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Stephen M. Phillips

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Article 40

The Unification of All Sacred Geometries & Its Implication for Particle Physics

Part 1

by

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Abstract

No doubt because of their fear of peer ridicule, mathematicians have taken little professional interest in the 'sacred geometries' of various cultures and mystical traditions. It may, therefore, surprise them to learn that Leibniz, the great German mathematician and philosopher who discovered calculus at the same time as Sir Isaac Newton, chose to analyze the table of hexagrams used in Chinese divination and found that it was a binary number representation of the vertices of a cube and the lines that join them. He did not pursue the matter further, and apart from studies by a few architects and mystically minded geometers, little cross-cultural study of sacred geometries like the Kabbalistic Tree of Life and the Tantric Sri Yantra have been made with the aim of elucidating what (if any) is the nature of the information hidden in these purported blueprints for all existence. Academics are loath to examine such a topic partly because of the far-reaching (indeed, paradigm-shifting) implications that they would have to confront if they ever found significant, but rationally unexplainable, similarities between purported representations of God's design for His universe, even though these geometrical systems are separated in provenance by thousands of miles and years!

This article reveals mostly in pictorial form what the archetypal nature of these sacred geometries is and what they imply for particle physics (for more details, refer to Articles 19-39 on the author's website). The latter has to be tentative as yet because this material summarizes work still in progress. The former, however, can be established with the rigour of a proof of a theorem of Euclidean geometry and does not require model-dependent interpretation. The present purpose is not to discuss the implications – religious and philosophical – of a demonstration that certain sacred geometries are isomorphic to one another. Rather, it is to assemble and then to compare research results from other articles by the author so as to identify the essential properties shared in different ways by the sacred geometries of the Tree of Life, the I Ching & the Sri Yantra. This research article will prove conclusively that the fundamental information that they contain relates to the superstring nature of matter. In his various books, the author proved beyond reasonable doubt that superstrings were described over 100 years ago with the aid of a yogic siddhi called 'anima.' This mental faculty generates highly magnified images of microscopic objects as they exist in real time, i.e., they are NOT merely symbolic. The information embodied in the sacred geometries discussed here relate unequivocally to the details of this description of the basic constituents of matter. However amazing it may be, such a conclusion should come as no surprise, given that these geometries are isomorphic blueprints that determine the very nature of reality, including physical matter. A supersymmetric generalization of the Standard Model used by particle physicists will be hypothesized in order to interpret the common characteristics of these geometries. Previous articles by the author have shown that their properties also appear in the Catalan solid called the 'disdyakis triacontahedron,' which was found to be the polyhedral counterpart of both the Tree of Life and the Sri Yantra. Like them, this polyhedron possesses amazing properties that are unique to itself. However, it also shares with them equally amazing features that indicate that the essential meaning of its spectacular geometry is identical to theirs, as, indeed, it has to be because they are ALL expressions of the one universal blueprint. Despite differences in morphology, their essential similarity can only be regarded as irrefutable evidence of a universal archetype pervading the sacred geometries of East and West that can now be shown to manifest in the mathematics of music. Moreover, it is one that is beginning to appear in the research journals of particle physics. This is the **true** 'theory of everything,' for the blueprint in sacred geometries applies not only to matter but to ALL holistic systems.

Figure 1

At the heart of Kabbalah, the Jewish mystical tradition, is the glyph called the Tree of Life (Otz Chiim). It is a geometrical representation of Adam Kadmon (Heavenly Man), the divine paradigm forming the basis of all holistic systems.

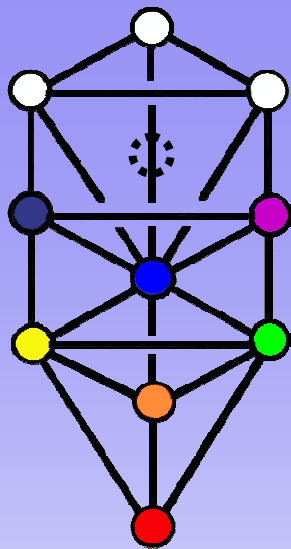
The Sri Yantra is the most famous and revered of the yantras used in India for meditation. So old that its origin is unknown, it depicts the nature of Creation.

The I Ching table has been used for hundreds of years in China for the purpose of divination. Its 64 hexagrams consist of pairs of trigrams, each a set of three parallel lines that are either Yang (unbroken) or Yin (broken).

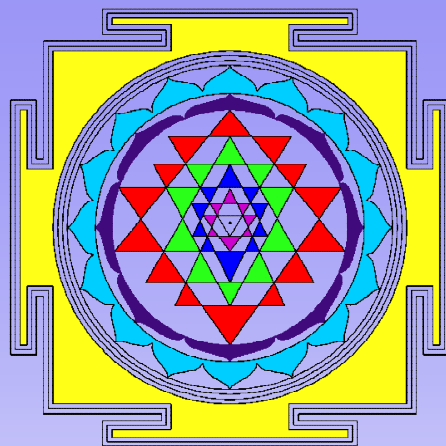
The disdyakis triacontahedron is the most complex of the Catalan solids – generated from the Archimedean solids by interchanging vertices and faces.

Over a century ago, the leading Theosophists, Annie Besant and C.W. Leadbeater, claimed to observe with a yogic siddhi the subatomic unit of matter. The author has proved that this object is the $E_8 \times E_8$ heterotic superstring constituent of up and down quarks.

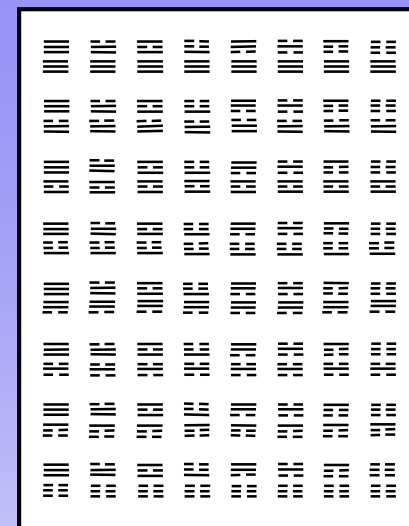
Article 40 shows that the Tree of Life, the Sri Yantra, the I Ching table and the disdyakis triacontahedron embody the same universal blueprint as that which manifests in the smallest subatomic particle.



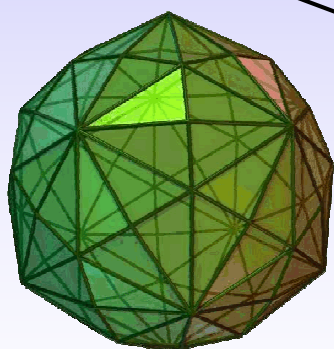
Tree of Life



Sri Yantra

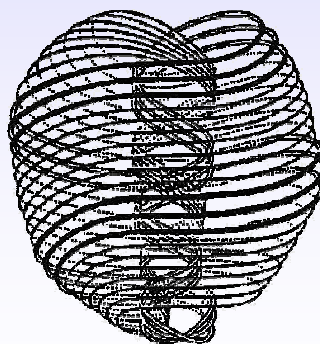


I Ching



Disdyakis

Triacontahedron



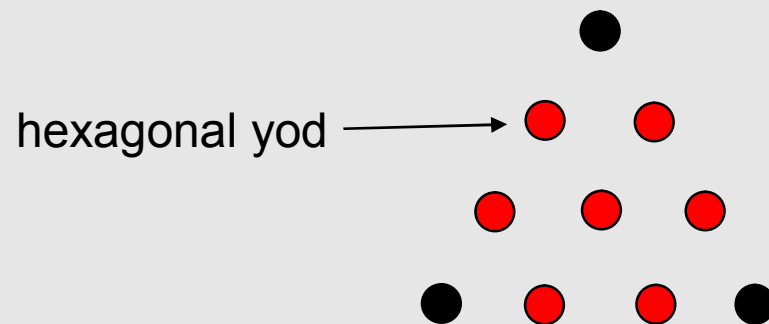
$E_8 \times E_8'$ heterotic superstring
constituent of up and down quarks

Figure 1

Figure 2

The tetractys is at the core of the mathematical philosophy of Pythagoras. A symbol of holistic systems, it is equivalent to the Kabbalistic Tree of Life. Each of its ten yods symbolizes one of the ten Sephiroth of the Tree of Life. The three yods at its corners symbolize the Supernal Triad (the triple Godhead). The seven hexagonal yods at its centre or at the corners of a hexagon denote the seven Sephiroth of Construction.

Figure 2



Tetractys

The triangular array of 10 dots (called “yods,” after the tenth letter of the Hebrew alphabet) is known as Pythagoras’ tetractys. It symbolizes the 10-fold nature of holistic systems. But it is far more than a symbol. Used as the template for constructing objects possessing sacred geometry, it turns their forms into numbers that have universal (and therefore scientific) significance.

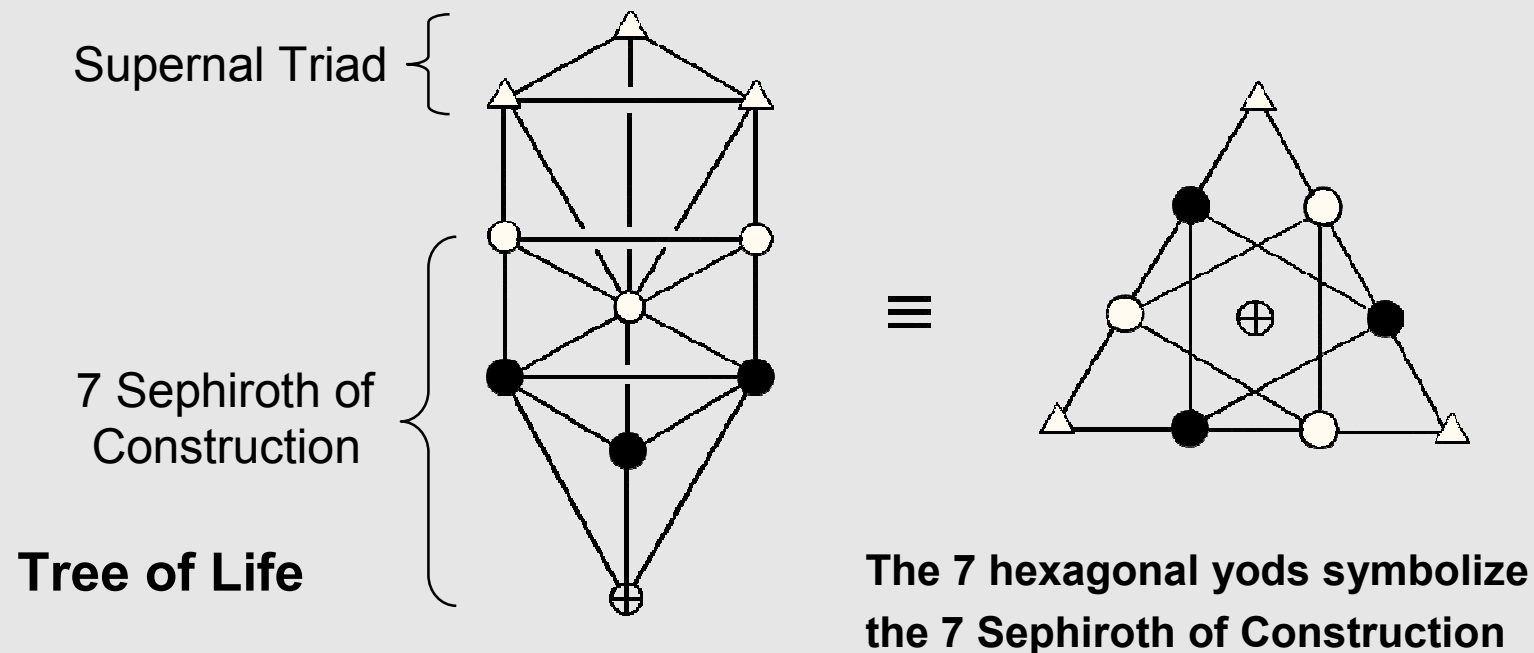
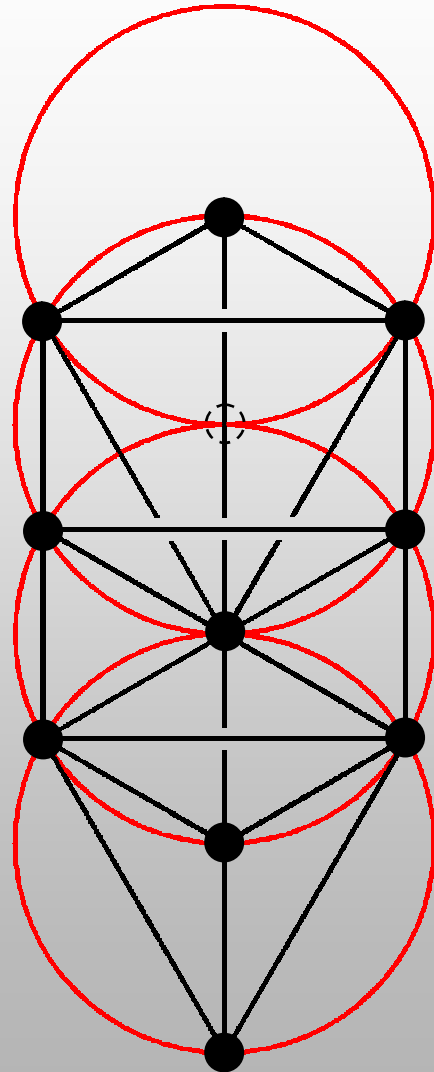


Figure 3

Four circles arranged in a vertical line and overlapping centre-to-circumference generate the positions of the ten Sephiroth of the Tree of Life.

Figure 3

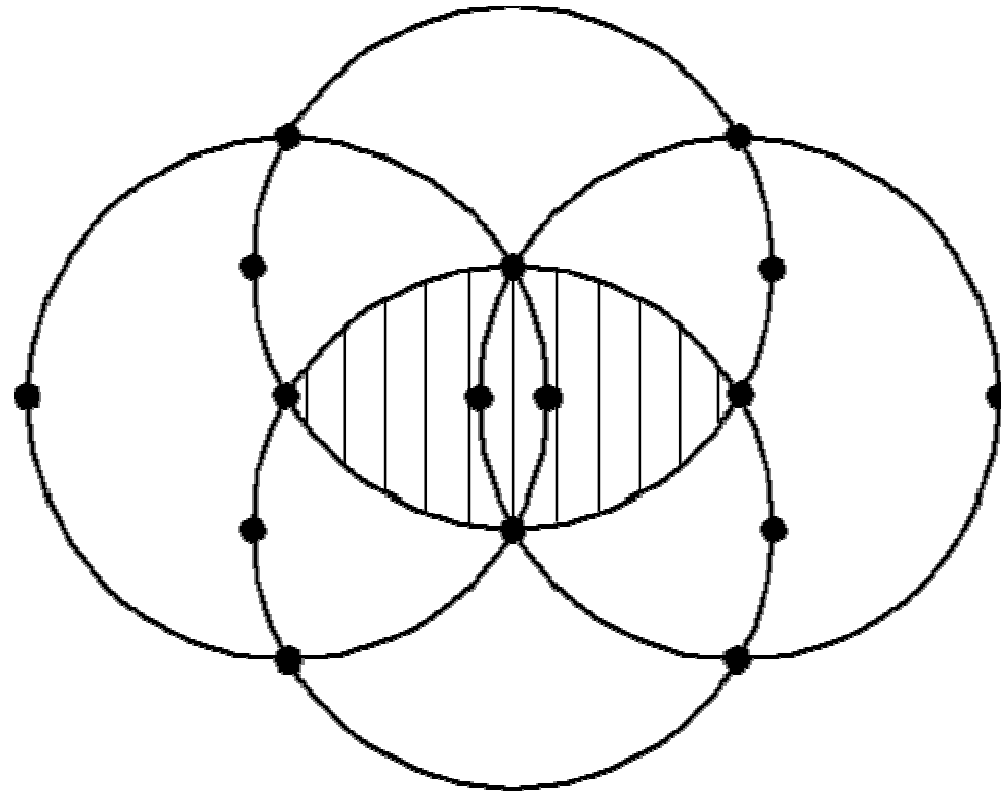


**Four overlapping
circles generate
the outer form of
the Tree of Life.**

Figure 4

Two similar, overlapping circles form a Vesica Piscis (shown shaded) as their region of overlap. Its apices are the centres of two new circles. The four circles have 18 centres and endpoints of their vertical and horizontal diameters. The 16 points shown are sufficient to generate the “inner” form of the Tree of Life.

Figure 4

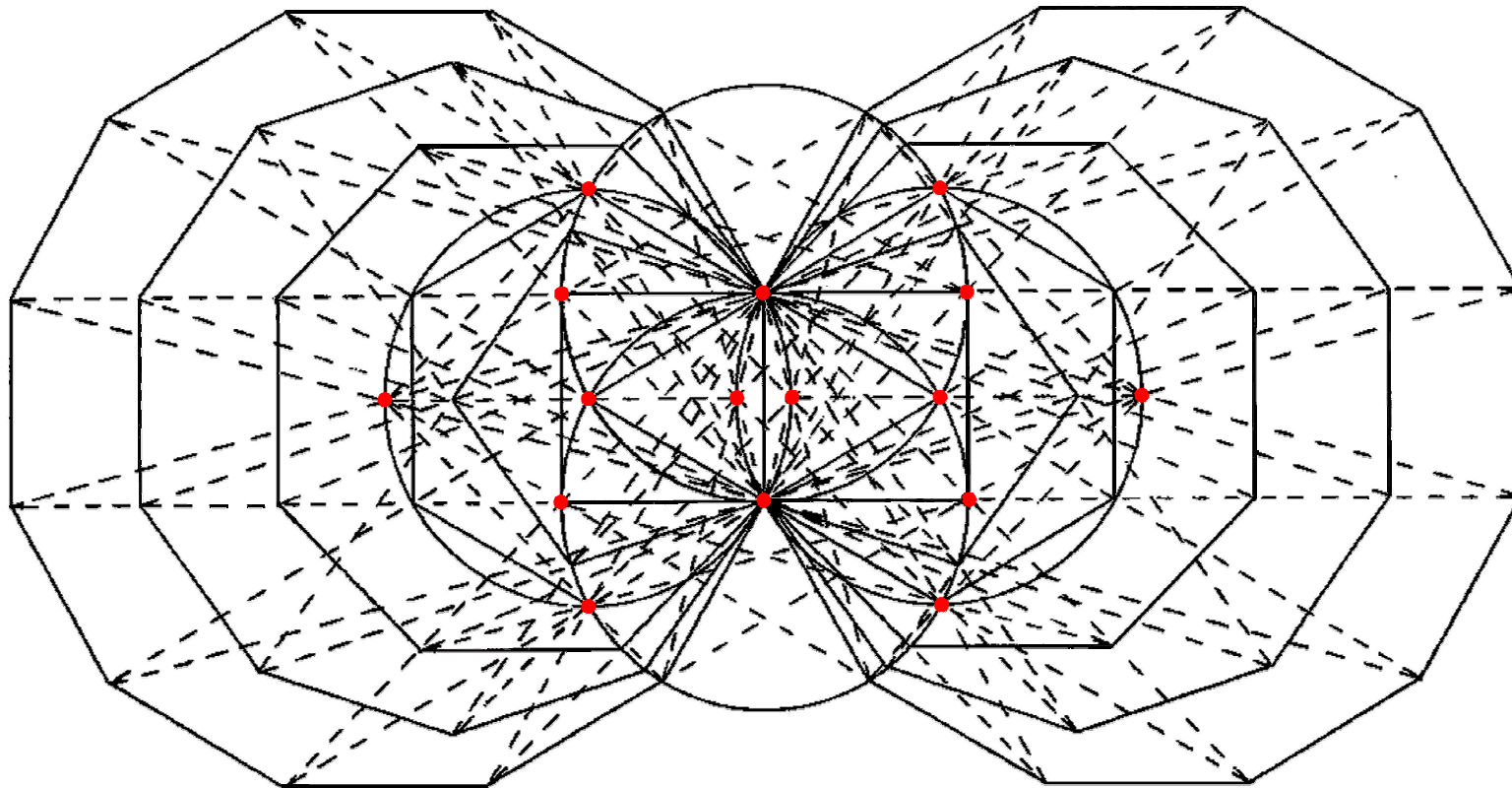


Points at the centres of four overlapping circles or at the ends of their vertical and horizontal diameters.

Figure 5

The inner form of the Tree of Life is generated from its outer form by joining pairs of points that belong to the set of 16 points. Straight lines joining pairs of points intersect at the 70 corners of two similar sets of seven regular polygons enfolded in one another and sharing one edge (the “root edge”).

Figure 5

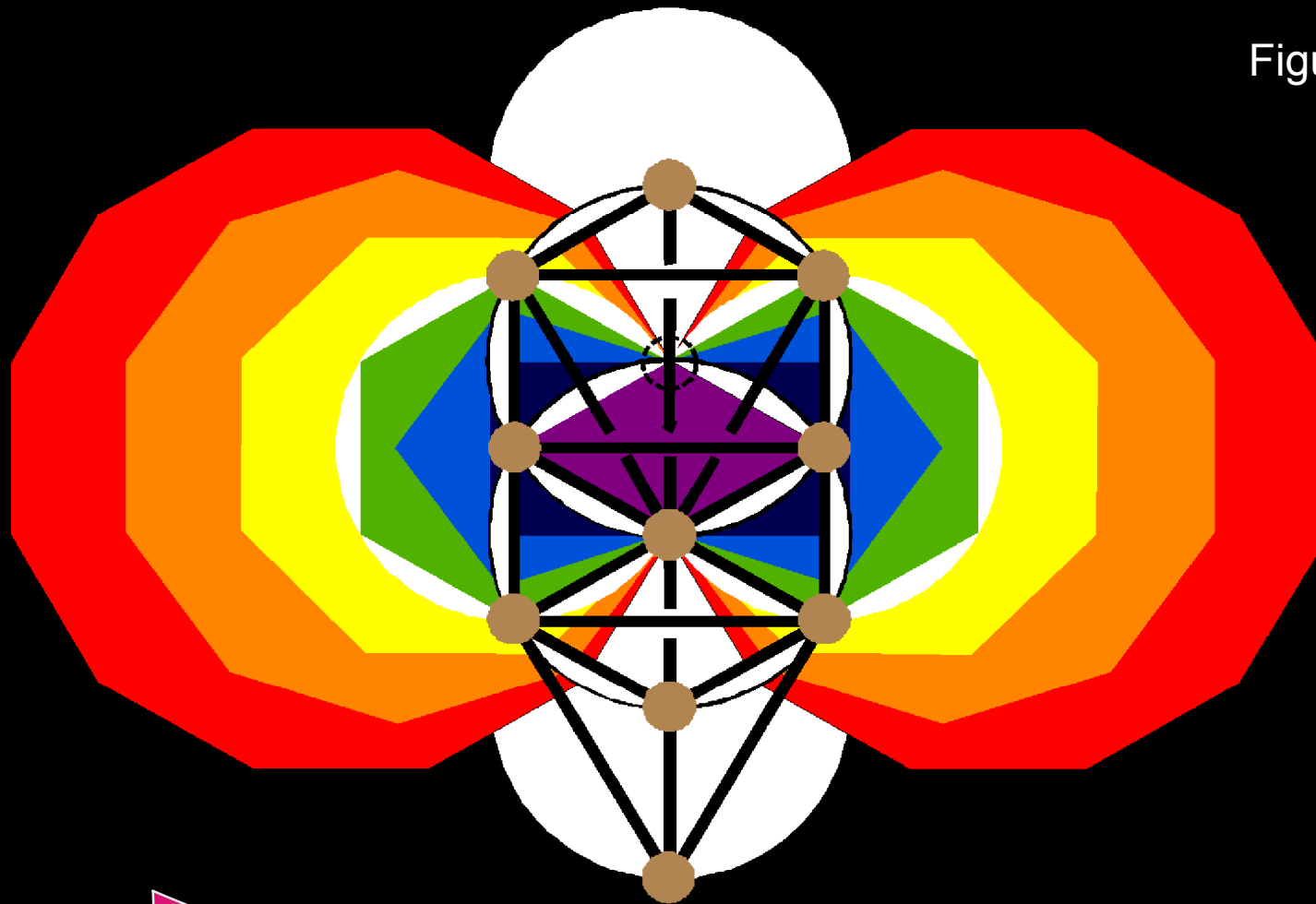


Lines joining pairs of points (●) intersect at the 70 corners of (7+7) enfolded, regular polygons.

Figure 6

The 3-dimensional, outer Tree of Life can be projected onto the plane of the two sets of enfolded polygons that constitute its inner form. Six of their corners outside their shared edge are shared with Sephiroth of the Tree of Life. Their corners on the root edge coincide with Tiphareth and with Daath. The outer and inner forms of the Tree of Life have 73 corners, where 73 is the number value of Chokmah (“Wisdom”), the second Sephirah.

Figure 6

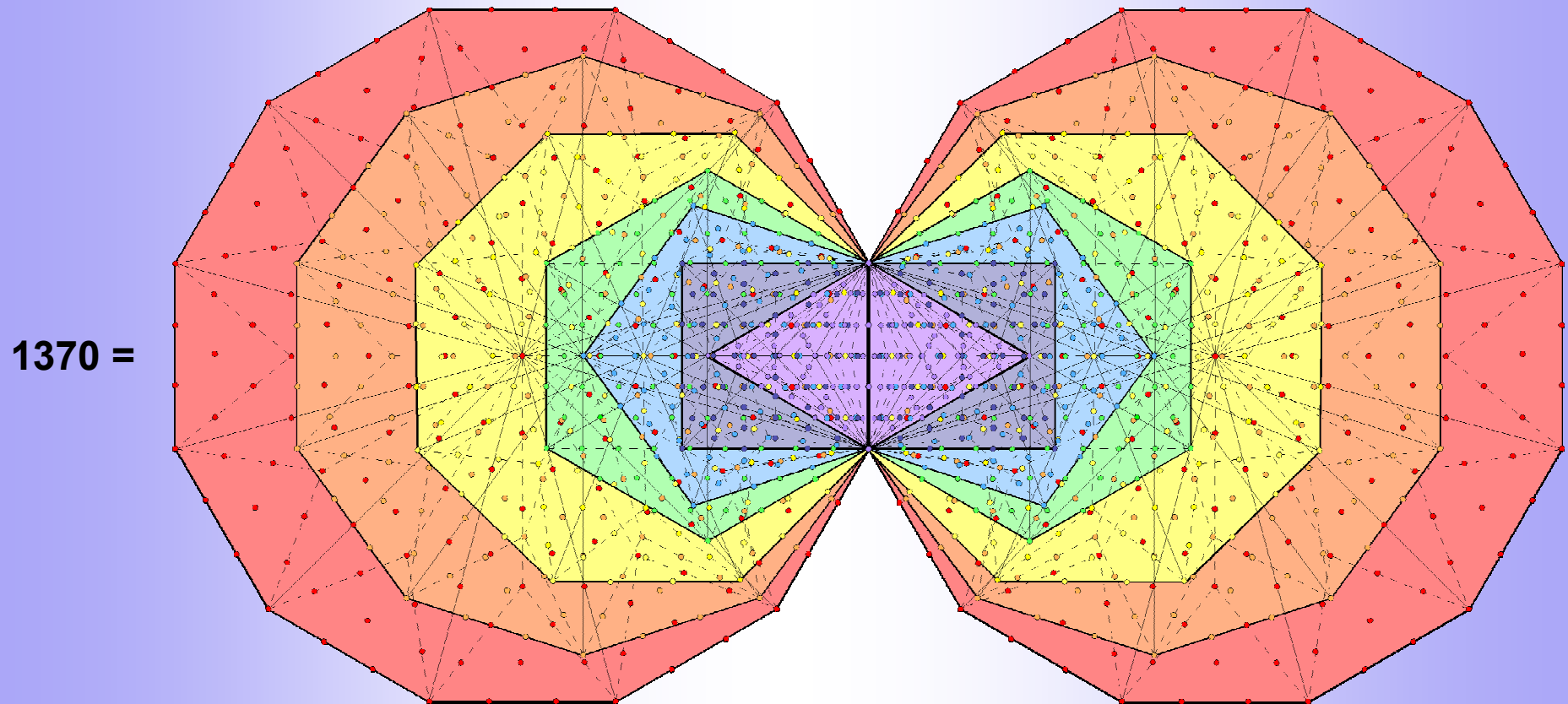


The Inner and Outer
Forms of the Tree of Life

Figure 7

The number 137 is embodied in the blueprint of the inner Tree of Life. Its $(7+7)$ enfolded polygons have 94 sectors. When they are each divided into three tetractyses, 1370 yods are generated, that is, the number of yods in 137 tetractyses. This proves beyond question that the number 137 is a basic structural parameter of the Tree of Life, in keeping with its central status in physics as a number which determines one of the fundamental constants of nature – the fine-structure constant, whose magnitude sets the scale of the energies of electrons in atoms.

Figure 7



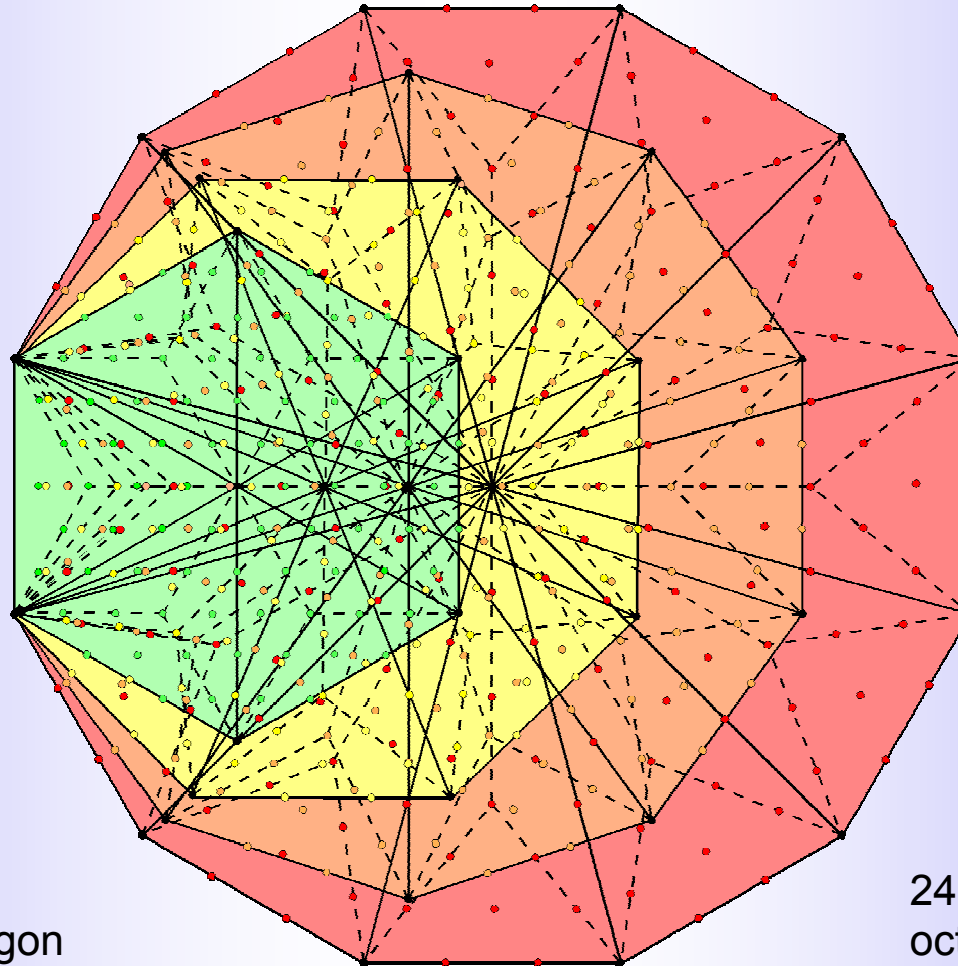
With its 94 sectors of polygons constructed from three tetractyses, the inner Tree of Life embodies the number 137 defining the ‘fine-structure constant,’ which is known in physics to determine properties of atoms.

Figure 8

496 extra yods are needed to turn into three tetractyses each sector of the last four enfolded polygons of the inner Tree of Life. 248 yods belonging to the hexagon and dodecagon symbolize the 248 particles transmitting the superstring force with the symmetry of the gauge symmetry group E_8 and 248 yods belonging to the octagon and decagon symbolize the 248 particles associated with E_8' (identical to E_8). The remarkable, natural division of the yod populations of the four polygons into two sets of 248 shows how the universal blueprint of the inner Tree of Life embodies the dynamics of the $E_8 \times E_8'$ heterotic superstring. It also illustrates how the Pythagorean Tetrad (4) defines parameters of scientific significance, for it is the last *four* regular polygons that embody the number 496 characterizing the unified force between superstrings.

Figure 8

496 =



248 yods in the
hexagon & dodecagon
→ dimension of E_8

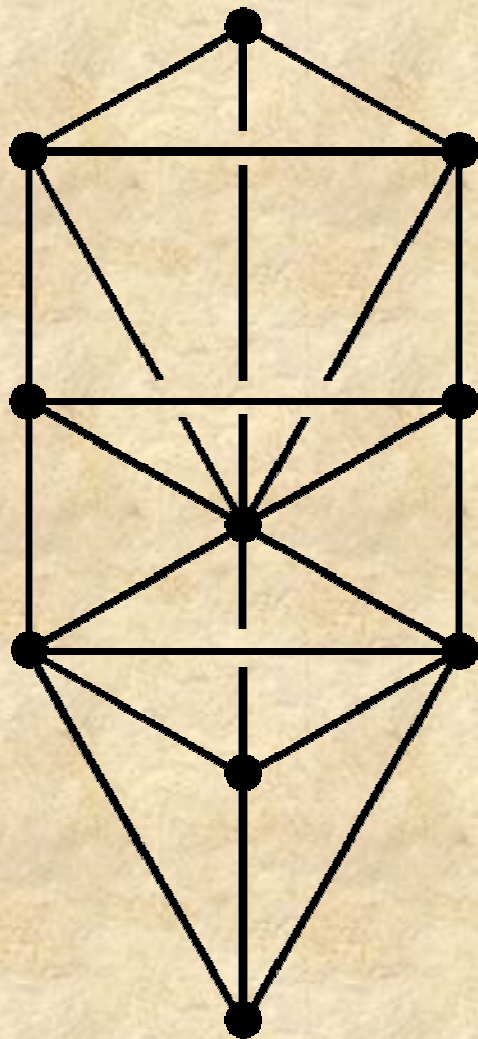
248 yods in the
octagon & decagon
→ dimension of E_8'

The last four polygons of the inner Tree of Life have 496 yods other than their centres and corners. They denote the (248+248) Yang-Mills gauge fields of the $E_8 \times E_8$ heterotic superstring symmetry group.

Figure 9

16 separate triangles with 48 corners join together to form the Tree of Life. It has 10 vertices and 22 edges of 16 triangles – a total of 48 geometrical elements. The re-appearance of this number is not a coincidence because it is a basic parameter of all geometrical structures that conform to the archetypal Tree of Life pattern and possess sacred geometry. 48 is the number value of Kokab, the Mundane Chakra of Hod.

Figure 9



16 triangles with 48 corners

The Tree of Life has:

10 vertices

22 edges

16 triangles

Total = 48

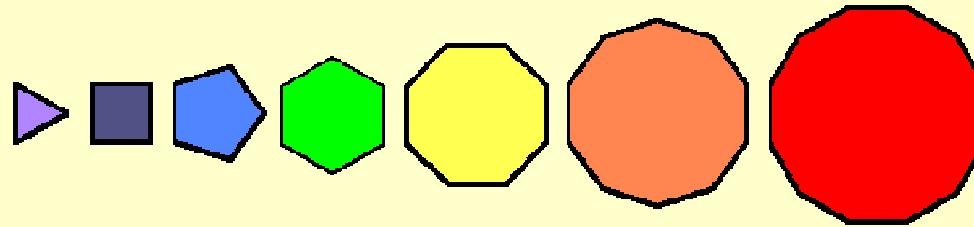
**The Tree of Life is both generated from and
composed of 48 geometrical elements.**

Figure 10

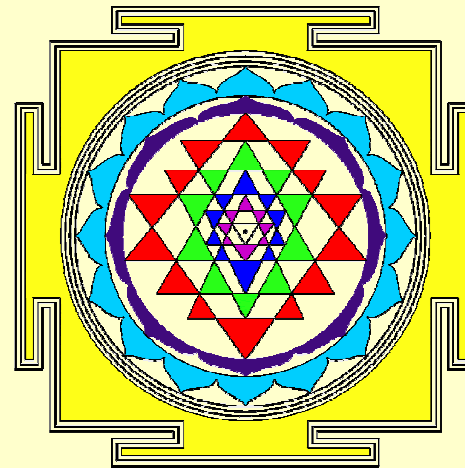
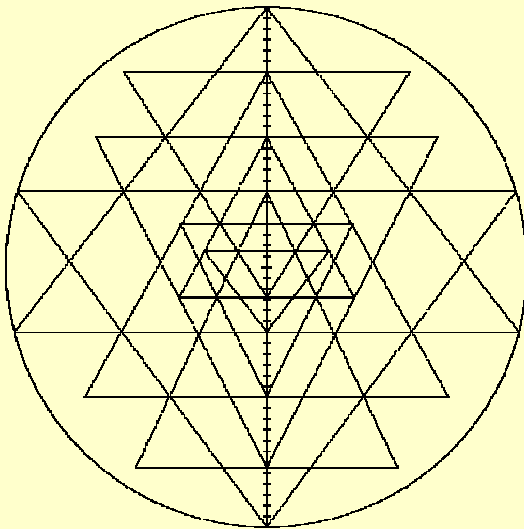
The number 48 also characterises the inner form of the Tree of Life. It is a set of seven enfolded polygons which, when separate, have 48 corners.

The Sri Yantra is difficult to draw accurately because a slight error at any stage of its construction can seriously distort the diagram. A template that achieves this task is a vertical, straight line with 48 spaces of the same width marked out on it.

Figure 10



The seven separate, regular polygons have 48 corners.

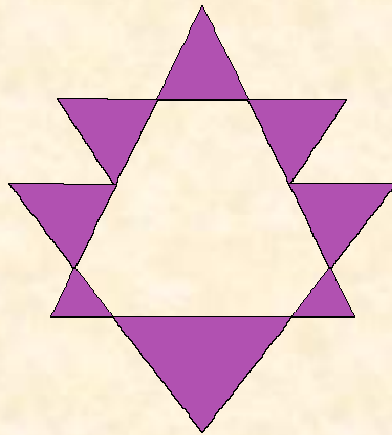


Sri Yantra

48 spaces need to be marked out on a vertical, straight line in order to draw the Sri Yantra.

