this process of evolution Plato more and more returned to archaic currents of Greek thinking, going back even to the thought of Old Orient; and in doing so he at the same time anticipated Hellenistic syncretism (Stenzel p. 111 f.).

As we may add, Plato's idea of worldly rule as a reflexion of heavenly rule has in a way survived in Byzantium and even until 1917 in Russia in connection with Christian doctrine, and in China until 1912, of course without any association with Christian doctrine. The assumption that earthly power must be derived from a transcendental realm is really present already in Plato's *Republic* where it is reflected in the idea of the king-philosopher and the philosopher-king. There exists, further, a parallel between the abnegation, inherent in the situation of the « just » who are to govern the state, according to *Republic*, and the Byzantine idea that the monarch must prostrate himself before Christ. In the same dialogue Plato is not at all sure that the just ruler will always be understood and hailed by the members of the community and, as is known, he has even the prophetic vision of the just who, after having suffered many pains, will be crucified.

Viewed from our side Plato is, of course, a thinker of restorative

character. What he tries to overcome and to surpass is the idea of humanism, taking this word in its narrow sense of « deifying humanity » or, as Protagoras said, making the man « the measure of all goods ». This idea was in a way present also in Socrates, yet connected with the idea of a subjectively perceived higher Good; in so far Plato, in his early dialogues, opposed Socrates to the sophists. But afterwards he transcended the Socratic point of view; he noted even with some pleasure the strict regulation, in Egypt, to which human affairs were submitted on a religious basis — and among these affairs was music.

What matters in the present connection is that, in accordance with the conceptions of late Plato, mathematics and music necessarily assume — or they reassume — a great importance in philosophical thought: *number* being by itself symbol of order and *sound* connecting in a unique way the outer world with the inner, the physical with the perceptual, the rational with the sensuous. It is indeed a peculiarity of the perception of sound that those sound relations appear most natural, obvious or fundamental, that correspond to the simplest arithmetical ratios (and relations between sounds determine the character of the single sound as perceived by us). Of course this correspondence between the character of sound relations and the form of numerical ratios is independent of the fact whether we do or do not know those ratios; as Ptolemy said (*Harmonics* I 10): our perception is just « crying out », when it meets with the consonance of the fifth as soon as the ratio 3:2 is realized on the monochord.

It is therefore logical that mathematical demonstrations and musical exemplifications are found especially in the later works of Plato; and, we may add, it is also logical that J. Stenzel, the historian who devoted to this phase of Plato's thought a special attention, dealt so much with the mathematical conceptions of Plato; it is true he did not concern himself with Plato's ideas about music, but both fields are closely connected in themselves. It is clear that an investigation of these mathematical and musical passages in Plato is as important in understanding him, as for the history of mathematics and music. But these passages still present many obscure points. In this paper we deal with that passage in *Timaeus* (35 A ff.) where a scale, musical as well as cosmical, is expounded.

We must note that even before Stenzel L. Robin had thrown much light upon this side of Plato's thinking. I have in mind his dissertation *La théorie platonicienne des idées et des nombres d'après Aristote* (1908) and that excellent chapter devoted to late Plato in *La pensée grecque et les origines de l'esprit scientifique*, (1923).

II.

THE BUILDING OF THE WORLD SOUL; STAGE 1. AND 2.

As he often does, Plato unfolds his thought through a fable. He tells us that God the Creator made the World Soul by taking the Indivisible substance or the Same, and the Divisible or the Other and mixing them into a Third substance.

What may be meant by those two primary substances is not so easy to determine. Perhaps Plato blends the two main currents of pre-Platonic thought: that of the Eleatics who saw everywhere one unchangeable and immovable essence, and that of Heraclitus who saw nothing but motion and change. Probably there is also a connection with the Pythagorean opposition of « limit » and « unlimited », of which the mixture yielded the « limited ». We must also remember the opposition of the « One » to the « unlimited Two » which occurs in Plato elsewhere (see below, p. 15 f.). I invite the reader to consult the learned survey contained in Taylor's commentary, p. 109-136, to see how those notions have been interpreted and how they may be interpreted. I shall only note here that there exist also musical analogies. Thus there are commentators who have assumed that Plato meant by the Same the number 1 and by the Other the number 2, both yielding the relation $2:1 = \text{octave} = \text{the harmony « par excellence »}$. Plutarch in his commentary on *Timaeus* (Ch. 27) also gives to that opposition a musical turn: he identifies the Same or One with the tone and the Other or Divisible with the interval, according to which the third essence would be the melody. This is not only a beautiful variation to the theme proposed by Plato, but represents at the same time a reconciliation of the two opposed tendencies in ancient musical psychology, namely, the Platonic and Pythagorean which hold the tone in first place, and the Aristoxenian which gives the prominence to the interval. From the point of view of musical psychology we could further propose to parallel the One and the Same with the octave and the Other with the twelfth or fifth, in so far as in our tonal system (and, in reality, in every system) the octave means repetition of the same tone quality, whereas the twelfth or fifth is the basis of tone variety, since all the different tone qualities result from the different degrees in which this interval is juxtaposed — in other words, from the « cycle of fifths ».

The question, then, is to build the World Soul or the principle of the whole. Having mixed the two primary principles into a third, the Creator again mixed these three into one essence; and then the latter was subjected to mathematical organization. He first took one part of the whole, then the double of this part, then one and a half of the second, or the triple of the first, then the double of the second, the triple of the third, the octuple of the first, and 27 times the first. Thus

the seven parts were to each other in these ratios: 1 2 3 4 9 8 27; as a matter of course, these numbers are taken to represent musical tones, or more exactly, the ratios between numbers are taken as tone intervals.

As we see, these numbers stand for the first three powers of two (2, 4 and 8) and for those of three (3, 9 and 27), 2 and 3 being the first of even and odd numbers respectively. The reason why Plato goes as far as the third power and not farther is, as it has been assumed, that the third power and the third dimension represent the limit of corporal things or, as a commentator has said, the soul must proceed into bodies (we shall return to that question below, p. 31). Stenzel explains that « musical harmony » does not need more than three powers (p. 103), but we shall see that it really needs more. Yet in fact already Plato's seven numbers include *four* powers of 2 and 3 respectively, as 1 is equal to 2^0 and to 3^0 ; whether or not Plato himself may have been conscious of this fact, it is important for the musical interpretation of the scale.

As has already been indicated, in constituting our tone system (and, in reality, every tone system) we must distinguish between the octave (ratio $2:1$) and the twelfth or fifth (ratios $3:1$ and $3:2$) in this sense that the latter and not the former is the constitutive interval, since the fifth produces by juxtaposition or multiplication all kinds of tone qualities, whereas the octave is in this respect meaningless, producing always the same quality in different position or « site ». Yet we must not proceed too hurriedly: as we see, our series means juxtaposition of thrice the octave and thrice the twelfth, but whether the juxtaposition is by ascending or descending we do not yet know.

We shall note that our series includes intervals yet other than the octave and the twelfth: consecutive tones (or numbers) yield the fifth $3:2$, the fourth $4:3$ and the whole tone $9:8$. We can also establish the existence of intervals composed of those noted, $8:3$ being, e. g., composed of an octave and a fourth. But we must now turn to something which would be more like a scale, i. e., a series composed of « neighbouring » tones.

Plato achieves the first step in this direction by stating that the Creator filled up the « double » and the « triple » intervals, i. e., the octaves and the twelfth by two « means ». He does not say expressly which means, but there can be no doubt that we have to interpose within either of these ratios or intervals the *arithmetical* and the *harmonical* mean; none of Plato's commentators has understood it otherwise.

As is known, the arithmetical mean is that which is equidistant from both outer terms (or, taken quantitatively, it is half their sum), whereas the harmonical mean must conform to the condition that the first outer term be to the second, as the difference between the first and the mean is to the difference between the mean and the second (which is the case, e. g., in the series 6: 4: 3).

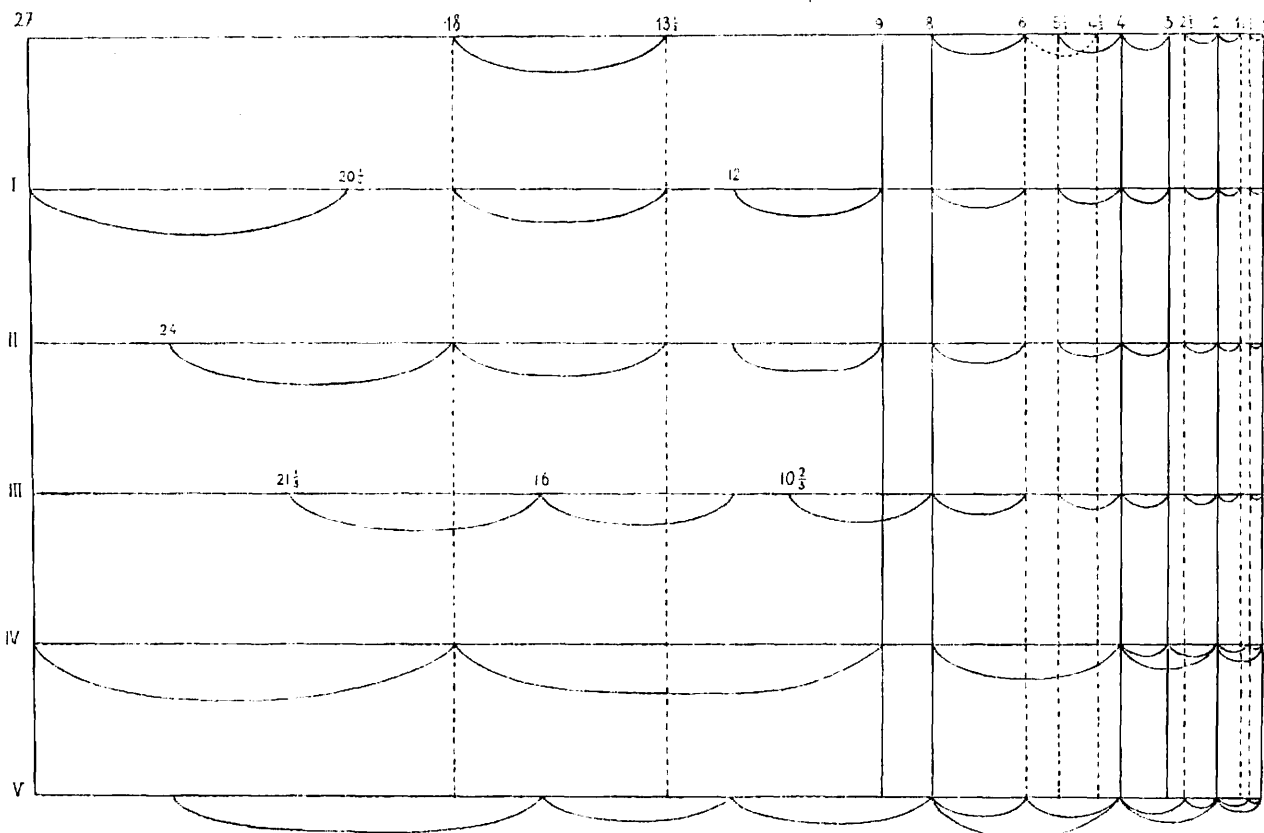
By dividing octaves and twelfths in these two ways, we obtain within the octave two tones distant from the outer tones respectively by a fourth (12: 9: 8: 6, the famous composite of four numbers which combines in itself the series' 4: 3: 2 and 6: 4: 3), and within the twelfth two tones distant from each outward tone by a fifth (6: 4: 3: 2, this being a combination of 3: 2: 1 and 6: 3: 2). As we see, both double divisions are symmetrical as to tone distance (or as to the magnitude of the ratios); in either case the smaller interval is in the middle (whole tone and fourth respectively) and the larger interval at both sides (fourth and fifth respectively).

