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Mathematics and the Imagination: A Brief Introduction

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"[I]f mathematics is the study of purely imaginary states of things, poets must be great mathematicians."

—Charles Sanders Peirce¹

I prove a theorem and the house expands:
the windows jerk free to hover near the ceiling,
the ceiling floats away with a sigh.

—Rita Dove²

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1. Charles Sanders Peirce (1839–1914), "The Essence of Mathematics," in *The World of Mathematics*, vol. 3, ed. James R. Newman (New York: Simon & Schuster, 1956), pp. 3:1773–1783.

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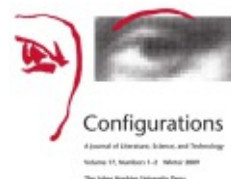
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Barry Mazur has demonstrated.⁴ “Of all escapes from reality,” wrote mathematician Giancarlo Rota, “mathematics is the most successful ever.”⁵

Not that literature is about escaping from reality, of course. It and all the visual and performing arts, as well as every discipline in the humanities and sciences for that matter, often share with mathematics a common goal: that of describing and/or addressing the “really real.” Questions of reality, truth, and certainty are at the core of the philosophy of mathematics: Does mathematics afford us entry into reality and truth? Does it provide us with certainty? Contrary to Platonist beliefs about the ability of mathematics to give us these things is the stance that mathematics is actually about multiple realities, relative truths, complexities, and ambiguities. In essence, doing pure mathematics (not merely doing computations) is an exercise in imagination—and imagination, an exercise in abstraction.

One need only recall a few key moments in the history of mathematics—the discovery of irrational numbers by the ancient Greeks; the development of non-Euclidean geometry; Kurt Gödel’s findings concerning undecidable propositions and the “incompleteness of mathematics”; chaos theory—as well as the debates and controversies surrounding these topics, to see how closely mathematics dances with uncertainties. Perhaps what mathematics shows us is that the “really real” is, in fact, a whirl of ambiguity, and that, as mathematician William Byers has written, mathematics requires thinking in terms of contradiction and paradox.⁶ Or perhaps, as Edwin Hutchins and others have argued, we exteriorize thinking through our physical environment, through marks, instruments, and the physical configurations of objects in our built environment.⁷ A related claim, but one that is vigorously contested, is the notion that George Lakoff and Rafael Núñez have proposed: that mathematics exists only because the human brain does—it is a product of it, just as metaphors and anything we make are.⁸ And so, the age-old question remains:

4. Barry Mazur, *Imagining Numbers (particularly the square root of minus fifteen)* (New York: Farrar, Straus and Giroux, 2003).

5. Giancarlo Rota, in N. G. Cooper, *From Cardinals to Chaos* (Cambridge: Cambridge University Press, 1989), p. 26.

6. William Byers, *How Mathematicians Think: Using Ambiguity, Contradiction, and Paradox to Create Mathematics* (Princeton, NJ: Princeton University Press, 2007).

7. Edwin Hutchins, *Cognition in the Wild* (Cambridge, MA: MIT Press, 1995).

8. George Lakoff and Rafael Núñez, *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being* (New York: Basic Books, 2000).

Are mathematical objects and concepts transcendental entities, Platonic ideas hovering in the out-there to be discovered, or are they invented by the human mind?

Thinking about mathematics inspires many kinds of questions: metaphysical questions on the status (existence) of mathematical objects and concepts; epistemological questions on how we “know” mathematics to be true and verify it as such; semiotic questions about the nature of mathematical language; as well as questions regarding how the mind visualizes, categorizes, systematizes, abstracts, and articulates entities that are imagined, or, in many cases, are barely imaginable at all. Among the early landmark thinkers in the philosophy of mathematics there have been those, like Plato and Kant, who have seen pure mathematics as part of the world of forms and only accessible via reason; Aristotle, who saw mathematics not as something separate from the world of sensation but as related to the way the mind performs its thought; Giordano Bruno, who believed mathematics (geometric thinking in particular) to be the link between the human and celestial worlds; Galileo and Kepler, who proposed that the universe was written in mathematics; Descartes, who built a philosophy around the certainty he believed mathematics offered; and Gottfried Wilhelm Leibniz, who saw mathematical propositions as not true of particular eternal objects or of idealized objects resulting from abstraction but as true because their denial would be logically impossible.