Figure 11

The parameter 48 characterising sacred geometry is displayed in the Sri Yantra, the I Ching table and the tetrahedron. The first layer of triangles in the Sri Yantra consists of eight triangles joined corner to corner. They have 16 vertices, 24 edges and eight triangles, totally 48 geometrical elements as two sets of 24. The eight hexagrams (shown black) along the diagonal of the I Ching table have 48 unbroken and broken lines as two sets of 24. The tetrahedron – the simplest Platonic solid – has four faces. When each face is divided into its three sectors and the latter then turned into tetractyses, there are 48 hexagonal yods in the 12 tetractyses. They comprise 24 red hexagonal yods inside each face and 24 black hexagonal yods on the edges of the tetrahedron. This division into two sets of 24 of the 48 degrees of freedom embodied in the generative, or germinal aspect of a holistic system is characteristic of such systems.



1st layer of triangles
in the 3-d Sri Yantra

16 vertices (= 8×2)

24 edges (= 8×3)

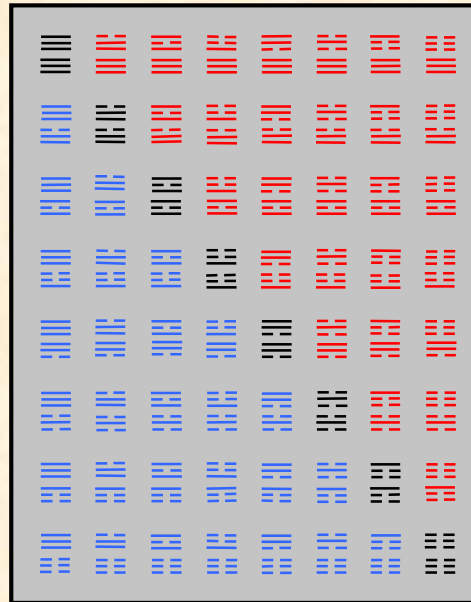
8 triangles (= 8×1)

Total = $8 \times [(1+2) + 3]$

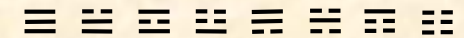
= $8 \times (3+3) = 24 + 24 = 48$

8 diagonal pairs of
→ trigrams with 24 lines &
24 broken lines

→ tetrahedron with (24+24)
hexagonal yods



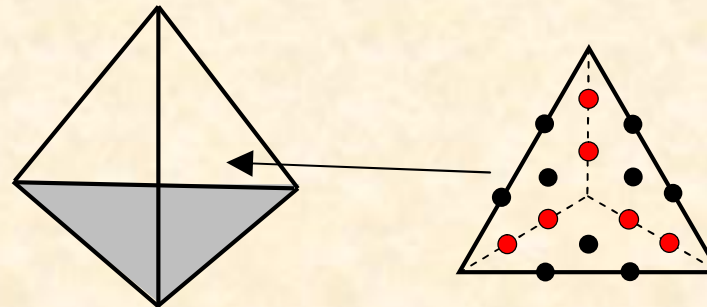
I Ching table



The 8 trigrams

The 8 diagonal hexagrams in the
8x8 array of hexagrams of the I
Ching table are made up of
[$8 \times (3+3) = 24 + 24 = 48$] lines &
broken lines

Figure 11



24 (●)

24 (●)

A tetrahedron has 48 hexagonal yods in its faces

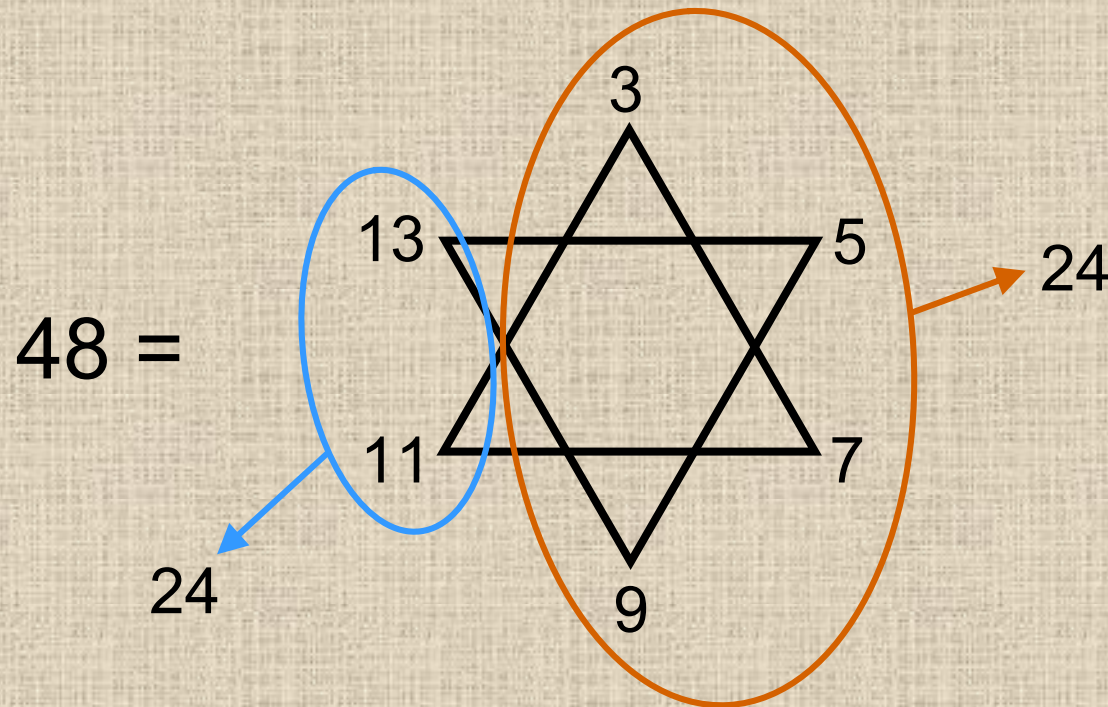
Examples of the generative character of the number 48 in the I Ching,

the Sri Yantra and the tetrahedron. In all cases, it divides into 24 & 24. 24

Figure 12

The number 48 is the sum of the first six odd integers after 1. 24 is the sum of the first four odd integers after 1. Arithmetically, therefore, the number 48 divides naturally into a pair of number 24s. The encircled selections are the only ones that add up to 24. As we shall see, this division *always* appears in holistic systems that display sacred geometry.

Figure 12



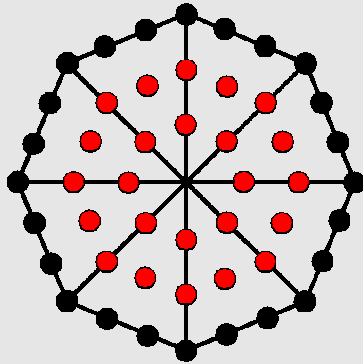
As the sum of the first 6 odd integers after 1, the number 48 naturally divides into 24 and 24. This division is displayed by all holistic systems embodying the divine archetypes.

Figure 13

The octagon constructed from tetractyses displays the same 24:24 division of the 48 yods surrounding its centre. There are 24 yods on its boundary and 24 inside it surrounding its centre.

When constructed from “2nd-order” tetractyses, in which each yod of a tetractys is replaced by a tetractys, the octagon has 496 hexagonal yods. They symbolize the 496 spin-1 particles that superstring theory predicts transmit the unified superstring force. Surrounding the centre of the octagon are 80 corners of tetractyses. 80 is the number value of Yesod, the penultimate Sephirah of the Tree of Life. 496 is the number value of Malkuth, the final Sephirah.

Figure 13



48 yods are needed to construct an octagon, starting from its centre. Their grouping into 24 boundary yods and 24 internal yods reflects the basic division of the parameter 48 into two sets of 24 and demonstrates how the tetractys is the natural means of deciphering information encoded in sacred geometry. The yods symbolize the 24 pure rotations and the 24 rotations/reflections of the octahedral group.

$$48 = 24 (\bullet) + 24 (\bullet)$$

The number 48 defines the octagon. Its scientific significance is that 496 hexagonal (coloured) yods are needed to construct the 8 sectors of an octagon from 2nd-order tetractyses. They symbolize the 496 spin-1 gauge bosons of both $O(32)$ and $E_8 \times E_8'$, the two possible symmetry groups of dimension 496 predicted by superstring theory to describe superstring interactions. Each coloured yod denotes one of these gauge bosons transmitting the unified superstring force.

496 =

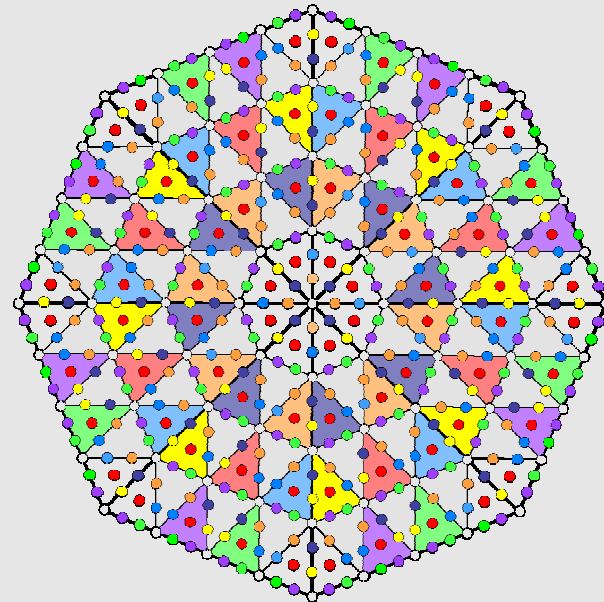
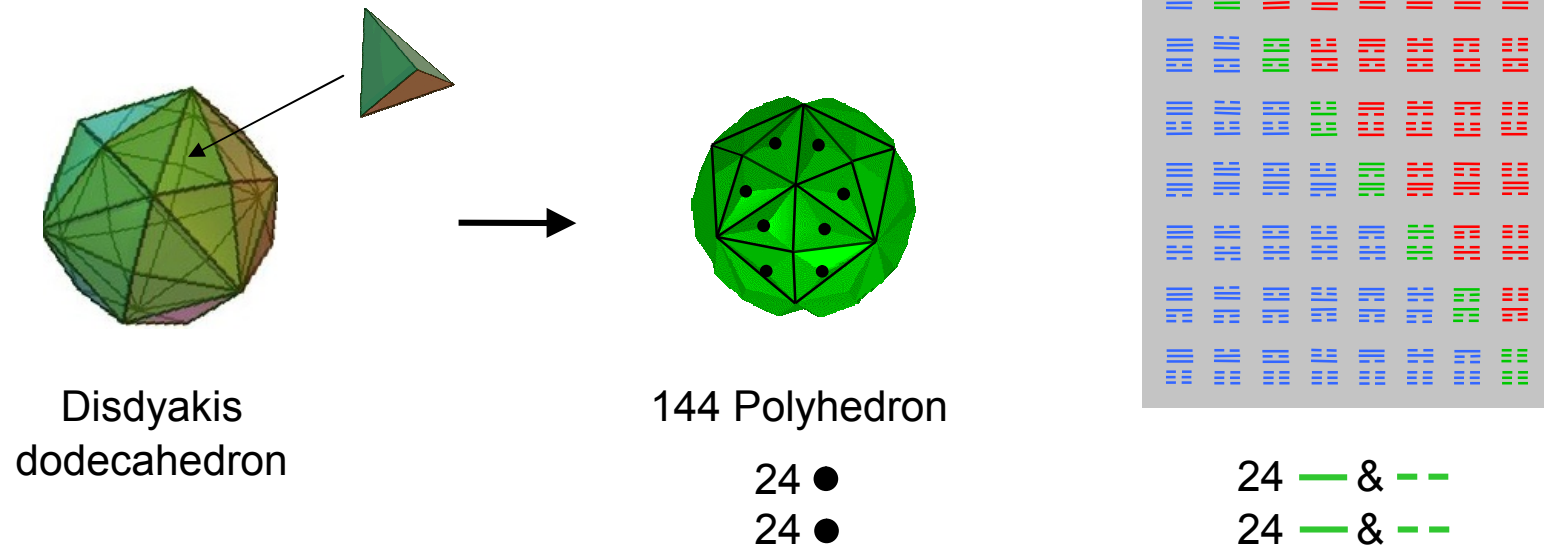


Figure 14

The holistic parameter 48 exists in the 144 Polyhedron (see Article 23 for details), the polyhedron which – together with the disdyakis triacontahedron – constitutes the polyhedral version of, respectively, the inner and outer forms of the Tree of Life. It is generated by attaching tetrahedra to the 48 faces of the disdyakis dodecahedron, which has 26 vertices and 72 edges. The latter is generated from the rhombic dodecahedron by sticking rhombic pyramids onto its 12 faces. The resulting 48 faces comprise 24 faces and their 24 mirror images. This means that the 48 peaks of the tetrahedra consist of 24 vertices and their 24 mirror images. This is how the 144 Polyhedron displays the 24:24 division of the parameter 48. Their counterpart in the I Ching table are the 24 lines & broken lines in the eight upper and eight lower trigrams making up the eight diagonal hexagrams. Each half of a disdyakis dodecahedron consists of six sets of four faces, i.e., four sets of six faces. The grouping of the 48 faces into eight sets of six means that 48 of the $(48+26=74)$ vertices of the 144 Polyhedron are similarly grouped. They correspond to the eight diagonal hexagrams of the I Ching table, each with six lines/broken lines, and to the eight sets of geometrical elements making up the eight triangles in the first layer of the Sri Yantra. They are related to the 24 rotational symmetries and the 24 rotation/reflection symmetries of the octahedral group.



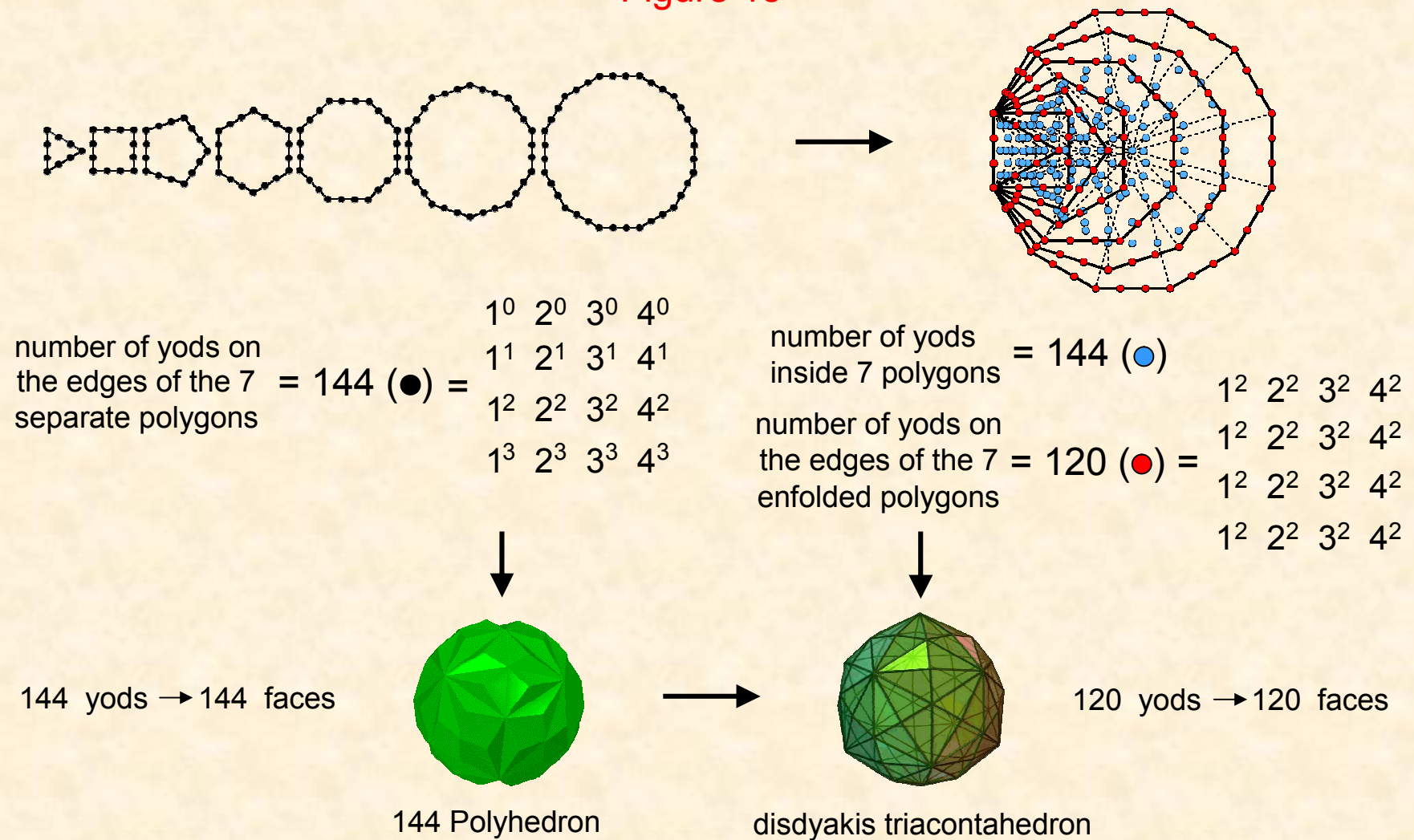
Attaching tetrahedra to the 48 faces of the disdyakis dodecahedron with 26 vertices generates the 144 Polyhedron with $(48+26=74)$ vertices. The 8 groups of 6 peaks of these tetrahedra are the counterparts of the 8 diagonal hexagrams of the I Ching diagram, each with 6 lines/broken lines. The 24 peaks in each half of the 144 Polyhedron correspond to the 24 lines/broken lines in one set of 8 trigrams and their Yang/Yin opposites in the other set making up the 8 diagonal hexagrams. They signify the $(24+24=48)$ symmetries of the octahedral group.

Figure 15

The correspondence between the polygonal and polyhedral versions of the Tree of Life is revealed here. As the generator of the disdyakis triacontahedron, the 144 Polyhedron has 144 faces creating its shape. This is the number of yods needed to create the 48 form-generating edges of the seven *separate*, regular polygons of the inner Tree of Life when the sectors of the latter are turned into tetractyses. It is also the number of yods *inside* the seven enfolded polygons and surrounding their centres. This corresponds to the 144 Polyhedron being the inner form of the polyhedral Tree of Life. It should therefore be thought of as inside the disdyakis triacontahedron — the outer form of the polyhedral Tree of Life. Just as there are 120 faces shaping the latter, so there are 120 yods on the 42 edges of the seven enfolded polygons that delineate their shapes. The polyhedral version of the outer and inner Trees of Life displays the *same* 144:120 division as their polygonal counterpart. This is because they are both isomorphic representations of holistic systems.

The Pythagorean Tetrad (4) defines the numbers 120 and 144 as square arrays of powers of the integers 1, 2, 3 & 4 symbolized by the four rows of dots in the tetractys.

Figure 15



The 7 separate regular polygons with 48 corners become enfolded, defining both the 144 Polyhedron and the disdyakis triacontahedron as the inner and outer forms of the polyhedral Tree of Life.

Figure16

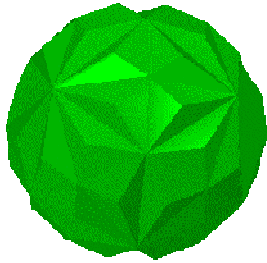
There are 84 yods up to the level of the lowest Tree of Life when it is constructed from tetractyses. Of these, ten are Sephiroth, leaving 74 yods other than Sephiroth. This is the number of non-Sephirothic degrees of freedom hidden within the outermost form of the Tree of Life. They comprises 26 yods down to the level of Daath and 48 yods below it. These yods correspond, respectively, to the 26 vertices of the disdyakis dodecahedron underlying the 144 Polyhedron and to the 48 vertices of the tetrahedra attached to its 48 faces.

The inner form of the lowest Tree of Life is two similar sets of seven enfolded polygons with 70 corners. They share seven corners with Sephiroth of the Tree of Life and one corner with Daath, leaving 62 corners. These 62 independent degrees of freedom correspond to the 62 vertices of the disdyakis triacontahedron. The 31 corners of each set of polygons are the counterpart of the 31 vertices of this polyhedron and their mirror images. The 30 corners of the two sets of pentagons, hexagons & dodecagons correspond to the 30 A vertices, the 12 corners of the two octagons correspond to the 12 C vertices (vertices of an icosahedron). The 20 corners of the square & decagon correspond to the 20 A vertices (vertices of a dodecahedron).

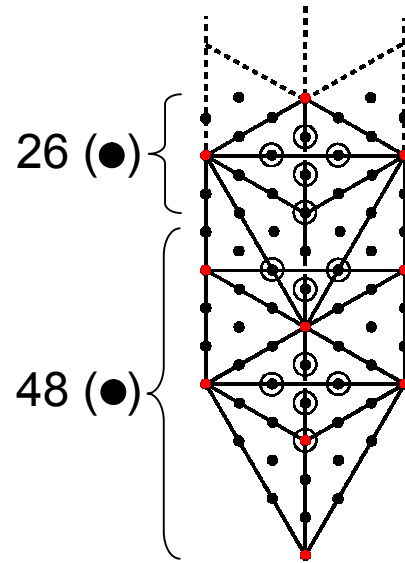
The 144 Polyhedron and the disdyakis triacontahedron are therefore implicit in the geometry of the outer and inner forms of the Tree of Life as their polyhedral counterparts.

Figure16

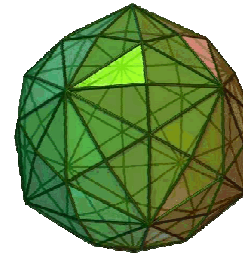
144
Polyhedron



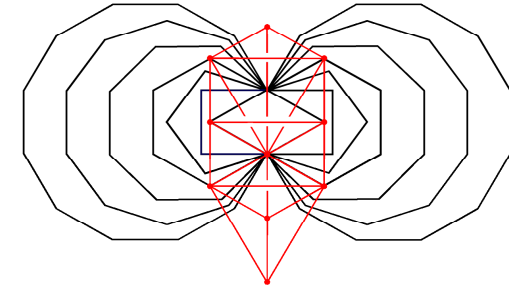
(26+48=74)
vertices



Disdyakis
triacontahedron

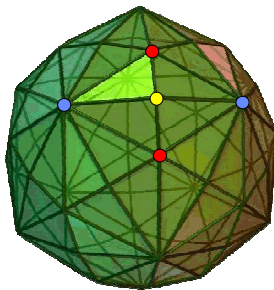


62 vertices

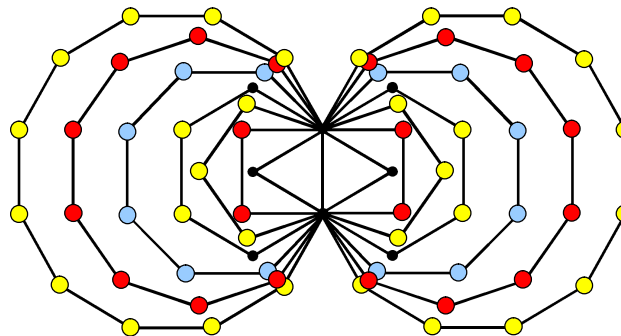


62 corners of 14 polygons
unshared with the lowest tree

74 (●) yods not Sephiroth up
to the top of the lowest tree



≡



30 A ●

12 B ●

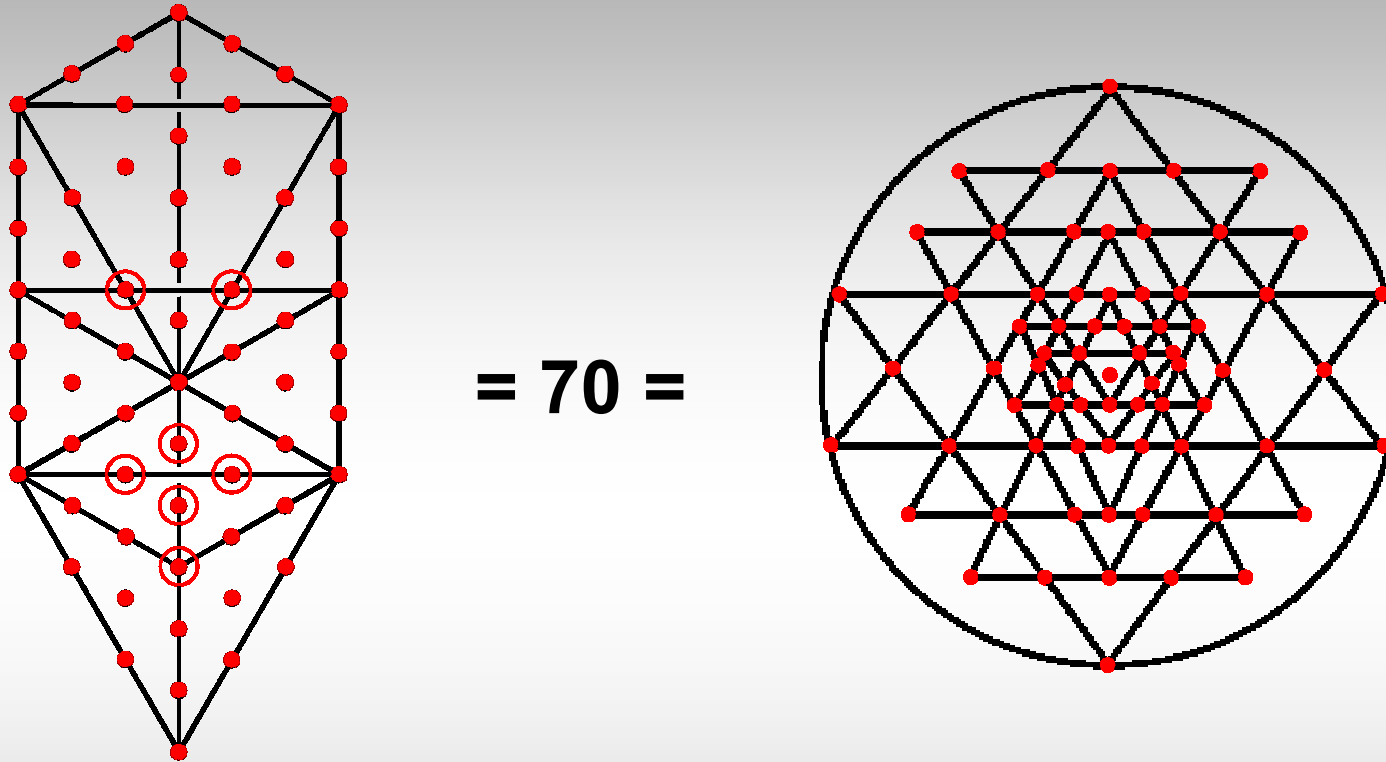
20 C ●

**The isomorphism between the polygonal and
polyhedral versions of the Tree of Life.**

Figure 17

Turned into tetractyses, the 16 triangles of the Tree of Life have 70 yods. This is the number of vertices (including the central bindu point) of the 2-dimensional Sri Yantra. This demonstrates the isomorphism between these representations of holistic systems.

Figure 17



The 70 vertices of the 2-dimensional Sri Yantra correspond to the 70 yods of the Tree of Life, demonstrating that holistic systems are characterized by 70 degrees of freedom.

Figure 18

The four groups of triangles in the 2-dimensional Sri Yantra are composed of 236 geometrical elements. (The central triangle does not count formally as a triangle because of the bindu, or point, at its centre).

Figure 18

	Vertices	Number of edges	Number of triangles	Total
Bindu	1	0	0	1
▽	1	3	0	4
Subtotal	2	3	0	5
▲	$4 + 8 = 12$	$8 \times 3 = 24$	8	44
▲	$6 + 10 = 16$	$10 \times 3 = 30$	10	56
▲	$2 + 10 = 12$	$10 \times 3 = 30$	10	52
▲	28	$14 \times 3 = 42$	14	84
Subtotal	68	126	42	236
Total	70	129	42	241

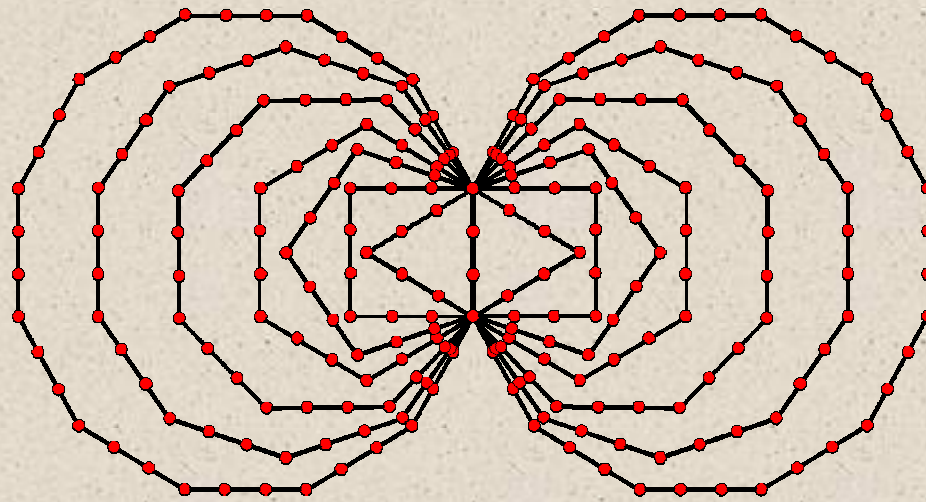
There are 236 geometrical elements in the four groups of triangles of the 2-d Sri Yantra.

Figure 19

236 yods lie on the 83 edges of the two identical sets of seven enfolded polygons constituting the inner Tree of Life. This the number of geometrical elements that compose the 42 triangles of the 2-dimensional Sri Yantra. Different representations of holistic systems embody the same structural parameter.

Figure 19

236 (●) =



The number of yods forming the *shape* of the inner Tree of Tree is the number of geometrical elements in the four layers of the 2-d Sri Yantra. The same structural parameter quantifies different representations of holistic systems.

Article 40

The Unification of All Sacred Geometries & Its Implication for Particle Physics

Part 2

by

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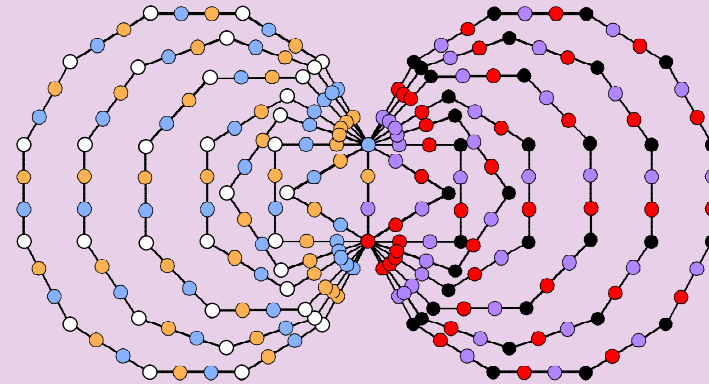
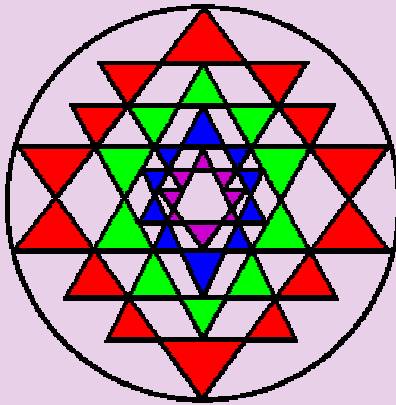
E-mail: stephen@smphillips.8m.com

Website: <http://www.smphillips.8m.com>

Figure 1

The 34 vertices of the 2-dimensional Sri Yantra and their 34 mirror images correspond to the 34 corners of the seven enfolded polygons outside their root edge and their 34 mirror images in the other set of polygons. The 42 edges and their 42 mirror images in the first three layers of triangles are the counterpart of 42 hexagonal yods on the edges of one set of polygons and their mirror images in the other set. The 42 edges & triangles in the fourth layer and their 42 mirror images are the counterpart of another 42 hexagonal yods on the edges of one set of polygons and their 42 mirror images in the other set.

Figure 1



(34+34) vertices

(42+42) edges in 1st three
layers of triangles

(21+21) edges in 4th layer

(21+21) triangles

168

34 (●) + 34 (○) corners outside root edge

42 (●) & 42 (○)
42 (●) & 42 (○) } 168

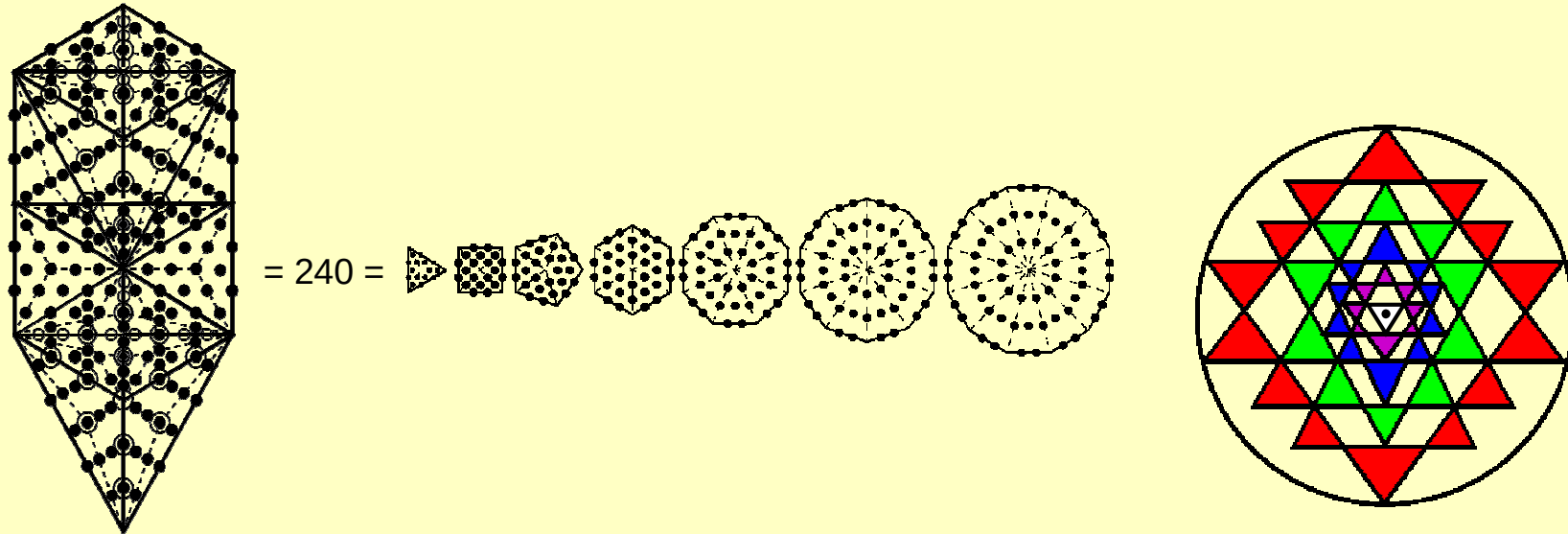
**Correspondence between the geometrical composition
of the four layers of the 2-d Sri Yantra and the boundary
yods of the 14 polygons of the inner Tree of Life.**

Figure 2

Just as 240 geometrical elements are needed to construct the 2-dimensional Sri Yantra, starting with the central bindu point, so 240 extra yods are needed to construct the 19 triangles of the lowest Tree of Life from tetractyses and 240 extra yods are needed to construct the sectors of the polygons in the inner Tree of Life from tetractyses.

Figure 2

(circles are yods
behind other yods)



The lowest Tree of Life needs 240 extra yods to construct each of its 19 triangles from three tetractyses. The inner Tree of Life needs 240 extra yods to turn its 48 sectors into tetractyses.

The 2-d Sri Yantra has 240 geometrical elements surrounding the central bindu.

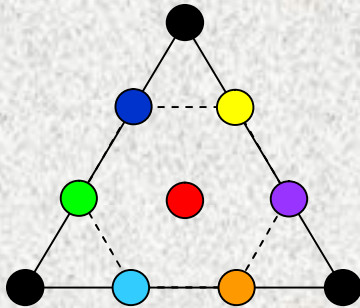
The number 240 is a structural parameter of the Sri Yantra, the Tree of Life and its inner, polygonal form.

Figure 3

When constructed from tetractyses, the first three Platonic solids have 240 hexagonal yods, as do the icosahedron and the dodecahedron. The number 240 is a structural parameter of holistic systems like the five possible, regular polyhedra.

Figure 3

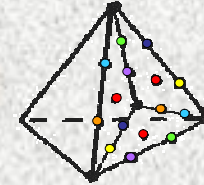
Coloured yods are
hexagonal yods



The structural parameter 240
defining holistic systems is
found in the Platonic solids as
the number of hexagonal yods
in the faces of the icosahedron
or dodecahedron when they
are tessellated with tetractyses

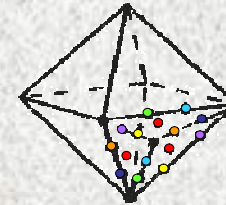
Number of
hexagonal yods

Tetrahedron



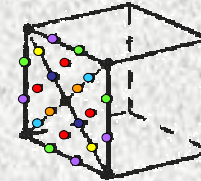
48

Octahedron



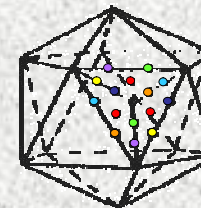
96

Cube



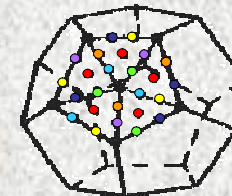
96

Icosahedron



240

Dodecahedron



240

240

Figure 4

There are 13 semi-regular polyhedra (Archimedean solids). They have 13 duals – the Catalan solids – in which each vertex is replaced by a face and vice versa. The two tables list the number of vertices, edges & triangles in their faces. The most complex of the Catalan solids is the disdyakis triacontahedron. 2400 corners, edges & triangles surround an axis through any two opposite vertices when its faces are divided into three triangles and 1680 geometrical elements when its faces are single triangles. This is ten times the corresponding numbers for the triakis tetrahedron, the simplest Catalan solid. The disdyakis triacontahedron is the polyhedral counterpart of the inner Tree of Life, embodying the structural parameter 240. As shown later, 168 and 1680 are also parameters embodied in any manifestation of the Tree of Life blueprint.