In applying this double division to every one of the octaves and the twelfths present in the primary series, we obtain:

$$1 \quad 1\frac{1}{2} \quad 1\frac{1}{2} \quad 2 \quad 2\frac{2}{3} \quad 3 \quad 4 \quad 4\frac{1}{2} \quad 5\frac{1}{3} \quad 6 \quad 8 \quad 9 \quad 13\frac{1}{2} \quad 18 \quad 27$$

(the numbers of the primary series are written in italics). Of course some of the numbers have been obtained in different ways, as, e. g., $1\frac{1}{2}$ which is the arithmetical mean of 2 and 1 and the harmonic of 3 and 1. We could easily do away with fractions by multiplying the whole series with 6. Musically speaking the main feature of our series, although not expressed by Plato, is the fact that it includes *five* different powers of 3 (which holds good whether we may multiply it with 6 or not), that is, five different degrees of the series of twelfths or fifths, or five different tone qualities (characters), as 27 and $13\frac{1}{2}$ correspond to 3^3 , 9 and 18 and $4\frac{1}{2}$ to 3^2 , 3 and 6 to 3^1 , 1 and 2 and 4 and 8 to 3^0 , $1\frac{1}{2}$ and $2\frac{2}{3}$ and $5\frac{1}{3}$ to 3^{-1} . Here we may remember that five tone qualities, built up on this basis, constitute the pentatonic musical series (e. g., c d e g a).

As we have noted, already the primary series contained the fifth, the fourth and the whole tone, besides the octave and the twelfth; and the double division applied to the octave and the twelfth produces further such intervals. Among these it is especially the fourth whose rôle is constitutive in the building of a scale, and that for the simple reason that



it is, as concerns tone « distance », the least of consonant intervals (or simple ratios), that which is first attained if, starting from one tone, we proceed by « steps ». Also 9:8 is important — the whole tone which represents, as to distance, precisely the normal « step » needed; being present already in the primary series, it is further produced by the double division of octave as 12: 9: 8: 6 and, in the latter sense, we understand why it is so often defined as the difference between fifth and fourth. Secondly, we have obtained even the (minor) third, $5\frac{1}{3}: 4\frac{1}{2}$ being equal to 32: 27; as we see, it is the result of the two overlapping fourths $5\frac{1}{3}:4$ and $6:4\frac{1}{2}$ (or, e. g., *d a* and *e h*), but we do not need it as a constitutive element. By juxtaposing or adding the intervals obtained we should get still others, but that too does not matter in this connection.

Now Plato continues: the Creator having realized the double division of the octaves and the twelfths, took the whole tone interval 9:8, and with it filled all the 4: 3, i. e., the fourths; by this the mixture of the soul substance was used up.

By saying that the Creator filled all the fourths with whole tone intervals, Plato conveys the impression that his series is already continuously divided by fourths (4:3); but that is not the case, as we see by the figures above p. 8, where the fourths are marked out by brackets. As has been said, two of these fourths, $5\frac{1}{3}:4$ and $6:4\frac{1}{2}$, interfere with each other; on the other hand there is a large space not filled in by fourths, i. e., the space included between the figures 27 and 8, or 27 and 9; this outer twelfth, lying outside of the three octaves 8:1, contains the fifths $13\frac{1}{2}:9$ and 27: 18 with a fourth between them.

Thus we understand the difficulties with which Plato's commentators have been faced when trying to form a definite idea of a scale agreeing with his instructions. They made their task easier by disregarding Plato's implicit statement about a frame of fourths, established *before* filling in the whole tone intervals. Proclus in his commentary (194 F) is really taking matters lightly when, in the face of Plato's instruction to fill in the fourths, he speaks of filling in « every fourth or fifth »: that is so much the more at variance with Plato, as the latter says explicitly that when the whole tone is inserted twice, there remains the « limma » or diatonic half-tone. On the other hand modern commentators have been perhaps induced to skip the frame of fourths because in the modern sense a scale is more or less determined by octave periodicity. But in the ancient Greek conception the division by fourths

was preeminent. At any rate Boeckh has not dared to disregard that statement of Plato; he was not only a philologist but also well acquainted with Greek musical theory. Taylor has, as we shall see, a series of fourths, but it does not accord with Plato's division.

Yet the question of establishing a frame of fourths is not easily solved and here, already, begins our doubt whether Plato really had in mind a definite scale with its details, or he merely stated the general principles of one.

For visualizing the situation I ask the reader to look at the diagram accompanying this paper and, first, at the uppermost row. This illustrates just the series of numbers given above, which presents the result of the double partition of our octaves and twelfths. We ought always to distinguish within this series between the primary numbers (1, 2, 3, 4, 8, 9, 27) and those which are the result of division ($1\frac{2}{3}$, $1\frac{1}{2}$, $2\frac{2}{3}$, $4\frac{1}{2}$, $5\frac{1}{3}$, $6, 13\frac{1}{2}$, 18).

The first thorny question is that of the interfering fourths $5\frac{1}{3}:4$ and $6:4\frac{1}{2}$. Are we to conserve both, as Boeckh has done, or to drop one of them, and if so, which? We could imagine that Plato had in mind those overlapping fourths of the Greek « modulating » scale (*systema metabolon*), which, being filled in similarly (e. g., as *a b c d* and *h c d e*), produce the tones *b* and *h*. Yet I am not at all sure that Plato had in mind such a concrete detail and there is even another detail which would not be in harmony with this: as we shall see, *b* and *h* coexist in *different* octaves of our scale according to several of the possible variants, and therefore it would not seem logical to have them, apart from that, in the same octave and produced by another method. However, that argument does not rule out the overlapping fourths conclusively, any more than the argument urged by Taylor (p. 142) in accordance with Proclus, i. e., that the coexistence of *b* and *h* as neighbours would produce the « apotome » *b-h* as distinct from the normal « diatonic » half-tone *a-b*, while Plato does not mention the apotome; as to this I should observe that the coexistence of *b* and *h* in a « modulating » scale does not mean that both tones are concretely connected. I think we must say modestly that we shall drop one of the competing fourths only for the sake of simplicity; and in this case I think it will be rather $6:4\frac{1}{2}$ than $5\frac{1}{3}:4$, as the latter fits better in the octave periodicity; and — though it is not always realized in music and we may perhaps not fully realize it in the present case — octave periodicity is inherent in the idea of a scale.

In consequence the tone $4\frac{1}{2}$ will have lost its position as a point of support in the framework.

In fact at this point I am already hesitating whether I ought to continue. But with the good intention of arriving at an illustration, at least, of what Plato may have meant, I go on and now it is the question of having fourths everywhere. Again I ask the reader to look at our diagram, and this time at rows I, II and III where some of the possibilities are outlined.

I). The disposition most closely adapted to that of the uppermost row would be I. In fact here all the tones of the uppermost row (excepted $4\frac{1}{2}$) preserve their rank as points of support, that is to say they are defining intervals of fourths. Yet $20\frac{1}{4}$ and 12 are added to the Platonic tones. In this case beginning from 6, there would always be fourths separated by a whole tone; this means that we should obtain within this space fifth periodicity, replacing octave periodicity. Or, to put it more precisely, octave periodicity would be ruled out from 8 onwards, as in the « normal » way the next fourth after 8:6 would have to be $10\frac{2}{3}$:8. In consequence the octave repetitions of $5\frac{1}{3}$, i. e., the tones $10\frac{2}{3}$ and $21\frac{1}{3}$, are lacking; or to put it still otherwise, the fourth 12:9 appears as an octave repetition of $5\frac{1}{3}$:4 shifted by a whole tone; $18:13\frac{1}{2}$ is in the same position when compared to 8:6; $27:20\frac{1}{4}$ is shifted by one whole tone in relation to 12:9 and, therefore, by two whole tones in relation to $5\frac{1}{3}$:4. From the musical point of view, looking at the tone qualities or the powers of 3 present, we see there are not less than six, which is precisely a consequence of the fact that octave repetition is so largely disregarded: the tones 1, 2, 4 and 8 mean 3^0 ; $1\frac{1}{2}$, $2\frac{2}{3}$ and $5\frac{1}{3}$ mean 3^{-1} ; $1\frac{1}{3}$, 3, 6 and 12 mean 3^1 ; 9 and 18 mean 3^2 ; $13\frac{1}{2}$ and 27 mean 3^3 ; $20\frac{1}{4}$ means 3^4 . Here we may note that the sixth power of 3, 729, assumes a special importance with Plato in another connection (*Republic* 587 D), where he takes it to represent the ideal period of solar revolution (729 days and nights = $364\frac{1}{2}$ whole days instead of ca. $365\frac{1}{4}$, a magnitude already known at that epoch); but of course I should not take this as a « proof » that we must follow this method, because, I think, there is nothing to be proved here.

II). Here the fourth 12:9 is, as in case I, shifted by a whole tone in relation to the octave repetition of $5\frac{1}{3}$:4, but the shifting goes no farther, as the fourth 24:18 is contiguous to $18:13\frac{1}{2}$ and therefore an octave repetition to 12:9. In fact the only difference between I and II is that in the latter

the first tetrachord from left is joined to the second, instead of being separated from it by a whole tone. The effect of this is, precisely, that octave periodicity is less violated than in case I. In our case, again, two tones are added which were not in the uppermost row, 12 and 24, but as they are in octave relation, they may appear as substantially one tone. One of the tones of the primary series, 27, has been left outside the « skeleton ». The last note being 24, the tonal range of the series is 4 octaves and a fifth instead of 4 octaves and a major sixth; in this connection we may remember that sometimes 24 and 1 are given as limits of cosmic harmony (Macrobius II 1, cf. Aristotle *Metaphysics* XIII 6). Perhaps the fact that this disposition does not fill out the whole range of 27:1 seemed tolerable because in the typical two octave-scale of the Greeks there was a note, the Proslambanomenos or « added », which was considered to be in a way outside the structure by a whole tone. This series gives five different powers of 3, since 1, 2, 4 and 8 mean 3^0 , $1\frac{1}{2}$, $2\frac{2}{3}$ and $5\frac{1}{3}$ mean 3^{-1} , $1\frac{1}{3}$, 3, 6 and 12 mean 3^1 , 9 and 18 mean 3^2 , and $13\frac{1}{2}$ means 3^3 . Thus the number of tone qualities is the same as in the Platonic series (uppermost row) and even the tones are the same (apart from different octave localisation, as the tone quality denoted by 3^1 has two more representatives, 12 and 24, and the 3^3 quality one less, 27).

III). In dealing yet more freely with the terms of the uppermost row we can concede to the « law » of octave periodicity its full sway. In this case we continue after 8 exactly the same kind of division which goes on within the three octaves 2:1, 4:2 and 8:4. That would mean interpolating first 16 which gives the next octave, then, between 8 and 16, the numbers $10\frac{2}{3}$ and 12, and beyond 16 the number $21\frac{1}{3}$. In this case the uniform division of the octave would extend to $21\frac{1}{3}$, i. e., to 4 octaves and a fourth; the extreme tone of the primary series, 27, would remain outside, as it is separated by two whole tones from $21\frac{1}{3}$. The disadvantage of this arrangement is that no less than four tones, $10\frac{2}{3}$, 12, 16 and $21\frac{1}{3}$, would be added to the Platonic series in the uppermost row. Two among the tones of the primary series, 9 and 27, would be dropped, as well as two further tones contained in the uppermost row, $13\frac{1}{2}$ and 18, and the fourth $18:13\frac{1}{2}$ would not be preserved. From the musical point of view, looking at the tone qualities, there would be only three, as 3^0 , 3^{-1} and 3^1 occur in the different octaves, 3^0 being

represented by 1, 2, 4, 8 and 16, 3^1 by $1\frac{1}{3}$, $2\frac{2}{3}$, $5\frac{1}{3}$, $10\frac{2}{3}$ and $21\frac{1}{3}$, 3^2 by 3, 6 and 12.

Now I think it is useless and hopeless to determine which of the three methods Plato has had in mind. Leaving a gap that can be filled in different ways is just characteristic of his attitude: he puts down a principle and does not much care how it will be carried through. Plato even fails to give prominence to the difference between 2 and 3 which is musically essential, for the octave corresponding to 2, serves only for delimiting spaces in the scale, whereas the twelfth or its octave equivalent, the fifth, corresponding to 3, produces different and ever more different tone qualities. Nor does Plato notice that these two constitutive principles of scale construction are in some way antagonistic or, at least, irreducible, as never will a power of 2 coincide with one of 3 and therefore never will a tone resulting from octave juxtaposition exactly coincide with one resulting from the juxtaposition of fifths (hence the « Pythagorean comma ») — although, of course, in practical perception a tone will always be able to take over the value of another one so near to it as to be scarcely distinguishable.

III.

THIRD STAGE

But the fable continues. Whichever we may choose of the variants I, II or III, we have not as yet a scale. As we have intimated, Plato goes on to tell us that the Creator filled all the fourths (the 4:3 intervals) with the whole tone (9:8), every fourth containing two of these and a rest interval, equal to $256:243$ or $2^8:3^5$.

Now for a survey we ought to fill out all the fourths in row I, II and III. But as that would take much time, we can restrict ourselves in every case to the first and the last fourth.

I). The fourth $1:1\frac{1}{3}$, will become $1:\frac{9}{8}:\frac{27}{64}:\frac{81}{64}$ and the fourth $20\frac{1}{4}:27$ will become $\frac{27}{8}:\frac{27}{4}:\frac{27}{2}:\frac{27}{1}$. That is precisely the « diatonic » form of the fourth or the diatonic « tetrachord », if we take the word « diatonic » to mean the combined use of whole and half tones, a half-tone being interposed after every two or three whole tones. It is true we might doubt whether 9:8 has to be apposed to the smallest number, as we have just done, or to the greatest which would produce, respectively,

$\frac{9}{8}:\frac{27}{64}:\frac{81}{64}:1$ and $27:24:\frac{81}{8}:\frac{81}{4}$. But that does not matter because, as we shall see, we are faced with two reciprocal methods of computing, one of which takes numbers as frequencies (vibration numbers) while the other takes them as string lengths; and in this aspect $1:\frac{9}{8}:\frac{27}{64}:\frac{81}{64}$ will yield the same result on the basis of string lengths as $\frac{9}{8}:\frac{27}{64}:\frac{81}{64}:1$ on the basis of frequencies, and vice-versa. In fact the wording of Plato does not even exclude a third possibility which would locate one whole tone beside the one limit of the fourth and another beside the other limit, thus putting the half-tone in the middle; again we see that Plato does not enter into concrete details. It is clear that having filled out all the fourths we obtain a greater variety of fraction numbers than before; and this we can again rule out by multiplication.

II). In this case the first tetrachord would be as in case I, and the last $18:\frac{81}{4}:\frac{27}{2}:24$.

III). In this case the first tetrachord will be again as in case I, and the last $16:18:\frac{81}{4}:\frac{81}{2}$.

But now we are faced with the question which is practically foremost. Before being able to represent our numbers by sound-names we must determine whether they are to mean relative frequencies or relative string lengths. As is known, these two are in reverse proportion one with another. All commentators, from Boeckh onwards, but with the exception of Thimus, have decided in favour of lengths, taking this as a matter of course. But it is not a matter of course, as, e. g., Plutarch, who was a very good Platonist, supposes frequency numbers in his commentary (Ch. 18) and Plato himself in *Timaeus* 67 B ascribes to the low tone the lesser rapidity and consequently the smaller number. (It is true Plato speaks of rapidity of locomotion, not rapidity of vibration, cf. *Der Toncharakter* p. 136 and 139-141; that may be an error or a kind of illustrative speaking; in either case the question is of « quantity of movement »).

Again I think that it is rather in the style of Plato to let the question open: what did matter for him was the correspondence between tones and numbers, or tone relations and numerical relations; and, whether the reader would choose one or the other among those reverse proportionals, was to him a secondary question. Nor can we take him literally with regard to the compass of his scale, because 4 octaves plus a major sixth is a compass unheard of in Greek musical theory. As to reciprocity of numbers, we may yet remember the notion of the « unlimited

Two», mentioned above as occurring in Plato, and which, according to Stenzel (34, 51 f. and. 59 f.), would signify the Two as a principle independent of whether it is taken as multiplier or divider, or, in other words, as positive or negative power; indeed, in this case it would be the «undefined Two» rather than the «unlimited», but with the Greek word both meanings are compatible. We have to remember also what Plato (*Republic* 525 D f.) says about fractional numbers: instead of fractioning the One, mathematicians would rather multiply it, or in other words, instead of saying, e. g. $\frac{2}{5}$, they rather say that a definite magnitude, taken five times, will be equal to three units. In all this there is a tendency to lay stress more on the formal than on the material character of number, to take number as a principle rather than as the measure of concrete quantity. (As is known, the idea of continuous quantity has found its extreme representation with differential calculus created by Leibniz; and yet it was this great thinker himself who, in regard to musical tones, most decidedly insisted upon the importance of integer numbers and, especially, the first prime numbers).

Yet as modern people we crave for concretization and, in this case, we have to choose. We choose — for the time being and with all necessary reserve — string lengths, in order to be more in accordance with the majority of modern commentators; and we follow them also in their method of inserting the whole tone, that is to say, we juxtapose two whole tones by starting from the smaller number which is, according to our first assumption, the higher tone of the fourth; in consequence the rest interval (the half-tone) will be placed at the lower end, producing the «Dorian» species of diatonic tetrachord (different from the «Lydian» which has the half-tone at the higher end and the «Phrygian» which has it in the middle — it is true, these names as applied to *tetrachords* are not quite authentic).

Now at last we can associate with numbers our favourite tone letters. But here we must yet remember that we should take them not, as is usual, as absolute degrees of pitch measured by vibration frequencies per second, but as a system of relative pitches or tone relations which, by principle, can be based on any concrete pitch. In reality even our tone letters had originally this meaning in the «Gregorian» system, and only by the development of instrumental music and ensemble playing was the relative (the musically real) meaning of the tone letters superseded by the concrete (the musically abstract).

Therefore if we take on the monochord the whole string length as representing 27 and its twenty seventh part as representing the number 1 — which gives an interval of 1 octave and a major sixth —, the question whether we shall denote these two tones as G_1 and e''' , or C_1 and a'' , or F_1 and d'' , or D_1 and h'' , etc., may cause some hesitation. We must answer that we are not interested in the concrete pitch of the tone taken as starting point, and that therefore we do not use the letters in the modern fixed sense. But taking them in the old, relative sense, would there be a reason for preferring one denomination to the other? Of course, a reason there exists, and the criterion which must determine our choice is the same as that observed in the old «Gregorian» system. In terms of «musical theory», it may be stated thus: «avoid accidentals as much as possible» or, in terms of musical psychology: «ascertain the tone qualities involved (i. e., the degrees of the series of fifths) and dispose them with d as centre» — so that a series of five tones will take the form, $c g d a e$, a series of seven tones the form, $f c g d a e h$, a series of nine the form, $b f c g d a e h f a$ (supposing in the latter case that b and $f a$ are real qualities coexisting with the others, not mere «chromatic» extensions or «modulating» features). It is true, this supposes an odd number of tone qualities, for in the case of an even number the centre would fall between two qualities, say between d and g , or d and a ; but actually systems with an odd number of terms have always been predominant and we can even doubt whether others (hexatonic, octatonic) have existed in the full sense.

Now Boeckh has taken the highest tone, the one corresponding to $\frac{1}{27}$ of the length, to be Nete diezcuemenon, which is e' in our transcription; that would mean that the tone corresponding to the whole string is G_2 , a tone located more than two octaves below the lowest tone of our piano. In order to avoid such extreme depths we should shift this series by three octaves, replacing G_2 and e' by G and e'''' .

We shall now see what kind of scale our three rows will produce on these premises. I hope the reader will forbear with me if this time I invert the order for the sake of convenience.

III). The third disposition would produce this scale:

G 27 A H c d e f g a h c' d' 9 e' 8 f' g' a' h' c'' d'' e'' 4 f'' g'' a'' 3
 h''' c''' d''' e''' 2 f''' g''' a''' h''' c'''' d'''' e'''' 1.

tones are never present in one octave and they are rather an assembly of disparate octaves. Or to put it otherwise, it is a «modulating» system, and modulating twice, always from one heptatonic to the other. In this sense it could be doubted whether we have a reason for changing the transcription of the whole from *h* — *es*, asymmetrical, to *fis* — *b*, symmetrical; indeed, it could be argued that they do not form a whole. But even if modulating, i. e., passing from one system to another, it is logical to have that one which is in the centre, written «without accidentals».

1). Given always the same premises, our first skeleton would produce:

G 27 *As B c d es f g a b c' d' 9 e' 8 f' g' a' h' c'' d'' e'' 4 f''*

g'' a'' 3 h'' c''' d''' e''' 2 f''' g''' a''' h''' c'''' d''''

e'''' 1.