Case A: triangular face as tetractys

C = number of corners

E = number of edges

F = number of faces

N = number of corners, edges & triangles surrounding the axis (case A)

N' = number of corners, edges & triangles surrounding the axis (case B)

Case B: triangular face as 3 tetractyses

Figure 4

Tables of properties of the Archimedean and the Catalan solids

N'	F	E	C	Archimedean solid
244	8	18	12	truncated tetrahedron
322	14	24	12	cuboctahedron
490	14	36	24	truncated cube
490	14	36	24	truncated octahedron
646	26	48	24	rhombicuboctahedron
802	38	60	24	snub cube
802	38	60	24	snub cube (chiral partner)
808	32	60	30	icosidodecahedron
882	26	72	48	truncated cuboctahedron
1228	32	90	60	truncated icosahedron
1228	32	90	60	truncated dodecahedron
1618	62	120	60	rhombicosidodecahedron
2008	92	150	60	snub dodecahedron
2008	92	150	60	snub dodecahedron (chiral partner)
2458	62	180	120	truncated icosidodecahedron

Catalan solid	F	E	C	N	N'
triakis tetrahedron	12	18	8	168	240
rhombic dodecahedron	12	24	14	-	324
triakis octahedron	24	36	14	336	480
tetrakis hexahedron	24	36	14	336	480
deltoidal icositetrahedron	24	48	26	-	648
pentagonal icositetrahedron	24	60	38	-	816
pentagonal icositetrahedron (chiral partner)	24	60	38	-	816
rhombic triacontahedron	30	60	32	-	810
disdyakis dodecahedron	48	72	26	672	960
triakis icosahedron	60	90	32	840	1200
pentakis dodecahedron	60	90	32	840	1200
deltoidal hexacontahedron	60	120	62	-	1620
pentagonal hexacontahedron	60	150	92	-	2040
pentagonal hexacontahedron (chiral partner)	60	150	92	-	2040
disdyakis triacontahedron	120	180	62	1680	2400

2400 geometrical elements surround the axis of the disdyakis triacontahedron constructed from triangles. This is 10 times that for the triakis tetrahedron, the simplest Catalan solid. It illustrates how the number 240 characterizes holistic systems like the disdyakis triacontahedron.

Figure 5

The Tree of Life parameter 240 appears in superstring physics as the number of non-zero roots of the Lie algebra of E_8 , the superstring gauge symmetry group. The parameter 168 appears as the number of non-zero roots of E_8 that are not non-zero roots of its exceptional subgroup E_6 .

The 240 non-zero roots of the superstring gauge symmetry group E_8 .

The roots of the E_8 algebra are described in terms of eight orthonormal unit vectors $\{u_i\}$.

Figure 5

Eight zero roots correspond to points at the centre of the root diagram and 240 non-zero roots all have length $\sqrt{2}$. They are given by

$$\pm u_i \pm u_j \quad (i, j = 1, 2, \dots, 8)$$

and

$$\frac{1}{2}(\pm u_1, \pm u_2, \dots, \pm u_8) \quad (\text{even number of '+'s})$$

Their explicit forms as 8-tuples and their numbers are listed below:

$(1, 1, 0, 0, 0, 0, 0, 0)$ and all permutations. Number = $\binom{8}{2} = 28$;	$\left. \begin{array}{c} \\ \\ \\ \\ \end{array} \right\} 168$	$\left. \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \right\} 240$
$(1, -1, 0, 0, 0, 0, 0, 0)$ and all permutations. Number = $\binom{8}{2} = 28$;		
$(1, 1, 0, 0, 0, 0, 0, 0)$ and all permutations. Number = $2 \times \binom{8}{2} = 56$;		
$(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ and all permutations. Number = $\binom{8}{2} = 28$;		
$(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ and all permutations. Number = $\binom{8}{2} = 28$;		
$(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ and all permutations. Number = $\binom{8}{4} = 70$;	$\left. \begin{array}{c} \\ \\ \end{array} \right\} 72$	
$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. Number = 1;		
$(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$. Number = 1.		

Figure 6

There are 260 vertices, edges & triangles in the 3-dimensional Sri Yantra. 168 geometrical elements are in the first three layers of triangles.

Figure 6

	Vertices	Edges	Triangles	Total
Bindu	1	0	0	1
	3	3	1	7
Subtotal	4	3	1	8
	$2 \times 8 = 16$	$3 \times 8 = 24$	8	48
	$2 \times 10 = 20$	$3 \times 10 = 30$	10	60
	$2 \times 10 = 20$	$3 \times 10 = 30$	10	60
	$2 \times 14 = 28$	$3 \times 14 = 42$	14	84
Subtotal	84	126	42	252
Total	88	129	43	260

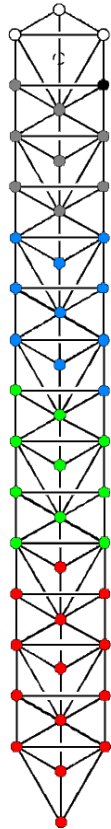
} 168

Geometrical composition of the 3-d Sri Yantra.

Figure 7

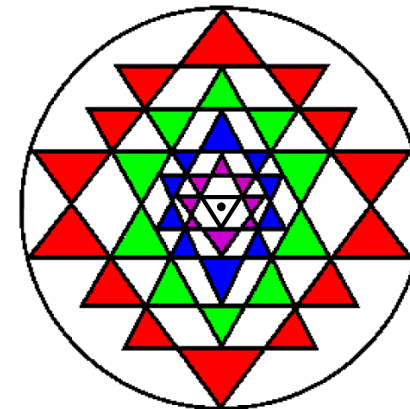
As confirmation that the 2-dimensional Sri Yantra is the counterpart of seven overlapping Trees of Life, compare their geometrical compositions. The seven overlapping Trees of Life are composed of 260 vertices, edges, triangles & tetrahedra. The 3-dimensional Sri Yantra has 260 vertices, edges & triangles. They are *both* composed of 260 geometrical elements. The seven overlapping Trees of Life are a map of the seven planes of consciousness. The number value 26 of YAHWEH, Godname of Chokmah, prescribes these two maps of the seven planes.

Figure 7



Number of vertices of triangles in n Trees of Life = $6n + 4$
 Number of edges of triangles = $16n + 6$
 Number of triangles = $12n + 4$
 Number of tetrahedra = $n + 1$

The 3-d Sri Yantra is the counterpart of 7 Trees of Life mapping the 7 planes of consciousness.



Number of vertices = 46
 Number of edges = 118
 Number of triangles = 88
 Number of tetrahedra = 8
 Total = 260

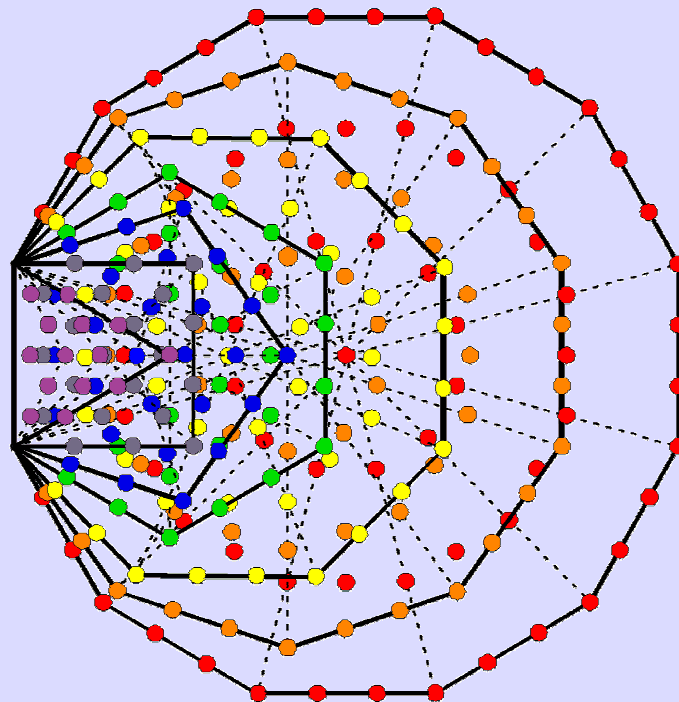
Number of vertices = 88
 Number of edges = 129
 Number of triangles = 43
 Total = 260

Figure 8

Constructed from tetractyses, the seven enfolded polygons have 260 yods outside their shared root edge. Each yod symbolizes one of the geometrical elements composing the 3-dimensional Sri Yantra. This demonstrates that the inner Tree of Life and the Sri Yantra are equivalent representations of holistic systems.

Figure 8

260 =



The inner Tree of Life is composed of 260 yods outside its root edge. This demonstrates its identity to the 3-d Sri Yantra, which comprises 260 geometrical elements. Each yod symbolizes an element.

Figure 9

The 260 geometrical elements of the Sri Yantra comprise the eight elements making up the central bindu point and innermost triangle and the 252 elements of the eight layers of triangles. Their counterparts in the inner Tree of Life are the eight yods that are either centres of polygons or Sephiroth of the Tree of Life and the 252 other yods outside the shared root edge of the polygons.

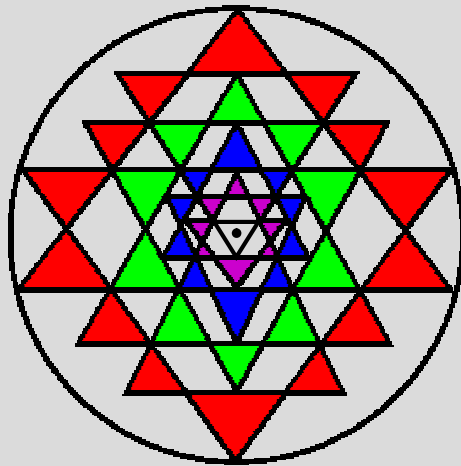
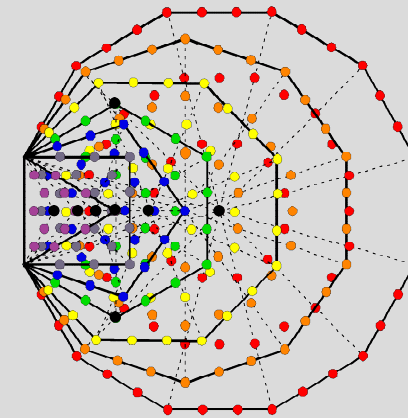


Figure 9

= 260 =



3-d Sri Yantra

Bindu + innermost triangle:

4 vertices + 3 edges + 1 triangle
= 8 geometrical elements

4 layers of triangles:

84 vertices + 126 edges + 42 triangles
= 252 geometrical elements

Total = 260

Inner Tree of Life

8 yods (●) are either centres of polygons or locations of Sephiroth of the outer Tree of Life;

has 252 coloured yods

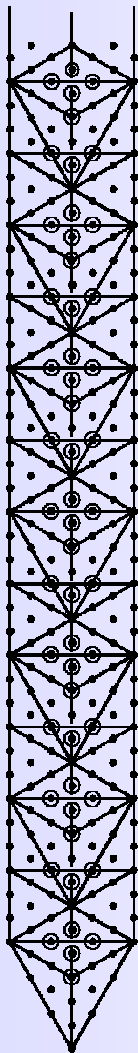
Total = 260

The correspondence between the Sri Yantra and the inner Tree of Life.

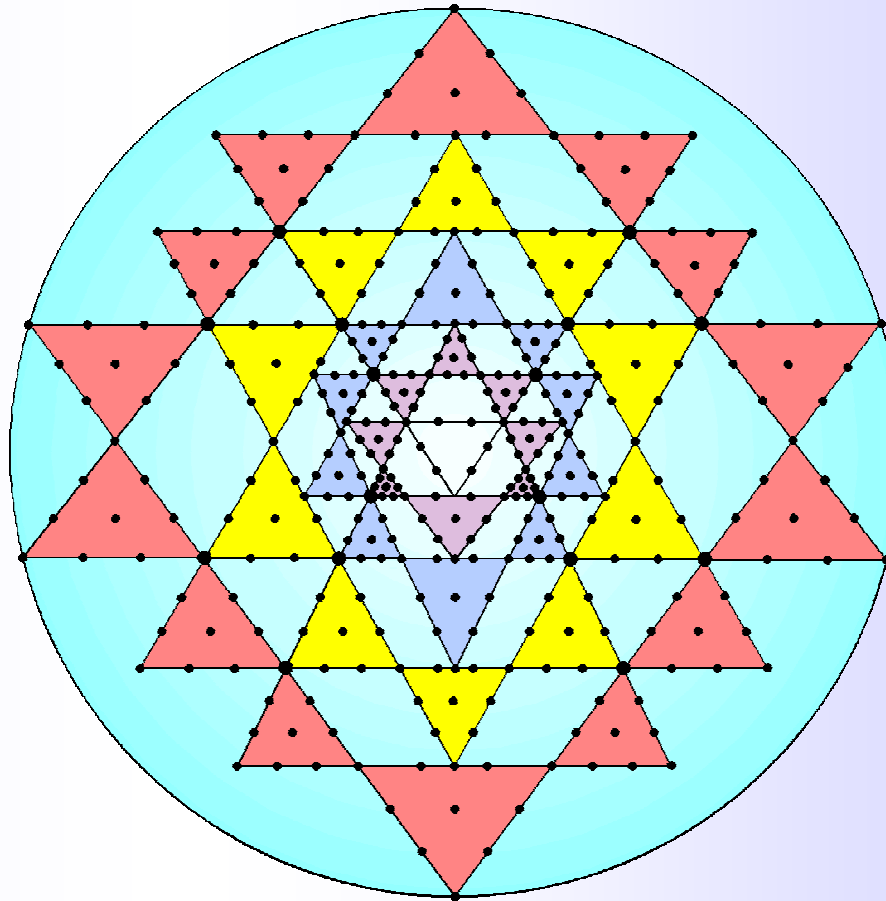
Figure 10

When their triangles are turned into tetractyses, there are 384 yods up to the top of the seventh, overlapping Tree of Life. The 42 triangles of the Sri Yantra have 378 yods, whilst the central triangle has six hexagonal yods on its edges, a total of 384 yods. This numerical correlation is not an accident but indicates that the Sri Yantra and the seven Trees of Life are isomorphic representations.

Figure 10



= 384 =



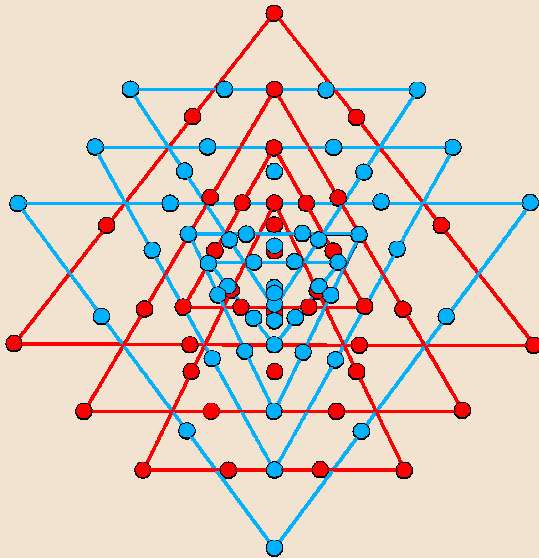
The Sri Yantra is equivalent to the lowest 7 Trees of Life because it comprises as many yods as there are yods up to the top of these trees.

Figure 11

According to Plato, the celestial sphere was designed in the proportions of the squares of the numbers 1, 2 & 3. Arranged in the shape of the Greek letter lambda, this set of seven integers (the so-called “Lambda”) is but two sides of a tetractys array of 10 integers (let us call it the “Lambda Tetractys”) that add up to 90. The four integers 1, 3, 9 & 27 on one side of the array add up to 40. The sum of the remaining integers is 50. The Sri Yantra is formed from five downward-pointing triangles expressing the Shakti (feminine) aspect of creation and four upward-pointing triangles expressing its Shiva (masculine) aspect. If each triangle is considered a tetractys, the number of yods in the nine tetractyses is 90, which is the sum of the integers in the tetractys extension of Plato’s Lambda. The four Shiva triangles/tetractyses have 40 yods and the five Shakti triangles/tetractyses have 50 yods. The source of the Sri Yantra therefore conforms to the Lambda Tetractys pattern, confirming its archetypal quality.

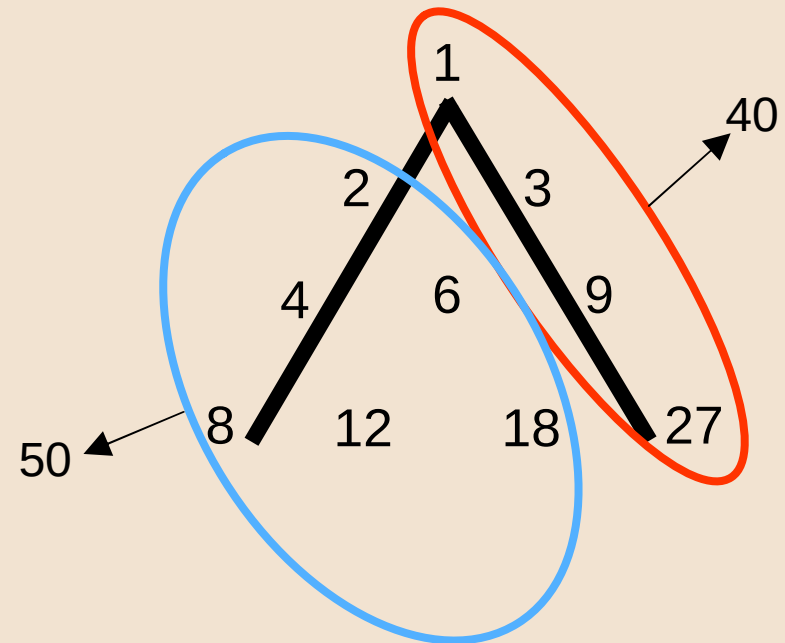
The four integers adding to 40 are all odd integers. The six integers adding to 50 are all even. The Pythagoreans regarded even integers as female and odd integers as male. This is consistent with the five triangles that embody the Shakti creative energy (female principle) having 50 yods and the four triangles that embody the Shiva energy (male principle of creation) having 40 yods. The division of the Lambda Tetractys into even and odd integers matches precisely its counterpart as five Shakti triangles and four Shiva triangles. The Lambda Tetractys is an arithmetic expression of the paradigm underlying different sacred geometries.

Figure 11



The 50:40 division of the Platonic Lambda Tetractys corresponds in the Sri Yantra to the 50:40 division of yods in the 5 Shakti triangles/tetractyses representing the feminine aspect of the creative process and the 4 Shiva triangles/tetractyses that represent the male aspect. 40 is the sum of the four odd integers and 50 is the sum of the six odd integers. This is consistent with the ancient Pythagorean view of the odd integers as male and the even integers as female.

50 (●) in 5 Shakti triangles/tetractyses
40 (●) in 4 Shiva triangles/tetractyses



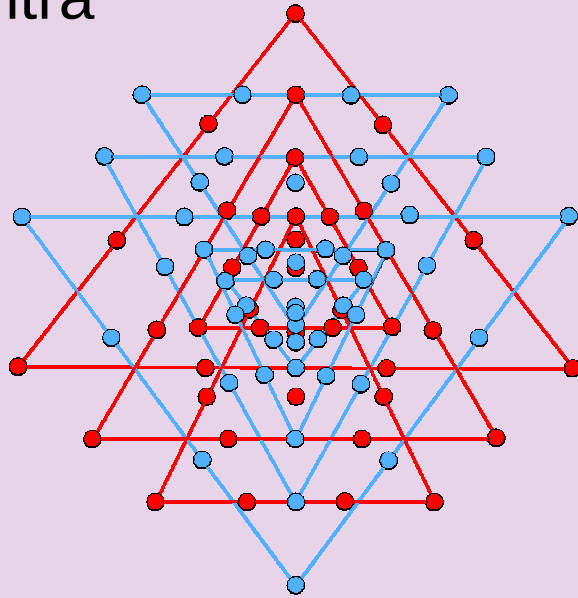
Lambda Tetractys

Figure 12

As tetractyses, the nine primary triangles generating the Sri Yantra have 27 yods at corners and nine central yods, i.e., 36 yods. The sum of the integers at the corners of the Lambda Tetractys is 36 (the largest of its integers is 27). The nine tetractyses have 54 hexagonal yods. The sum of the seven integers in the Lambda Tetractys arranged at the corners and centre of a hexagon is 54. This further demonstrates that the Lambda Tetractys arithmetically expresses the geometrical origin of the Sri Yantra. The integers 1 and 8 at two corners of the Lambda Tetractys denote, respectively, the central yod of the unpaired Shakti tetractys (the smallest, downward pointing, blue tetractys and the eight central yods of the four pairs of Shiva & Shakti tetractyses forming Stars of David. The integer 27 at the third corner of the Lambda Tetractys is the number of yods at the corners of the nine tetractyses.

Figure 12

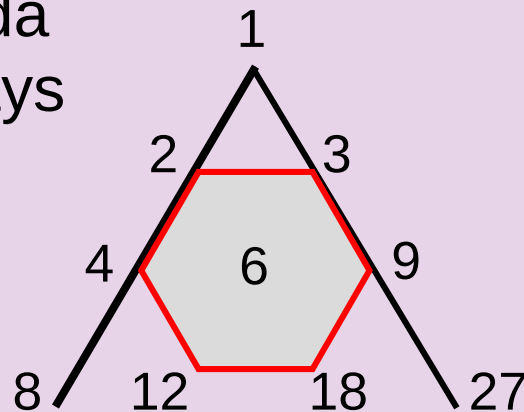
Sri Yantra



27 corners and 9 centres = 36

54 hexagonal yods in 9 tetractyses

Lambda Tetractys



Sum of integers at corners = $27 + \overbrace{8 + 1}^9 = 36$

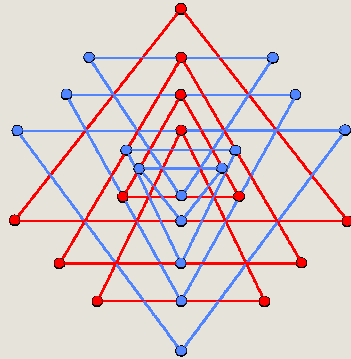
Sum of 7 integers in hexagon = 54

The Lambda Tetractys arithmetically defines the creation of the Sri Yantra from 9 triangles/tetractyses.

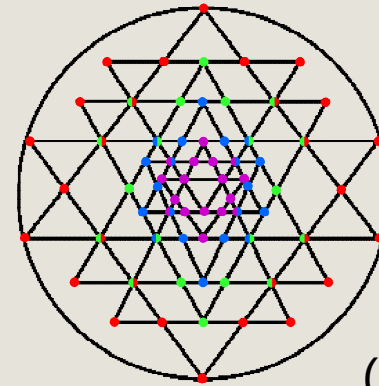
Figure 13

The nine triangles with 27 vertices overlap to form the 3-dimensional Sri Yantra whose 43 triangles have 87 vertices. 60 *new* vertices are generated. This is comparable with the Tree of Life when its 16 triangles with 10 vertices are turned into tetractyses made up of 70 yods: 60 *new* yods appear.

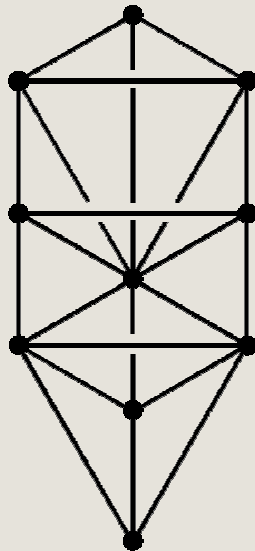
Figure 13



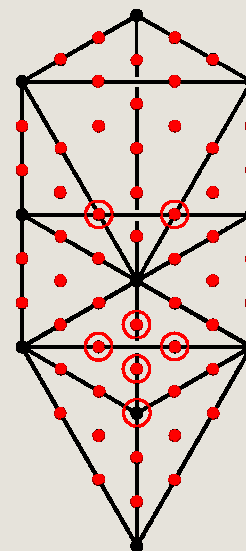
27 vertices



(27+60=87) vertices



10 (●)



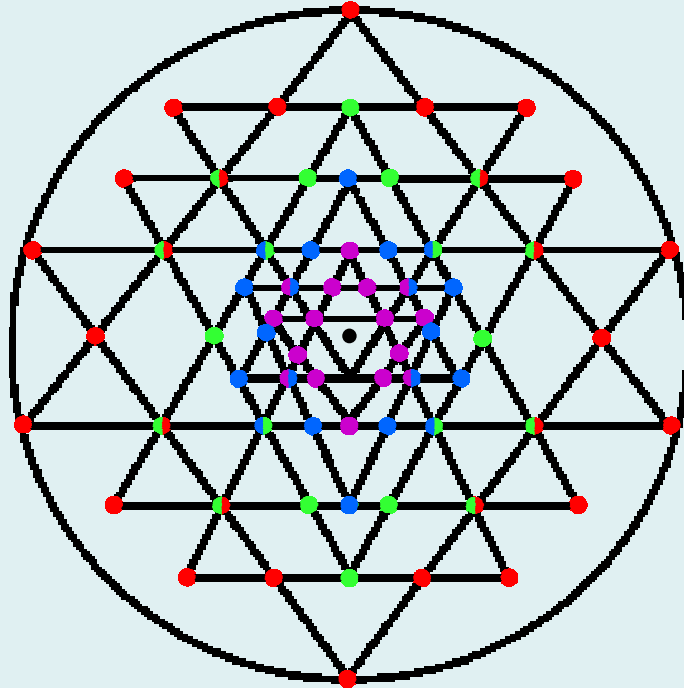
10 (●) + 60 (●)

60 new vertices are generated when 9 triangles with 27 vertices form the 3-d Sri Yantra with 87 vertices in its 43 triangles. Likewise, 60 yods are created by turning the 16 triangles of the Tree of Life into tetractyses.

Figure 14

The four layers of triangles in the 3-dimensional Sri Yantra have 84 vertices. This is the number of yods surrounding the centre of the 2nd-order tetractys. The 24 triangles of the outer two groups have 48 vertices. This is the number of hexagonal (brown) yods in the seven tetractyses arranged in a hexagon that surround the centre. The two inner groups of triangles have 36 vertices. 36 yods in the 2nd-order tetractys do not belong to these tetractyses. These correlations show how the Sri Yantra is equivalent to the 2nd-order tetractys – a higher differentiation of the tetractys symbolizing the 10-fold nature of Divine Unity.

Figure 14

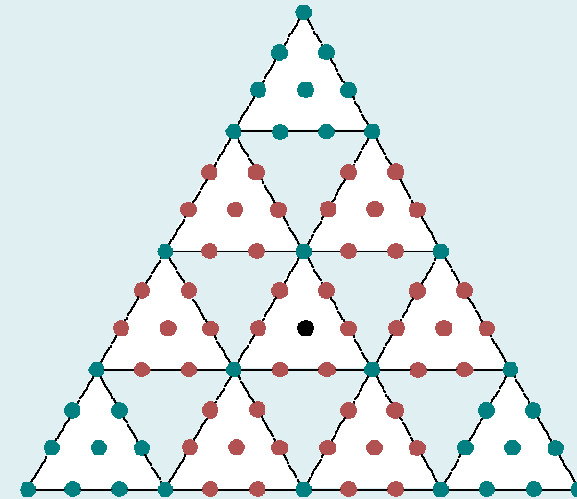


48 (● & ●)

36 (● & ●)

*(coloured semicircles denote two vertices,
one of which lies directly above the other)*

≡



2nd-order tetractys

48 (●)

36 (●)

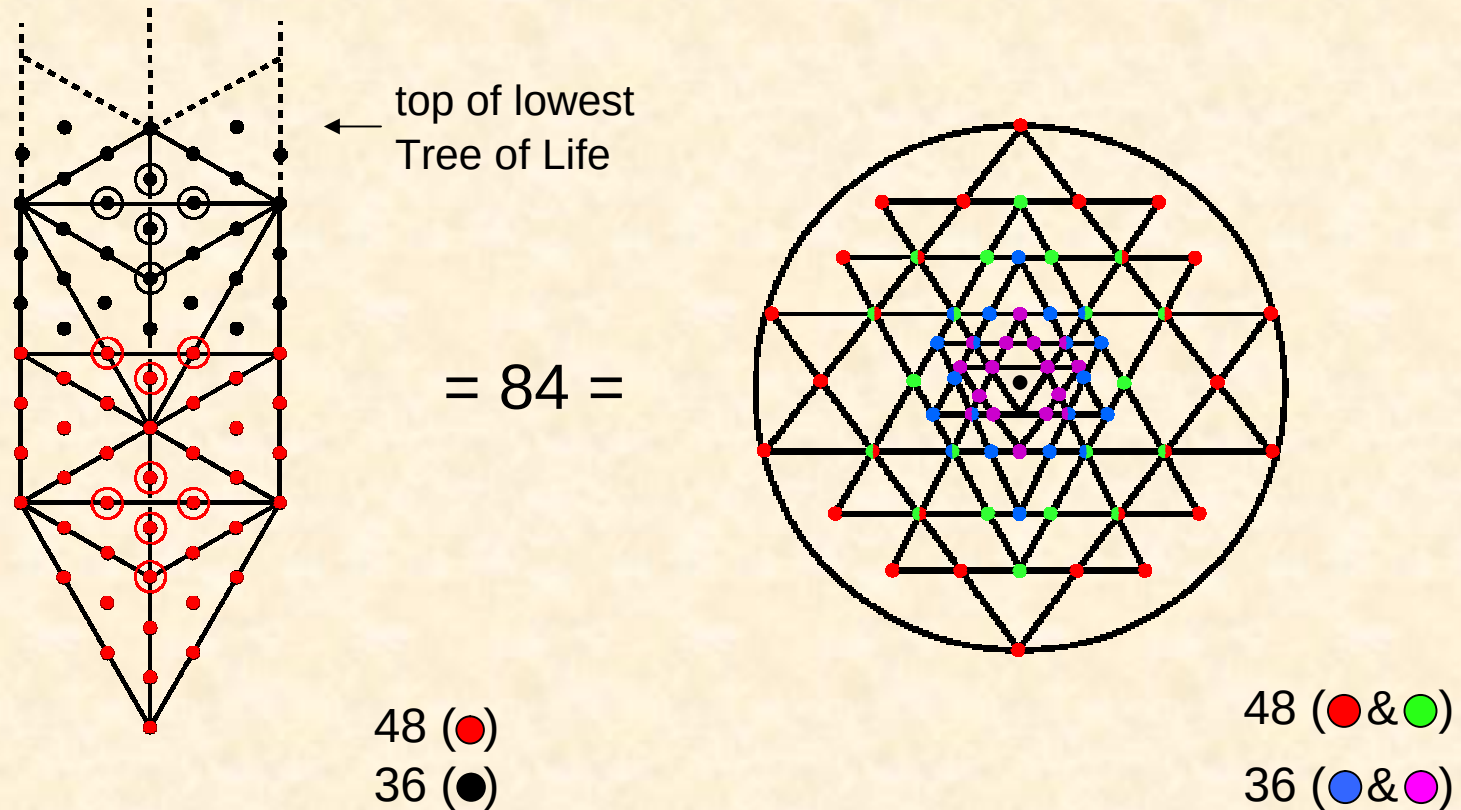
Correspondence between the 84 vertices in the 4 layers of triangles of the 3-d Sri Yantra and the 84 yods surrounding the centre of the 2nd-order tetractys.

Figure 15

When the lowest Tree of Life is constructed from tetractyses, there are 84 yods up to the top of the lowest Tree of Life. They correspond to the 84 corners of the 42 triangles of the Sri Yantra. The third and fourth layers of triangles have 48 corners corresponding to the 48 yods up to Chesed, the first Sephirah of Construction, and the first and second layers have 36 vertices corresponding to the 36 yods between Chesed and the top of the lowest tree.

Figure 15

The 84 yods up to the top of the lowest Tree of Life correspond to the 84 vertices of the four layers of triangles in the 3-d Sri Yantra. The 48 yods up to Chesed, the first Sephirah of Construction, correspond to the 48 vertices in the 3rd & 4th layers of triangles, and the 36 yods above Chesed correspond to the 36 vertices in the first two layers of triangles.



Correspondence between the Tree of Life and the 3-d Sri Yantra.

Figure 16

336 yods lie on the edges of the 42 triangles of the Sri Yantra. 168 yods form the edges of each half. 168 yods also lie on the edges of the 14 triangles in the lowest layer.

Figure 16

Numbers of yods in the Sri Yantra

	Vertices	Hexagonal yods on edges	Yods on boundaries of triangles
	$8 \times 2 = 16$	$8 \times 3 \times 2 = 48$	$16 + 48 = 64$
	$10 \times 2 = 20$	$10 \times 3 \times 2 = 60$	$20 + 60 = 80$
	$10 \times 2 = 20$	$10 \times 3 \times 2 = 60$	$20 + 60 = 80$
	$14 \times 2 = 28$	$14 \times 3 \times 2 = 84$	$28 + 84 = 112$
Total	84	252	$84 + 242 = 336$

168

336 yods lie on the 126 edges of the 42 triangles of the 3-d Sri Yantra. 168 yods lie on the edges of one half and 168 yods lie on the edges of the other half. Remarkably, the *same* number of yods also lie on the edges of the 14 triangles in the lowest layer of the Sri Yantra, whilst 168 hexagonal yods lie on the edges of the 28 triangles in its first 3 layers.

Figure 17

When the 42 triangles of the Sri Yantra are converted into tetractyses, 168 yods lie on the edges of the 28 triangles in the first three layers. 168 yods lie on the edges of the 14 triangles in the fourth layer.

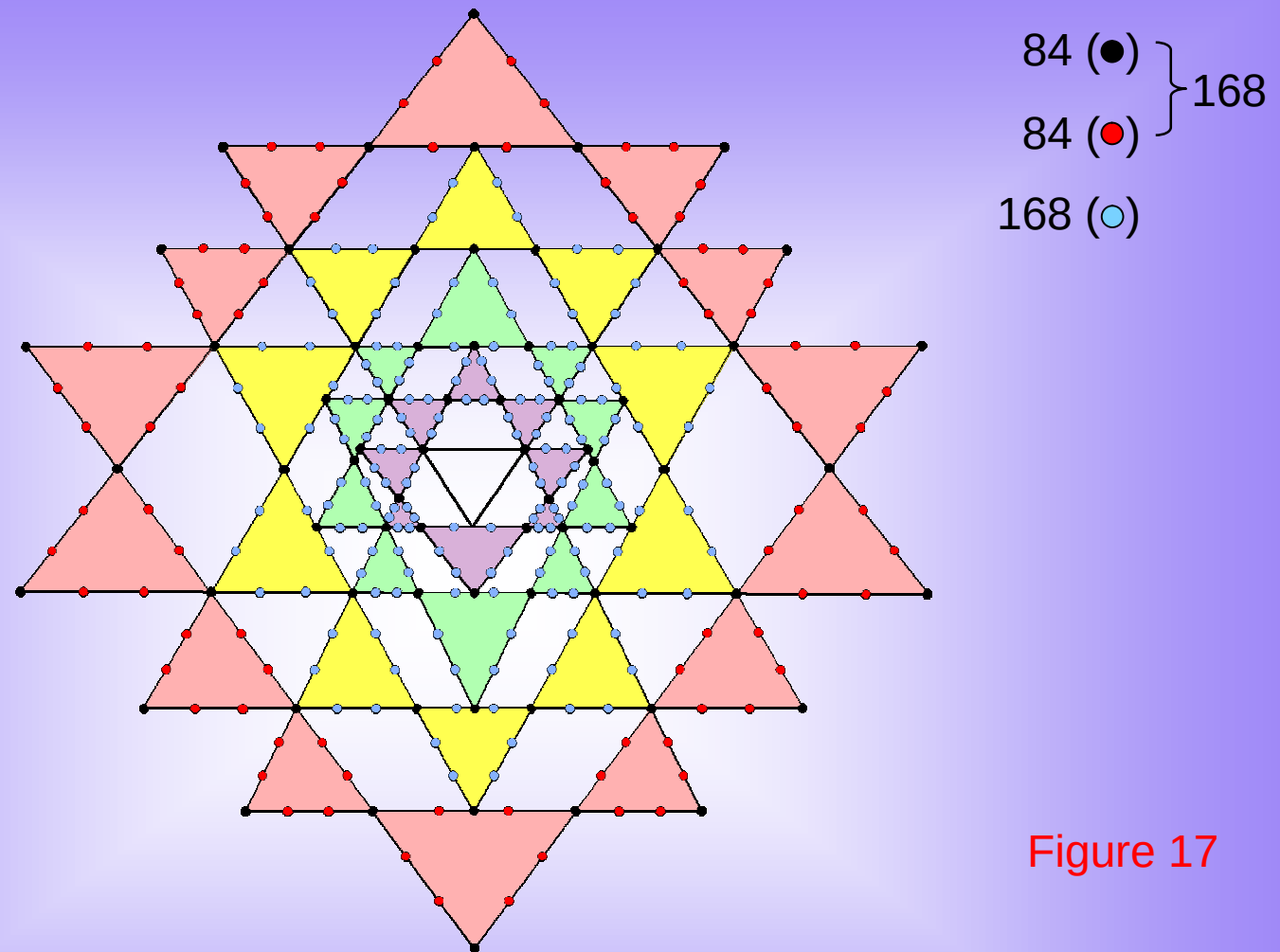


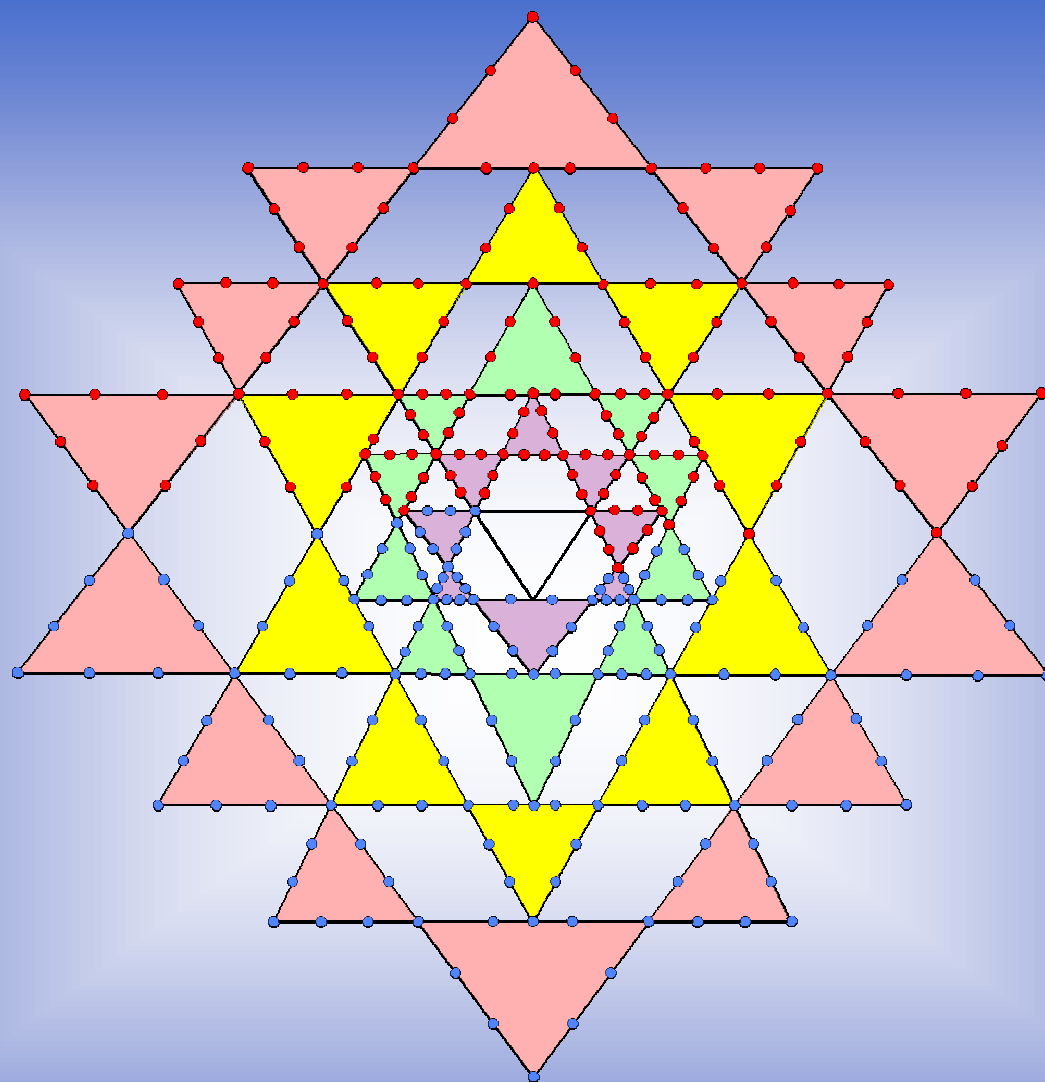
Figure 17

168 yods lie on 42 edges of 14 triangles in the 4th layer.

168 yods lie on 84 edges of 28 triangles in the 1st, 2nd & 3rd layers.

Figure 18

336 yods lie on the edges of the 42 triangles of the 3-dimensional Sri Yantra. 168 yods lie on the edges of the 21 triangles in one half and 168 yods lie on the edges of the 21 triangles in the other half.



168 (●)

168 (●)

Figure 18

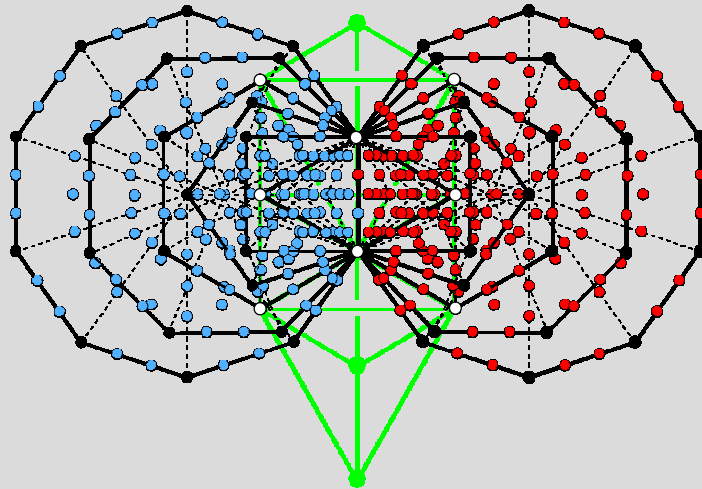
The superstring structural parameter 336 is the number of yods on the 126 edges of the 42 triangles of the 3-d Sri Yantra.

Figure 19

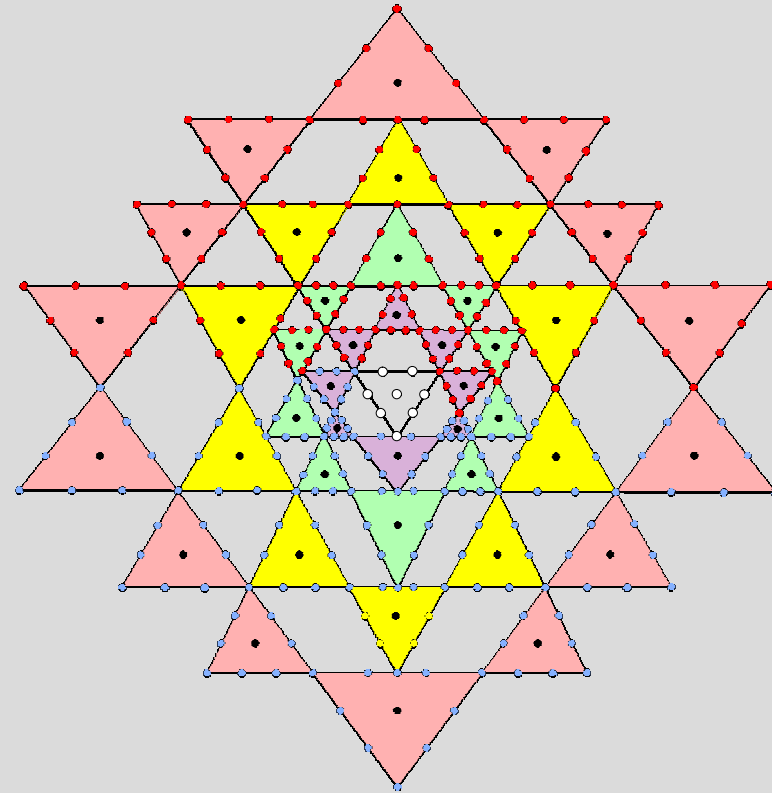
The first (6+6) enfolded polygons of the inner Tree of Life have 42 corners that do not coincide with Sephiroth of the outer Tree of Life. The eight corners that *do* coincide are shown as white yods (although not a Sephirah, Daath can be formally treated here as a Sephirah because it is Yesod of the next higher, overlapping Tree of Life). Each set of six polygons have 168 yods (red or blue) that are not corners. Compare this with the 42 centres of triangles and the 168 yods on the edges of each half of the Sri Yantra. The correspondence is complete if we include the seven hexagonal yods of the central triangle (corresponding to the seven Sephiroth whose positions coincide with corners) and the bindu, which corresponds to Daath.

Figure 19

8 (○) yods are shared with the Tree of Life



≡



168 (●) 8 (○)
 42 (●) 168 (●)

168 (●) 8 (○)
 42 (●) 168 (●)

**Correspondence between the inner
Tree of Life and the 3-d Sri Yantra.**

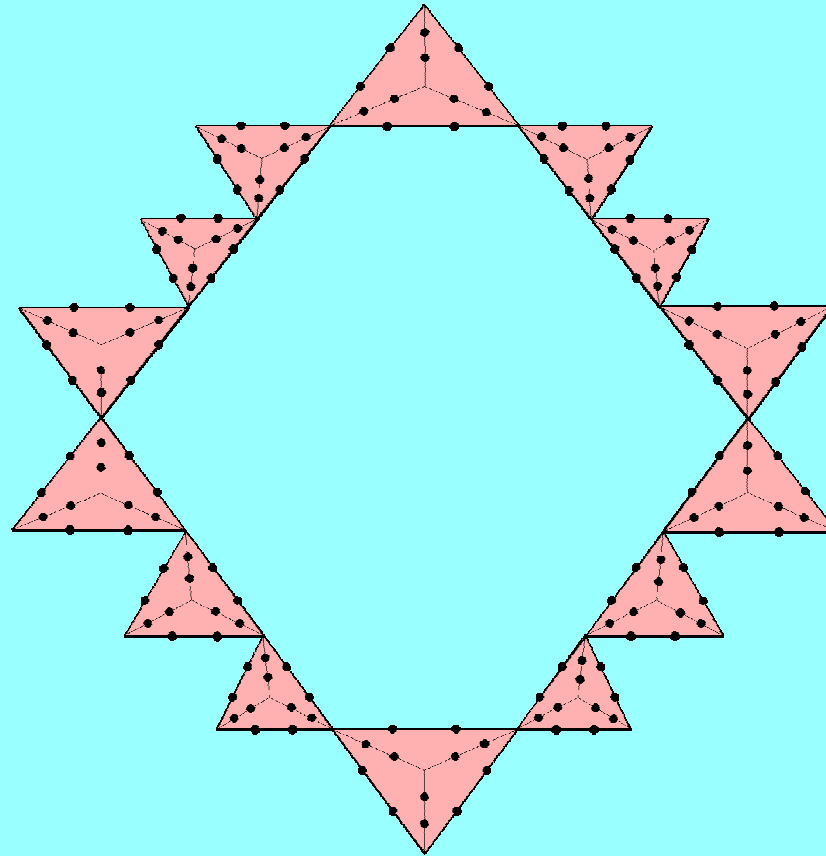
Figure 20

When the triangles of the Sri Yantra are each constructed from three tetractyses, its outermost 14 triangles have 168 hexagonal yods on the edges of their 42 tetractyses.

Figure 20

14×12

168 (●) =

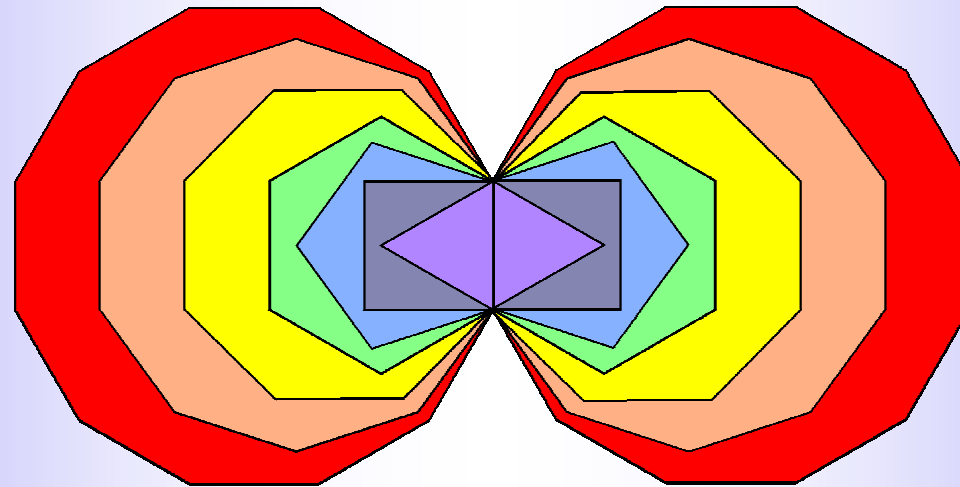


The 14 outermost triangles of the Sri Yantra embody the superstring structural parameter 168 as the number of hexagonal yods on the edges of their 42 tetractyses.

Figure 21

The dodecagon is the seventh and last of the regular polygons in the two sets that constitute the inner Tree of Life.

Figure 21



The inner Tree of Life

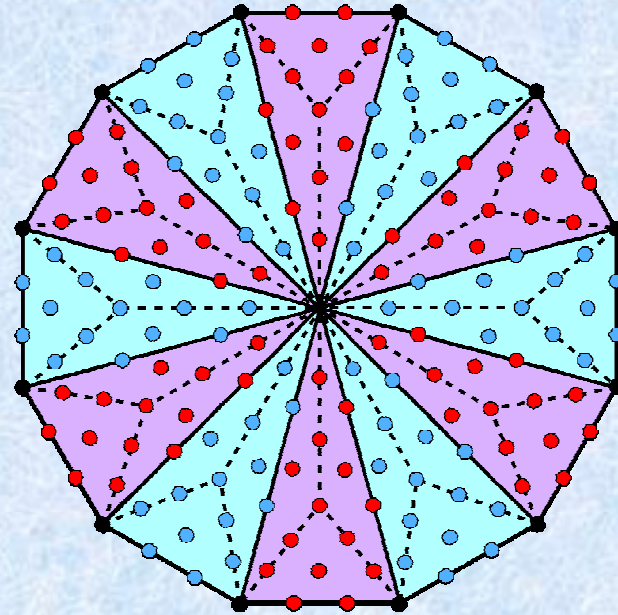
Figure 22

The dodecagon requires 168 extra yods when its sectors are divided into three tetractyses. 14 yods are added per sector. Therefore, six sectors have 84 extra yods. As the dodecagon is two hexagons rotated through 30° , the number 168 embodied in the dodecagon divides naturally into 84 and 84. This is the counterpart of the 84 vertices of the 42 triangles of the Sri Yantra and the 84 hexagonal yods on the edges of its outermost 14 triangles.

Figure 22

$$12 \times 14 = 168$$

$$84 (\bullet) + 84 (\circ) = 168 =$$



168 extra yods are needed to construct each of the sectors of the dodecagon from three tetractyses.

Article 40

The Unification of All Sacred Geometries & Its Implication for Particle Physics

Part 3

by

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Figure 1

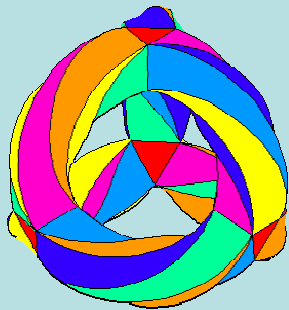
The quartic equation:

$$x^3y + y^3z + z^3x = 0$$

was studied by the mathematician Felix Klein. He showed that its Riemann surface is mapped onto itself (hence “automorphisms”) by 168 analytic transformations. These symmetries are mapped onto a surface of genus 3, i.e., the 3-torus. In fact, this is the maximum number of symmetries for a surface of this genus. This Riemann surface can be represented in the hyperbolic plane by the Klein Configuration. For the $\{7,3\}$ tiling, it has 168 red, hyperbolic triangles denoting the 168 automorphisms and 168 blue triangles denoting the 168 anti-automorphisms. The 168 red or blue triangles are grouped 24 to each of seven sectors, forming 24 heptagons. Three heptagons meet at each of its 56 vertices. Each sector has two slices. Each of the 14 slices shown numbered has 12 red triangles and 12 blue triangles. The 168 automorphisms belong to the group $\text{PSL}(2,7)$. The 168 automorphisms and 168 anti-automorphisms belong to the double-cover group $\text{SL}(2,7)$.

Figure 1

$$14 \times 12 = 168$$

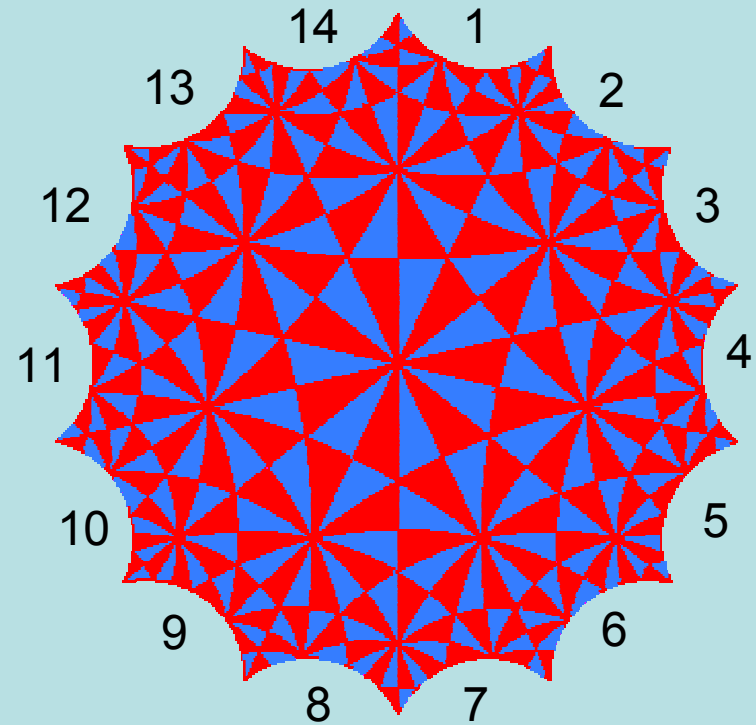


3-torus

$$168 (\triangle) =$$

Klein quartic:

$$x^3y + y^3z + z^3x = 0$$



$\{7,3\}$ mapping of the 168 symmetries of the Klein quartic

The Klein configuration is the mapping on the hyperbolic plane of the 168 automorphisms (red hyperbolic triangles) of the Klein quartic. The 168 blue triangles denote its 168 anti-automorphisms.

Figure 2

In their book *Occult Chemistry*, published in 1908, Annie Besant and C.W. Leadbeater recorded their observations of what they believed were atoms of all the elements. The basic unit of matter, or “ultimate physical atom” (UPA), has two varieties, one the mirror image of the other. In the positive type, the ten closed curves, or “whorls,” spiral clockwise $2\frac{1}{2}$ times around the axis about which the particle spins, then wind back $2\frac{1}{2}$ times in a narrower helix back to the top of the particle. Each of the ten helices is a closed curve with 1680 turns. There are 336 turns in each of the five revolutions of each whorl, that is, 168 turns in a half-revolution. The ten whorls make 3360 turns in each revolution. The author proved that these particles are constituents of up and down quarks and has interpreted them as $E_8 \times E_8'$ heterotic superstrings.

Figure 2

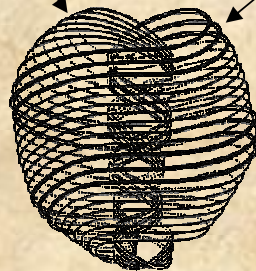


Annie Besant
(1847-1933)

C.W. Leadbeater
(1847-1934)

minor whorl

major whorl



positive UPA



negative UPA

A whorl is a helical coil with 1680 turns. It winds 336 times in each of five revolutions around the spin axis of the positive or negative UPA.

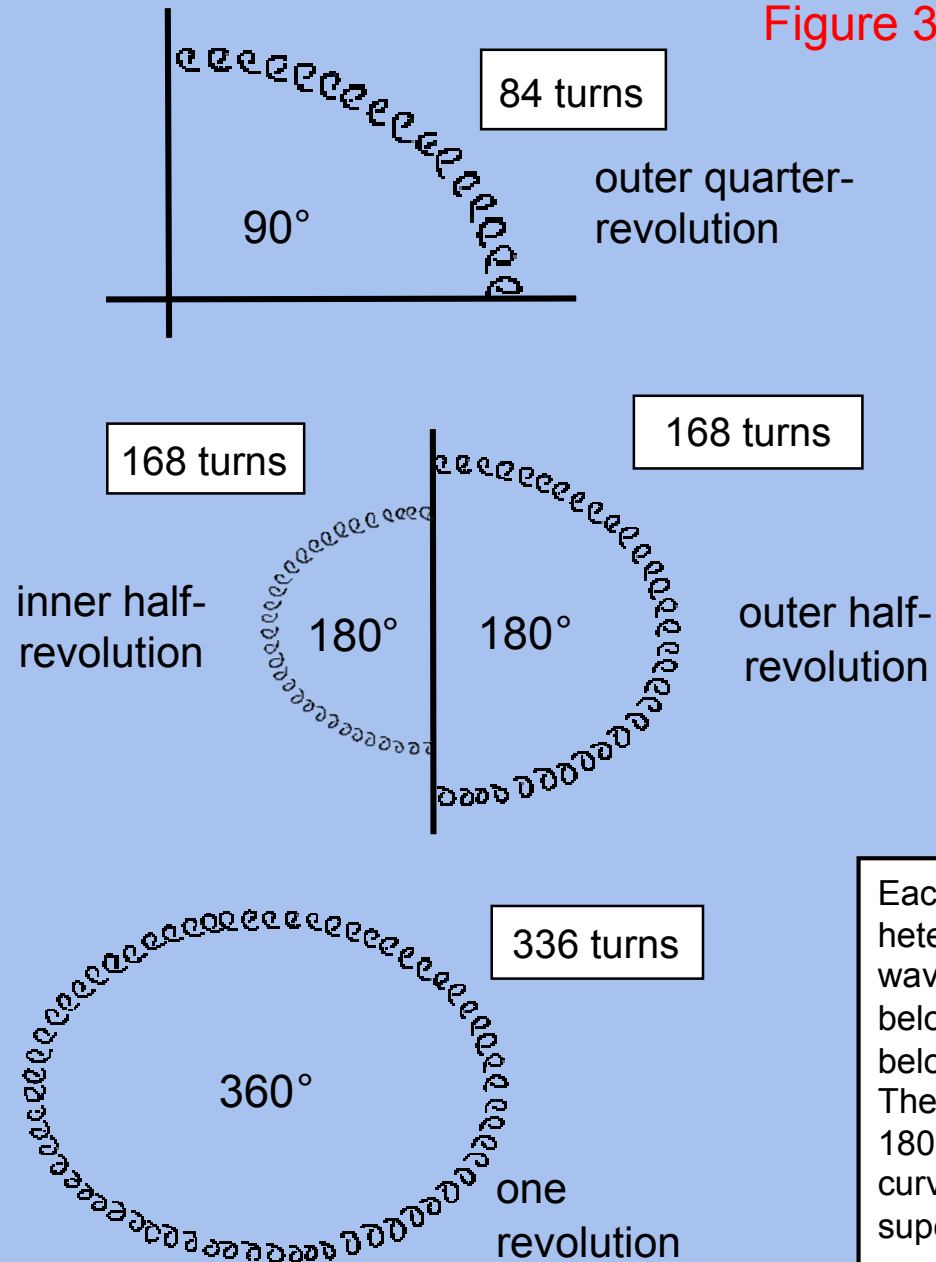
The two types of UPAs ('ultimate physical atoms') making up matter are mirror images of each other. They are $E_8 \times E_8$ heterotic superstring constituents of quarks. They consist of 10 separate, never touching, closed curves, or 'whorls,' each winding 1680 times around a torus. In the positive UPA, the whorls spiral clockwise; in the negative UPA, they spiral anticlockwise. Each curve makes five revolutions but does NOT form a knot either with itself or with other curves. The twisting of the three major whorls at the bottom of the UPA depicted in the 3rd edition of Besant's & Leadbeater's book "Occult Chemistry" was a diagrammatic error, as it would imply that the 1st and 3rd major whorls are the single edge of a Mobius strip, i.e., that there are 9 (not 10) curves, contrary to what they stated. Instead, the three major whorls form what is, topologically speaking, a 'link' with the seven minor whorls.



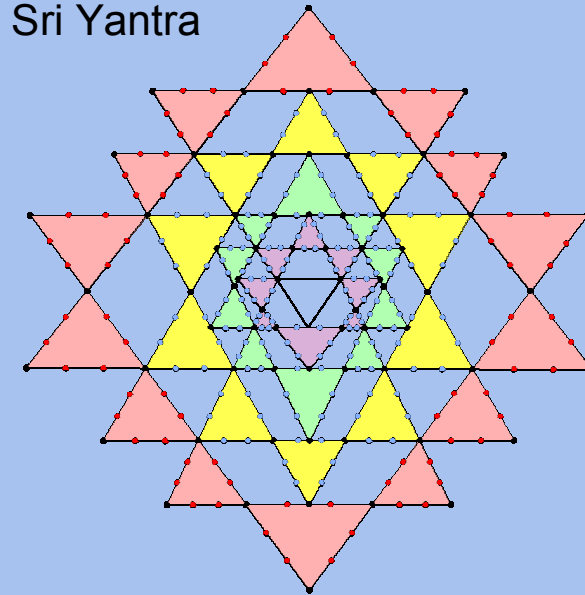
Figure 3

The numbers 84, 168 and 336 appear in the Sri Yantra. Its 42 triangles have 84 vertices (black yods), 84 (red) hexagonal yods on the edges of its 14 outermost triangles and 168 (blue) hexagonal yods on the edges of the 28 triangles in its first three layers. 336 yods therefore line its 42 triangles, delineating its *shape*. 168 yods mark out each half. The Sri Yantra therefore represents the number of turns in one revolution of each helical whorl of the UPA. If one assigns the number 10 (the Pythagorean Decad symbolized by the tetractys) to each yod, the Sri Yantra generates the number 3360. This is the number of turns in one revolution of all ten whorls of the UPA. If one assigns the number value 50 of the Godname ELOHIM to each yod, the Sri Yantra generates the 16800 turns in all ten whorls of the UPA, a heterotic superstring. Created out of the number value of ELOHIM, the Godname of the third member of the Supernal Triad, the Sri Yantra generates the *form* of the fundamental subatomic particle making up the atoms of the elements. The Godname YAHWEH of the second member of the Supernal Triad with number value 26 prescribes the 26 dimensions of space-time from which the 10-dimensional superstring emerges.

Figure 3



3-d Sri Yantra



84 (●)
84 (●) } 336
168 (○)

Each of the ten closed curves (whorls) of the $E_8 \times E_8$ heterotic superstring has 1680 circularly polarised waves or oscillations running along it. 840 waves belong to the outer $2\frac{1}{2}$ revolutions and 840 waves belong to the $2\frac{1}{2}$ inner revolutions around its spin axis. There are 84 waves in a 90° twist, 168 waves in an 180° twist and 336 waves in one revolution of each curve. These structural parameters of the heterotic superstring are embodied in the 3-d Sri Yantra.

Figure 4

The I Ching table, used for thousands of years by the Chinese for purposes of divination, is an 8×8 array of 64 hexagrams, each a pair of trigrams. There are eight hexagrams in its diagonal and 28 hexagrams (56 trigrams with 168 lines and broken lines) on either side of it. The table is equivalent to the Klein Configuration because the 168 red lines and broken lines of the 56 trigrams on one side of the diagonal correspond to the 168 red hyperbolic triangles denoting the 168 automorphisms, whilst the 168 blue lines and broken lines of the 56 trigrams on the other side of the diagonal correspond to the 168 blue hyperbolic triangles denoting the 168 anti-automorphisms.

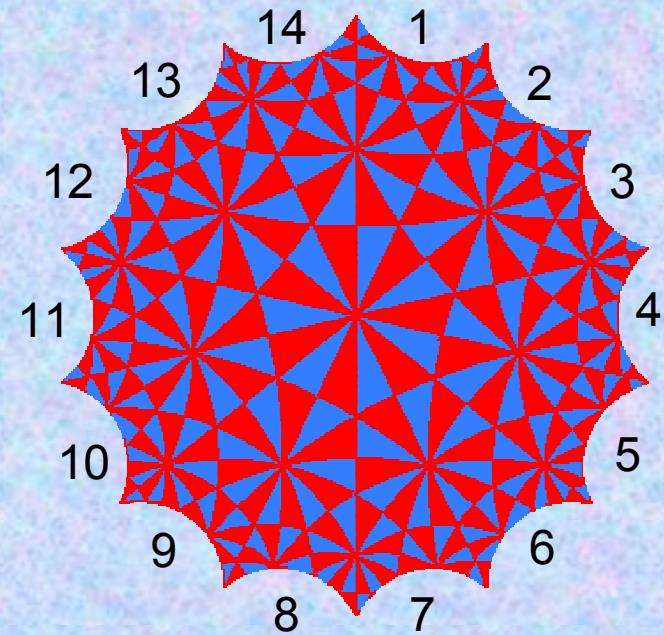
Figure 4

I Ching table

168 —&— —

168 —&— —

Klein configuration



168 ▲ (automorphism)

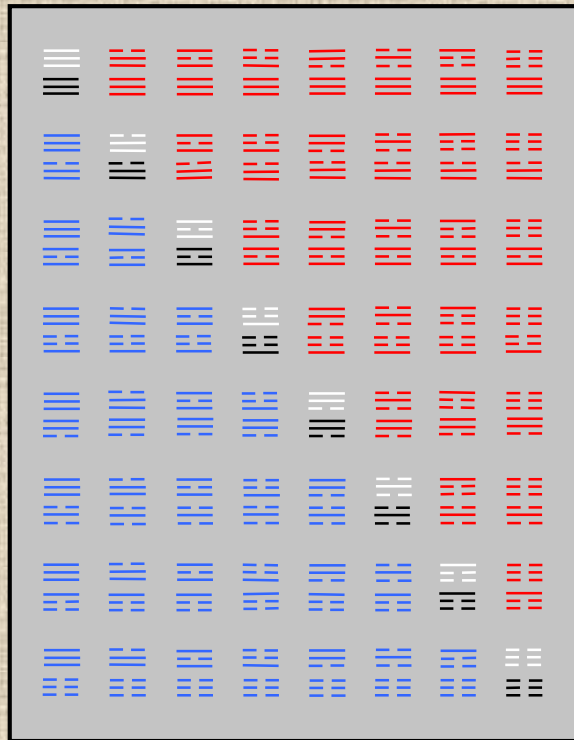
168 ▲ (anti-automorphism)

Figure 5

In the $\{3,7\}$ tiling of the Riemann surface of the Klein quartic, 56 triangles with 168 vertices tessellate completely the hyperbolic plane. The vertices denote the 168 automorphisms of the Klein quartic. The triangles correspond in the I Ching table to the 56 red trigrams on one side of the diagonal with 168 lines and broken lines, each one corresponding to an automorphism of this equation well-known to mathematicians.

Figure 5

The correspondence between the (168+168) lines/broken lines of the (56+56) off-diagonal trigrams and the (168+168) symmetries of the Klein quartic



56 trigrams

56 trigrams

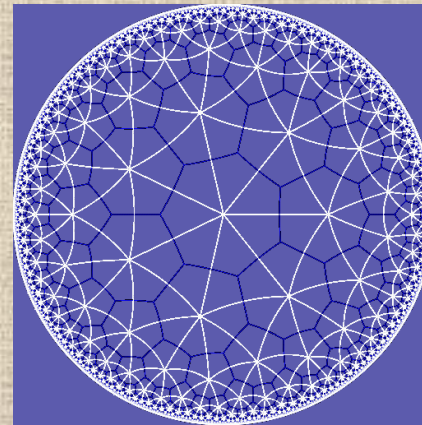
$$56 \times 3 = 168$$

168

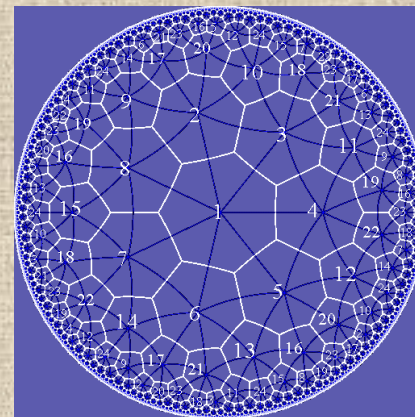
anti-automorphisms

168

automorphisms



$$56 \times 3 = 168$$



$$24 \times 7 = 168$$

{3,7} tiling

56 triangles with
168 vertices are
needed to tessellate
the 3-torus.

Poincaré
dual

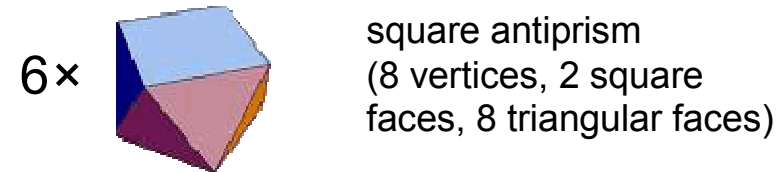
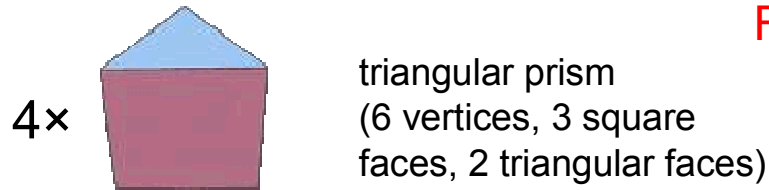
{7,3} tiling

24 heptagons with
168 vertices are
needed to tessellate
the 3-torus.

Figure 6

The $\{3,7\}$ tiling onto the 3-torus of the 168 automorphisms of the Klein quartic represented by the 168 vertices of the 56 triangles of the Klein configuration requires seven colours to colour the 56 triangles so that no two triangles sharing an edge have the same colour. The 3-torus may be deformed into a tetrahedral structure formed by sticking the 12 square faces of six square antiprisms onto the 12 square faces of four triangular prisms and then twisting each antiprism. This creates 56 triangles (eight triangular faces of the four prisms + 48 triangular faces of the six square antiprisms). The 56 triangular faces define the 56 vertices of seven cubes. As a cube has 24 different rotations, the 168 automorphisms of the Klein quartic correspond to seven copies of 24 symmetries of the octahedral group. Its 168 anti-automorphisms are represented by the 168 vertices of the 56 triangular faces of the tetrahedral structure after it has been turned inside out. These triangles, too, define the 56 vertices of seven cubes that generate seven copies of the 24 symmetries/inversions of the octahedral group. These make up the 168 anti-automorphisms of the Klein quartic.

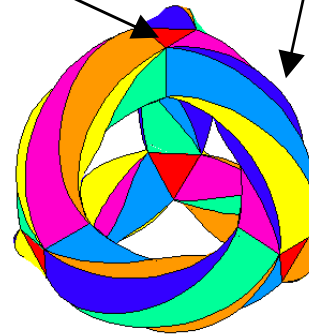
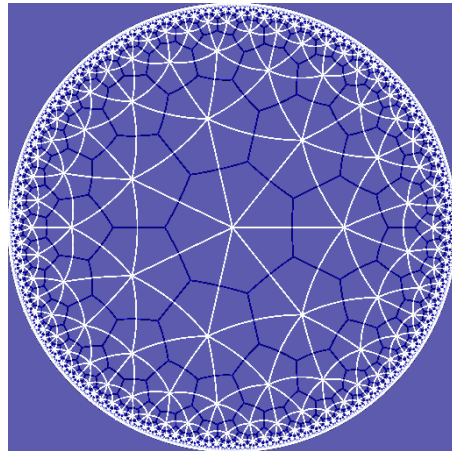
Figure 6



4×2=8 triangles

6×8=48 triangles

Attach a triangular prism to the 4 vertices of a tetrahedron. Then stick each square face of an antiprism onto a square face of each triangular prism, giving it a twist.



8 triangular faces of
4 triangular prisms

+

8×6=48 triangular faces
of 6 square antiprisms

=

56 triangles

Klein configuration

{ 3,7 } 56 triangles

{ 7,3 } 24 heptagons



168
automorphisms

7-colour tiling of the Klein
configuration on the 3-torus

24 rotations of tetrahedron×7-
fold rotational symmetry of
Klein configuration = 168
symmetries of PSL(2,7)

The 336 symmetries
consist of 7 copies of the
(24+24=48) symmetries of
the octahedral symmetry
group shown by the cube.
This is because the 56
triangular faces define the
56 vertices of 7 cubes.

The mirror reflections of the 56 triangles with 168 vertices
define the 168 anti-automorphisms of the Klein quartic.

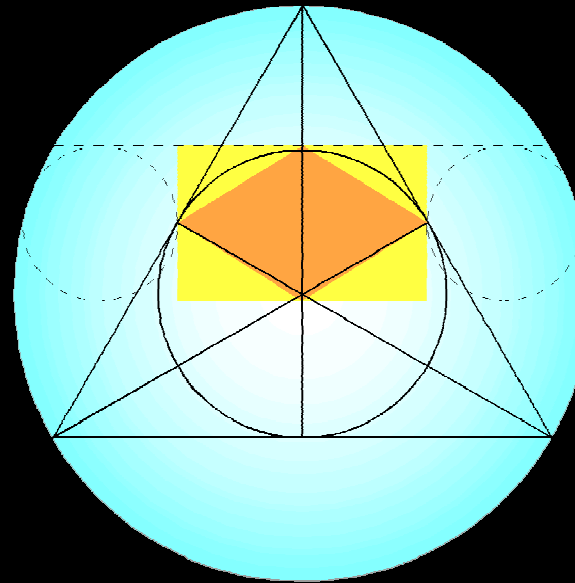
Figure 7

The 168 automorphisms of the Klein quartic belong to $\text{PSL}(2,7)$. This group is isomorphic to $\text{PSL}(3,2)$, whose 168 symmetries are displayed by the Fano plane, the simplest projective plane. Implicit in its geometry is a Golden Rectangle, the ratio of whose adjacent sides is the Golden Ratio $\varphi = 1.6180\dots$, and a Golden Rhombus, the lengths of whose diagonals are in the same proportion. The 60 edges of a rhombic triacontahedron form 30 Golden Rhombi that are the bases of pyramids with four faces, creating the 120 faces of the disdyakis triacontahedron. This shows how the ideal, aesthetically pleasing proportion of the Golden Ratio, used throughout history by artists and architects, manifests in the form of the disdyakis triacontahedron. Beauty is exhibited in the shape, as well as in the mathematical properties, of the polyhedral version of the inner Tree of Life.

Figure 7



disdyakis triacontahedron with a
Golden Rhombus formed by edges



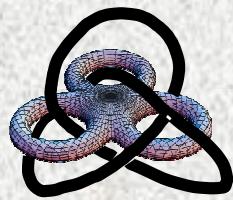
Fano plane with a Golden Rectangle
and a Golden Rhombus

The Fano plane has the 168 symmetries of $SL(2,3)$, which is isomorphic to $PSL(2,7)$, the group of symmetries of the Klein quartic. The Golden Rhombic shape of the faces of the rhombic triacontahedron underlying the disdyakis triacontahedron is implicit in the geometry of the Fano plane, which represents the algebra of the octonions.

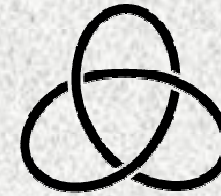
Figure 8

A continuous, logical link connects the (2,3) torus knot to the quantitative description given in 1908 by Annie Besant and C.W. Leadbeater of the form of the heterotic superstring constituent of up and down quarks.

Figure 8



(2,3) torus knot



(2,3) torus knot winds on 3-torus

3-torus is Riemann surface of Klein quartic with 168 symmetries of $\text{PSL}(2,7)$ & 336 symmetries of $\text{SL}(2,7)$

$\text{PSL}(2,7)$ is isomorphic to $\text{SL}(3,2)$

Fano plane has symmetry group $\text{SL}(3,2)$ of order 168

Fano plane represents octonion algebra

Octonion algebra is isomorphic to E_8 Lie algebra

E_8 is gauge symmetry group of unified, superstring force

$E_8 \times E_8$ heterotic superstring is UPA

Helical whorl of the UPA winds 168 times on torus in half a revolution

Figure 9

The polyhedral inner form of the Tree of Life is the 144 Polyhedron, which has 74 vertices, 216 edges and 144 faces. It generates another polyhedron with 62 vertices, 180 edges and 120 triangular faces. The latter is the disdyakis triacontahedron and the former is the disdyakis dodecahedron with tetrahedra added to its 48 faces. Their $(24+24=48)$ peaks correspond to the 24 rotational symmetries of the octahedral group and the 24 rotations/inversions. This is a subgroup of $SL(2,7)$, and seven copies of it define the 336 symmetries of $SL(2,7)$, the double-cover group of $PSL(2,7)$, which is isomorphic to $SL(2,7)/Z_2$ and describes the 168 automorphisms of the Klein quartic. This symmetry manifests in the 168 edges of the disdyakis triacontahedron above and below the equator that is perpendicular to the axis joining two diametrically opposite vertices of this polyhedron. The centre of $PSL(2,7)$ is Z_3 , the cyclic group of order 3 that is isomorphic to the three primitive roots of 1, namely, 1, $\exp(2\pi i/3)$ & $\exp(4\pi i/3)$. The fact that the dimension of $SL(2,7)$ is the number of edges above and below the equator of the disdyakis triacontahedron is evidence that $PSL(2,7)$ is embodied in this polyhedron as a fundamental symmetry.