There are 34 tones as before. Taking again the scale as a whole, we see that the series from *h* to *as* corresponds to *ten* tone qualities — an even number. It may therefore be shifted in two ways, the center falling in either case between two notes: if it is written as ranging from *fis* to *es*, the center would be between *d* and *g*, if from *cis* to *b*, the center would be between *d* and *a*; in the former case the lowest and the highest tone would be *D* and *h''''*, in the latter *A* and *fis''''*. (Among these two representations we should perhaps prefer the former, because the Greek conception was inclined more towards «flats» than «sharps», as exemplified by their «modulating» scale which connects not *f* and *fis*, but *b* and *h*). Again we see how the breach of the octave periodicity, present in the «skeleton», is equivalent to inner contradiction. Whereas in the skeleton stage we had 3^{-1} (*fis* according to the transcription preferred by us), 3^0 (*h*), 3^1 (*e*), 3^2 (*a*), 3^3 (*d*) and 3^4 (*g*), now to them are added 3^5 (*c*), 3^6 (*f*), 3^7 (*b*) and 3^8 (*es*). In accordance with this we shall again shift the mathematical representation from $3^{-1} = fis$, $3^0 = d$ and $3^1 = es$ to $3^{-4} = fis$, $3^{-1} = d$ and $3^0 = es$. We can even say that the proper representation would be to give to *fis* not «minus four» as power exponent, but «between minus five and minus four», to *d* «between minus one and zero», and to *es* «between four and five»; nay, even the tone which we have marked by *d* is, after all, not *d* but «between *a* and *d*», etc. Yet all this is rather theoretical, as in fact the ten qualities are not operating

together (and in so far they are not real qualities). Here we have, in the terms of Cleonides, a «composite system with four Mesai», the Mese being shifted from *a* to *e* in the high region and to *d* and *g* in the low (according to the transcription we have preferred). Again, if we assume frequencies instead of string lengths, the order would be inverted:

$$es = 3^{-8}, b = 3^{-7}, f = 3^{-6}, c = 3^{-5}, g = 3^{-4}, d = 3^{-3}, a = 3^{-2}, e = 3^{-1}, h = 3^0, fis = 3^1.$$

But we have yet to add an observation. There is still a method by which we could fill out the fourths diatonically without compromising octave periodicity (and, consequently, unity of diatonicism). As we saw, the breach of diatonicism, i. e., the surpassing of the number of seven different tones in I and II, was caused by the fact that, while in case III there were only two fourths repeated from octave to octave (say *h* — *e* and *e* — *a*) and these were limited by three different tones, in case II there were four, and in case I even five (the number of limiting tones being always greater by one than the number of fourths). All these fourths, when juxtaposed in one row, can be denominated, e. g., as *h e* — *e a* — *a d* — *d g* — *g c*. It is clear that when going beyond two fourths the result is breach of diatonicism if the fourths are filled out in an *uniform* manner: say, if they were all «Dorian» as in our previous examples, or if they would be all «Phrygian» or all «Lydian». But it is understood that we could avoid the breach by giving to the different fourths *different* diatonic forms, e. g., if having given to *h e* and *e a* the «Dorian» form with half-tone in the lower part, we should give to *a d* and *d g* the «Phrygian», with half-tone in the middle, and to *g c* the «Lydian», with half-tone in the upper part. Yet that would only mean transferring the contradiction to another plane: in this case the uniformity of disposition which also is inherent to the idea of scale would be repudiated.

It is interesting to note in this connection that commentators have not always been in agreement in deciding for the «Dorian» form, though such, in general, has been the case with modern commentators. In fact, as we shall see, Pseudo-Timaeus, Proclus and Psellus leave the question open between Dorian and Lydian. Plutarch who, as we have said, supposes frequency numbers, has the Lydian tetrachord (see Ch. 18 of his commentary), precisely that which seems the most natural from the modern point of view ($1: \frac{9}{8}: \frac{11}{8}: \frac{4}{3} = c d e f$ or $g a h c$). And still more interesting,

he tells us (Ch. 19) that other people used the tetrachord with the half-tone in the middle.

It is understood that in all this the supposition seems to be that one species of fourth is uniformly applied and that it is not a question of finding an expedient for safeguarding diatonicism throughout. Yet whether the breach of diatonicism may be accepted or not, it can be maintained that the « Phrygian » division of fourth, with the half-tone in the middle, appeals in a special manner to our sense of logic. By filling up the fourth by the « Dorian » method we add two tones which are, as to their « character » or their position within the series of fifths, wholly at *one* side of the two tones given (e. g., e and $a = 3^2$ and 3^1 would be joined by $f = 3^2$ and $g = 3^1$, supposing, of course, frequencies), and the « Lydian » tetrachord means adding to the given notes two notes at the *other* side (e. g., g and $c = 3^1$ and 3^2 would be joined by $a = 3^1$ and $h = 3^2$); whereas, according to the Phrygian method, both added notes are symmetrical on both sides of the given tones (e. g., to $a = 3^1$ and $d = 3^2$ there would be added $h = 3^2$, two degrees at « right » from a , and $c = 3^2$, two degrees at « left » from d). In short, whereas by filling out the fourth by the Dorian method we displace the centre of our group to « left » or, as I should say, to the « masculine » side, and in the case of the Lydian tetrachord to « right » or to the « feminine » tone qualities, by using the Phrygian method the centre is not shifted.

Of course that holds good also when we combine two, three or more fourths, and therefore I should prefer to fill out our frame of fourths by the Phrygian tetrachord, whether taken from row I, II, or from III. In case I we should obtain:

C 27 D E s F G A B c d e f g 9 a 8 h c' d' e' e' fis' g' a' 4 h' c' d' ' ' 3
| _____ | _____ | _____ | _____ | _____ |
e' ' fis' ' g' ' a' ' 2 h' ' c' ' ' d' ' ' e' ' ' ' fis' ' ' g' ' ' a' ' ' ' 1.
| _____ | _____ | _____ | _____ |

or, in case of frequency numbers:

$a''' 27 \quad g''' \quad fis''' \quad e''' \quad d''' \quad c''' \quad h''' \quad a''' \quad g''' \quad f''' \quad e''' \quad d''' \quad 9 \quad c''' \quad 8 \quad b''$
 $a' \quad g' \quad f' \quad es' \quad d' \quad c' \quad 4 \quad b \quad a \quad g \quad 3 \quad fes \quad dc \quad 2 \quad B \quad A \quad G \quad F \quad Es \quad DC \quad 1.$

In case II:

C 27 D E F G A B c d e f g 9 a 8 h c' d' e' fis' g' a' 4 h' c' d' 3
 e' fis' g' a' 2 h' c' d' e' fis' g' a' 1.

or, in assuming frequency numbers:

$e''' 27 d'''' c'''' h''' a''' g''' fis''' e''' d''' c''' h'' a'' 9 g'' 8$
 $f'' e'' d'' c'' b' a' g' 4 f' e' d' 3 c' b a g 2 f e d c B A G 1.$

In case III:

F 27 *G A H c d e f g a h c' 9 d' 8 e' f' g' a' h' c'' d'' 4 e'', '' g'' 3*
a'' h'' c''' d''' 2 e''' f''' g''' a''' h''' c'''' d'''' 1.

or, in case of frequency numbers:

$h''' 27 a''' g''' f''' e''' d''' c''' h'' a'' g'' f'' e'' 9 d'' 8 c''$
 $h' a' g' f' e' d' 4 c' h a 3 g f e d 2 c H A G F E D 1.$

After what has been said above, I shall not explain further why I have adopted one method of transcription rather than another. At any rate I repeat that, with the six series noted, it is only a question of illustrating Plato's thought and not of deciding what he must have represented to himself.

The only thing we must still note in this connection is that III, taken in both significations, presents a disadvantage: one of the primary tone, 9, stands not as a « frame » tone delimiting a tetrachord, but only as a tone included in a tetrachord; that is the price for the advantage of octave periodicity. In looking again at our diagram we see that in I and II all the fourths contained in the Platonic division are made use of and all the Platonic points of support stand again as such (of course with exception of the fourth $6:4\frac{1}{2}$ and the tone $4\frac{1}{2}$), which is not the case

in III — the difference between I and II being only that the primary tone 27 is in I, employed as delimiting tone, and not in II. The form I, which so largely replaces octave periodicity with fifth periodicity, is much like the scale of *Musica Enchiriadis*.

Summarizing our argument, we can say that Plato not having contented himself with dividing the octave but having divided also the twelfth, has offended against octave periodicity, and the consequence is that if we followed him strictly, we should transgress the limits of diatonicism. It can be doubted whether Plato was aware of all the consequences of what he had said.

IV. Some Commentators

As I have said, most of the commentators have neglected to establish a frame of fourths to be filled in, but, having given their allegiance to the primary series 1 2 3 4 8 9 27, they filled in whole tones (9:8) and limmata (256:243), forming fourths and fifths rather as they pleased.

Boeckh is, as has been intimated, in this respect an exception. It is true he does not present the division by fourths in his scheme on p. 161, but such a division is presupposed. His disposition is the same as that of our row II, but with two differences: 1) he includes the fourth $6:4\frac{1}{2}$ concurrently with $5\frac{1}{3}:4$; 2) he includes ~~does not present the division by fourths in his scheme on p. 161, but such a division~~ the tone 16; why? for obtaining, as he says, the fourth octave from 1, but in fact he wishes also to agree with Pseudo-Timaeus, as we shall see forthwith. The first produces, as has been said above (p. 11), a tone «chromatically» opposed to one contained within $5\frac{1}{3}:4$; the consequence of the second — not made evident by Boeckh — is that there is virtually inserted a new tetrachord ($12: \frac{27}{2}: \frac{243}{16}: 16$) which again interferes with one already present ($\frac{27}{2}: \frac{243}{16}: \frac{2187}{128}: 18$). In this way Boeckh obtains 36 tones (instead of 34), a number upon which stress has been laid by some ancient commentators. Of course, like nearly all modern authors, he is convinced that Plato supposes string lengths, and therefore his series is *descending* from 1 to 27 (or from 384 to 10368, as he, in agreement with ancient authors, puts it, in order to avoid fractions); Boeckh, like most modern authors, is also of the opinion that it must be the «Dorian» tetrachord. We should have, then, his scale if in the one noted above (p. 19) we inserted *c* and *b'*, if we united by brackets *c* with *a* and *a'* with *d'*, and if we transposed the whole series three octaves lower. We see that the number of differing notes remains the same, nine; and as they range from *h* to *es*, they must, as has been observed, be shifted to *fis* - *b*.

The earliest commentary on *Timaeus* come down to us, though not the earliest known, is attributed to *Timaeus*, one of the interlocutors of Plato's dialogue itself: a forgery intended to give the impression that it had served as model for Plato's dialogue, whereas it is really an abstract from the latter. (It has been published in some of the editions of Plato's works as an appendix to the *Timaeus*, as, e. g., in the

fourth volume of C. F. Hermann's edition). R. Harder (in Pauly-Wissowa) distinguishes between two versions of this treatise, one dating from the first century B. C. and one from the first century of our era; according to him, the claim of Timaeus' authorship is put forth only in the second version, while the first was an unpretentious paraphrase of Plato's dialogue.

This treatise gives a very short account of what is supposed to be Timaeus' system of psychogony. It states that the first number (Plato's 1) must be represented by 384 and that, taking the «double» and «triple» intervals with their partitions and filling in the whole tones, there will arise 36 terms, the sum total of which is 114695. Now that is also the sum of Boeckh's 36 terms. But Pseudo-Timaeus does not say by which means he derived his terms from the primary numbers. Of course he also leaves in suspense the question of string lengths or frequencies, as well as that of the tetrachord species.

Now in Hermann's edition of that treatise a *scheme* is added on p. 410 which gives the whole series, still not as divided by tetrachords but only distributed among the seven primary numbers. As this scheme is presented in the edition not clearly and even incorrectly, I shall reproduce it here in order to show how the thing was represented in ancient times:

First number 384, 432, 486 (h) 512, 576, 648, 729 (h)

Double of first number 768, 864, 972 (h) 1024

Triple 1152, 1296, 1458 (h)

Quadruple 1536, 1728, 1944 (h) 2048 * 2304, 2592, 2916 (h)

Octuple 3072

Ninefold first number 3456, 3888, 4374 (h) 4608, 5184, 5832 (h) 6144 *

Ninefold second number 6912, 7776, 8748 (h) 9216

Twentysevenfold first number 10368.

(by h we indicate the half-tones or limmata). It is curious that 18 which is not one of the primary numbers inaugurates a special row, namely, as «Ninefold second number»; the reason is perhaps purely graphical, as the line beginning with «Ninefold first number» was exceedingly long. There are 34 terms and they are counted as such; their sum is 105947. Yet at the places indicated by * there are inserted above the line the numbers 2187 and 6561 which are to their neighbours in the relation of Apotome and Limma. As they are not counted with the others, they must be a later addition and therefore this scheme could not originally have belonged to the treatise. Taking the 34 terms with the two interpolated, there are 36 which produce as sum 114695; we see that the interpolation makes the scheme agree with what is said in the text of the treatise. And this interpolated series is precisely Boeckh's (yet the 34 terms are not the same as those obtained by excluding from Boeckh's series the superfluous tetrachord and the added tone which we discussed above p. 24, i. e., the terms derived from our row II; for the present scheme has as sixth term from the end 6144, while there it was 6561).

The number of differing notes (tone qualities) contained in this scheme is eight. In supposing string lengths, we should write it as $G - e''''$, with *h* within the three

octaves $e'''' - e'$ and with b below; supposing frequencies, it would be $F - d''''$ with b in the three lower octaves and b in the two higher. The two interpolated notes would augment the number of tone qualities by one; they would be e and b' in case of string lengths, producing of course Boeckh's scale which, however, we have preferred to shift to $D - b''''$, with fs and b ; they would be fs'''' and b' in case of frequencies and in this case we should preserve the notation as $F - d''''$.

The next commentary, chronologically, is *Plutarch's* (about 100 A. D.) who, as has been said above p. 15, supposes frequency numbers and the « Lydian » tetrachord. He jumps from the primary series straight to the division of the fourth as such, without establishing the relative sizes of the tetrachords (I refer to Ch. 16 ff. of his commentary). This procedure may seem very general and too cautious. Yet we must acknowledge the very fact that Plutarch occupied himself with the fourth; and as to establishing a frame of fourths, he may have been too well aware of the thorny difficulties which Plato poses in this way.

Proceeding chronologically (though a complete survey is not here proposed), the commentary of *Proclus*, the great Alexandrine of the 5th century, comes next. Proclus discusses the Timaeus scale at great length (191 E - 198 B, 211 F and 212 E).

We have already mentioned (p. 10) that Proclus does not care about establishing a frame of fourths and he alters Plato's « fourths » to « fourths and fifths ». Now we must in detail see how he proceeds. But first the text must be established in the passage 196 E - 197 B, where the terms are enumerated. We see in the latest edition (1964) that in 197 B six words have been bracketed and six others added, with the result that the term 1374 is replaced by 4096; in a foot-note the editor says candidly that Proclus himself may have erred. Now it is amazing that the editor did not see that this passage needed no emendation and that another passage is evidently wrong, namely 197 A, where the text runs: « adding to 1536 the next term by 9.8 we obtain 1728, adding to this number 9.8 would be 1944, to this 9.8 is 2187, to this limma is 2048, etc. ». Evidently the words which we give in italics and which, by the way, are missing in one of the manuscripts, should be cancelled, because 2048 is limma to 1944 and not to 2187. With this correction, and leaving the former passage as it stands, we obtain the number of terms and their sum as required, i. e., 34 and 105947; moreover we are in accordance with the scheme added to the Pseudo-Timaeus treatise.

Now we shall follow Proclus in his procedure. The disposition of his scale, as far as he indicates it, may be seen in row IV of our scheme.

Proclus makes a good start (194 B) by exposing the double division, arithmetical and harmonic, of all Plato's octaves and twelfths (according to row I in our diagram). This series is transformed into integer numbers and given as $6 \dots 162$. Then, having stated that, to insert the smaller intervals, we must replace 1 (or 6) by 384 (194 C), Proclus (194 D) puts down the first octave as follows:

384 432 486 (b) 512 576 648 729 (b) 768

(what this division of that octave is we shall see later on). Proclus then (194 E-F) gives again the double partition of every octave and every twelfth. There follows a discourse about Limma, Apotome and even Comma (is it really Proclus' or an in-

terpolation?), and then (196 E) he turns to the question of constructing the scale, mentioning a diagram but supplying us with none.

In this passage Proclus again expounds the terms of the first octave, as given above and this time he establishes its division, yet not, as we should expect, into fourths with a whole tone between them, but into a fourth and a fifth (384 - 512 - 768). Nevertheless, Proclus, like a too conscientious philologist, repeats that 512 and 576 are the two means. In adding the fifth 768 - 1152 he states that the limit of the first twelfth has been attained, after which (197 A) the fourth 1152 - 1536 completes the second octave. The terms of the third octave, 1536 - 3072, are stated without subdivision; within this series occurs the contradiction that I have tried to remove by text emendation. In going on to the next term, 3456, Proclus (197 B) states that the limit of the second twelfth is attained. From 3456 the series goes on to 6912; that is an octave and again its division is not stated (in this passage, the text is, as we have seen, *not* in need of emendation); this octave, with the adjoining fifth 6912-10368, constitutes the last (third) twelfth. As we see, the last (fourth) octave is not adjoining to the third which was 1536 - 3072, but is separated from it by a whole tone; instead of 3072 - 6144 it is 3456 - 6912, and thus the last octave is subordinated to the last twelfth. That is in accordance with the primary series of Plato who, although the number 16 is virtually present within the range 1 - 27, disregards the fourth octave (8: 16 = 3072: 6144).

The series as given is confirmed by Proclus (197 C) by the statement that it contains 24 whole tones and 9 limmata. It is further confirmed by the passage 212 E where Proclus establishes the sum of his 34 terms as 105947. And it is furthermore supported by its agreement with the scheme quoted above in connection with Pseudo-Timaeus which produces the same sum; the latter scheme may therefore serve to supply the diagram missing in the text of Proclus.

The text of Pseudo-Timaeus is opposed by Proclus (197 D - E) who maintains that the number of terms is not 36 but 34; Proclus' argument is that Plato does not mention Apotome but only whole tones and limmata (see above p. 11). But it is curious that elsewhere Proclus appears to be much more conciliating. In 211 F he says that the diagram contains 34 or 36 terms and that, as 34, 36 may be considered as appropriate to the Soul. He even contradicts himself (195 D - E) by quoting the term 2187 which, as we have seen, is not rightly included in his series, and by adding: « we shall need it in the diagram ». It is impossible not to see in this the hand of a corrector adapting Proclus' text by alteration and interpolation to the number of 36 terms, as required in the treatise of Pseudo-Timaeus (also the discourse about Apotome and Comma 195 D - 196 C, mentioned above, is superfluous if Apotome is not admitted in the diagram).

The surprisingly tolerant attitude of our text towards the number of 36 terms appears even in the same passage where Proclus opposed this number: in 197 D there is said that other interpreters who wanted to have in the diagram not only limma but apotome, have added two terms; and 197 F - 198 A these are specified as 2187 and 6561, the first being in apotome relation to 2048 and the second to 6144. We see these additional terms are exactly the same as those inserted in the scheme added to Pseudo-Timaeus.

As could be expected, Proclus does not say whether he means string lengths or frequencies, yet the tetrachords occurring in his series show at least that he places the limma at the limit of the fourth, i. e., as lowest or highest. I see that Vincent, p. 178, ascribes to Proclus a constructive method that would be rather complicated, though consistent in itself: having filled out the first twelfth with tones proceeding from the division of the octave (the series from 384 to 768 as above, but continued to 1152), Proclus would treble the single terms for obtaining those between 1152 and 3456 (≈ 3 and 9 in the primary series), and would treble them once more for obtaining those contained between 3456 and 10368 (≈ 9 and 27 of the primary series). This I can not find in Proclus' commentary and, moreover, it would not produce the sum 105947.

The scale set forth by Proclus is, as has been said, the same as that of the scheme added to Pseudo-Timaeus which contains eight different tone qualities; but while in the latter no division is marked except that by the seven primary tones, Proclus' division is that represented in our row IV. However, this division is, from the point of view of a musical scale, not logical: we do not see why the first octave is divided as fourth and fifth and the second as fifth and fourth; in the following octave no division is established; and the next octave which is again given without division, is, as we have noticed, «disjunct», i. e., separated by a whole tone from the foregoing, a feature unheard of in the history of musical scales. On the whole we have the impression that Proclus is not much concerned about musical reality, and this may be characteristic of him as Neoplatonist. We must further notice the way how he urges upon us the purely intelligible character of musical harmony 199 D ff., where the intelligible is emphatically opposed to the perceivable. It is the same decayed form of Platonism which strikes us in S. Augustine (but, curiously enough, not in the Greek fathers as Basilus, both Gregories, and Maximus); indeed in the conception of Augustine, follower of the Neoplatonist Plotinus, intelligible and real music appear as infinitely distant from each other.

We have now to mention the small treatise devoted to Plato's Psychogony by the Byzantine Michael Psellus, late in the 11th. century, and edited by Vincent, 316-332. Psellus quotes Proclus at length, but only the preliminaries of Proclus' dissertation, up to 195 A, which include the modelling of the first octave and the double division of the octaves and twelfths, finishing with the statement that there are 24 whole tones and 9 limmata. To this Psellus abruptly adds his diagram; but this is at variance with Proclus' scale. Probably Psellus found it too laborious to follow Proclus in the details of his exposition, or was he discouraged by the contradictions of Proclus' text; therefore he proceeded as a compiler, inserting what seemed to him plausible. His diagram differs from Proclus' scale, in so far as it has 2187 instead of 2048 (yet 2048 is inserted above the line), and also the division is different (see row V in our diagram). There is more of musical logic in his than in Proclus' arrangement. From the tone 384 (≈ 1) onwards there are first two octaves divided as fourth and fifth, then two octaves divided as fifth and fourth; the beginning of the next octave is again a fifth, followed by a whole tone. We see that there is some symmetry in this grouping. The division of each of the octaves by the fifth on equal terms with the fourth is Byzantine (we remember that Proclus had already had an

eye out for the fifth, and this may be significant, as Alexandrine is pre-Byzantine). The symmetry of Psellus' construction appears also in the fact that, in supposing string lengths, the upper octaves will contain *h* and the lower *b*, or in supposing frequency numbers it will be the reverse. There is nothing to show us whether Psellus may have thought of string lengths or frequencies; yet it is, as in the case of Proclus, clear that the tetrachords are supposed to be either «Dorian» or «Ly-dian».