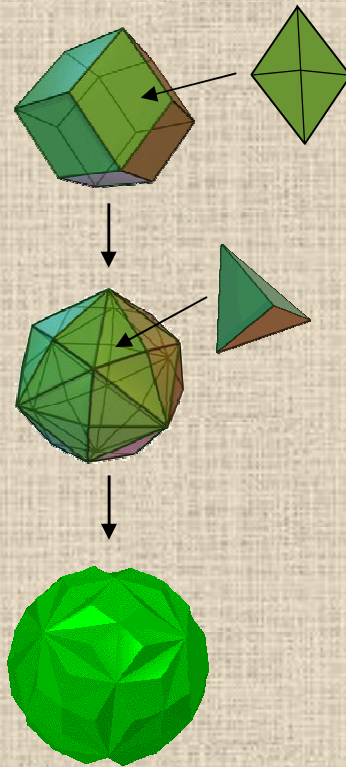
The algebra of the seven unit imaginary octonions is represented by the Fano plane, which has the symmetry of $SL(3,2)$, a group that is isomorphic to $PSL(2,7)$. It can alternatively be represented by assigning them to the vertices and centre of an octahedron. This suggests a connection between the disdyakis triacontahedron and the octonions. It is to be expected because the octonions form a basis of E_8 , the superstring gauge symmetry group (see Figs. 7 & 8).

Figure 9

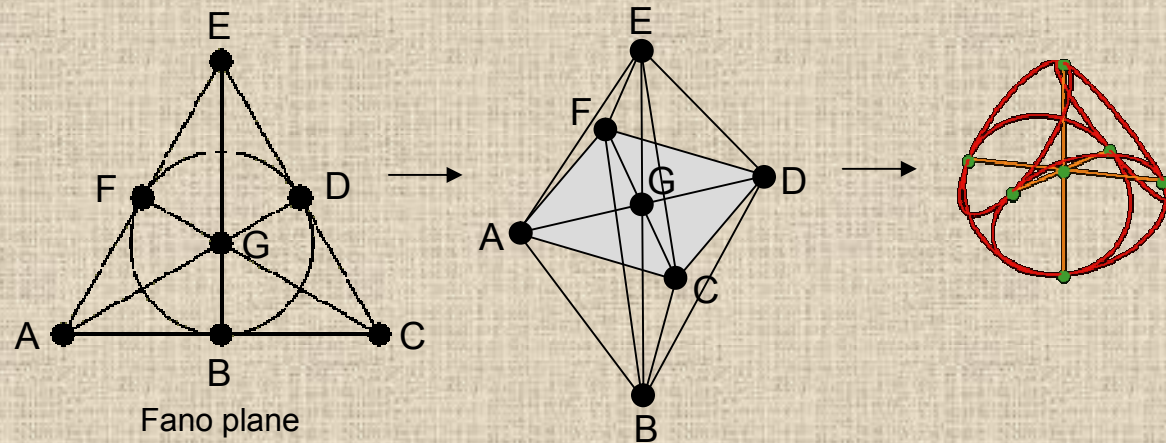
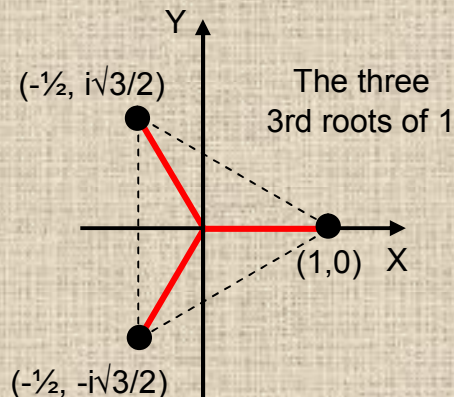
Rhombic dodecahedron
14 vertices
24 edges
12 faces

Disdyakis dodecahedron
26 vertices
72 edges
48 faces

144 Polyhedron
74 vertices
216 edges
144 faces



48 of the 74 vertices of the 144 Polyhedron are peaks of tetrahedra that can be stuck on the 48 faces of a disdyakis dodecahedron. As the latter are the faces of rhombic pyramids attached to the 12 faces of a rhombic dodecahedron, they form 12 sets of four faces, that is, 24 faces and their 24 mirror images. The corresponding peaks of the tetrahedra correspond to the $(24+24=48)$ symmetries of the octahedral group. This is a subgroup of $SL(2,7)$ and seven copies of it define the 336 symmetries of $SL(2,7)$, the double-cover group of $PSL(2,7) \sim SL(2,7)/Z_2$ describing the 168 automorphisms of the Klein quartic. They manifest in the disdyakis triacontahedron as its 168 edges above and below the equatorial plane parallel to its 7 sheets of vertices. The octahedral group is primary because an octahedron and its centre can represent the Fano plane defining the multiplication of the unit imaginary octonions and having the 168 symmetries of $SL(3,2)$, which is isomorphic to $PSL(2,7)$. The 168 symmetries of $PSL(2,7)$ comprise $(8 \times 6 = 48)$ rotations by $(1/7-6/7)$ turns of the central 8 heptagons in their $\{7,3\}$ tessellation on the hyperbolic plane. The centre of $PSL(2,7)$ is Z_3 , the cyclic group of order 3 that is isomorphic to the three primitive 3rd roots of 1: $1, \exp(2\pi i/3) \text{ \& } \exp(4\pi i/3)$.



The 7 lines of the Fano plane representing the seven 3-tuples of unit imaginary octonions become the 3 diagonals of the octahedron plus the 4 circles which circumscribe 4 of its 8 faces.

Figure 10

If a Yang (unbroken) line of a trigram denotes a face of a cube orientated along one of the three perpendicular, positive directions and a Yin (broken) line denotes a face facing a negative direction, the eight trigrams symbolize the sets of three perpendicular faces that intersect at the eight corners of a cube. As a pair of different trigrams, a hexagram defines the straight line joining two corners of a cube. The pairs of identical trigrams in the eight hexagrams in the diagonal of the I Ching table define the eight corners themselves. The ordering of pairs of trigrams signifies the directions of the lines regarded as arrows. This means that the hexagrams below the diagonal signify arrows connecting different corners that point in the opposite direction to those connecting corners signified by hexagrams above the diagonal. The 28 hexagrams above the diagonal have 168 lines/broken lines denoting the 168 faces (84 positive, 84 negative) defined by the endpoints of the 28 lines joining different corners. Each combination of three faces appears seven times, creating seven copies of each corner, i.e., 56 copies of the eight corners, totalling 64 corners. These are the counterparts of the 56 hyperbolic triangles with 168 vertices in the $\{3,7\}$ tiling of the automorphisms of the Klein quartic on the hyperbolic plane, each triangle in fact defining one of the corners of seven cubes. The 56 hyperbolic triangles mapped onto a 3-torus turned inside-out, whose 168 vertices denote the 168 anti-automorphisms of the Klein quartic, similarly correspond to the 56 trigrams with 168 lines/broken lines in the 28 hexagrams below the diagonal, each trigram defining a corner of seven copies of a cube - the seven cubes whose 56 corners are defined by these 56 triangles. The Yang/Yin nature of each line/broken in the I Ching table signifies whether the faces of the cube are orientated towards the positive or negative directions of each perpendicular axis. These 168 bi-polar degrees of freedom manifest in the disdyakis triacontahedron as the 168 edges other than the 12 edges in its equatorial plane perpendicular to an axis joining two diametrically opposite vertices. Regarded as arrows that can point in two opposite directions, the edges of the polyhedron point in 168 directions above the equatorial plane and 168 directions below it. The 168 pairs of oppositely directed edges are the counterpart of the 168 automorphisms and the 168 anti-automorphisms of the Klein quartic and the 168 lines and 168 broken lines in the 56 off-diagonal hexagrams of the I Ching table.

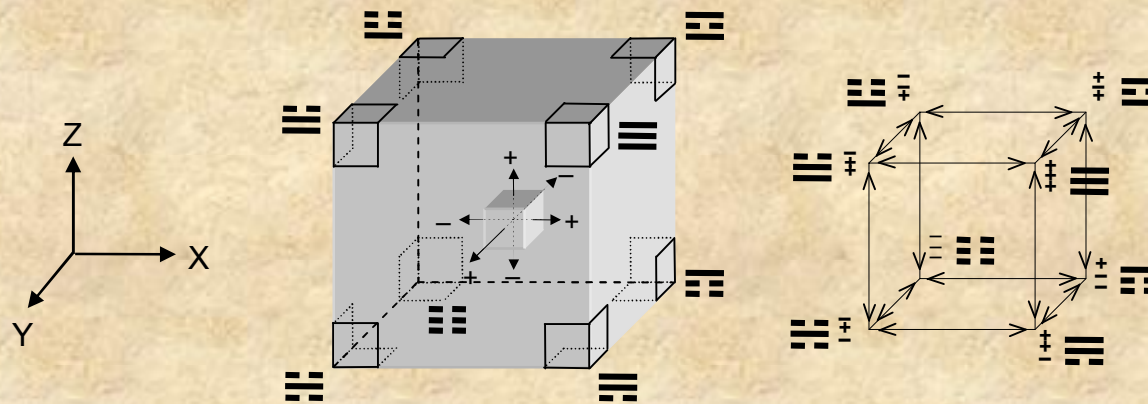
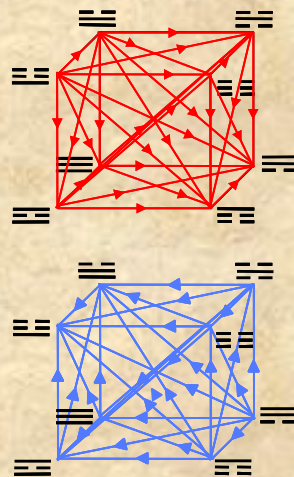
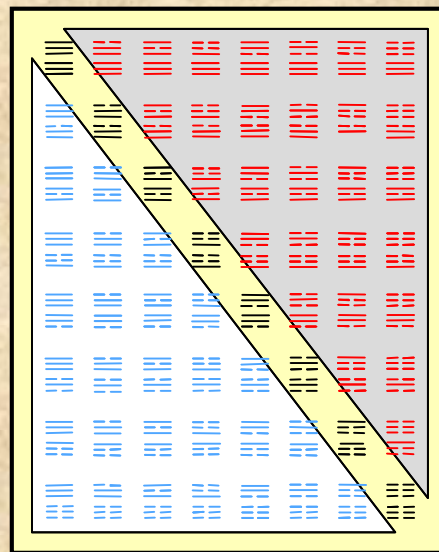


Figure 10

The eight trigrams define the eight corners of a cube because their three yin/yang lines correspond to the three orthogonal faces (positive or negative) that intersect at these corners.



8 diagonal hexagrams → 8 corners
 28 upper, off-diagonal hexagrams → 28 directed lines →
 28 lower, off-diagonal hexagrams → 28 oppositely directed lines →

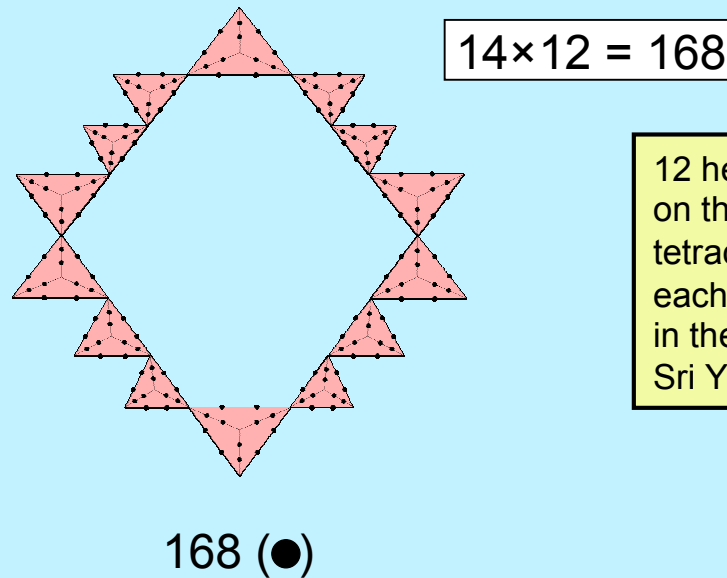
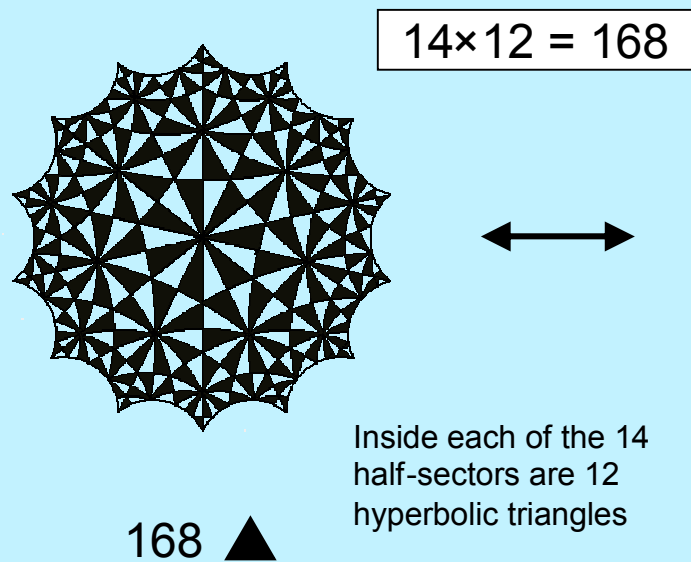
The ordered pairing of trigrams into hexagrams signifies the joining of the corners of a cube by arrows. The diagonal hexagrams correspond to the corners themselves.

That the full octahedral group with 48 elements is relevant to the understanding of the formation of the disdyakis triacontahedron from the '48 beams of light' is indicated by the fact that they emanate from the centres of the 48 faces of the disdyakis dodecahedron, the Catalan solid that displays the symmetries of the octahedral group. Furthermore, the 168 edges of the disdyakis triacontahedron above and below its equator correspond to the 168 Yang/Yin lines in the 28 hexagrams on either side of the diagonal of the I Ching table. The eight trigrams define the corners of a cube, each line or broken line denoting a 'positive' or 'negative' face. Their pairing into the 28 hexagrams defines the 28 straight lines joining pairs of its corners. The ordering of their pairing signifies the direction of the joining of pairs of corners by arrows. The eight diagonal hexagrams with 48 lines/broken lines denote the eight corners. Their 12 Yang lines denote the 12 positive faces and the 12 Yin lines denote the 12 negative faces needed to define the 8 corners.

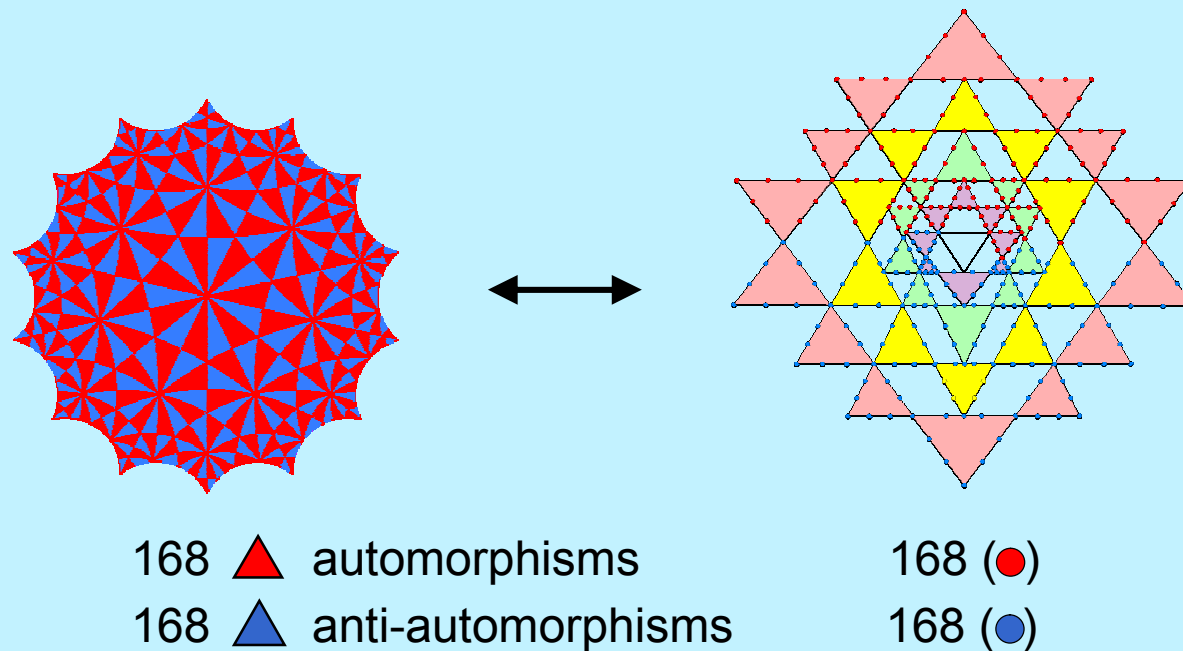
Figure 11

In its $\{7,3\}$ tiling, the Klein configuration (KC) is divided into seven identical sectors. A sector is composed of two half-sectors, each with 12 hyperbolic triangles, so that KC is divided into 14 sectors with 168 triangles. Compare this with the 14 triangles in the fourth layer of the Sri Yantra. When each triangle is constructed from three tetractyses, there are 12 hexagonal yods on their six edges. This means that $(14 \times 12 = 168)$ hexagonal yods line on the $(14 \times 6 = 84)$ edges of the $(14 \times 3 = 42)$ tetractyses comprising the 14 triangles. KC is isomorphic in this sense to the last layer of triangles of the Sri Yantra. To establish such isomorphism, however, we need to consider only each triangle as a tetractys, for 168 yods symbolizing the 168 automorphisms line the 63 edges of the 21 triangles in one half of the Sri Yantra and 168 yods symbolizing the 168 anti-automorphisms line the 63 edges of the 21 triangles in its other half. Unlike, however, in the case of the disdyakis triacontahedron, one half of the Sri Yantra is *not* the exact mirror image of its other half.

Figure 11



12 hexagonal yods lie on the 6 edges of the 3 tetractyses making up each of the 14 triangles in the 4th layer of the Sri Yantra.

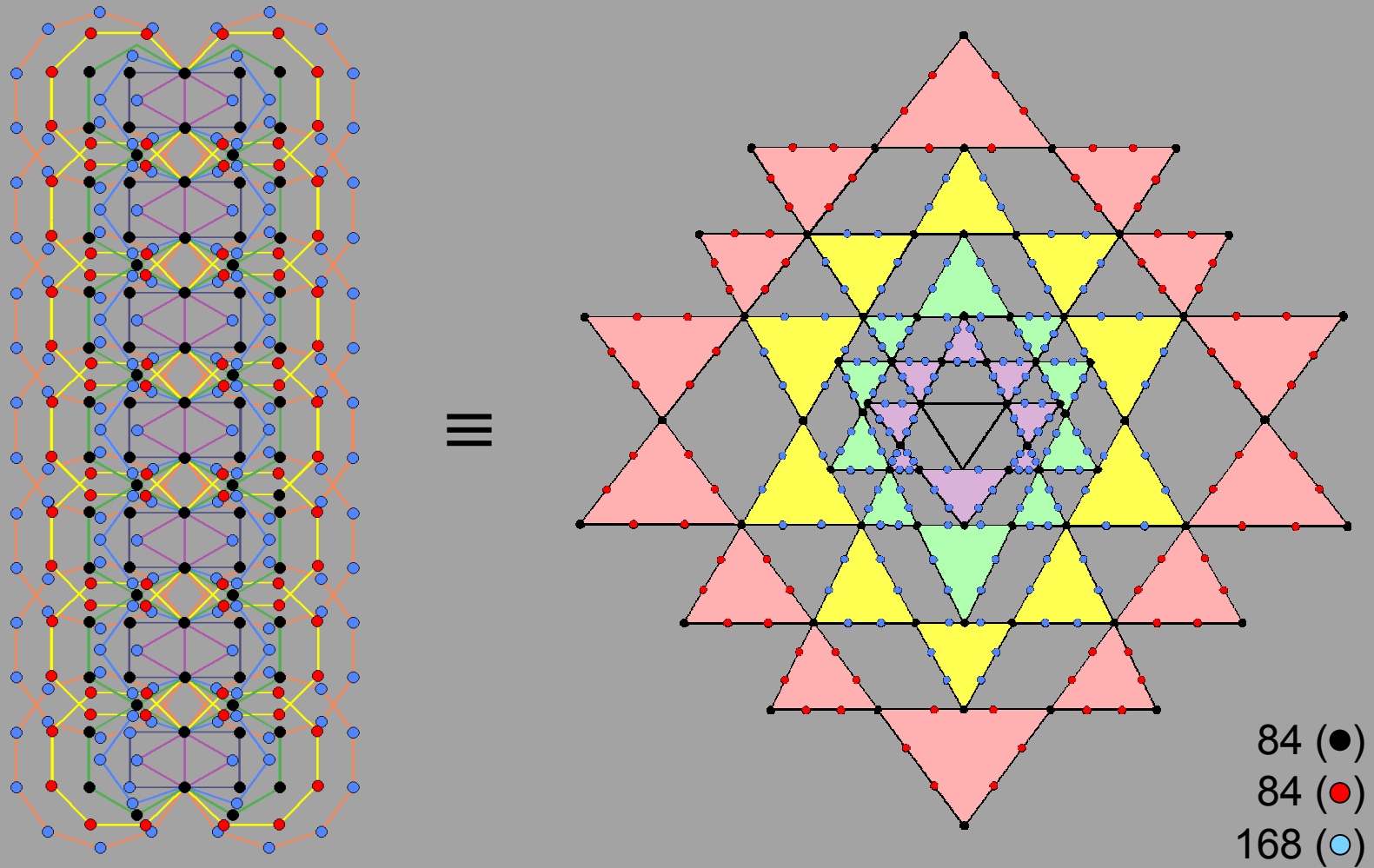


The isomorphism between the Klein configuration and the Sri Yantra. The higher order transformation of the 4th layer of triangles is equivalent to the lower-order transformation of *all* 4 layers.

Figure 12

It was shown in Part 2 (pp. 21, 22) that the Sri Yantra and the lowest seven overlapping Trees of Life are equivalent. Here is displayed the isomorphism between the Sri Yantra and the inner form of seven Trees of Life – or rather, the first six polygons enfolded in each tree, as these two sets of polygons constitute a Tree of Life pattern in themselves. There are 168 corners associated with the 42 polygons enfolded in seven overlapping Trees of Life on each side of its central pillar. They correspond to the 168 yods lying on edges of the 21 triangles in each half of the Sri Yantra. The 42 (black) vertices associated with each set of 21 triangles correspond to the 42 (black) corners associated with the squares and hexagons in each set of six polygons. The 42 (red) hexagonal yods on edges of each set of seven triangles in the fourth layer correspond to the 42 (red) corners of the octagons outside their shared edges in each set of polygons. The 84 (blue) hexagonal yods on the 42 edges of the 14 triangles in each half of the first three layers correspond to the 84 (blue) corners of the triangle, pentagon & decagon in each set of polygons outside their root edges.

Figure 12

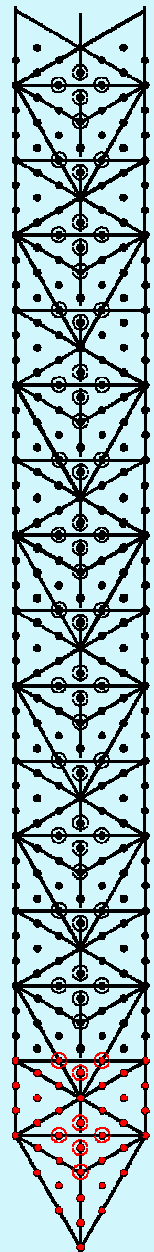


The Sri Yantra is isomorphic to the inner form of 7 Trees of Life.

Figure 13

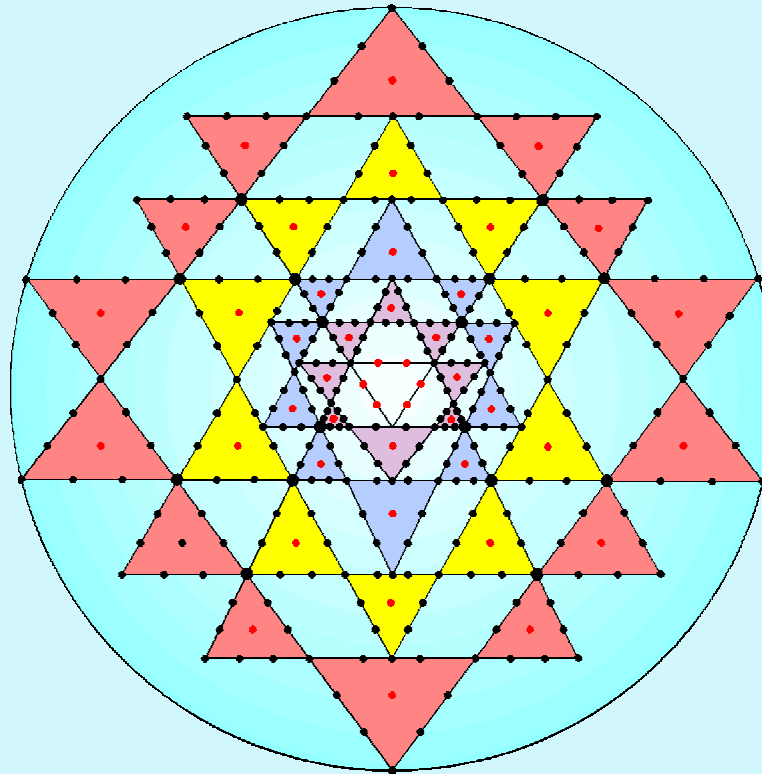
Of the 384 yods in the tetractyses forming the lowest seven Trees of Life up to the top of the seventh tree, 48 (red) yods are in the lowest tree up to the level of Chesed, the first Sephirah of Construction. The counterpart of this in the Sri Yantra is that there are 48 (red) yods which are either centres of triangles or hexagonal yods on edges of the central triangle. The 336 (black) yods on the edges of the 42 triangles correspond to the 336 (black) yods in the lowest seven trees above Chesed of the lowest tree.

Figure 13



(a circle denotes a yod behind another one)

≡



The 48 red yods that are either centres of tetractyses or hexagonal yods in the innermost triangle correspond to the 48 red yods up to Chesed of the lowest Tree of Life.

48 (●)

336 (●)

Total = 384

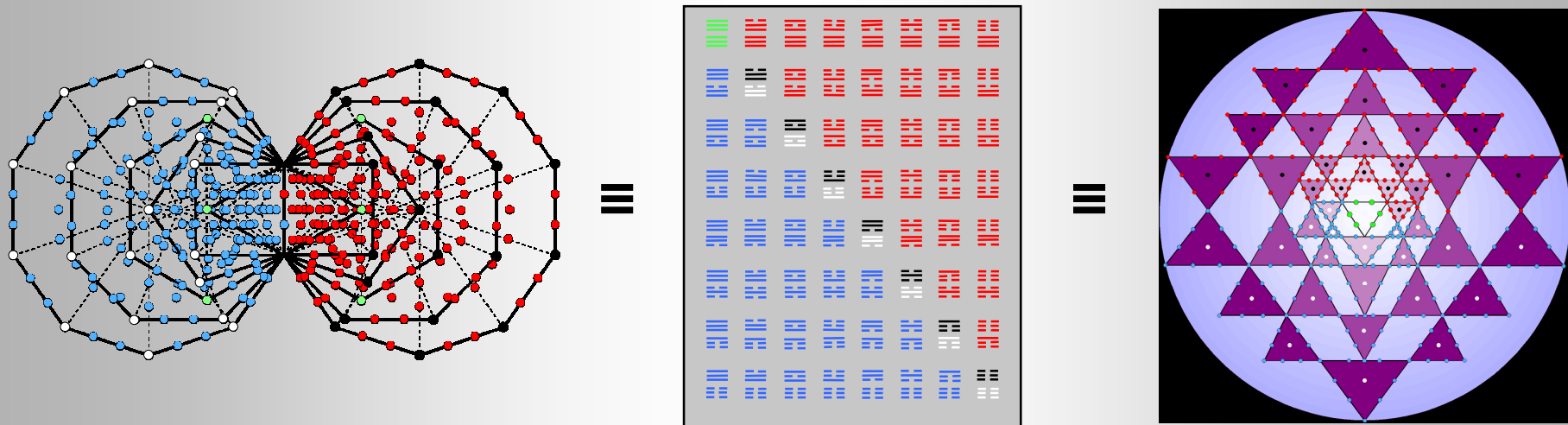
The Sri Yantra is isomorphic to the outer form of 7 Trees of Life.

Figure 14

This displays the corresponding features of the inner Tree of Life, the I Ching table and the Sri Yantra as isomorphic representations of holistic systems.

1. The six green yods either at the external corners of the triangles or at the topmost and lowest corners of the hexagons coincide with Sephiroth. They correspond to the six lines of the pair of Heaven trigrams in the top left-hand corner of the table and to the six green hexagonal yods on the edges of the central triangle of the Sri Yantra.
2. The 21 black yods denoting corners of one set of the first six polygons that lie outside their shared edge correspond to the 21 black lines/broken lines in the upper trigrams of the seven remaining diagonal hexagrams and to the black centres of the 21 triangles in one half of the Sri Yantra. The 21 white yods denoting external corners of the other set of polygons correspond to the 21 white lines/broken lines in the lower trigrams of the seven hexagrams on the diagonal of the table and to the white centres of the 21 triangles in the other half of the Sri Yantra.
3. The 168 red yods in one set of six polygons that are not corners correspond to the 168 red lines/broken lines in the 28 off-diagonal hexagrams of half of the table and to the 168 red yods on the 63 edges of the 21 triangles in one half of the Sri Yantra. The 168 blue yods in the other set of six polygons that are not corners correspond to the 168 blue lines/broken lines in the 28 off-diagonal hexagrams of the other half of the table and to the 168 blue yods on the 63 edges of the 21 triangles in the other half of the Sri Yantra.

Figure 14



6 (●)
 168 (○) 21 (●) 168 (●)
 21 (○)
 Total = 384

6 —
 168 —& — 21 —& — 168 —& —
 21 —& —
 Total = 384

6 (●)
 168 (○) 21 (●) 168 (●)
 21 (○)
 Total = 384

**The isomorphism between the inner Tree
of Life, the I Ching & the Sri Yantra.**

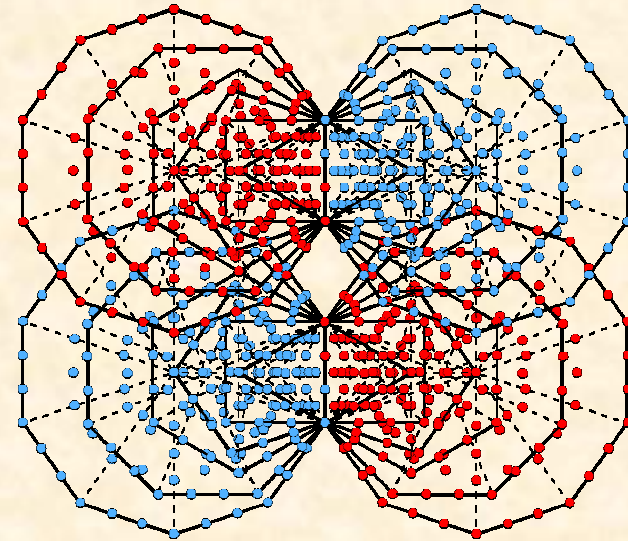
Figure 15

192 yods are associated with each set of the first six enfolded polygons of the inner Tree of Life. They are intrinsic to that set because none of them is shared with the six polygons enfolded in the next higher Tree of Life, which also have 192 such yods. The 192 yods associated with each set of six polygons correspond to the 192 lines/broken lines in each half of the I Ching table.

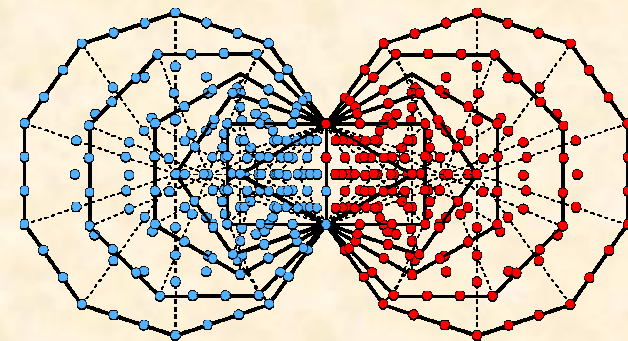
Figure 15

Of the 193 yods associated with each set of 6 enfolded polygons, one yod (the topmost corner of the hexagon) is shared with the polygons enfolded in the next higher Tree of Life because it coincides with the lowest corner of the hexagon in this set. Therefore, 192 yods are intrinsic to each set of polygons. Successive sets of (6+6) polygons require 384 yods. The number 384 therefore characterizes a holistic system (in this case the two sets of 6 enfolded polygons).

$$2 \times 384 =$$



$$384 =$$



192 (●)

192 (●)

384 yods are intrinsic to each inner Tree of Life.

Figure 16

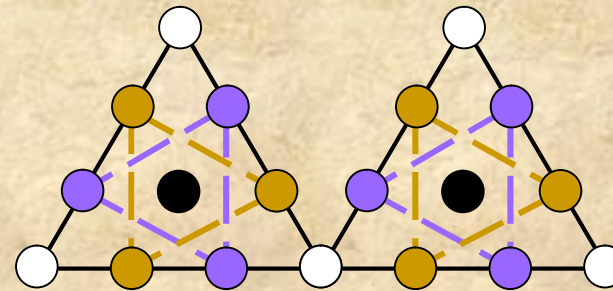
Considered as tetractyses, the 21 triangles in one half of the Sri Yantra have 21 black centres and 21 white pairs of vertices, i.e., 21 triplets of one white yod and two black yods. They also have 21 purple triplets of hexagonal yods and 21 brown triplets of hexagonal yods, each pair of triplets forming a Star of David. The 189 yods associated with the 21 tetractyses in each half of the Sri Yantra comprise three sets of 21 triplets of yods, i.e., 63 triplets.

The 21 centres of tetractyses in each half of the Sri Yantra comprise 12 centres of the tetractyses in the third and fourth layers and nine centres of tetractyses in the first and second layers. The significance of this will be discussed on pages 41 & 42.

21:

Half of Sri
Yantra

21 tetractyses have:



42 vertices + 21 centres

21 purple triplets

21 brown triplets

Figure 16

Half of Sri
Yantra

189 yods =

$$\begin{array}{ccccccc}
 & \bullet & \circ & \text{brown} & \text{purple} \\
 & 21 & 21 \times 2 & 21 \times 3 & 21 \times 3 \\
 & \underbrace{\hspace{1cm}} & & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} \\
 = & 63 & + & 63 & + & 63
 \end{array}$$

21 (●) $\begin{cases} \rightarrow 12 \text{ centres of 3rd \& 4th layers} \\ \rightarrow 9 \text{ centres of 1st \& 2nd layers} \end{cases}$

**The 189 yods in each half of the Sri
Yantra comprise 63 triplets of yods.**

Figure 17

Selection of successive notes of the Pythagorean scale as the starting note (tonic) generates seven possible musical scales. These sequences of intervals (T) and leimmas (L), the counterpart of the modern semitone, define the musical modes of the Roman Catholic Church. The four “authentic modes” are the Dorian mode (D scale), the Phrygian mode (E scale), the Lydian mode (F scale) & the Mixolydian mode (G scale). They have four “hypo” modes separated by the interval of a perfect fourth from their authentic counterparts – the Hypodorian (A scale), the Hypophrygian (B scale), the Hypolydian (C scale) & the Hypomixolydian (D scale). In terms of notes, the lattermost is identical to the Dorian, differing in the choice of the ‘dominant’ and ‘reciting note.’

The table displays the ‘tone ratios’ of the notes in the seven musical scales. These are the frequencies or pitches of the notes relative to that of the tonic. The Pythagorean musical scale is the C scale (Hypolydian mode). Tone ratios of notes belonging to this scale are written in black; non-Pythagorean tone ratios are written in red. The seven scales have 26 Pythagorean notes between the tonic and the octave, showing how YAHWEH, the Godname of Chokmah with number value 26, prescribes the seven musical scales. Their notes have 14 different tone ratios, i.e., there are 12 distinct types of notes between the tonic and octave. The sequence of 14 notes is split into the first seven, primary notes and their seven “complements” (notes whose tone ratios are the interval between its corresponding primary note and the octave). They comprise the eight notes of the Pythagorean scale and six non-Pythagorean notes. The 12 types of notes between the tonic and octave comprise six Pythagorean notes and six non-Pythagorean notes.

Sequences of intervals in the 7 musical scales

$$T = 9/8, \quad L = 256/243$$

- | | | | | | | | | | |
|----|---|---|---|---|---|---|---|---|--------------------------------|
| 7. | | T | T | L | T | T | L | | Hypolydian (mode 6, C scale) |
| 6. | | L | T | T | L | T | T | | Hypophrygian (mode 4, B scale) |
| 5. | | T | L | T | T | L | T | | Hypodorian (mode 2, A scale) |
| 4. | | T | T | L | T | T | L | | Mixolydian (mode 7, G scale) |
| 3. | | T | T | T | L | T | T | L | Lydian (mode 5, F scale) |
| 2. | L | T | T | T | L | T | T | | Phrygian (mode 3, E scale) |
| 1. | T | L | T | T | T | L | T | | Dorian (mode 1, D scale) |

Figure 17

Musical scale	Tone ratio							
D scale	1	9/8	32/27	4/3	3/2	27/16	16/9	2
E scale	1	256/243	32/27	4/3	3/2	128/81	16/9	2
F scale	1	9/8	81/64	729/512	3/2	27/16	243/128	2
G scale	1	9/8	81/64	4/3	3/2	27/16	16/9	2
A scale	1	9/8	32/27	4/3	3/2	128/81	16/9	2
B scale	1	256/243	32/27	4/3	1024/729	128/81	16/9	2
C scale	1	9/8	81/64	4/3	3/2	27/16	243/128	2

Notes

C		D		E		F
1	256/243	9/8	32/27	81/64	4/3	1024/729

Complements of notes

G		A		B		C'
3/2	128/81	27/16	16/9	243/128	2	

Figure 18

There are 28 intervals between the eight notes of a musical scale. Excluding the octave, there are ($7 \times 27 = 189$) intervals between the notes of the seven varieties of scales. They all have the values of the tone ratios of the 12 types of notes between the tonic and the octave found in the seven scales. The second table lists the number of intervals of each type. The number in the brackets next to each one is the number of times the *note* with that tone ratio appears in the scales. Some of the notes and intervals can be paired with their complements. Their numbers are listed in the third table, together with the numbers of intervals without complements. There are 21 notes and 42 intervals that have complements. They form two sets of 63 intervals, one set paired with its complementary set, creating 63 pairs. There are 63 unpaired intervals. The 189 intervals therefore divide into three sets of 63 intervals, that is, 63 triplets, each triplet comprising a pair of notes or intervals and an unpaired interval. They comprise 21 notes made of the second, third & fourth notes of the seven scales and 168 intervals made up of the 21 complements of these notes, 42 intervals and their 42 complements and 63 unpaired intervals.