As the system of Proclus (and that of the scheme added to the Pseudo-Timaeus), that of Psellus contains eight different notes (or powers of 3) which we should represent as ranging from *b* to *h*. That is, in terms of Cleonides, a «composite system with two Mesai» and it would correspond to the Greek «systema metabolon» (modulating system), but with the difference that in the latter *b* and *h* are alternative notes within one octave, while with Psellus they are located in different octaves.

As to modern commentators we have already referred to Boeckh. Vincent (p. 176 ff.) should also be adduced as a thorough commentator and connoisseur of Greek musical theory. He has, as to the Timaeus scale, his own theory which is not devoid of logic, although it is not adapted to the division of octaves and twelfths as given by Plato. His meaning, in the way it is formulated, is rather difficult to understand; still, it is evidently the following. He subdivides the first octave strictly in accordance with Plato's division as $1:1\frac{1}{2}:1\frac{1}{2}:2$, but then he proceeds independently, without looking at the (incomplete) frame presented by Plato. Having filled out the fourths $1:1\frac{1}{2}$, and $1\frac{1}{2}:2$ in the normal way and obtained eight terms in the first octave, he doubles these terms for filling the second octave, and he doubles them once more for obtaining the terms of the third octave. Then the interval $1:3$ is taken and again subdivided according to Plato as $1:1\frac{1}{2}:2:3$, which produces twice the fifth and once the fourth; and now, contrarily to Plato's instruction which provided only the filling out of fourths but in accordance with Proclus and Psellus, the fifths within this twelfth are filled out with whole tones leaving a limma, as in the case of the fourth; these twelve terms are tripled for obtaining those from 3 to 9 and again tripled for obtaining those from 9 to 27. We see that Vincent takes the first octave and the first twelfth as a nucleus which he subjects to a constructive, but rather theoretical idea of his own. Of course, some of the terms obtained result as well from the division of the octave as from that of the twelfth. On the whole there are 39 terms, and apotome occurs rather often (chromaticism in the modern sense). There are ten different tone qualities or powers of 3, as in the case of our row I when it was filled out. Of course Vincent supposes string lengths. The result would be a scale extending from *D* to *h'''*, with notes ranging from *es* to *fis*. But in the realization of his idea Vincent is not consistent. Here intervenes the idea of the «conjunct» (or modulating) Greek system with eight different notes ranging from *b* to *h*; and therefore in the sector of two octaves which is represented on p. 183, two of the tone qualities (or powers of 3) are omitted. It is curious that Vincent was not aware of Boeckh's interpretation.

Although it is manifestly wrong, we shall now consider the explanation given by A. von Thimus, because he is a distinguished thinker and because he bases himself on the continuation of Plato's story which is in itself interesting to us. In

fact Plato, after having said that the Creator filled all the 4:3 with 9:8, producing in every one a rest interval of 256:243, goes on to connect his musico-arithmetic series with cosmology. We remember that in this method of thinking astronomy has always been associated with music and mathematics; and the latter being diversified as arithmetic and geometry, there results the fourfold group: arithmetic, geometry, music, astronomy, which later on constituted the so-called Quadrivium. Plato says that the mixture having been used up (as we have seen), the Creator split up the whole structure longitudinally in two strips, then putting the middle above the middle, as in the Greek letter Chi (X), he bent both of these strips making each of them into a circle; and making one circle the outer, the other the inner, he conferred to the outer movement — the circles are, then, rotating — the nature of the Same and to the other that of the Other; then leaving the outer circle unsplit he split the inner sixfold; thus he produced seven circles distant by double and triple intervals (as we see, the series 1 2 3 4 3 9 27 comes again into play), these being the orbits of the seven planets disposed according to their distance from the earth, and that distance being taken as 1 in the case of the Moon and 27 in the case of Saturn. This had been correctly interpreted already by Boeckh. But Timaeus does not agree (II 213 ff. and I 157 ff.). In spite of Plato's clear statement, he fails to realize the double division of octave and twelfth. He starts from the two strips just mentioned, putting down one of them as 1 2 4 8 and the other as 1 3 9 27, and completing both by reciprocal fractional numbers in this way:

$$\begin{array}{ccccccc} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} \end{array}$$

To these rows Timaeus applies the crosswise superposition and he joins them by crosswise multiplication, obtaining the series:

$$\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}, \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}, \frac{1}{4} \cdot \frac{1}{5} = \frac{1}{20}, \frac{1}{5} \cdot \frac{1}{6} = \frac{1}{30}, \frac{1}{6} \cdot \frac{1}{7} = \frac{1}{42}, \frac{1}{7} \cdot \frac{1}{8} = \frac{1}{56}, \frac{1}{8} \cdot \frac{1}{9} = \frac{1}{72}$$

which, musically speaking, is nothing other than the diatonic (heptatonic) system, or the series of seven tone qualities arranged in fifths: f, c, g, d, a, e, h , which can be symbolized also by $3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$. The result of this system is indeed the same as that of Plato's, that is, gradation by fifths, but Plato does not state it so nakedly. These *fifths* are, in Timaeus' opinion, the «double and triple intervals» to be divided arithmetically and harmonically (because the fifth is equal to 3 divided by 2), and this double division gives major and minor chords in the modern sense (cf. below, p. 38); only the *thirds* contained in these chords would have to be filled out by 9: 8. That is interesting but very aprioristic. By assuming frequency numbers Timaeus is again at variance with all the other modern commentators, but in this we can not wholly disapprove of him.

Finally, there is the learned commentary given by Taylor. Having transformed the primary series, as the ancient authors do, in 384 - 768 - 1152 - 1536 - 3072 - 3156 - 10368, Taylor (p. 133-135) proceeds directly to realize the division by whole tones and limma without keeping to the skeleton series which Plato derives by arithmetic and harmonic partition. The basis on which he applies his division by whole

and half tones is just the idea of diatonic scale which juxtaposes within the octave two fourths and one whole tone. Taylor even narrows down this idea, without apparent reason, to an octave with the fourths as extremes and the whole tone in the middle. In consequence he has a frame of fourths, but it is his own; as it subjects itself to octave periodicity, it coincides practically with our row III (the least orthodox of the three). Taylor then goes on to establish what was exactly Proclus' disposition. He determines it in starting from the sum 105947 as given by Proclus, and the result is right, but it is attained by comparison of that sum with Taylor's series, not, as we should expect, with the scheme added to Pseudo-Timaeus' treatise. Again when polemizing with Pseudo-Timaeus, i. e. *pro* 34 terms and *contra* 36, Taylor does not avail himself of the scheme. He explains the difference between Proclus' scale and his own by assuming that Proclus was induced by the wording of Plato to continue inserting whole tones so far as it was arithmetically possible (always supposing 384 to be the first term and admitting only integer numbers); but this, I fear, is a misinterpretation, as Plato does not say «as far as possible» but «as far as possible within the fourth» and, moreover, Plato does not put down the terms in the multiplied form from 384 to 10368. After all that has been said, I think Taylor is too confident when he is «sure that the series intended by Timaeus (the interlocutor in Plato's dialogue) is either that of Proclus» or his.

1. AN APPRECIATION OF PLATO'S STANDPOINT.

As we have seen, Plato is not scrupulous about concrete details. In a rather imprecise way he behaves as if by dividing his octaves and twelfths harmonically and arithmetically he would already possess the frame of a scale divided by fourths; and as to the filling out of the fourths, he says only that there must be two intervals of 9:8 and one of 256:243. In reality it is not a scale which Plato sets forth but only the *principles* of a musical scale, and these are: 1) the division of the series by agency of the consonance and, especially, the fourth, 2) diatonicism, this, of course, being not a feature proper to every scale, but to the normal one; at least that which we consider as normal and which the Greeks considered as such, notwithstanding their use of the «chromatic» and the «enharmonic» genus, with many intermediate shades, all of which are probably to be interpreted as «colourings» of diatonicism. (In fact it is testified that the diatonic genus seemed to them the most natural and the most easy, and that it was the most common).

As observed above, p. 7, the reason why Plato's seven primary numbers do not go beyond the third power is not a musical but rather a symbolical one. It is probably his tendency to geometrical representation: as this is limited by three dimensions, it can not represent more than cube numbers or, in general, more than num-

bers composed of three factors; in this the geometrical representation of tone is more limited than the arithmetical. We have further noticed (p. 27) that 16, the fourth power of 2 and the fourth octave, is virtually included within the range 1 — 27, but Plato does not set it off, and that probably for the sake of aesthetic symmetry, as 1 3 9 27 is in a way one side of a triangle and 1 2 4 8 the other. But in reality, as we have observed (p. 7), already the seven terms as put down by Plato include four powers of 2 and of 3, as the number 1 00 is a power of 2 and 3. We had further noticed that Plato does not engage in a comparative appreciation of the rôle of his constitutive elements, the Two (octave) and the Three (fifth), of which the latter stands foremost in the aspect of real musical qualities, nor does he notice that they are in a way antagonists (see above, p. 14 and p. 24). We have also observed that further powers of Two, and especially of the decisive Three, are admitted by Plato *implicitly* (p. 18). All this is, I think, a sign that Plato proceeds as an artist who presents his thought in the form of a fable.

In other words, Plato does not go on to *concrete* what he has said. But this does not mean that what he has said has no real significance. It is the principle of a *real* scale, considered as normal, which he exposes, and here he is right in establishing first the general principle of consonance, then a first division of primary consonances which, curiously enough, is pentatonic (see above, p. 8); then, in a leap, he purports to have erected a frame composed of fourths, and these fourths he divides quite normally by inserting whole tones. Surely those were principles of scale construction which had been established long since.

It is, then, not at all a « phantom » (« Hirtgespinnst ») which Plato presents to us, as has been asserted in a rather sensational and miscellaneous book, published 25 years ago, titled *Plato und die sogenannten Pythagoræer*: a book which of course (of counsel) has exerted a great influence upon philosophers and writers on music, may even upon the flock, reputed to be cautious, of philologists.

Indeed concreteness is not reality: but that is just the confusion so widespread in our modern era. Plato looks at the real world as it is, and does not resort to abstractions like matter which is not form, or the idea of sensuous perception in which no element of reason is present. That which is fundamental to him is the correspondence between our apparatus of cognition and the object of this cognition (a correspondence which exists by principle and does not exclude error in a given case); and this he is not, as we are not, able to trace back to anything else than the creative act of a transcendent being. Precisely because that is Plato's meaning and because he thinks that idea and reality are in their inner-

most in accordance with each other, his example of a scale, as his examples in general, can not be a mere « phantom », contrived arbitrarily; he is, in other words, not a Kantian of the 19th century.