Types of intervals in
the 7 musical scales

1	2	$1 \times 2 = 2$
256/243	243/128	$256/243 \times 243/128 = 2$
9/8	16/9	$9/8 \times 16/9 = 2$
32/27	27/16	$32/27 \times 27/16 = 2$
81/64	128/81	$81/64 \times 128/81 = 2$
4/3	3/2	$4/3 \times 3/2 = 2$
1024/729	729/512	$1024/729 \times 729/512 = 2$

Interval	Number	Interval	Number
256/243	14 (2)	243/128	4 (2)
9/8	35 (5)	16/9	10 (5)
32/27	24 (4)	27/16	12 (4)
81/64	18 (3)	128/81	9 (3)
4/3	30 (6)	3/2	24 (6)
1024/729	4 (1)	729/512	5 (1)

Total = 125 (21)

Total = 64 (21)

Figure 18

Number of paired notes+intervals

Interval	Number	Interval	Number
256/243	2+2	243/128	2+2
9/8	5+5	16/9	5+5
32/27	4+8	27/16	4+8
81/64	3+6	128/81	3+6
4/3	6+18	3/2	6+18
1024/729	1+3	729/512	1+3

Total = 21+42=63

Total = 21+42=63

Number of unpaired intervals

Interval	Number	Interval	Number
256/243	10	243/128	0
9/8	25	16/9	0
32/27	12	27/16	0
81/64	9	128/81	0
4/3	6	3/2	0
1024/729	0	729/512	1

Total = 62

Total = 1

21

168

63

37

Figure 19

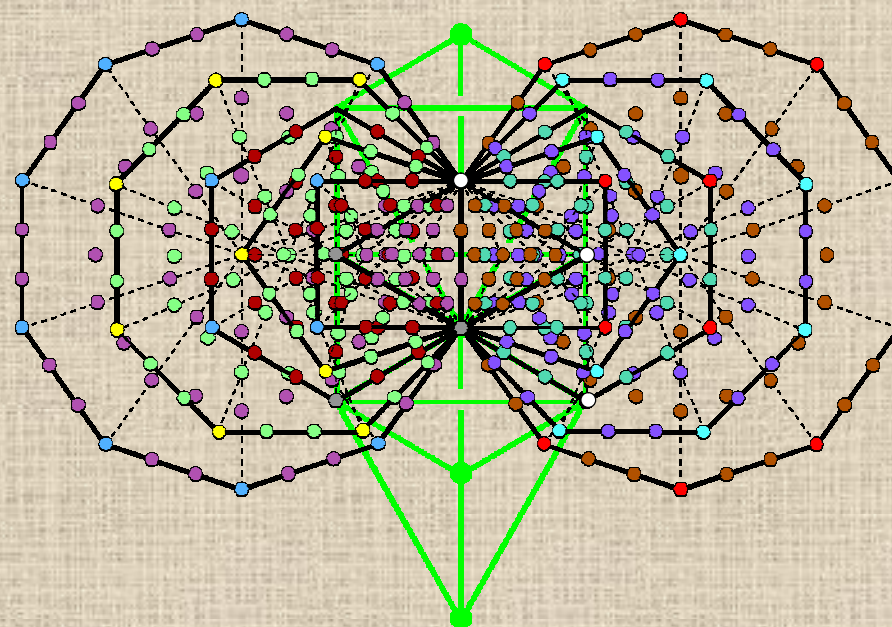
Between the notes of the seven scales, there are 189 rising intervals below the octave. They also have 189 falling intervals (this is the interval between the lower note and the higher note. It is the reciprocal of the corresponding rising interval). So there are 378 rising and falling intervals other than the octave. As the seven scales constitute a holistic system, their patterns of intervals are encoded in the inner form of the Tree of Life. The two sets of the first six enfolded polygons have 384 yods that are intrinsic to them. Six of them coincide with Sephiroth or Daath, leaving 378 yods that do not belong to the outer form of the Tree of Life. The 189 yods in one set of polygons denote rising intervals. Their mirror images in the other set denote falling intervals.

As evidence that this is not a coincidence, the yod populations of pairs of polygons correlate exactly with the numbers of different classes of intervals. The 21 corners of polygons that are outside their shared edge and which do not coincide with Sephiroth of the Tree of Life symbolize the 21 notes made up of the second, third & fourth notes of each scale. Their mirror images in the other set of polygons denote the complements of these notes (not necessarily in the same scale, of course). The 21 corners comprise 12 corners of the square, hexagon & decagon, and nine corners of the pentagon & octagon. The 42 yods in the square & hexagon symbolize the 42 intervals that are not notes. The 63 yods in the triangle & decagon denote the 63 complements of the notes and intervals. The 63 yods in the pentagon & octagon denote the 63 intervals lacking a complement.

This is the Tree of Life basis of the seven types of musical scales as a holistic system.

Figure 19

384 =



shared with outer Tree of Life 3 
 corners 12  + 9 
 square+hexagon 42 
 triangle+decagon 63 
 pentagon+octagon 63 
 $192 = 3 + (21+42) + 63 + 63$

shared with outer Tree of Life 3 
 corners 12  + 9 
 square+hexagon 42 
 triangle+decagon 63 
 pentagon+octagon 63 
 $192 = 3 + (21+42) + 63 + 63$

Tree of Life representation of the three sets of 63 rising & falling intervals between the notes of the seven musical scales.

Figure 20

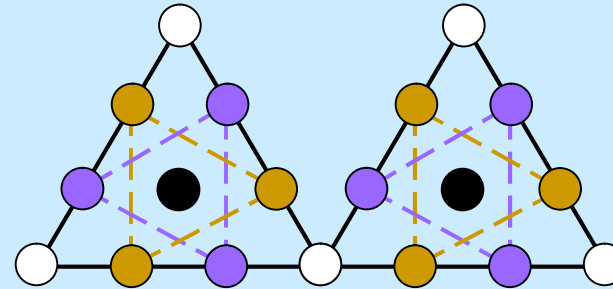
This correlation between the intervallic structure of the seven musical scales and the sacred geometry of the inner Tree of Life extends to the tetractys-transformed Sri Yantra as well. The 21 notes made up of the first three notes above the tonic in each scale determine the seven scales collectively because the 21 remaining notes below the octave are their complements and therefore are defined by them. They are symbolized as rising intervals by the yods at the centres of the 21 triangles/tetractyses in one half of the Sri Yantra, the centres of the 21 tetractyses in the other half denoting their corresponding 21 falling intervals (note: *not* their complements) . The tips of the 21 triangles denote the 21 complements of the notes. The tips of the 21 triangles in the other half of the Sri Yantra denote their falling interval counterparts. The 42 intervals and their complements are symbolized by the 42 hexagonal yods on each of the two edges of each triangle forming its tip. Their corresponding falling intervals are symbolized by the counterparts of these yods in the other half of the Sri Yantra. The 63 intervals without complements are symbolized by the 63 yods on the bases of the 21 tetractyses in one half, their falling interval counterparts being denoted by the 63 yods on the bases of the 21 tetractyses in the other half of the Sri Yantra.

The centres of the 21 tetractyses in either half of the Sri Yantra comprise the centres of the 12 tetractyses in the third and fourth layers and the centres of the nine tetractyses in the first and second layers. This 12:9 differentiation is the counterpart of the distinction between the 12 notes above the tonic made up of the second, third & fourth notes in the four authentic modes and the nine notes above the tonic made up of the second, third & fourth notes in the three plagal modes (the fourth, plagal mode is not a distinct octave species, the Hypomixolydian mode having the same sequence of notes as the Dorian). The uppermost first and second layers of the Sri Yantra therefore correlate with the three plagal modes that have different sets of notes, whilst its third and fourth layers correlate with the four authentic modes.

C scale: C D E F G A B C'

$\underbrace{\quad\quad\quad}_{3 \text{ notes}} \quad \underbrace{\quad\quad\quad}_{3 \text{ complements}}$

7 scales: $(7 \times 3 = 21)$ notes $(7 \times 3 = 21)$ complements



21 notes $\rightarrow$ 21 centres (●) of 21 tetractyses

21 complements of notes $\rightarrow$ 21 tips (○) of tetractyses

Figure 20

42 intervals + 42 complements $\rightarrow$ (42+42) hexagonal yods on sides of 21 tetractyses

63 unpaired intervals $\rightarrow$ 63 yods on bases of 21 tetractyses

4 Authentic modes have $(4 \times 3 = 12)$ notes

3 Plagal modes have $(3 \times 3 = 9)$ notes

21 (●) $\begin{cases} \rightarrow 12 \text{ centres of triangles in 3rd \& 4th layers} \\ \rightarrow 9 \text{ centres of triangles in 1st \& 2nd layers} \end{cases}$

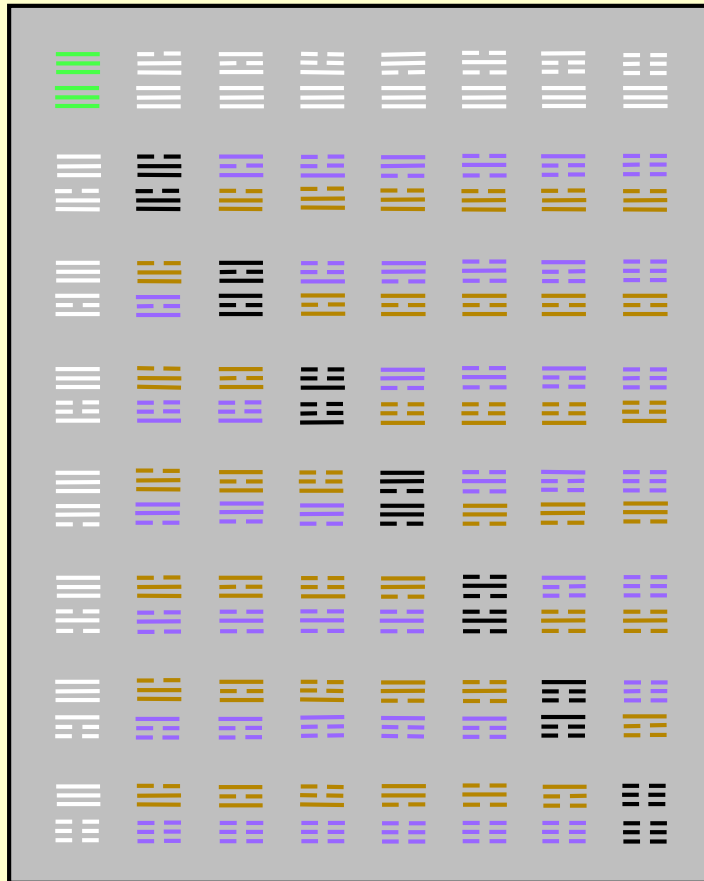
Therefore,

1st & 2nd layers of triangles $\rightarrow$ Plagal musical modes

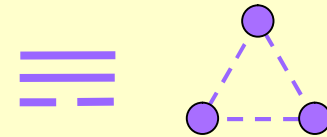
3rd & 4th layers of triangles $\rightarrow$ Authentic musical modes

Figure 21

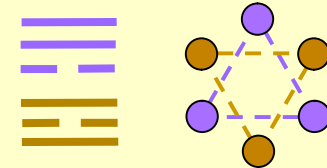
Arranged in a Star of David in each tetractys of the Sri Yantra, the two triads of hexagonal yods correspond to the two trigrams in each hexagram of the I Ching table. The six green, unbroken lines of the two Heaven trigrams at the upper left-hand corner of the table correspond to the six hexagonal yods arranged as two triangles in a Star of David in the central triangle (see Fig. 14). The (21+21) black lines/broken lines of the seven remaining hexagrams in the diagonal correspond to the centres of the two sets of 21 tetractyses. The 84 white lines/broken lines of the 14 hexagrams containing the Heaven trigram correspond to the 84 corners of the 42 tetractyses. Each category 'marks out', so to speak, the boundary of their respective systems. The 21 purple trigrams on one side of the diagonal correspond to the 21 triplets of purple, hexagonal yods in one half of the Sri Yantra. The 21 purple trigrams on the other side of the diagonal correspond to the 21 triplets of purple, hexagonal yods in the other half of the Sri Yantra. Similarly for the brown, hexagonal yods. The I Ching table is a non-geometrical version of the Sri Yantra.



Trigrams → triplets of hexagonal yods



Hexagrams → pairs of triplets of hexagonal yods



Correspondence of the Yin/Yang lines and the yods in the Sri Yantra

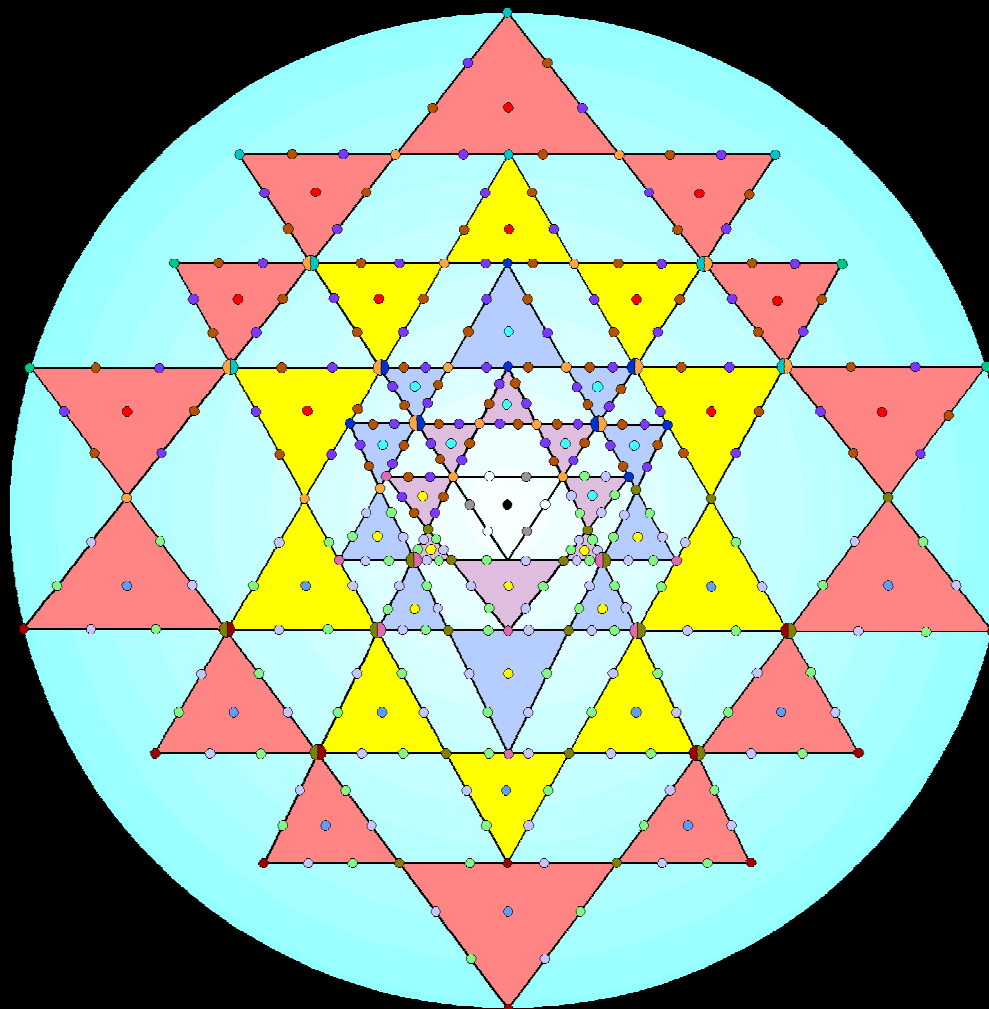
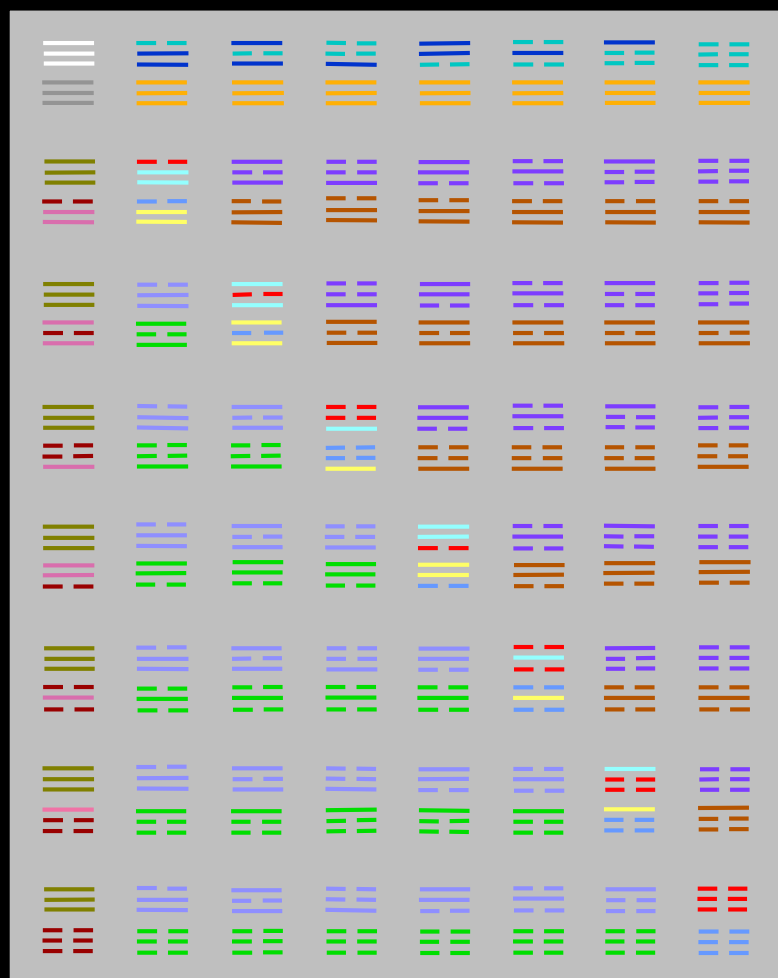
Figure 21

42	— & - -	→	42 centres (●) of tetractyses
126	— & - -	→	126 (●) hexagonal yods in triplets
126	— & - -	→	126 (●) hexagonal yods in triplets
84	— & - -	→	84 (○) corners of tetractyses

Figure 22

Displayed here is the detailed correspondence between different types of trigrams in the I Ching table and the triplets of hexagonal yods in the tetractys-transformed triangles of the Sri Yantra. Lines/broken lines and their counterpart yods have the same colour.

Figure 22

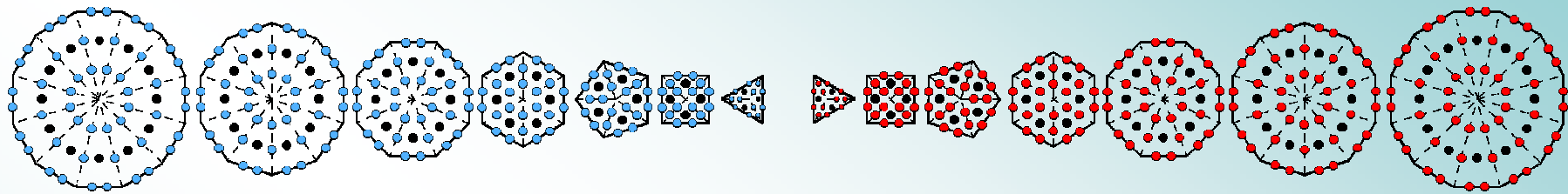


Correspondence between the I Ching table and the Sri Yantra.

Figure 23

The seven separate, regular polygons constituting the inner form of the Tree of Life comprise 295 yods when their 48 sectors are turned into tetractyses. In other words, starting with the seven polygons divided into their sectors, $(295 - 48 - 7 = 240)$ more yods are needed to transform their sectors into tetractyses. There are 48 centres of tetractyses. This leaves $(240 - 48 = 192)$ hexagonal yods arranged at the corners of six-sided polygons (i.e., at the tips of Stars of David). 192 yods are similarly arranged in the other set of polygons. In terms of the formal equivalence between the Tree of Life and the tetractys, the six yods at the corners of a six-sided polygon (it does not need to be a hexagon, of course) symbolize the six Sephiroth of Construction above Malkuth. These two sets of 192 hexagonal yods arranged at the corners of triangles are the Tree of Life counterpart of the 192 lines/broken lines grouped in trigrams in each half of the I Ching table. The 24 red lines/broken lines in the upper trigrams of the eight diagonal hexagrams correspond to the 24 red, hexagonal yods in one of the hexagons. The 24 blue lines/broken lines in the lower trigrams of these eight hexagrams correspond to the 24 blue, hexagonal yods in the hexagon belonging to the other set of polygons. As $24 = 4 \times 3 \times 2$ and as each sector of a polygon contributes four of these hexagonal yods, the counterpart of the two sets of four diagonal trigrams are the four hexagonal yods per sector multiplied, firstly, twice (a hexagon is two equilateral triangles rotated by 60°) and then thrice (pairs of hexagonal yods are arranged at each corner of a triangle). The trigrams are expressing the three-fold, rotational symmetry of an equilateral triangle – a sector of a hexagon. The eight-fold nature of trigrams corresponds to the fact that there are eight hexagonal yods on the edges of a pair of adjacent sectors in the hexagon. These are basic because the rotation of each one by 120° about the centre of the hexagon generates all its remaining yods.

Figure 23

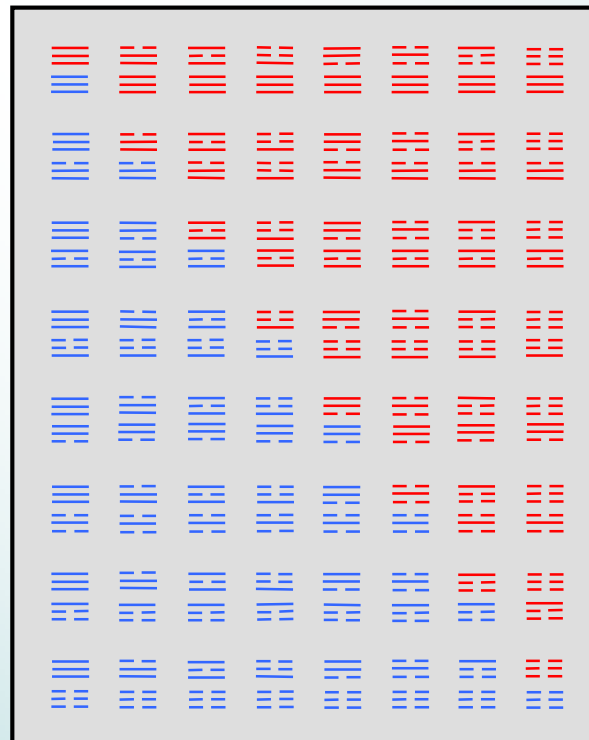


192 (●)

192 (●)

192 — & --

192 — & --

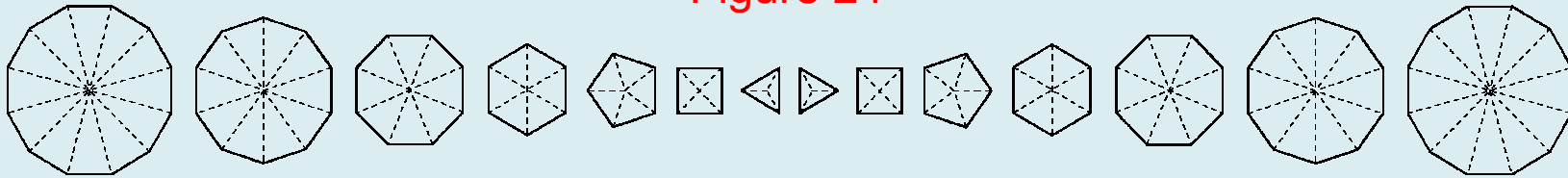


Correspondence between the I Ching table and the (7+7) separate polygons.

Figure 24

Divided into their sectors, the seven separate polygons have 48 corners, 96 edges and 48 triangles surrounding their centres, i.e., 192 geometrical elements. They correspond to the 192 lines/broken lines in one diagonal half of the I Ching table. The 192 geometrical elements in the other set of seven polygons of the inner Tree of Life correspond to the 192 lines/broken lines in the other half of the table. The 12 lines and 12 broken lines in the upper trigrams of the eight diagonal hexagrams correspond to the 12 edges and 12 corners/triangles of the sectors of the hexagon. Similarly, the 12 lines and 12 broken lines in the lower trigrams correspond to the 12 edges and 12 corners/triangles of the sectors of the hexagon in the other set of polygons. The 384 lines and broken lines in the table express the number of geometrical elements needed to construct the two sets of seven polygons making up the inner form of the Tree of Life. They symbolize the independent 'bits of information' needed to build a holistic system.

Figure 24



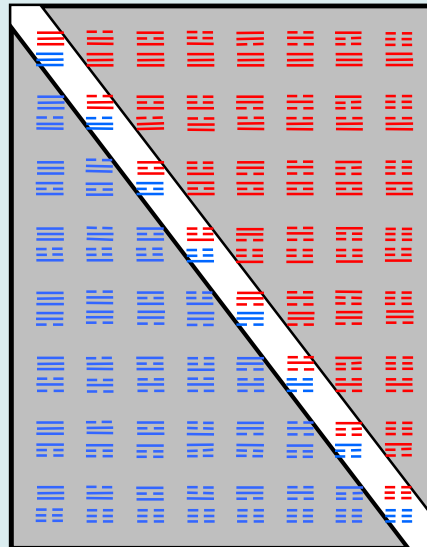
Number of corners = 48
 Number of edges = 96
 Number of triangles = 48
 Total = 192

Number of corners = 48
 Number of edges = 96
 Number of triangles = 48
 Total = 192

96 —
 96 - -

 Total = 192

12 lines & 12 broken lines of
 lower 8 trigrams in diagonal
 → 12 edges & 12
 corners/triangles of hexagon



96 —
 96 - -

 Total = 192

12 lines & 12 broken lines of
 upper 8 trigrams in diagonal
 → 12 edges & 12
 corners/triangles of hexagon

**The 384 lines & broken lines of the I Ching table
 denote the 384 geometrical elements composing
 the 14 regular polygons of the inner Tree of Life.**

Article 40

The Unification of All Sacred Geometries & Its Implication for Particle Physics

Part 4

by

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Figure 1

The graph shows the positions of the 62 vertices of the disdyakis triacontahedron projected onto the XY plane with the Z axis running through two diametrically opposite C vertices. 60 vertices are corners of polygons in 15 layers. Solid lines of the same colour connect vertices at the same height above the XY plane. Dashed lines of the same colour connect vertices at the same depth below the XY plane. Vertices at the same height form either triangles or 6-sided polygons, with six A vertices in the XY plane located at the corners of a hexagon. Suppose that the 60 sectors of these polygons are each constructed from three triangles. The latter share one corner at the centre of each sector. 15 of the corners of the 180 new triangles are the centres of the polygons, so that they have $(15+60=75)$ corners inside the polyhedron and 60 corners that are polyhedral vertices. Including the C vertices at its apex and nadir, there are $(75+60+2=137)$ vertices. This number is one of the most important numbers in physics because it defines approximately as its reciprocal the fine structure constant $e^2/\hbar c$ that measures the strength of the coupling of electrically charged particles to the electromagnetic field. Appropriately, the number determining the electron structure of atoms is embodied in the sheets of vertices of the disdyakis triacontahedron as the number of vertices needed to construct its faces and interior from triangles.

If the triangles are now turned into tetractyses, the transformation requires 840 new yods. This is the number of turns in each helical whorl of the heterotic superstring as it spirals $2\frac{1}{2}$ times around its spin axis, making half a complete circuit. The disdyakis triacontahedron embodies this structural parameter of the heterotic superstring.

Number of yods other than 15
centres & 60 polyhedral vertices

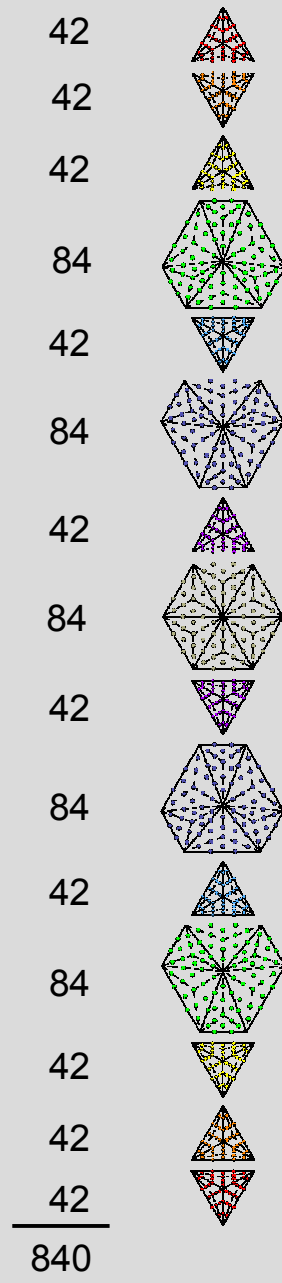
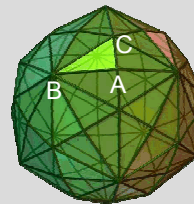


Figure 1

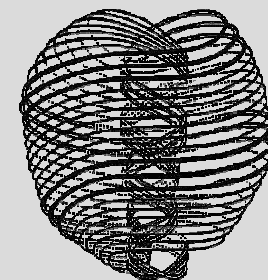
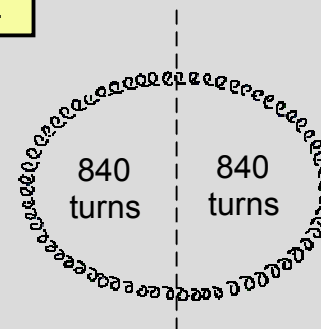
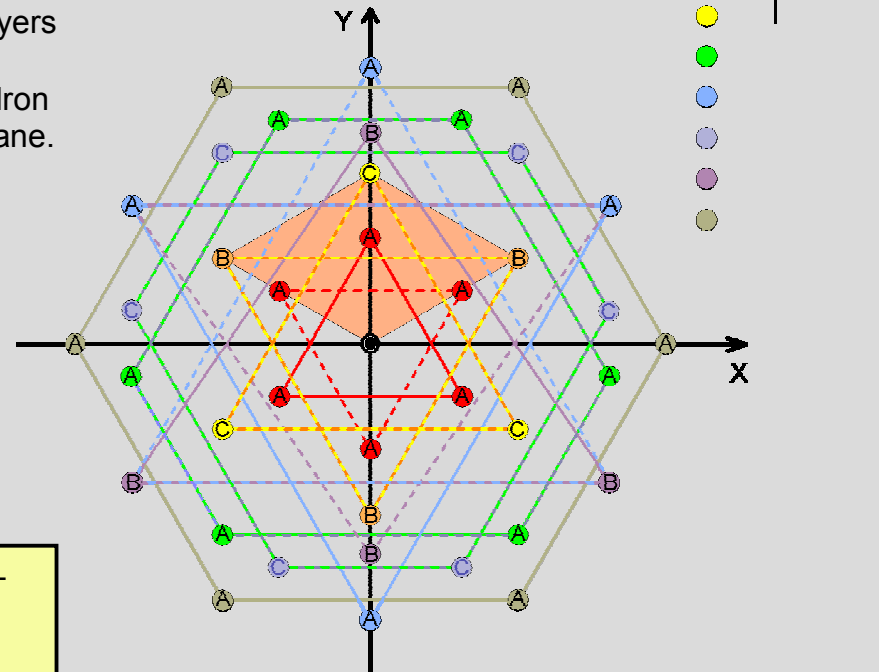
Projection of the 15 layers
of vertices of the
disdyakis triacontahedron
onto the central XY plane.



Disdyakis triacontahedron

Number of vertices = $62 + 15 + 60 = 137$. This number defines
as its reciprocal the fine
structure constant $e^2/\hbar c \approx 1/137$.

Each closed curve in the
heterotic superstring is a helix
with 1680 circular turns. It makes
840 turns in $2\frac{1}{2}$ revolutions
about the vertical axis.



$E_8 \times E_8'$ heterotic superstring

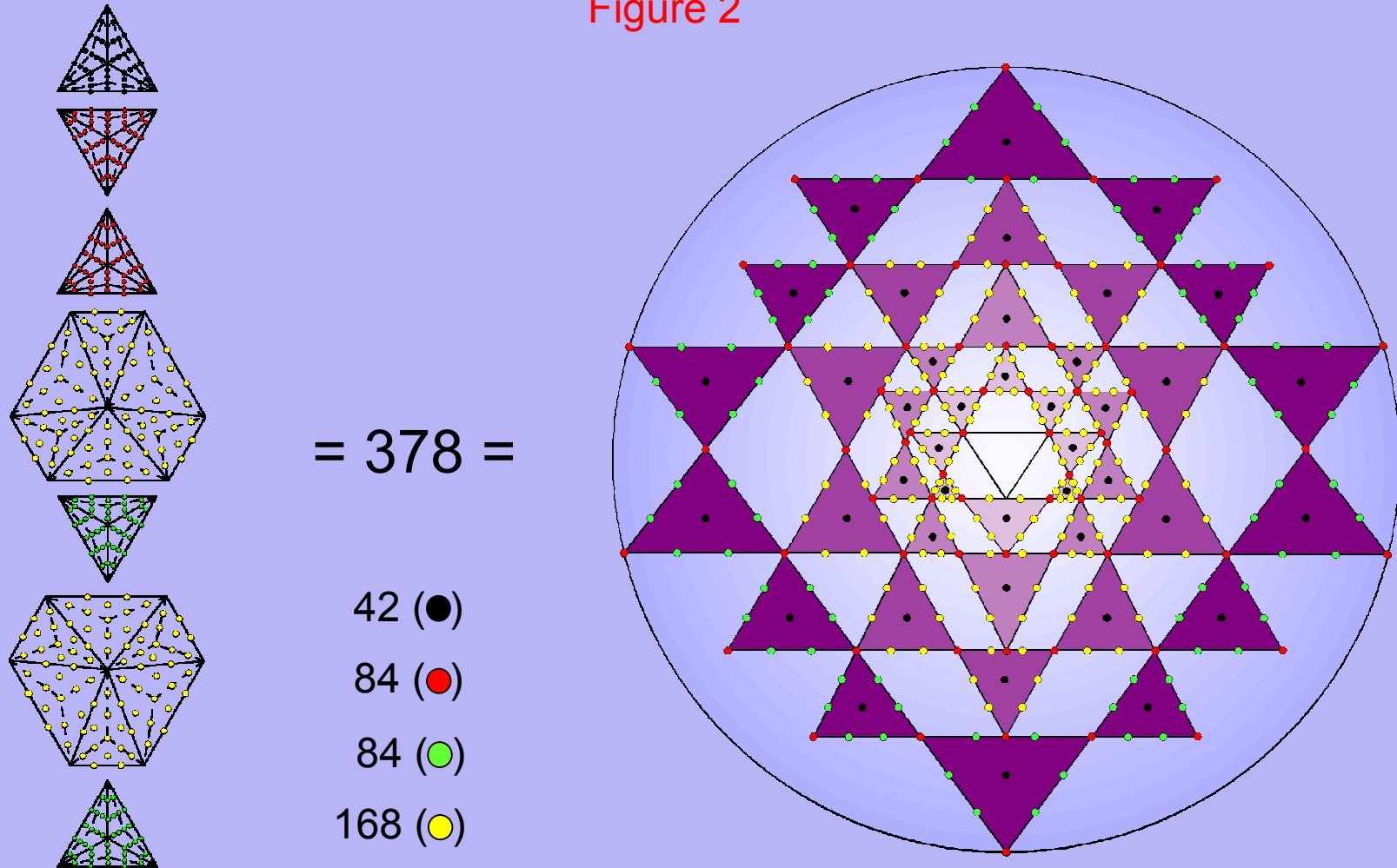
The 15 layers of vertices embody the superstring structural parameter
840 and the number 137 defining the fine-structure constant.

Figure 2

Suppose that the seven polygons above the XY plane formed by vertices of the disdyakis triacontahedron are divided into their sectors and the latter then constructed from three tetractyses. There are 378 yods other than polyhedral vertices surrounding the centres of the seven polygons. This is the number of yods needed to turn the 42 triangles of the 3-dimensional Sri Yantra into tetractyses. Their 42 black centres correspond to the 42 black yods surrounding the centre of the uppermost triangle that are not vertices of the polyhedron. The 84 red yods at corners of tetractyses correspond to the 84 similar red yods in the next two triangles. The 84 green, hexagonal yods on the 42 edges of the 14 tetractyses in the fourth layer of the Sri Yantra correspond to the 84 green yods in the other two triangles. The 168 yellow, hexagonal yods on the 84 edges of the 28 triangles in the first three layers correspond to the 168 yellow yods in the pair of six-sided polygons.

In either case, the 378 yods symbolize the 378 rising and falling intervals other than the octave between the notes of the seven musical scales.