The allegation of « phantom » is supported by another erroneous assertion which also has (of counsel) had a widespread echo: according to this the Timaeus scale (or quasi-scale) would be wrong in its root because it bases its structure only upon octave and twelfth (or fifth) and not upon the « natural » third, which implies the following prime number, 5, 4:5 representing the major third and 5:6 the minor — the tone relations so prominent in Western music of later times. Here the author of the « phantom » theory confounds what is peculiar to a given musical era and what can be stated generally. The « natural » third, which establishes a « direct » link between tones distant by a third, does of course impose itself as a consonance in a musical culture as ours, that adheres so exclusively to simultaneous harmony (cf. *Der Toncharakter* § 13), but it plays only an insignificant rôle in broader musical conceptions. Even as concerns ourselves, it can be demonstrated that, notwithstanding the apparent sweetness and attractiveness of this new consonance, the fifth is yet far more constitutive and fundamental: we must remember, 1) the famous experiments of C. Stumpf showing that even men of our culture are naively more apt to mistake as one tone a simultaneous fifth than a third, 2) the fact (on which I laid so much stress in *Der Toncharakter*) that in our perception the differences of tone quality (as distinct from pitch) are still in the main based on the series of fifths.

I am glad to note that at last the view that the « Timaeus scale » is quite arbitrarily conceived seems to loose its credit, as I see in a paper by B. L. van der Waerden in *Hermes*, LXXVIII, 1943. But on the other side a book about musical psychology which appeared in 1947 bears still witness to the havoc caused by the aforesaid book: not only does it repeat the wrong assertions referred to above (as well as another which we shall forthwith cite), but it combines them into a new error: that Aristoxenus was a champion of the third as a consonance (and as such — of course — a champion of musical progress).

The other wanton assertion of that book, and one which has found an applause even more unanimous, is this: Plato would do violence to music and mistake its substance and nature, since he considers tones as quantities and not as qualities, and only Aristotle and Aristoxenus would have done justice to the qualitative character of musical sound. It seems clear that

this assertion depends mainly on a one-sided conception of number, or rather on failure to grasp the Greek conception of it (without which we too really can not do). In that larger sense number is not only measure of quantity, indicating the more or less, but it indicates a structure, and the gradation of powers of 3 is, as we have seen, precisely in correspondence with the musical quality of sound.

This conception is further reflected in the ancient tendency to represent numbers as geometrical figures. The Greeks called « oblong » a number composed of two factors differing by one, e. g., $6 = 2 \times 3$, or $12 = 3 \times 4$, which is the counterpart of superparticular ratio (3:2, 4:3, etc.); this produced figures like: ; (cf. Th. Heath, *A history of Greek mathematics* I, 1921, 76 ff.). In this sense number was to them really « figure », i. e., form; and what they saw, thus, with their eyes, they perceived with their ears as the musical, qualitative relations of tones. Yet there was a restriction imposed on this craving for visual representation (see above, p. 31): numbers composed of more than three factors could not be represented geometrically. With us, it is by a survival of this ancient conception that we speak of « square » and « cube » numbers.

In fact we can not fail to be impressed by this correspondence which has something of a marvel: not only our perception of musical pitch and tone « distance » is rigorously (logarithmically) in accordance with frequency numbers, but also the structure of the number, apart from its magnitude, is reflected in the musical character of the tone, apart from its pitch; e. g., the tone *e* (which in a system with two « sharps » would be written as *fs*) is, as to its musical quality, precisely as remote from another given tone within the same system, as both are distant from each other according to the series of fifths. In this sense we can say that the tone is symbolized by a power of 3, as well as that it is itself the vivid symbol of the number (as to eventual claims of the number five to play a similar rôle as the Three, cf. *Der Toncharakter*, § 13).

It has become just a slogan that Platonism considers the musical tone quantitatively and that the qualitative conception of it was introduced first by Aristotle and Aristoxenus. We may just as well say the contrary: that according to the Platonic (and the Pythagorean) view, and in view of the Greek conception of number, musical quality appears as a matter of course, and that it is rather the Aristotelian and especially the Aristoxenian conception that can be suspected of quantitative tendencies. It is known that Aristotle and Aristoxenus, more than Plato, laid stress on the character of number as measure of quantity, homogeneous and unimixedly

compatible (see Stenzel 59); and it is exactly in correspondence with this conception of numbers when they are inclined to view musical sounds as exponents of a continuous gradation — the pitch gradation of course — rather than as qualities in the proper sense. Now this gradation is akin to one of intensity and, therefore, of quantity. Yet we ought to note that Aristotle was only the initiator of such conceptions which were to be developed and emphasized by Aristoxenus. Aristotle's intermediate and somewhat contradictory standpoint as a musical thinker would make a very interesting subject of study.

In a broader sense we may further note that Plato — at least late Plato — is not a thinker hostile to world beauty, as he is sometimes considered to be. The tendency of fleeing the world, as manifested in Neoplatonism during the late period of paganism, was not Plato's. It is true, beauty is not to him — as it can not be to us — a thing strictly and merely dependent upon sense perception; perceptible beauty depends upon a superior order of things shining through what the senses grasp. That precisely is aesthetic reality, and sense perception devoid of it would be a mere abstraction. As we have seen, the quality of a tone perceived musically is like that of a form or a figure, one quality always being interdependent with others. But the order, shining through our sensuous impressions and transforming them into aesthetic perceptions, points to an absolute Good standing behind all things and which is the source of being. That is why Plato, in his lecture about the Good, resorted to numbers, to mathematics and to the Pythagorean notion of « limit » — a method which, as we are told by Aristoxenus on the testimony of Aristotle, was surprising and disappointing to many of the hearers who had come in order to partake of « goods » like wealth, health and strength.

VI.

ABOUT PROPORTIONS, ARITHMETICAL, HARMONIC AND GEOMETRICAL

But having engaged in polemics with others I do not see why I ought not to seize the opportunity to polemicize against myself. There is in *Der Toncharakter* a section devoted to « harmonic partition » and there the « harmonic mean » has been treated in a rather peremptory way as having no real musical importance as compared with the « arithmetical

mean » (p. 211 f.). I think that is to some extent exaggerated and some defence may be made of the harmonic mean if we reflect on Plato's coordination of these two means. Actually the harmonic proportion is not quite so artificial as it appears from its definition (on this see above, p. 8) but it is in a way a reversal and a derivation of the arithmetical. Take an arithmetical proportion with one mean, as 2:3:4, and suppose it is to be reversed, yet not as a whole (4:3:2), but by parts, reversing first 2:3 and then 3:4. The first relation in the reversed form being 3:2 and the second 4:3 we must, for uniting them into one proportion, take the smallest common dividend of 2 and 4, which is 4, and accomplish the corresponding multiplication which gives 6:4 and 4:3, i. e., 6:4:3, precisely a harmonic proportion; as we see, the relation of the outer terms is also reversed in comparison to the arithmetical proportion with which we started. The relation between the two proportions may also be expressed as follows: the reciprocal value of the harmonic mean is equal to the arithmetical mean of the reciprocal values of both outer terms (denoting the harmonic mean by m , its reciprocal value $\frac{1}{m}$ is $= \frac{1}{a} + \frac{1}{b}$); or as Proclus says in his Timaeus commentary, 193 B, if in a fourfold proportion like $\frac{a}{b} = \frac{c}{d}$, b is the arithmetical mean between a and d , c is the harmonic, and vice-versa; or again, 196 F, if we take the double and the triple of one term, e. g., 1:2 and 1:3, the arithmetical mean of 1:2 will be the harmonic of 1:3, and the major term of 1:2 is the arithmetical mean of 1:3. In the same way as the arithmetical proportion 2:3:4 produces the harmonic 6:4:3, 4:5:6 will produce 15:12:10, etc. Now what does that mean musically? It means that, the interval between the extreme tones remaining the same, the dividing point is now in relation to the upper what it was in relation to the inferior; e. g., $c g c'$ is replaced by $c' f c$ or (admitting the « natural » third 4:5) $c e g$ is replaced by $g e c$: the component intervals are the same but their order is reversed.

It is easy to see that starting from two given outer terms which differ by one, e. g., from 1:2, in octave relation, or from 2:3, in fifth relation, we find in every case the arithmetical mean by multiplying these terms by two (necessarily so, since the arithmetical mean is, quantitatively, half their sum) and thus we obtain 2:3:4, or 4:5:6, etc. On the other hand we find the harmonic mean by multiplying the outer terms by the arithmetical mean: by 3 in the case of 1:2 (= 2:4) which produces 3:6, with 4 as harmonic mean; by 5 in the case of 2:3 (= 4:6) which produces 10:15, with 12 as harmonic

mean; thus, the harmonic proportion is in a way derived from the arithmetical. And again what is the musical meaning of it? As it preserves the same component intervals in reverse order, the harmonic proportion does not render things more « complicated » if we consider the component intervals, but it does so if we consider the whole; indeed in the latter respect 6:4:3 or 3:4:6 is more « complicated » than 2:3:4, and 15:12:10 or 10:12:15 more than 4:5:6.

Therefore it would seem preferable not to exclude the method of the harmonic mean from the scheme of partition established in *Der Toncharakter*, § 21 c). However it must take place behind the arithmetical mean, and we can still maintain that the partition by the latter is the true « harmonic partition ».

The connection existing between arithmetical and harmonic proportion is further illustrated by the fact that an interval sequence which represents an arithmetical proportion when supposing vibration frequencies, produces a harmonic one if string lengths are assumed, and vice-versa. Thus $c g c'$ is 2:3:4 as to frequencies and 6:4:3 as to lengths, $c f c'$ is 3:4:6 as to frequencies and 4:3:2 as to lengths; $c e g$ is 4:5:6 as to frequencies and 15:12:10 as to lengths, $c es g$ is 10:12:15 as to frequencies and 6:5:4 as to lengths. That is precisely the consequence of the fact that, in replacing frequencies by string lengths, we are reversing not only the relation between the outer terms but also that between every outer term and the mean.

Now just as there exists a preponderance of arithmetical proportion over the harmonic, we must concede to frequency numbers a preponderance over numbers representing string lengths. For this there is an evident reason which is, that string lengths are not so directly connected with sound production as are vibrations; therefore the representation of sounds by lengths is to be considered rather as derivative. We may add that string lengths are perceived by another organ, the eye, whereas our perception of vibrations is in a way a « continuation » of our auditory sense. It seems important to note in this connection that the eye is not capable of that spontaneous reaction to simple ratios as is the ear, i. e., it is not struck in the same way as the ear when faced with ratios as 1:2 or 2:3 (see above, p. 5). Thus we may find that the musical ear perceives more qualitatively than the eye; and yet colour as perceived by the eye is the classical instance of the notion of quality! This apparent contradiction is of course solved by recognizing that the notion of quality

may be a little different in both cases: one is the quality inherent in the member of a system in whose erection we take an active part (cf. above, p. 35; yet that does not mean that we may erect it arbitrarily); the other is a quality the foundations of which are infinitely more remote.

But whilst arithmetical proportion prevails over the harmonic, and frequency numbers prevail over string lengths, there is an affinity between arithmetical proportion and frequencies, as well as between harmonic proportion and lengths. It is not to be doubted that of the two tone relations $c g c^*$ and $c f c^*$ the former is spontaneously judged as more natural, smooth and « simple » than the latter, as well as $c e g$ when compared with $c e s g$; now in every case the first is producing the simpler ratios when expressed by *frequency numbers*; and these are in the form of arithmetical, not of harmonic, proportion.

Yet Zarlino (*Istituzioni* III 31) is very proud that in supposing string lengths, as he does, and taking the proportion to which the Greeks gave the beautiful name of « harmonic », he obtains the beautiful major chord (15: 12: 10 by string lengths), whereas the arithmetical proportion produces the minor chord (6: 5: 4). He is not aware that he is twice relying on the secondary: on arithmetical proportion as compared with harmonic, and on string lengths as compared with frequency numbers. He is, in a word, more humanist than thinker. Now as is known, the dispute between adherents and adversaries of harmonic « dualism » in the modern musical theory turns precisely around this question — whether the minor chord is to be considered as a counterpart of the major with equal rights, or rather as derived from it. I think this dispute could be settled by establishing that the relation is analogous to that between arithmetical and harmonic proportion as characterized above: indeed in both cases there is something of a reversal and something of a derivation. Yet I should not like to introduce here questions of « musical theory ». What we ought to be aware of is only that the comparison between major and minor chord (as between intervals and interval sequences in general) will be in its true perspective if we base it on frequency numbers, rather than if we assume string lengths.

Let us yet recall a curious parallel established by Boetius, *Arithmetics* II 45 (probably according to Nicomachus), as to arithmetical and harmonic proportion. He compares the arithmetical mean to the rule of a few men, i. e., oligarchy, because here the greater ratio is with the *smaller* numbers, e. g., in 2:3:4 2:3 is the greater ratio and 3:4 the smaller (we see that a quantitative conception of ratio is intruding here). The harmonic proportion is compared to aristocracy or the rule of the best men, because here the greater ratio is with the *larger* numbers, e. g., in 2:3:6 3:6 is the greater ratio.

But what about the *geometrical* mean? Boetius compares it to a « popular » or « equalized » state, because here the ratio is the same with the greater and the smaller numbers, as, e. g., in 4:6:9. Other writers have stated that the geometrical proportion is not made use of in music. That is surely right in a large measure. In fact when juxtaposing two equal intervals we never attain a higher unity, whether it may be two fifths which produce a ninth, or two whole tones of 8:9 which lead to an interval clearly not « contented with itself » but pointing to a further progression, 64:81. (In this sense the replacement of 64:81 by 4:5 — a third composed of two *unequal* whole tones, 8:9 and 9:10 — means that the third is « closing » on itself). This illustrates the general principle that *different* things, when being in inner agreement, form a harmony (as, e. g., the fifth and the fourth forming an octave); it is the reverse of the principle, touched on above, of « harmonic partition ».

As is known, the geometrical mean is equal to the square root of the product of the outer terms, and finding the middle of two given tones corresponds to extracting this root from the product of their numbers. As this product is, in general, not a square number, this will lead to irrational numbers; and this is exactly in correspondence with the fact that it would be a *tour de force* (yet not a musical feat!) to sing the precise middle of a fourth or a fifth. In practice geometrical proportion and, then, root extraction is applied only where a purely external compromise between tones is established in the sense of temperament. As in this case we start from the octave that has to be divided into twelve equal intervals, it means that we have a geometrical proportion with thirteen members, the outer terms being related as 1:2, i. e., it means the extraction of the root of twelfth degree from 2.

We must admit that Aristoxenus, with his assertion of twelve equal half-tones in the octave, is not far from that conception. (At all events I do not think there is much in it that is « qualitative »!). Yet if we consider what he says, we see that even he does not think of deriving the whole tone in this artificial manner (apart from the fact that he would not have been able to calculate this kind of root). In fact Aristoxenus derives the whole tone, not otherwise than done by Plato, by subtracting the fourth from the fifth, and this means that he is, after all, going back to consonant tone relations, with the only difference that, by some kind of idiosyncrasy, he does not like to connect them with numerical ratios. Therefore his statement that a whole tone is the sixth part of an octave, etc., must not be taken in a genetical sense, but only as an

external approximation: to him the whole tone *represents* the sixth of an octave, as the sixth part of the octave represents a whole tone. Indeed in proceeding from the whole tone to smaller intervals Aristoxenus is more in the pure line of distance equalization; he speaks of halves, thirds and fourths of a whole tone, a quarter of a tone being in his mind the *non minus ultra*. Yet it is not imaginable how he could have determined these smallest magnitudes otherwise than by approximation. Indeed in this realm of minim intervals and, accordingly, of complicated and not obvious ratios, we are more than in the domain of «normal» intervals reduced to pure distance appreciation; but it is rather a question of estimating than of determining, as in this case we have not the obvious points of support imposed by simpler ratios, upon which to rely. This is what could be said in answer to the question whether Aristoxenus knew of equalized temperament.

However there is, in the strictly musical sense, a field where procedure by equal steps takes place and is even essential: it is our series of fifths producing the different musical «qualities». Indeed here *a* differs as much from *d* by its quality as *e* from *a*. Yet that is not a musical complex in the proper sense but only a system *underlying* such: we may say that an infinite progression by uniform steps being given, we appropriate a sector of it and dispose it according to our symmetrical tendency (cf. above, P. 18). Here we may remember that it is the same with Plato's «three powers of 2 and 3» representing, in the opinion of Proclus 193 B, the geometrical proportion which «embraces» the harmonic and the arithmetical; this is a *background* of musical reality, as is our series of fifths which has been so much calumniated because misunderstood (as to the Two and its powers we can, as we have seen, disregard them in a strictly musical sense). We must agree with Greek theorists: «popular» or «equalized» rule is not compatible with musical reality. And yet, on the other hand, we must state that a single tone is nothing without a «community» in which it takes its place.

P. 5. I. This paper was completed and forwarded to the Editor when I got notice of a paper by Ottavio Tiby, *Note musicologiche al «Timeo» di Platone*, published in «Dioniso», Bollettino dell'Istituto Nazionale del Dramma Antico (XII, 1949), which treats the same subject. The Author introduces a new point of view in the discussion, as he believes that, in order to obtain the scale intended by Plato, we must keep in mind only the division of the three octaves (1 2 4 8), dismissing that of the twelfths (1 3 9 27) with its «disparate and perturbing» elements; in other words, the filing

out of the fourths, prescribed by Plato, would have to be applied only to the fourths resulting from octave division. I take this, from my point of view, as confirmation that Plato's directions are not to be taken too concretely. In the rest I am inclined to adopt, with regard to ancient thought, an attitude less «modern» than the author, i. e., less naturalistic or positivistic. As to the «number of Plato» referred to by Cicero as an example of obscurity, I think it is that about which Plato speaks in *Republic*, 546, a passage still much more involved than ours.

P. 5. II. As this paper is in some way a supplement to «Der Toncharakter», I may perhaps be permitted to add that this book has been curiously misunderstood in *The Musical Quarterly*, issue of July 1949, by a writer introduced there as «chairman of the Department of Psychology at Princeton University». According to this writer it is not worth while to discuss further what thinkers, not yet enlightened by modern neurology, have said in regard to the musical quality of sound, and he thinks «progress» can henceforth be achieved only by joining psychology to neurology or physiology — a method happily inaugurated, according to him, about 1925. I should indeed be grateful if he would indicate what this kind of research — surely valuable in other respects — has hitherto contributed to the problems tackled in my book. Although he is evidently well acquainted with it, the writer gives no hint as to this, and thus his attitude remains a purely negative one. He quotes the series *c g d a e h fis cis gis dis ais eis his* with the same misprint as in my book (i. e., without *h*) and charges me with having made this the basis of a tonal system with *d* as the «center of tonal gravity». But apart from the fact that a tonal system in my sense never extends to so much as 12 or 13 tones, I must state that I never took *d* as a «center of tonal gravity» or «fundamental» tone. These terms, used by the writer, seem to point to a specific conception of harmonics, whereas I have expressly distinguished — in superposing them one to the other — three conceptions: 1) the series of fifths, or rather a section taken from it, within which we naturally take *d* as «central» or «neutral» in respect of tone quality; 2) the «mode» in which any note may be the center of tone relations; 3) the major tonality with *c* as fundamental tone. The writer is at best unprecise when he ascribes to me the assertion that «Toncharakter» is more «fundamental» than any of the other tone attributes. I said clearly that I take it as not being so *elemental* as the others; and which of them we take as the more *important*, depends, of course, on our point of view. The writer — who really mentions Pythagoras as one of the authors from whom I draw my material — seems quite unaware of the fact that our tonal letters have a double meaning, the first and primary, qualitative and relative, denoting the tone character, the other, more recent: denoting «absolute» degrees of pitch — and that I am entitled to use the letters accordingly. We are in agreement in so far as we distinguish those two aspects of tone reality, but he puts more stress on the latter and I, on the former. Be that as it may, I have done without physiology and have therefore to bear the blame of the writer (who, however, could have mentioned that I gave some reasons for my attitude). But when seeing that the writer consigns Hornbostel and Stumpf to the same scrap-heap, I must advance in their defence precisely that for which I reproached them, i. e., that in the last resort even they fell into the error of taking the physiological process as cause or explanation of the psychic; in this

respect the writer fails to recognize the affinity he bears to them. Yet he also charges me with neglecting the *physical* conditions of tone; and here I have the real satisfaction to display a reviewer who represents the opposite extreme and charges me with indulging too much in acoustics (*Schweizerische Musikzeitung*, 1949). However, I hope that the writer will recognize that, even when I dealt with psychology, I was not bound to disregard history. He strives to give his review a witty close by speaking of « unconscious irony » but, alas, that supposes an arbitrary interpretation of the word « *Einführung* » as used in the subtitle of my book; an introduction is not necessarily a survey; we may, I think, try to « introduce » the reader to a subject from whichever side we choose and « *Einführung* » in die Tonpsychologie » is not the same as « *Musikgeschichte im Ueberblick* ». If I have failed in my undertaking — a thing quite possible —, the reason is surely not this.

But ambiguity in expression is a thing I deplore not less than these misunderstandings. What is this notion of a musical psychology (or an area of musical psychology) « chiefly influenced by Stumpf and von Hornbostel »? Is it possible for any musical psychology not to be so in a large measure and has not the writer himself borrowed from Hornbostel the notion of « Tonhöhe » (height) as an intrinsic (not metaphorical) tone property *which I did not?* (Cf. *The meaning of music*, 1931, 32-54; and compare *Journal of experimental Psychology* 1934, 585 ff., for another of these Hornbostelian phantoms taken over by one of the champions of psychological « progress » adduced by the writer). What signifies the allegation that my « factual material draws heavily » upon writers ranging from Pythagoras (!) to Hornbostel? It could mean that I took from them raw material and worked it over; and that would be wrong; or it could mean that I repeated what they have said and that would be wrong as well. The truth is that I have tried to understand the thinkers of the past (which is not, as the writer seems to believe, a matter of course) and to discuss the same problems. And why does the writer mention Stumpf's first great book, the one from 1883, and not the last one, from 1926, if not in order to reinforce the impression that my book is a priori obsolete?

I could add that the review of the same « Toncharakter », published in the *Journal of the American Musicological Society*, II, Fall 1949, represents a progress in comparison to that just discussed, in so far as its author, who has not in reality understood much more than his colleague, candidly confesses that he had difficulties to understand. But what is the reason of these difficulties? I am sure that it is an one-sided training rather than lack of training, and therefore a non-professional reader will, in general, come through much more easily.

J. H.