Figure 2

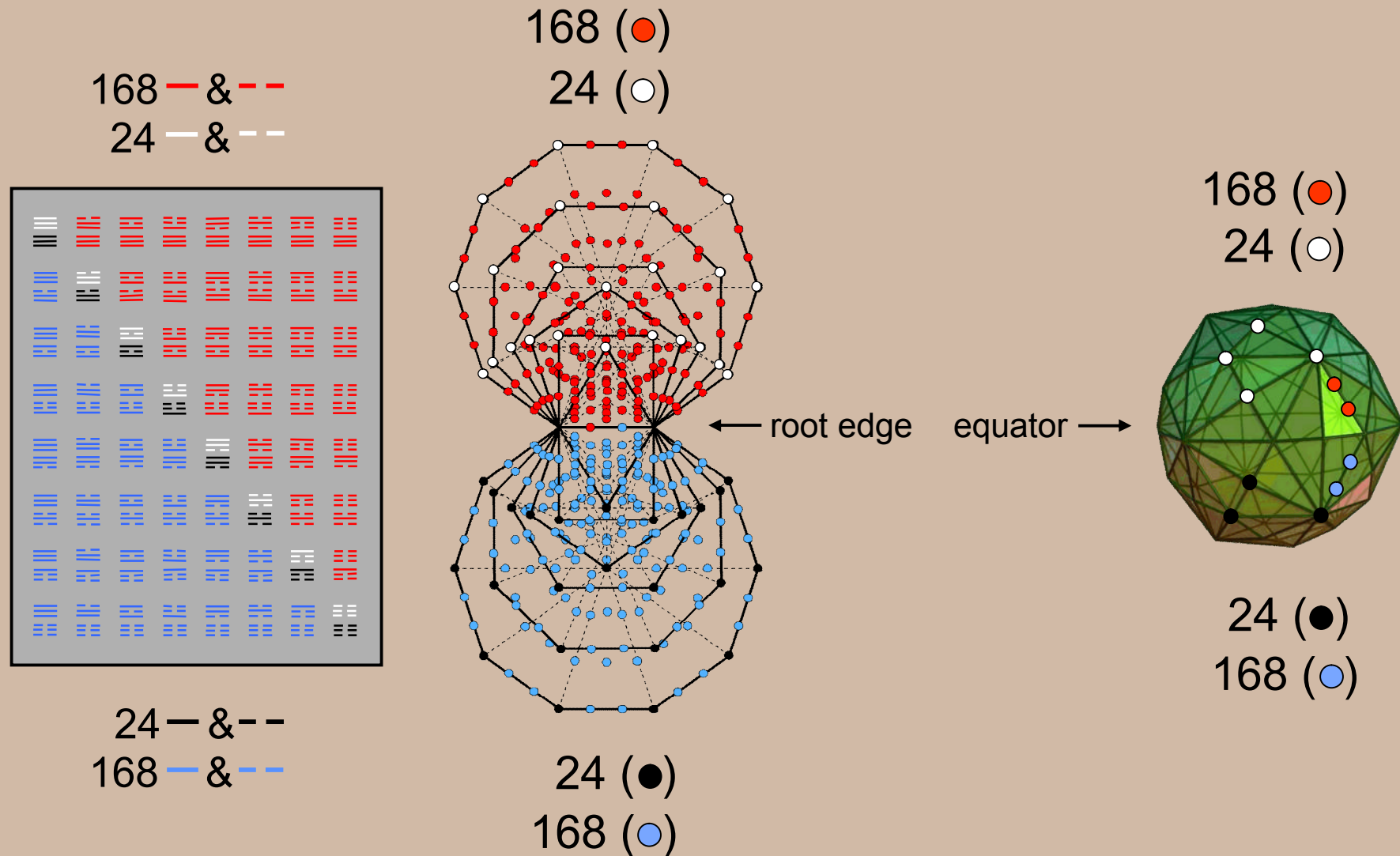


There are 378 yods other than polyhedral vertices surrounding the centres of the seven polygons above the equatorial plane of the disdyakis triacontahedron. This is the number of yods in the 42 triangles of the 3-d Sri Yantra. They symbolize the 378 rising and falling intervals below the octave of the notes in the seven musical scales. 5

Figure 3

The I Ching table, the inner Tree of Life and the disdyakis triacontahedron are equivalent representations of holistic systems, as now shown. Of the 62 vertices of the disdyakis triacontahedron, 12 vertices (four A, four B & four C vertices) are in the XY plane when the Z axis passes through two diametrically opposite A vertices. Hence, there are 50 vertices above and below this plane, i.e., 24 vertices lie between the XY plane and the uppermost or lowest A vertex. One set corresponds to the 24 lines/broken lines in the upper trigrams of the eight diagonal hexagrams, the other set correspond to the 24 lines/broken lines in the lower trigrams. 12 of the 180 edges of the polyhedron are in the XY plane, leaving 84 edges above this plane and 84 edges below it. Two hexagonal yods lie on each edge. $(2 \times 84 = 168)$ red hexagonal yods lie on edges above the plane and 168 blue hexagonal yods lie on edges below it. Each set of 168 hexagonal yods corresponds to the 168 yods other than corners associated with each set of six enfolded polygons and to the 168 lines/broken lines in the 28 hexagrams above or below the diagonal of the I Ching table.

Figure 3



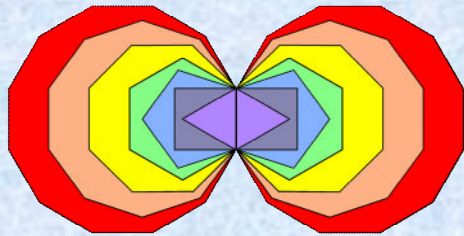
Isomorphism between the I Ching, the inner
Tree of Life & the disdyakis triacontahedron.

Figure 4

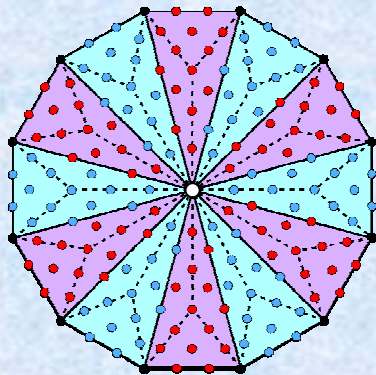
The outermost polygon in the inner Tree of Life is the dodecagon. 168 extra yods are needed (84 in six sectors) to construct each of its 12 sectors from three tetractyses. The disdyakis triacontahedron has 84 edges and 24 vertices and 60 triangles above the XY plane, that is, another 84 geometrical elements. Similarly below this plane, which contains 12 vertices and 12 edges. Hence, the 84:84 division of yods in the dodecagon has its counterpart in the 84:84 division of geometrical elements above or below the equator of the disdyakis triacontahedron.

The centres of the two dodecagons correspond to the A vertices at the top and bottom of the polyhedron. The 12 corners of each dodecagon correspond to the 12 vertices & edges in the XY plane and their mirror images. The two sets of 84 *new* yods in each dodecagon correspond to the 84 edges and 84 vertices/triangles above or below the XY plane. Each yod in the pair of dodecagons symbolizes a geometrical element composing the disdyakis triacontahedron. The yods denote the 'bits of information' needed to construct the polyhedral form of the inner Tree of Life.

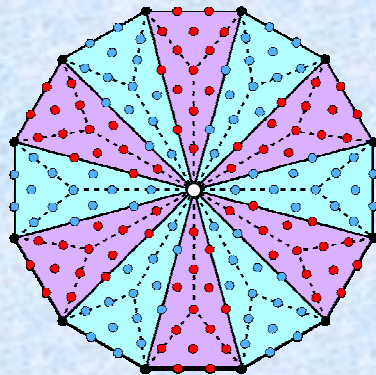
Figure 4



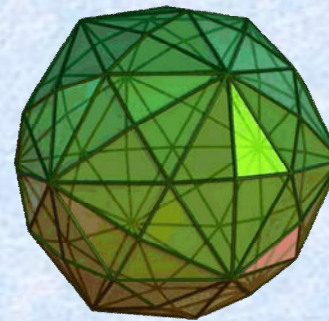
The outermost, regular polygon in the inner Tree of Life is the dodecagon.



1 (○)
 84 (●) } 168
 84 (●) }
 12 (●)



1 (○)
 84 (●) } 168
 84 (●) }
 12 (●)



disdyakis
tricontahedron

apex (A vertex)

84 edges
 24 vertices } 84 } 168
 60 triangles }

6 vertices + 6 edges } 12

6 vertices + 6 edges } 12

60 triangles } 84 } 168
 24 vertices }
 84 edges }

nadir (A vertex)

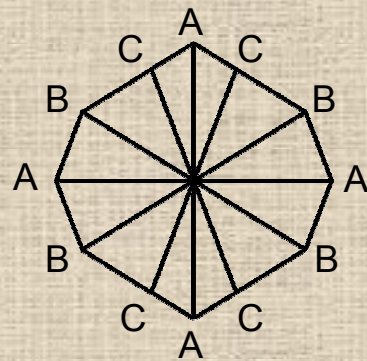
Isomorphism between the disdyakis triacontahedron and the pair of dodecagons of the inner Tree of Life.

Figure 5

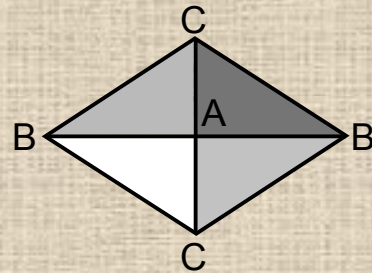
Each of the 120 triangular faces of the disdyakis triacontahedron has an A, B & C vertex. The polyhedron has 60 AB edges, 60 BC edges and 60 AC edges. The 12-sided polygon in the equator has four AB edges, four BC edges and four AC edges. This leaves 56 sets of three different types of edges above and below the XY plane, when the Z axis passes through two diametrically opposite A vertices, that is, 168 edges in 56 sets of three shape its 3-dimensional form.

Compare that property of the disdyakis triacontahedron with the 56 triangles with 168 vertices needed for the $\{3,7\}$ tiling in the hyperbolic plane of the 168 symmetries of the Klein quartic belonging to $PSL(2,7)$. Also notice that there are 24 vertices, 84 edges and 60 triangles, that is, 168 geometrical elements, between the XY plane and the apex of the polyhedron and 168 elements between this central plane and its nadir. These $(168+168=336)$ elements correspond to the 336 triangles needed in the $\{7,3\}$ tiling of the 168 automorphisms and 168 anti-automorphisms of the Klein quartic. The two sets of 168 symmetries correspond to the 168 geometrical elements either above or below the equator of the disdyakis triacontahedron between opposite vertices. The $PSL(2,7)$ and $SL(2,7)$ groups are embodied in the geometry of this polyhedron. $PSL(2,7)$ is isomorphic to $SL(3,2)$, the symmetry group of order 168 of the Fano plane that represents the algebra of the octonions, which form a natural representation of the Lie algebra of E_8 , the unified superstring gauge symmetry group. This indicates that a connection exists between the geometry of the disdyakis triacontahedron and superstring dynamics.

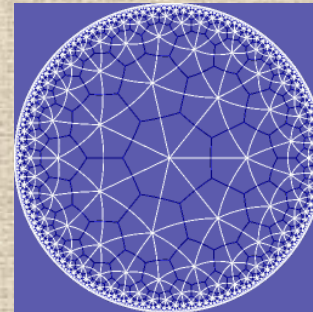
Figure 5



12-sided polygon
in equator

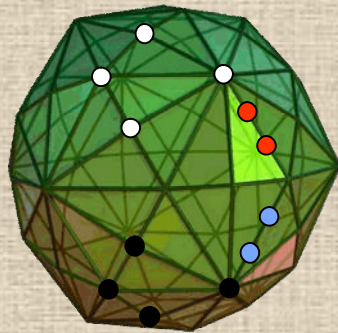


edges of faces



$\{3,7\}$ tiling of
Klein quartic

The 56 triplets of edges AB, BC & AC above & below the equator of the disdyakis triacontahedron correspond to the 56 triangles in the $\{3,7\}$ tiling in the hyperbolic plane of the 168 symmetries of the Klein quartic that belong to $\text{PSL}(2,7)$.



Above equator: 24 (○)
168 (●)

60 triangles
24 vertices
84 edges } 168

Equator: 12 vertices+12 edges } 24
12 interior edges+12 interior triangles } 24

Below equator: 24 (●)
168 (○)

60 triangles
24 vertices
84 edges } 168

The disdyakis triacontahedron is the polyhedral embodiment of $\text{SL}(2,7)$.

Figure 6

The I Ching table, the Sri Yantra, the inner Tree of Life, the Klein configuration and the disdyakis triacontahedron exhibit a universal 168:168 pattern in their geometries. It is highly implausible that this is a coincidence. Instead, it strongly indicates the relevance to theoretical physics (in particular, to string/brane theory) of the group $SL(2,7)$ of order 336, which is the double-cover group of $PSL(2,7)$ of order 168. It is further suggested by the fact that the basic constituent of matter described by Annie Besant and C.W. Leadbeater over a century ago consists of ten closed curves, each of which is a circularly polarized standing wave that makes 168 oscillations in half a revolution around the spin axis of the particle. Its inner and outer halves with 168 helical turns per half-revolution are the manifestation of the mirror/inversion symmetry of these isomorphic objects of sacred geometry or – in the case of the I Ching table – the Yin/Yang balance of the 168 unbroken lines and the 168 broken lines of its 64 hexagrams.

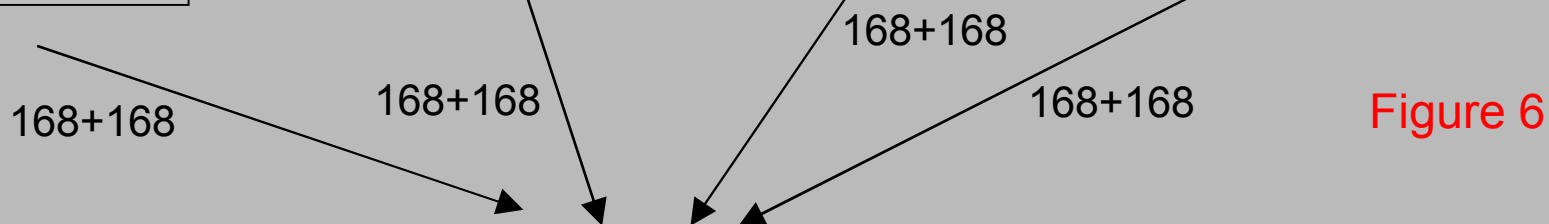
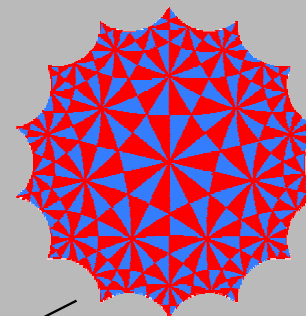
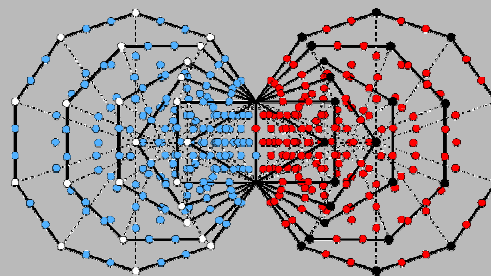
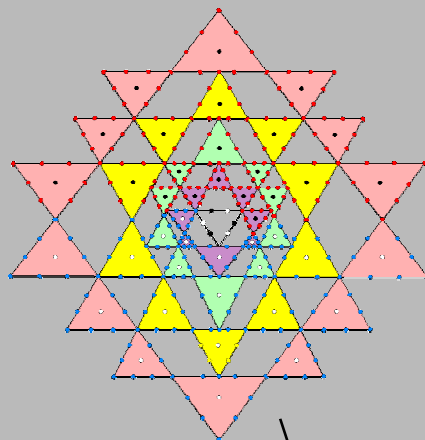
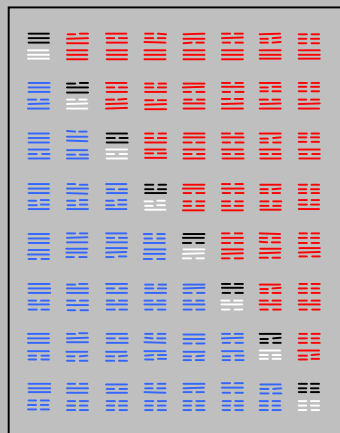
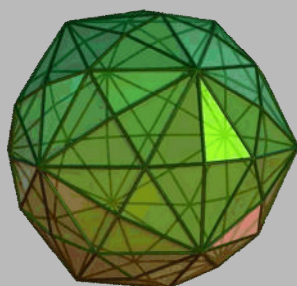


Figure 6

$SL(2,7)$ and $PSL(2,7) \cong SL(2,3)$

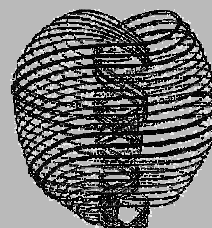


$168+168$

Fano plane

octonions

E_8 symmetry group of
superstring forces



$(168+168)$ turns in $(\frac{1}{2}+\frac{1}{2})$
revolution of a whorl

Figure 7

Each Sephirah has a corresponding Godname, Archangel, Angelic Order & Mundane Chakra. The Mundane Chakra of Malkuth, the last Sephirah of the Tree of Life signifying the physical universe, is called by Kabbalists “Cholem Yesodeth” in Hebrew. It means “breaker of the foundations.” Its gematraic number value is 168. The number value of Cholem is 78 and the number value of Yesodeth is 90.

Figure 7

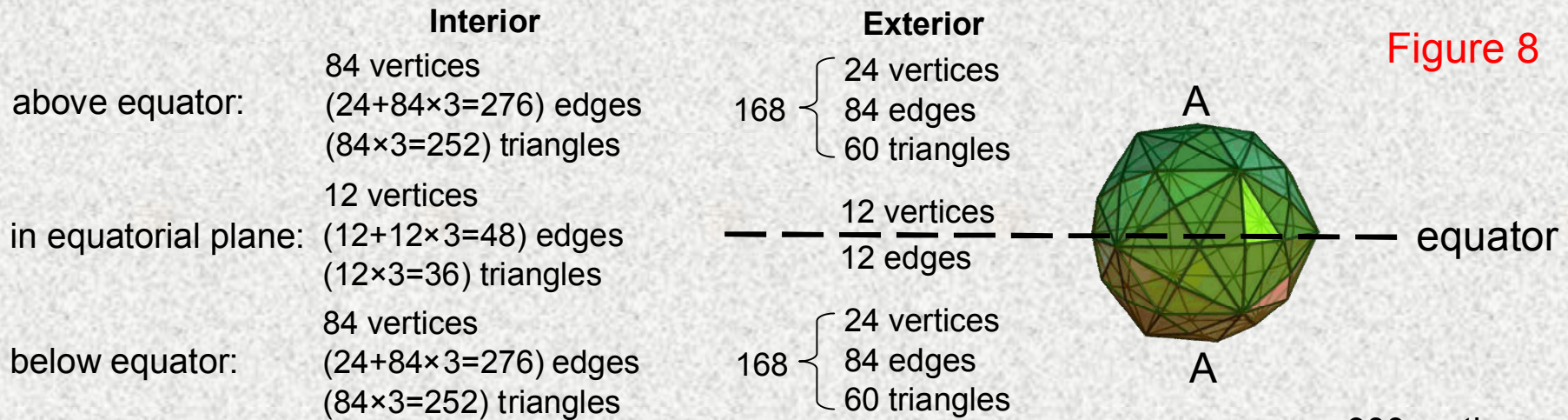
$$\begin{array}{rcccl} \text{יְסוּדוֹת} & \cdot & \text{חֵלֶם} & & \\ \text{T U D U S Y} & & \text{M L Ch} & & \\ 4 \leftarrow 400 & 6 & 4 & 6 & 60 & 10 & 40 & 30 & 8 \\ \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} & & \\ & 90 & & 78 & \\ & & & & = 168 \end{array}$$

***Cholem Yesodeth*, the Hebrew name of the Mundane Chakra of Malkuth (physical universe) has the gematria number value of 168.**

Figure 8

Consider the disdyakis triacontahedron constructed internally as well as externally from triangles. Each of its 180 edges is then the base of a triangle one of whose vertices is the centre of the polyhedron. Then suppose that these internal triangles are each divided into three triangles. This generates 168 vertices above and below the equator of the polyhedron with 'A' vertices (peaks of its 30 rhombic pyramids) at its apex and nadir. Surrounding the vertical axis joining these two vertices, there are 168 vertices & triangles above the equator, 168 vertices & triangles below the equator and 168 edges above and below it that are in its 120 faces. Surrounding the axis of the disdyakis triacontahedron are 660 triangles with 240 vertices, i.e., 900 geometrical elements, and 780 edges, making a total of 1680 geometrical elements. The split: $1680 = 780 + 900$ matches (apart from the tetractys/Tree of Life factor of 10) the number values of the Hebrew words "Cholem" and "Yesodeth" making up the Mundane Chakra of Malkuth. Moreover, there are 1680 turns in each helical whorl of the basic unit of matter described by Besant & Leadbeater over a century ago and now identified as the $E_8 \times E_8$ heterotic superstring! This extraordinary embodiment of the superstring structural parameter 1680 in the geometry of the disdyakis triacontahedron is very strong evidence that it is the cosmic blueprint defining both the dynamics and structure of superstrings. Further confirmation that this is not a coincidence is the fact that there are 900 geometrical elements in the polyhedron down to the equator and 780 elements below it. The two words "Yesodeth" and "Cholem" define the numbers of geometrical elements making up, respectively, half the polyhedron and the remainder of it.

Figure 8



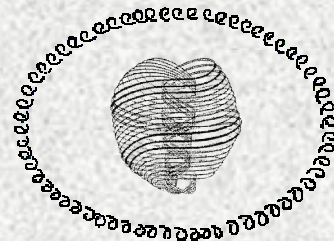
Geometrical composition of the disdyakis triacontahedron

	Vertices	Edges	Triangles	Total
above equator:	24 + 84 = 108	84 + 84×3 + 24 = 360	60 + 3×84 = 312	780
in equatorial plane:	12 + 12 = 24	12 + 12×3 + 12 = 60	12×3 = 36	120
to equator:	Subtotal = 132	420	348	900
below equator:	24 + 84 = 108	84 + 84×3 + 24 = 360	60 + 3×84 = 312	780
	Total = 240	780	660	1680

1680 { 900 vertices & triangles
780 edges

1680 { 900 elements to equator
780 elements below equator

(168+168) windings of each whorl in
($\frac{1}{2}+\frac{1}{2}$) revolution around a 1-torus



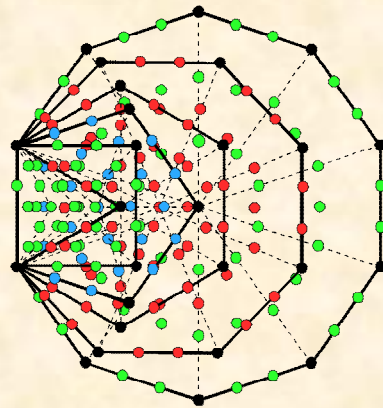
1680 windings of each whorl in 5
revolutions around a 1-torus

Number values of *Yesodeth* (90) & *Cholem* (78) define the numbers of geometrical elements in the disdyakis triacontahedron to, respectively, its equator and below its equator. ¹⁷

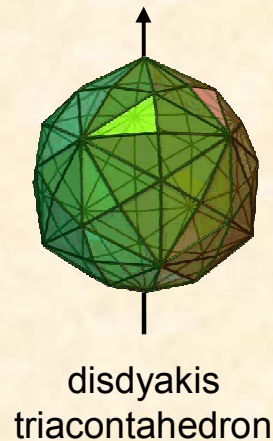
Figure 9

The gematria number value 168 of *Cholem Yesodeth* is the number of yods other than corners associated with each set of the first six enfolded polygons making up the inner Tree of Life. The pentagon, hexagon & octagon have 90 yods and therefore embody the number value of *Yesodeth*. The triangle, square & decagon have 78 yods, so that they embody the number value of *Cholem*. The polyhedral counterpart of this is the triakis tetrahedron. When the interior triangles formed by its centre and edges are each divided into three sectors, there are 90 vertices & triangles surrounding an axis passing through two opposite vertices and 78 edges. As the Catalan solid with the fewest number of edges (see the table on p. 9 in Part 2 of Article 40), it is the only semi-regular polyhedron with this property. The Catalan solid with the largest number of edges is the disdyakis triacontahedron. Similarly constructed, it has 900 vertices & triangles and 780 edges surrounding an axis joining two opposite vertices, that is, exactly *ten* times the corresponding numbers for the triakis tetrahedron. It is as if the disdyakis triacontahedron is the tetractys of the world of polyhedra and the triakis tetrahedron is one of its ten yods.

This hidden, ten-fold nature of the disdyakis triacontahedron is confirmed by the fact that, associated with the 60 polygons of the first six types enfolded in *ten* overlapping Trees of Life are 900 yods other than corners in the ten pentagons, ten hexagons and ten octagons and 780 yods other than corners in the ten triangles, ten squares & ten decagons (“associated” means that only one of the two hexagonal yods in each shared, root edge is included in the count, the other hexagonal yod being associated with the other set of polygons).

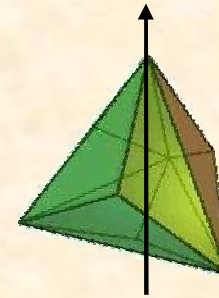
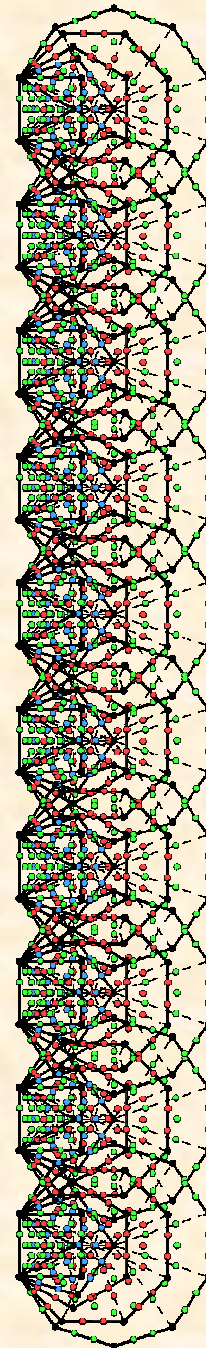


24 (●) + 78 (●) + 66 (●)
 pentagon triangle hexagon
 square octagon
 decagon
 Total = 168



Surrounding an axis
 of symmetry are:
 240 vertices
 780 edges
 660 triangles
 Total = 1680

disdyakis
triacontahedron



triakis tetrahedron

Surrounding an axis
of symmetry are:

24 vertices
 78 edges
 66 triangles

Total = 168

The correspondence between the
 disdyakis triacontahedron and the
 inner form of 10 Trees of Life. Each
 yod symbolizes a geometrical element
 of the polyhedron. Such precise
 correlation demonstrates the holistic
 nature of this Catalan solid. The
 polyhedron that is the counterpart of
 the inner form of a *single* Tree of
 Life is the triakis tetrahedron, the
 simplest of the Catalan polyhedra.

240(●) + 780(●) + 660(●)
 Total = 1680

Figure 9

Figure 10

Above or below the equator of the disdyakis triacontahedron between two opposite vertices are 168 geometrical elements. They comprise 84 edges & 60 triangles, i.e., 144 elements, and 24 vertices. This 24:144 division of the geometrical parameter 168 is found in the $\{3,7\}$ mapping onto a 3-torus of the 168 automorphisms of the Klein quartic, as now explained: the 3-torus is topologically equivalent to a tetrahedral array of four triangular prisms to whose 12 square faces is attached the 12 square faces of six antiprisms (see pp. 12 & 13 of Part 3 of Article 40). The four triangular prisms have eight triangles with 24 vertices; the six square antiprisms have 48 triangles with 144 vertices. So 56 triangles with $(24+144=168)$ vertices tile the 3-torus. The 24 vertices of the triangular prisms correspond to the 24 vertices above or below the equator of the disdyakis triacontahedron. The 144 vertices of the square antiprisms correspond to the 144 edges & triangles above or below the equator. The two halves of the polyhedron outside the equator correspond to the 3-torus mapping the 168 automorphisms of the Klein quartic and its turned inside-out version, which maps its 168 anti-automorphisms.

This correlation is further evidence that the disdyakis triacontahedron embodies the symmetry of $SL(2,7)$ displayed by the Klein quartic.

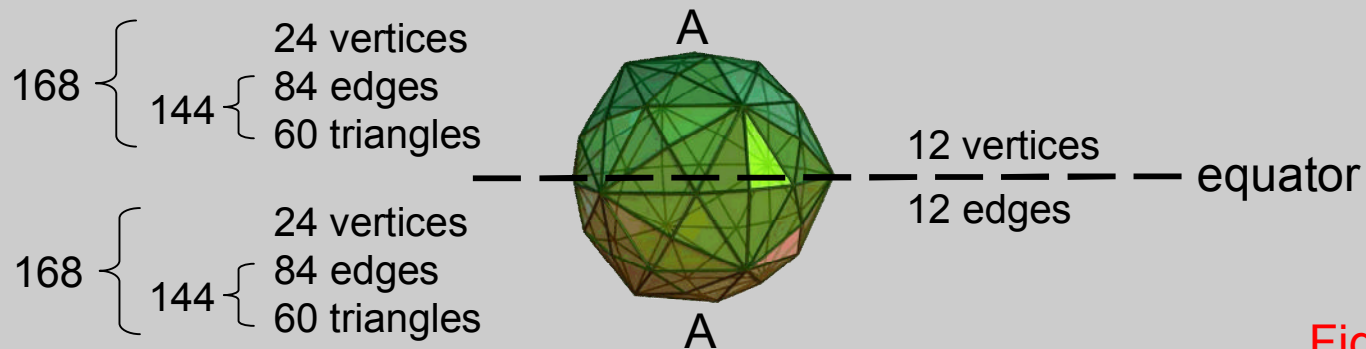


Figure 10

$$168 = 24 + 144$$

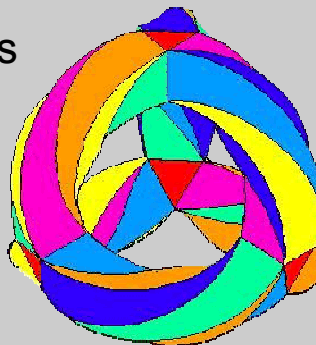
Triangular prism
(6 vertices, 3 square
faces, 2 triangular faces)

4×



8 triangles with 24 vertices

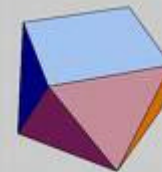
3-torus



56 triangles with (24+144=168) vertices

Square antiprism (8 vertices, 2
square faces, 8 triangular faces)

6×



48 triangles with 144 vertices

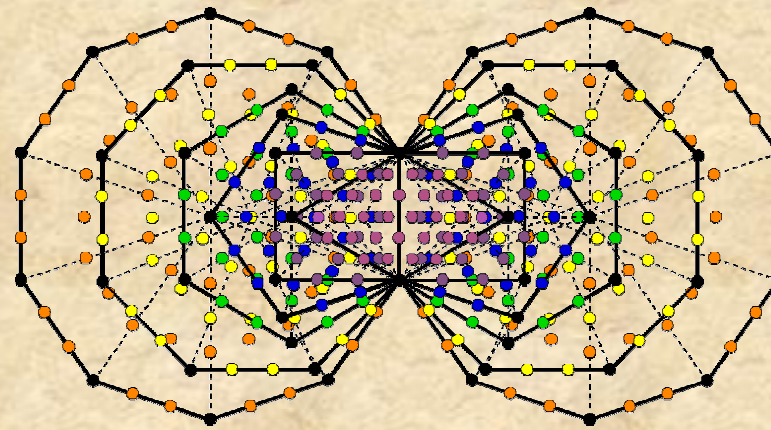
Isomorphism between the (24+144) geometrical elements of the disdyakis triacontahedron above & below the equator and the (24+144) vertices of the (8+48) triangles tiling the 3-torus and its turned inside-out form.

Figure 11

The same 24:144 division is displayed by the yods in the set of the first six enfolded polygons of the inner Tree of Life. The 168 yods other than corners associated with each set includes 24 that belong to the pentagon. So each of these 24 yods symbolize a vertex of the eight triangles in the four triangular prisms joining the six square antiprisms, whilst the 144 yods in the five other polygons denote the 144 vertices of the 48 triangles in the square antiprisms. As these vertices denote automorphisms of $PSL(2,7)$, one of which is the identity element, it is tempting to identify the single hexagonal yod in the root edge that is associated with one set of polygons as the identity I and the other hexagonal yod associated with the other set as $-I$. The two identical sets of six polygons, each with 168 yods other than corners, correspond to the $\{3,7\}$ tilings of the 3-torus by 56 triangles with 168 vertices mapping the 168 automorphisms and by the 56 triangles on its turned inside-out version mapping the 168 anti-automorphisms. Every yod in the six enfolded polygons that is not one of its corners symbolizes a symmetry of the Klein quartic. The two mirror-image sets of polygons correspond to the 3-torus and to a 3-torus turned inside out.

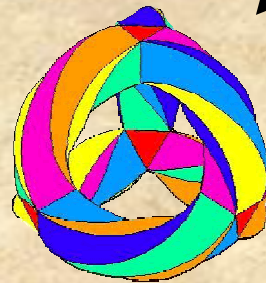
Figure 11

24 (●) in pentagon
144 (●,●,●,●,●)
Total = 168 yods

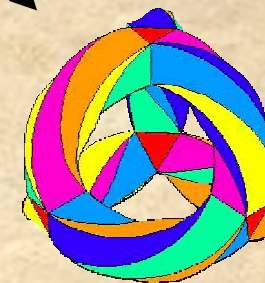


24 (●) in pentagon
144 (●,●,●,●,●)
Total = 168 yods

two yods in root
edge denote the
identity I & -I



turned-inside out
version of the 3-torus



3-torus

24 vertices of 8 triangles in 4 triangular prisms
+
144 vertices of 48 triangles in 6 square antiprisms
= 168 vertices

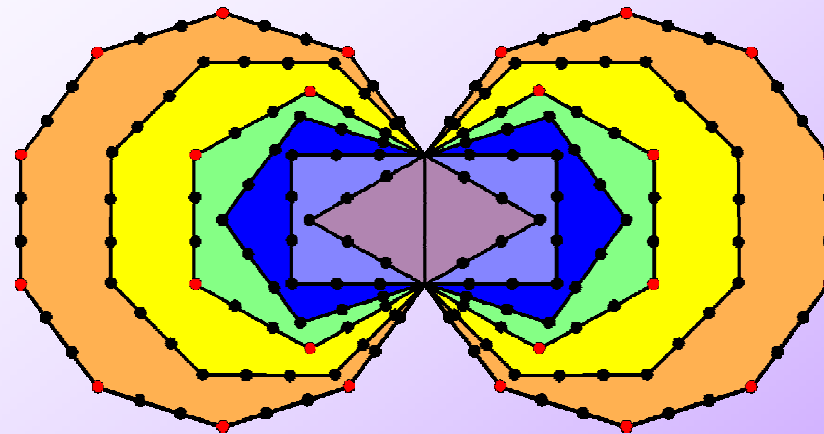
24 vertices of 8 triangles in 4 triangular prisms
+
144 vertices of 48 triangles in 6 square antiprisms
= 168 vertices

Each of the two sets of the first 6 enfolded polygons of the inner Tree of Life has (24+144) yods. One set is isomorphic to the (24+144) vertices of the 56 triangles in the {3,7} tiling on a 3-torus of the 168 automorphisms of the Klein quartic. The other mirror-image set is isomorphic to the (24+144) vertices of the 56 triangles on the turned inside-out 3-torus tiling the 168 antiautomorphisms.

Figure 12

As further confirmation of the remarkable isomorphism between the inner Tree of Life, the disdyakis triacontahedron and the tiling of the 168 symmetries of the Klein quartic onto a 3-torus, the *same* 24:144 division of the respective yods, geometrical elements and vertices of triangles is exhibited in the boundaries of the two sets of the first six polygons. 168 yods lie on their edges outside their shared root edge. Of these, 24 red yods are corners of the pair of hexagons and decagons outside their shared root edge. 144 black yods lie on the 60 external edges of the other eight polygons. The Godname EL of Chesed with number value 31 prescribes the first six enfolded polygons because they have 31 sides. The Godname YAH with number value 15 prescribes the enfolded hexagon and decagon with 15 edges.

Figure 12



24 (●)

144 (●)

Total = 168

The pair of 24:144 divisions of: 1. geometrical elements in the disdyakis triacontahedron, 2. yods in the first (6+6) regular polygons of the inner Tree of Life and 3. triangles in the {3,7} tiling of the 168 automorphisms and the 168 antiautomorphisms of the Klein quartic on a 3-torus is a fundamental property of holistic systems. This basic division appears not only in each set of 6 enfolded polygons but also in the *pair* of them. There are 24 corners of the two hexagons and the two decagons outside the shared root edge and 144 yods on the external edges of the two sets of polygons. In other words, 168 yods are needed to delineate their *shapes*, confirming the character of this number as a structural parameter of the heterotic superstring as the microscopic manifestation of the Tree of Life blueprint.

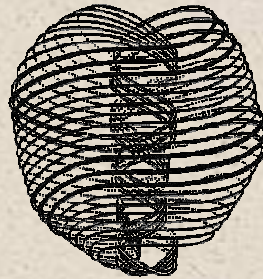
Figure 13

Each of the ten closed curves of the basic unit of matter described by Annie Besant and C.W. Leadbeater is a helix with 1680 turns. As each curve winds five times around the spin axis of the particle, it makes 336 helical turns in one revolution, that is, 168 turns in a half-revolution. This 168:168 division corresponds to:

1. the $(168+168)$ geometrical elements in the faces of the disdyakis triacontahedron above and below its equator;
2. to the 168 automorphisms and 168 anti-automorphisms of the Klein quartic as represented by the 168 vertices of the 56 triangles tiled on the 3-torus and the 168 vertices of the 56 triangles tiled on its turned inside-out version;
3. the $(168+168)$ yods other than corners in the first $(6+6)$ enfolded polygons;
4. the $(168+168)$ lines/broken lines in the 56 off-diagonal hexagrams of the I Ching table.

Figure 13

Each of the 10 closed curves of the heterotic superstring winds 336 times in a circle as it revolves once around its central axis.

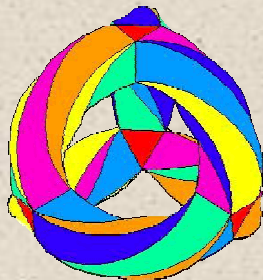


$E_8 \times E_8$ heterotic superstring

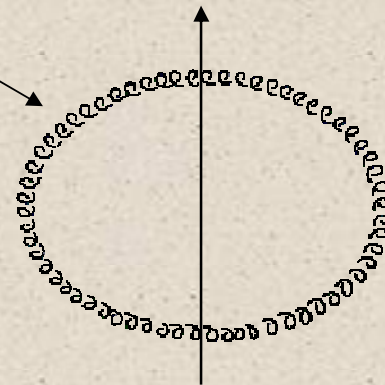
The (168+168) helical turns in each revolution of a closed curve are the counterpart of the (168+168) geometrical elements in the surface of the disdyakis triacontahedron, the 168 automorphisms & 168 antiautomorphisms of the Klein quartic, the (168+168) yods other than corners in the first (6+6) polygons of the inner Tree of Life and the (168+168) lines & broken lines in the 28 off-diagonal hexagrams of the I Ching diagram.

closed curve in $E_8 \times E_8$
heterotic superstring

168 helical turns in half-revolution

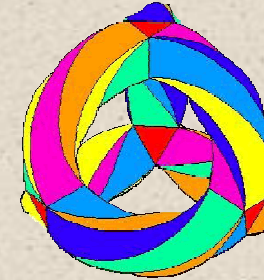


168 anti-automorphisms



turned inside out version

168 helical turns in half-revolution

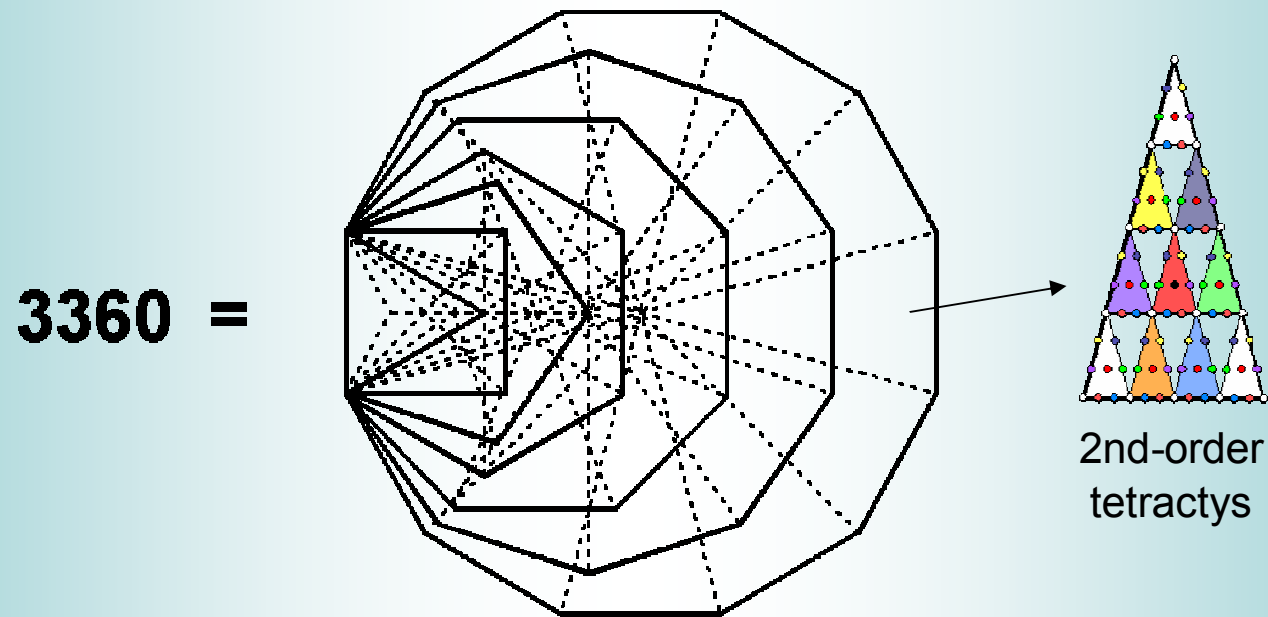


168 automorphisms

Figure 14

When the 47 sectors of the seven enfolded polygons are transformed into 2nd-order tetractyses (the next order above the tetractys in which yods are replaced by tetractyses), they contain 3360 yods. This is the number of helical turns made by all ten closed curves of the $E_8 \times E_8$ ' heterotic superstring when they make one revolution around the spin axis of the spin- $\frac{1}{2}$ particle. Each yod denotes a turn. Physically, it is an oscillation of the circularly polarized standing wave running around the curve. The Godname ELOHIM with number value 50 prescribes the fundamental constituent of quarks because its ten whorls make 50 revolutions about its spin axis. Assigning this number to each of the yods in the inner Tree of Life generates the number 16800 as the number of circularly polarized oscillations in the ten closed curves of the heterotic superstring.

Figure 14



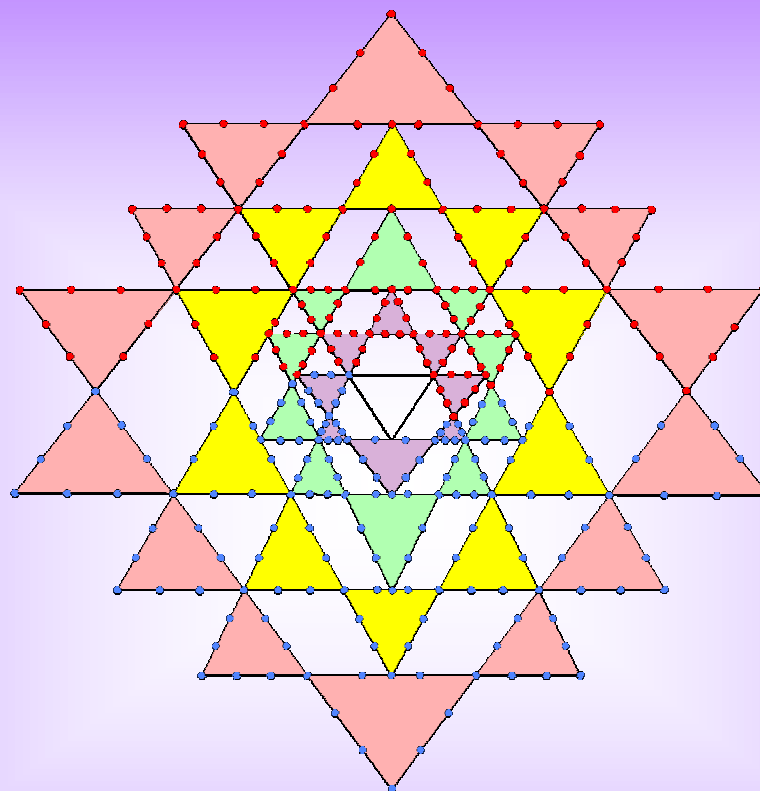
All ten closed curves of the $E_8 \times E_8$ heterotic superstring wind 3360 times in one revolution around its spin axis. This is the number of yods in the seven enfolded polygons of the inner form of the Tree of Life when their 47 sectors are each turned into 2nd-order tetractyses. It is astounding evidence that the superstring is the microscopic manifestation of the universal Tree of Life blueprint.

Figure 15

Assigning the number 10 (the Pythagorean Decad) to the 336 yods on the 126 edges of the 42 triangles of the 3-dimensional Sri Yantra generates the number 3360. This demonstrates how the Sri Yantra and the inner Tree of Life are equivalent representations of holistic systems like the heterotic superstring. Each half of the Sri Yantra with 168 yods defines the number (1680) of circularly polarized oscillations made by all ten closed curves of the superstring during half of a revolution around its spin axis. The inner and outer halves of the superstring correspond to the two halves of the Sri Yantra. They are prescribed by the Godname EHYEH with number value 21 because each half is made of 21 triangles.

Figure 15

3360 =



● = ● = 10

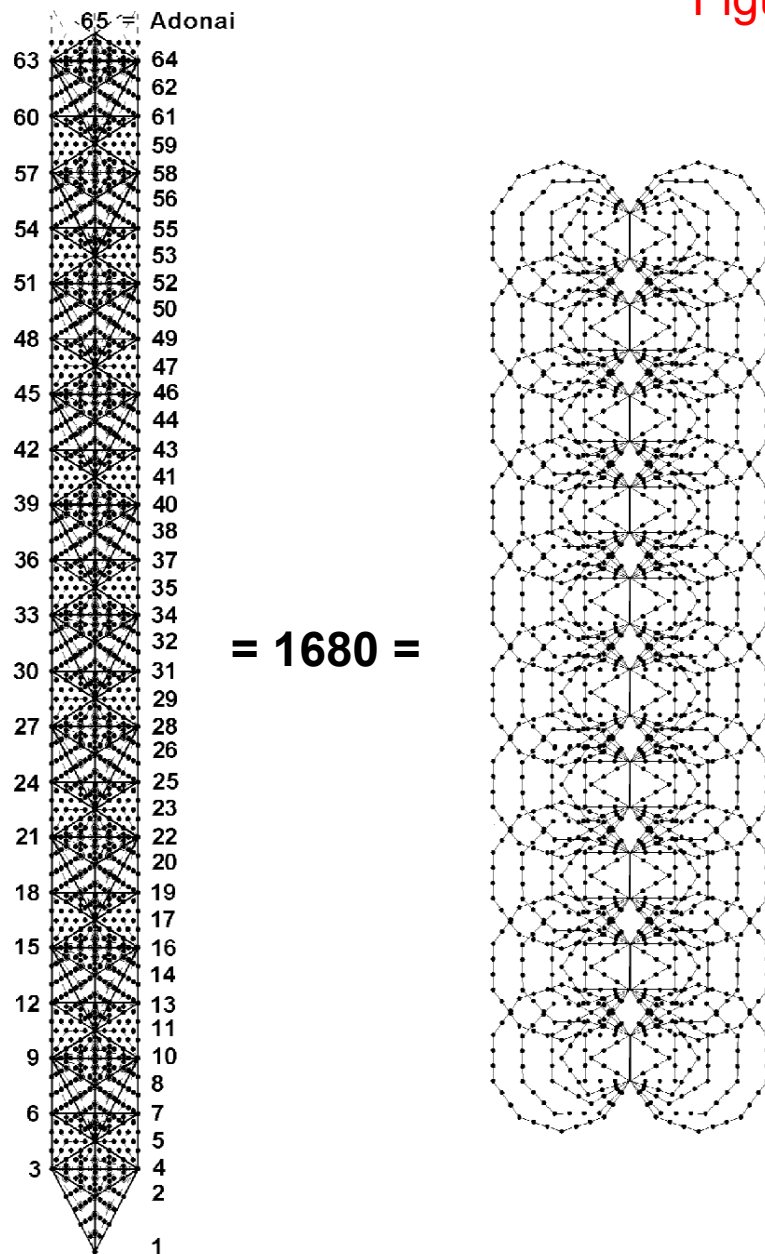
3360 is the sum of the numbers 10 assigned to each of the 336 yods on the 126 edges of the 42 triangles of the 3-d Sri Yantra.

Figure 16

The Godname of Malkuth is ADONAI. Its number value is 65. It prescribes the lowest ten overlapping Trees of Life because they have 65 Sephiroth. When their triangles are each constructed from three tetractyses, there are 1680 yods below the top of the tenth tree – the 65th Sephirah. Each yod symbolizes a circularly polarized oscillation in each closed curve of the heterotic superstring. It shows how ADONAI, the Godname of the Sephirah signifying the physical universe, prescribes the form of the basic unit of matter.

There are 1680 yods outside the shared edges that lie on the edges of the first six polygons enfolded in each of ten overlapping Trees of Life on either side of the central Pillar of Equilibrium. This demonstrates the shape-defining character of this structural parameter of the $E_8 \times E_8$ heterotic superstring.

Figure 16

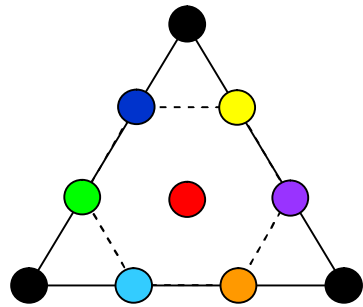


There are 1680 yods below the top of the tenth Tree of Life when each of its triangles is constructed from three tetractyses. The Godname ADONAI with number value 65 prescribes the ten trees mapping the ten dimensions of superstring space-time because they have 65 Sephiroth. There are also 1680 yods on the boundaries of the first six polygons enfolded in each of the ten trees on either side of the central Pillar of Equilibrium. The superstring structural parameter 1680 is embodied in the outer and inner forms of ten Trees of Life.

Figure 17

Suppose that the 50 faces of the five Platonic solids are constructed from tetractyses. The four faces of the tetrahedron have 48 hexagonal yods, the eight faces of the octahedron have 96 hexagonal yods, the six faces of the cube have 96 hexagonal yods, the 20 faces of the icosahedron have 240 hexagonal yods and the 12 faces of the dodecahedron have 240 hexagonal yods.

Figure 17



Coloured yods are
hexagonal yods

**The numbers of hexagonal
yods in the faces of the five
Platonic solids constructed
with the Pythagorean tetractys.**

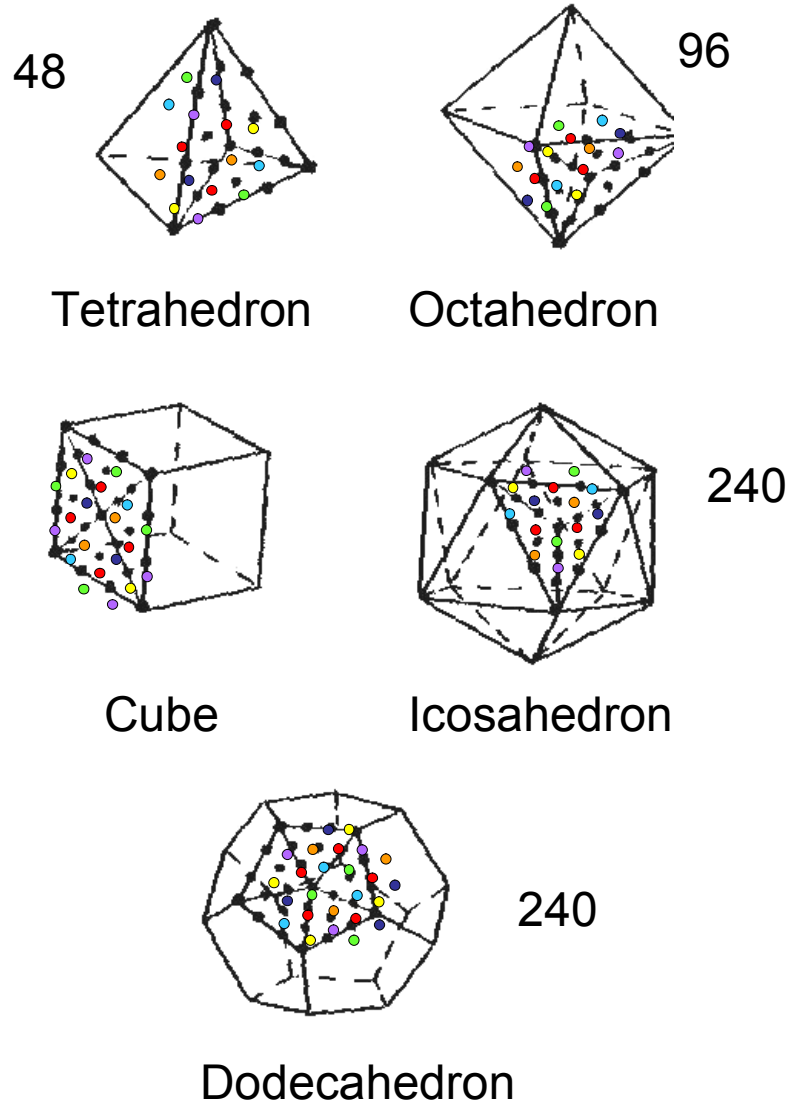


Figure 18

The 62 vertices of the disdyakis triacontahedron generate 10 tetrahedra, five octahedra, five cubes, one icosahedron, one dodecahedron, five rhombic dodecahedra and one rhombic triacontahedron. The 28 polyhedra contain 3360 hexagonal yods when their faces are constructed from tetractyses. The superstring structural parameter 3360 is embodied in the polyhedral potential of the disdyakis triacontahedron, demonstrating its remarkable archetypal character. Notice that the four Platonic solids representing the four elements of Earth, Water, Air and Fire have 1680 hexagonal yods. They comprise 1440 hexagonal yods in the solids representing Fire, Air & Earth and 240 hexagonal yods in the icosahedron representing Water. The same 240:1440 division is exhibited in the other seven polyhedra. It is another manifestation of the division discussed on pages 20-25 in the contexts of the disdyakis triacontahedron, the tiling on the 3-torus of the 168 symmetries of the Klein quartic and the yods in the first six enfolded polygons.

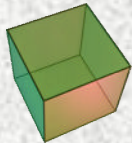
Figure 18



$$10 \text{ tetrahedra: } 10 \times 48 = 480$$



$$5 \text{ octahedra: } 5 \times 96 = 480$$



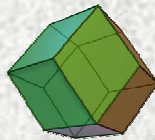
$$5 \text{ cubes: } 5 \times 96 = 480$$



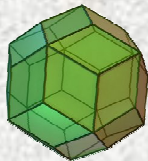
$$1 \text{ icosahedron: } 1 \times 240 = 240$$



$$1 \text{ dodecahedron: } 1 \times 240 = 240$$



$$5 \text{ rhombic dodecahedra: } 5 \times 192 = 960$$



$$1 \text{ rhombic triacontahedron: } 1 \times 480 = 480$$

$$\text{Total} = \underline{\underline{3360}}$$

3360 hexagonal yods are needed to construct from tetractyses the 28 regular and semiregular polyhedra generated by the vertices of the disdyakis triacontahedron

Figure 19

A Hypothesis

Hypothesize that each of the three generations of basic fermions has seven colour states, one defining leptons and six defining subquarks. Each generation consists of two weak isospin states, each isospin state consists of two supersymmetric states, each supersymmetric state is a particle and its antimatter counterpart and each particle exists in left-hand and right-hand parity states. This implies that there are 336 different particles made up of 168 left-handed states and 168 right-handed states. This would correspond to the 168 geometrical elements in each half of the disdyakis triacontahedron between opposite vertices and the equator. The inversion symmetry of the polyhedron would naturally correspond to the distinction between left-handed particles and their mirror image right-handed counterparts. Moreover, the 24 left-handed leptons would correspond to the 24 vertices above the equator, the 24 right-handed leptons would correspond to the 24 vertices below the equator and the 144 edges & triangles in each half of the polyhedron would correspond to the 144 left-handed and 144 right-handed subquarks. The 24 triangles in each of the seven segments of the $\{7,3\}$ tiling of the Klein quartic would signify the 24 left-handed or right-handed particles in each of the seven colour states.

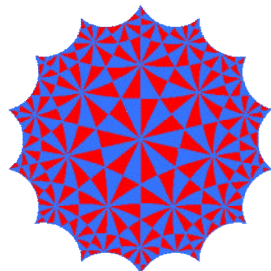
As we have seen (pp. 20-25), the inner Tree of Life, the I Ching table, the Sri Yantra and the disdyakis triacontahedron also embody the number 168 and its division into 24 and 144. The physical meaning of this would be that the former is the number of leptons of a given handedness ($3 \times 2 \times 2 \times 2$) and that the latter is the number of subquarks of the same handedness ($3 \times 6 \times 2 \times 2 \times 2$). Do, therefore, the 168 automorphisms and 168 anti-automorphisms of the Klein quartic have a physical interpretation in terms of these states of matter fields? Is the field of order 7 in $\text{PSL}(2,7)$ the seven colour states?

Figure 19

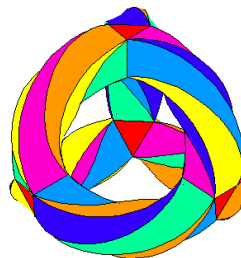
Hypothesize 7 colour states per generation (one for leptons, six for subquarks)

×3 generations	→	21 colour states	
×2 (weak isospin)	→	42 states	
×2 (supersymmetry)	→	84 states	
×2 (matter/antimatter)	→	168 states	dimension of $\text{PSL}(2,7) = 168$
×2 (left/right parity states)	→	336 states	dimension of $\text{SL}(2,7) = 336$

Geometry of the disdyakis triacontahedron represents the 168 left-handed particles and the 168 right-handed particles



Klein
Configuration
(KC)



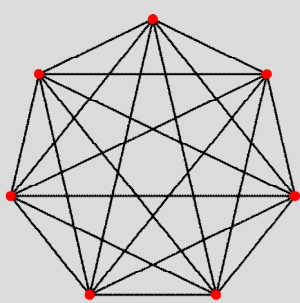
$168 = 7 \times 24$ (7-fold rotational symmetry of KC $\times$ (12+12=24) symmetries of T_d)

24 automorphisms & 24 anti-automorphisms per sector of the heptagonal KC → 24 left-handed & 24 right-handed superstrings in each colour state.

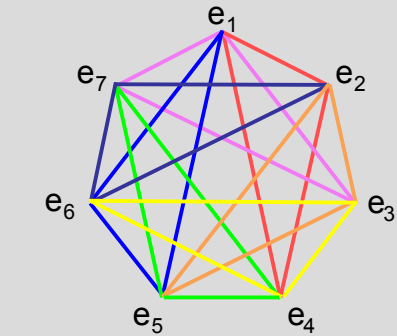
Figure 20

It requires seven colours to colour a map drawn on a 2-torus. K_7 , the complete graph of order 7, can be embedded on a 2-torus. This is the topology of each whorl of the unit of matter described by Besant & Leadbeater. The seven vertices of K_7 could symbolize the seven colour states of each generation of superstring, the 21 straight lines joining its vertices corresponding to the 21 colour states of the three generations of superstrings. Turning the straight lines into bi-directional arrows generates $(7+7=14)$ arrows pointing along the edges of the graph and $(14+14=28)$ arrows pointing inwards, that is, 42 arrows. This correlates with the 42 triangles of the Sri Yantra, the 14 arrows corresponding to the 14 triangles in the fourth layer and the 28 arrows corresponding to the 28 triangles in the first three layers. It would seem that the correlation, if not accidental, means that the Sri Yantra is defining the seven-fold nature of holistic systems, e.g., the musical intervals above the tonic, the seven musical scales, the seven unit imaginary octonions and the seven colour states of superstrings.

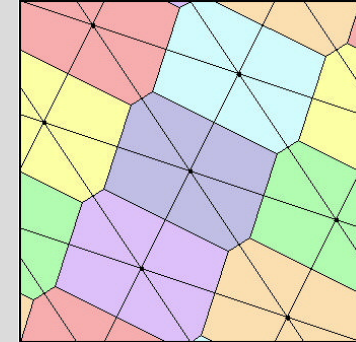
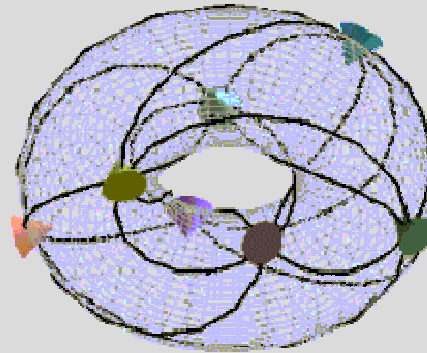
Figure 20



K_7



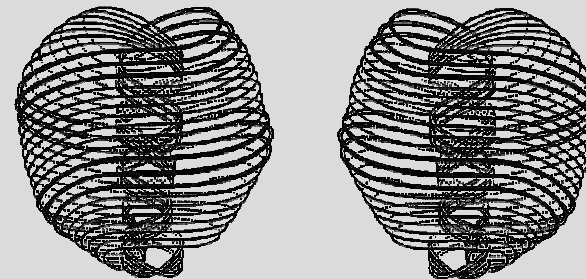
The seven 3-tuples of unit imaginary octonions are vertices of seven triangles



Embedding of K_7 on a 1-torus.

The seven vertices of the complete graph K_7 symbolize the 7 colour states of each generation of superstring, the 7 unit imaginary octonions e_i ($i=1-7$) and the 7 notes above the tonic in each of the 7 musical scales.

The 21 edges of K_7 represent the 21 colour states of the three generations of basic superstring, the 21 octonion products $e_i e_j$ ($i \neq j$) and the 21 intervals between the 7 notes in a musical scale above the tonic.



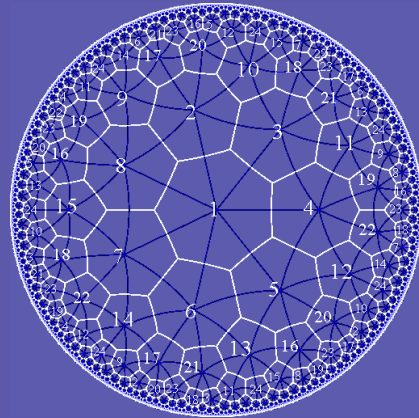
The two types of superstring of opposite chirality (depicted here are subquarks) are the left-handed and right-handed states defined by the 168 automorphisms and 168 anti-automorphisms of $SL(2,7)$.

Figure 21

The 168 vertices of the 24 heptagons in the $\{7,3\}$ mapping of the 168 automorphisms of the Klein quartic may denote the 168 particles in 24 sets of seven colour states. The 168 vertices of the 56 triangles in the $\{3,7\}$ mapping of the 168 automorphisms may denote the 168 particles made up of three generations of 56 states.

Figure 21

$\{7,3\}$ tiling

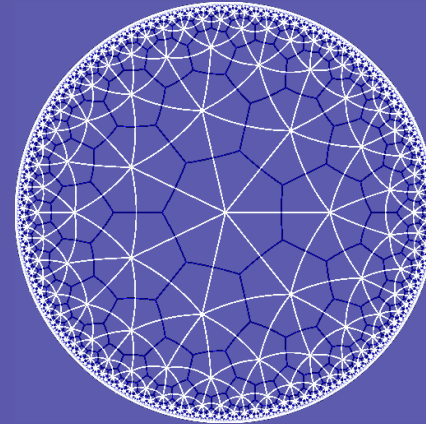


168 vertices of
24 heptagons



168 particles in 24 sets
of 7 colour states

$\{3,7\}$ tiling



168 vertices
of 56 triangles



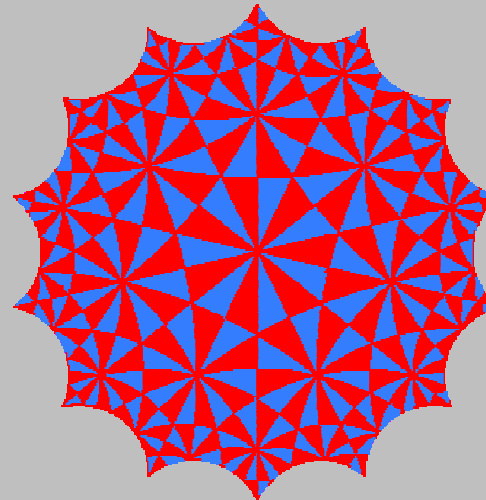
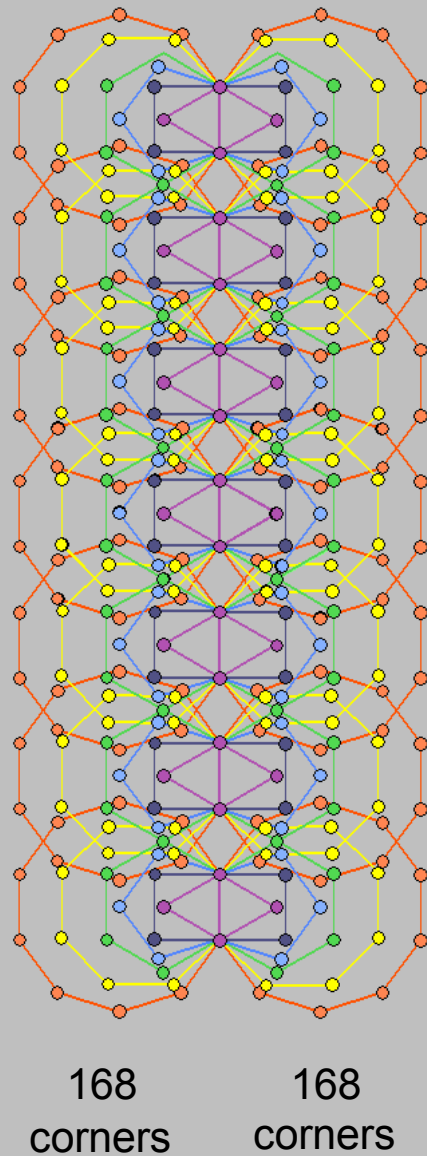
168 particles in 3
generations of 56 states

A physical interpretation of the Klein configuration.

Figure 22

The first six enfolded polygons have 26 corners. One corner on the root edge is associated with one set of polygons and the other corner is associated with the other set. The topmost corner of the hexagon coincides with the lowest corner of the hexagon enfolded in the next higher tree. This means that the 42 polygons enfolded on each side of seven overlapping Trees of Life have 168 corners associated with them that are truly intrinsic to them because none belong to the polygons enfolded in the next higher tree. The number of independent degrees of freedom represented by these corners is therefore $(168+168=336)$. They correspond in the $\{7,3\}$ hyperbolic mapping of the 168 automorphisms and 168 anti-automorphisms of the Klein quartic to, respectively, the 168 red hyperbolic triangles and the 168 blue hyperbolic triangles. The counterpart of the 24 triangles in each of the seven segments of the Klein configuration is the set of 24 corners associated with each set of polygons. In each case, the 24 degrees of freedom may denote the 24 particles in each of the seven colour states. If this true, then the seven colour states are the manifestation of the seven Sephiroth of Construction. The colour state defining leptons would correspond to Malkuth and the six colour states defining subquarks would correspond to the two triads of Sephiroth of Construction above Malkuth.

Figure 22



The pair of sets of the first 6 polygons enfolded in 7 overlapping Trees of Life have $(168+168=336)$ corners. This is the Tree Life counterpart of the $\{7,3\}$ Klein configuration. Each of the 7 sectors with 24 red triangles representing automorphisms of the Klein quartic corresponds to a set of 6 polygons with 24 corners associated with them. Each of the 7 sectors with 24 blue triangles representing its anti-automorphisms corresponds to the mirror image of this set. Pairs of corners that are mirror images of each other may denote left-handed and right-handed states of fundamental particles.

The Tree of Life counterpart of the Klein configuration.

Figure 23

Earlier, we found that the I Ching table is isomorphic to the $\{3,7\}$ mapping of the 168 automorphisms of the Klein quartic. The 56 trigrams of the 28 off-diagonal hexagrams in one half of the table correspond to the 56 hyperbolic triangles whose 168 vertices represent automorphisms. Just as there are 56 sets of three rows of lines/broken lines, so there are 56 particle states in each of three generations. The three rows of a trigram define the three possible generations of superstrings. The eight different trigrams denote the $(2^3=8)$ states for each colour: weak isospin doublet $(2) \times$ supersymmetric doubling $(2) \times$ particle/antiparticle (2) . The 168 lines and broken lines in the 28 hexagrams in one half of the table denote left-handed, superstring matter fields. The 168 lines and broken lines in the 28 hexagrams in the other half of the table denote right-handed particles.

Amongst the 90 intervals below the octave between the 14 types of notes found in the seven musical scales, there are 24 pairs of intervals and their complements. Three pairs of intervals are intervals between notes belonging to two different musical scales. They are not notes of these scales. This leaves 21 pairs of intervals that *are* notes of the seven scales. They correspond to the 21 basic, colour states of the three generations of superstrings and their antimatter, supersymmetric or weak isospin-doublet partners.

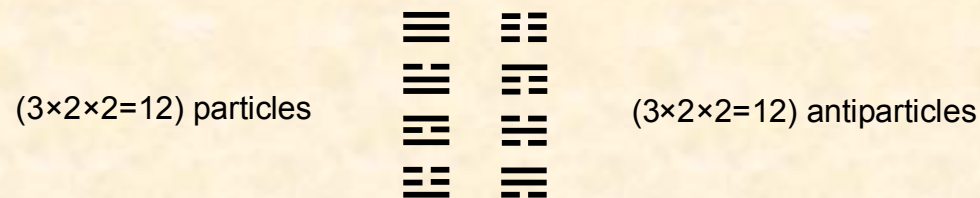


Figure 23

The three rows of a trigram denote the three generations of superstrings. The Yang/Yin duality of the line or broken line in each row signifies the three types of bipolarity in matter: third component of weak isospin $T_3 = \pm 1/2$, matter/antimatter & supersymmetric pairing of spin states. The 12 lines & 12 broken lines in the 8 trigrams denote the 12 particles and 12 antiparticles in each colour state. The pairing of trigrams into a hexagram signifies a particle and its supersymmetric partner. The 168 lines & broken lines in the 28 hexagrams on one side of the diagonal of the I Ching table denote the 168 left-handed particles and antiparticles. The 168 lines & broken lines on the other side denote the 168 right-handed particles and their antiparticles.

The 56 trigrams in each half of the table are the counterpart of the 56 triangles in the {3,7} tiling of the 3-torus mapping the 168 automorphisms of the Klein quartic and the 56 triangles mapping its 168 anti-automorphisms. They correspond to the 168 geometrical elements above the equator of the disdyakis triacontahedron and to the 168 elements below it. The 56 triangles define the 56 vertices of seven cubes, each with 24 pure, rotational symmetries. The 56 triangles mapping the 168 anti-automorphisms define seven 'anticubes', each with 24 mixed, rotation/reflection symmetries.

These (24+24=48) symmetries of the octahedral group, which is a subgroup of $SL(2,7)$ and whose seven copies generate the 336 symmetries of $SL(2,7)$ have their musical counterpart in the fact that the seven types of notes and their complements making up the seven musical scales have 90 intervals below the octave that include 24 pairs of intervals and their complements:

tone ratio		tone ratio		number of pairs	
L	256/243	T^5L	243/128	2	$T = 9/8$ = tone interval of the Pythagorean scale $L = 256/243$ = the Pythagorean leimma
L^{2*}	6536/59049	T^{5*}	59049/32768	1	
T	9/8	T^4L^2	16/9	2	$T^5L^2 = 2$
TL	32/27	T^4L	27/16	4	
TL^{2*}	8192/6561	T^{4*}	6561/4096	2	
T^2	81/64	T^3L^2	128/81	3	
T^2L	4/3	T^3L	3/2	6	
T^2L^2	1024/729	T^3	729/512	4	(Asterisked intervals are intervals between notes belonging to two different musical scales and therefore not part of the basic set of 14 notes found in the 7 musical scales.)
				Total = <u>24</u>	

The 168 symmetries of $PSL(2,7)$ or its isomorphic group $SL(3,2)$ correspond to the 84 rising and 84 falling intervals that are repetitions of the basic set of six notes and their six complements between the tonic and octave of the seven musical scales. They manifest geometrically in the disdyakis triacontahedron as the 84 edges above the equator and the 84 edges below it, the (6+6) edges in the equator denoting the six basic notes and their six complements listed above that are between the tonic and the octave of the seven musical scales.

Figure 24

The 24:168 division that, as we have seen, is displayed by various sacred geometries appears in the sequence of tone ratios of the notes of the Pythagorean scale. The perfect fifth of the fifth octave has the tone ratio 24. It is the tenth overtone and the 33rd note in the sequence. Indeed, every 33rd note increases in pitch by a factor of 24 and becomes the tenth overtone relative to the starting note. There are 24 overtones up to the note with tone ratio 192. So 14 overtones span a tone ratio difference of 168 between the note with tone ratio 24 and that with tone ratio 192. The notes of the seven musical scales have 189 intervals below the octave. They include 21 notes that are the second, third and fourth notes of each scale, leaving 168 intervals. Including the tonic, the octave and the unit interval between a note and itself, there are 192 intervals. They comprise 24 intervals (tonic, octave, unit interval and the 21 notes) and 168 intervals made up of the 21 complements of the 21 notes and 147 intervals between notes above the tonic (21 in each scale). Both the seven musical scales and the Pythagorean scale display the 24:168 division found for the inner Tree of Life, the I Ching table and the Sri Yantra.

Figure 24

Tone ratios of the notes in the Pythagorean musical scale

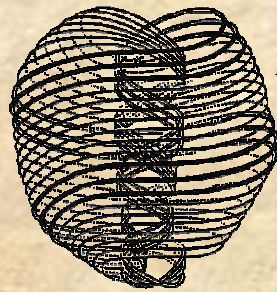
	C	D	E	F	G	A	B	Number of overtones
1	1	9/8	81/64	4/3	3/2	27/16	243/128	0
2	2	9/4	81/32	8/3	3	27/8	243/64	2
3	4	9/2	81/16	16/3	6	27/4	243/32	4
4	8	9	81/8	32/3	12	27/2	243/16	7
5	16	18	81/4	64/3	24	27	243/8	11
6	32	36	81/2	128/3	48	54	243/4	15
7	64	72	81	256/3	96	108	243/2	20
8	128	144	162	512/3	192	216	243	26

Note G of the eighth octave (its perfect 5th) has the tone ratio 192. It is the 24th overtone. Note G of the fifth octave has the tone ratio 24. It is the tenth overtone. So 14 overtones span a tone ratio difference of 168 between the 24th overtone and the tenth overtone. Including the tonic, octave and unit interval common to the seven musical scales, there are 192 intervals between the notes of the seven scales. They comprise 24 intervals (tonic, octave, unit interval and the 2nd, 3rd & 4th notes of each scale) and 168 intervals. We have seen how the number 192 has appeared in various sacred geometries as a measure of a holistic system. For example, there are 192 yods associated with the first six enfolded polygons, of which 24 yods are their corners. There are 192 lines/broken lines in each half of the I Ching table, of which 24 lines/broken lines make up the eight basic trigrams, leaving 168 lines/broken lines in the 28 hexagrams above or below the diagonal. There are 192 yods in each half of the Sri Yantra (including the central triangle). They comprise the three hexagonal yods on the edges of the central triangle, the centres of the 21 triangles and the 168 yods on the edges of these triangles, i.e., they divided into a set of 24 yods and a set of 168 yods. Finally, there are 192 geometrical elements surrounding the centres of the seven separate, regular polygons. They comprise the 24 elements surrounding the centre of the hexagon and the 168 elements surrounding the centres of the other six polygons. The intervals between the notes of the seven musical scales have the same 24:168 division, demonstrating that they are elements of a holistic system.

Figure 25

42 triangles of the Sri Yantra surround the central one. Suppose that their 126 sectors are each constructed from three triangles that are then turned into tetractyses. Each triangle is composed of 46 yods. It comprises seven vertices, 15 edges and nine triangles, i.e., 31 geometrical elements, showing how the Godname EL of Chesed with number value 31 prescribes the most elementary shape when constructed from tetractyses. The value 1 of the Hebrew letter aleph (E) denotes the vertex at the centre of the triangle and the value 30 of the Hebrew letter lamed (L) denotes the 30 geometrical elements that surround it. As the nine triangles making up each triangle of the Sri Yantra have 15 edges, the Godname YAH of Chokmah with number value 15 prescribes these structural units, whilst the Godname EHYEH of Kether with number 21 prescribes them because there are 21 vertices and edges surrounding the centre of a triangle. 36 yods lying on edges of the nine tetractyses surround the centre, where 36 is the number value of ELOHA, Godname of Geburah. This is also the extra number of yods that have to be added to change the triangle from a tetractys with ten yods into one with 46 yods. Each triangle has 39 hexagonal yods and one centre (shown as black yods). There are $(42 \times 40 = 1680)$ of these yods in the Sri Yantra. Alternatively, there are 1680 yods other than outward tips of triangles and interior corners of triangles. They symbolize the 1680 helical turns in each closed curved of the heterotic superstring. As a geometrical paradigm of wholeness, the Sri Yantra represents the form of the basic unit of matter.

Figure 25



whorl

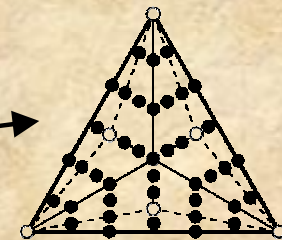
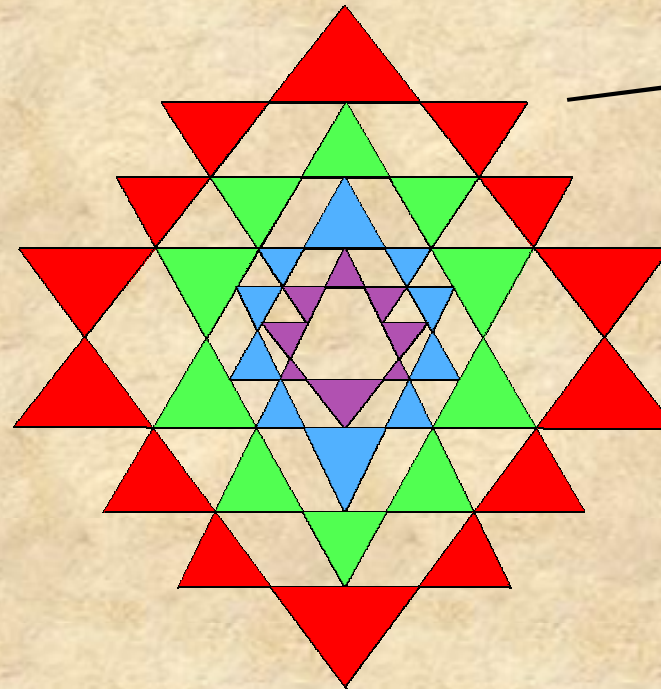
Heterotic
superstring



whorl

= 1680 =

$$1680 = 42 \times 40$$



40 (●)

The 1680 (●) yods in the 42 triangles of the Sri Yantra symbolize the 1680 turns in each helical whorl of the heterotic superstring.