In the nineteenth and early twentieth centuries, discourse around the philosophy of mathematics developed considerably. George Boole, Gottlob Frege, Charles Sanders Peirce, Bertrand Russell, Alfred North Whitehead, and Rudolf Carnap (among many others) investigated the relationship between mathematics and logic (and how to reduce mathematics to logic), asking questions about topics such as certainty, consistency, equivalence, and the nature of axioms, propositions, and proofs. Thus formed what is known as the mathematical foundations movement, or “logicism,” which continues to thrive as a theory of and method for thinking about mathematics, as well as being much debated. Work on the relationship between mathematics and language—formal and natural—also increased its activity at the beginning of the twentieth century, inspiring later work in semiotics, linguistics, computer science, and cognitive science by Wittgenstein, Tarski, Montague, Chomsky, Hofstadter, and Rotman. Around the beginning of the twentieth century, the “formalist” view of mathematics emerged, which views mathematics as axiomizable symbol-entities of Peano arithmetic and ultimately comprised of strings of “game” rules (i.e., Hilbert, Curry, Bourbaki). Contrary to

this position was (and is) the “intuitionist” position, which seeks to establish mathematics as a system of mental constructs that arise from what a finite human can conceive of and prove without the use of indirect demonstrations derived from contradiction (i.e., Brouwer). And apart from adherents to the three major schools in the philosophy of mathematics (logicism, formalism, and intuitionism) are those who continue to tend toward the Platonic model, in which mathematical objects exist independently of our minds. During this last century of great mathematical advances and discussion on the nature of mathematics as a project, mathematicians Henri Poincaré, Edward Kasner and James Newman, David Hilbert, Hermann Weyl, Jacques Hadamard, and François Le Lionnais, among others, penned important biographical studies, as well as treatises, on the doing of mathematics—that is, the processes of imagining, inventing/discovering, calculating, verifying, and so on.⁹

The philosophy of mathematics, like mathematics itself, continues to flourish.¹⁰ During the last half-century, W. V. O. Quine and Hilary Putnam have proposed a notion of “mathematical empiricism,” or “naturalism” (building, to some degree, on that of J. S. Mill and on intuitionism), in which the ontology of mathematical entities and mathematical truth is based in human experience and reveals itself to be indispensable to scientific theory. Studies by Brian Rotman, Sha Xin Wei, and Andrew Pickering have examined the gestural, performative, and technical (and technological-related) aspects involved in doing mathematics: actions such as creating notations, methods, proofs, diagrams, and digital simulations.¹¹ The

9. Henri Poincaré, *Science and Hypothesis*, trans. George Bruce Halsted (New York: Science Press, 1905); Edward Kasner and James Newman, *Mathematics and the Imagination* (New York: Simon & Schuster, 1940); David Hilbert and S. Cohn Vossen, *Geometry and the Imagination*, trans. P. Nemenyi (1932; reprint, New York: Chelsea Publishing, 1952); Hermann Weyl, *The Philosophy of Mathematics and Natural Science* (Princeton, NJ: Princeton University Press, 1945); Jacques Hadamard, *An Essay on the Psychology of Invention in the Mathematical Field* (Princeton, NJ: Princeton University Press, 1945); François Le Lionnais, ed., *Great Currents of Mathematical Thought: Mathematics in the Arts and Sciences*, 2 vols., trans. H. Kline and C. Pinter (1948; reprint, New York: Dover, 1971).

10. See, for example, Paul Benacerraf and Hilary Putnam, eds., *Philosophy of Mathematics: Selected Readings* (Englewood, NJ: Prentice-Hall, 1964); Stephan Körner, *The Philosophy of Mathematics* (1968; reprint, New York: Dover Publications, 1986); Morris Kline, *Mathematical Thought from Ancient to Modern Times* (Oxford: Oxford University Press, 1972); Thomas Tymoczko, ed., *New Directions in the Philosophy of Mathematics* (Boston: Birkhäuser, 1986); Stewart Shapiro, *Thinking about Mathematics: The Philosophy of Mathematics* (Oxford: Oxford University Press, 2000).

11. Brian Rotman, *Ad Infinitum: The Ghost in the Turing Machine: Taking God Out of Mathematics and Putting the Body Back In—An Essay in Corporeal Semiotics* (Stanford, CA:

work of philosophers of mathematics such as Imre Lakatos and of mathematicians such as Reuben Hirsch is closely connected to social and cultural phenomena.¹² Social constructivists may characterize their approach as thoroughly materialist and not mentalist or logicist at all, while at the opposite end of the philosophical spectrum, metamathematics (the study of mathematics through the use of mathematics) allows us to distinguish mathematical entities from mathematical operations, examining the peculiar reflexive status of those mathematical theorems and proofs that can be used as a mathematical entity in another theorem or proof. The “structuralism” of Stewart Shapiro considers mathematics as something that describes positions of entities, rather than describing entities as objects, within a given mathematical structure (which is itself abstract and Platonic in its existence). The philosopher Alain Badiou has returned to Plato in order to reclaim what he views as the genuinely ontological capacity of mathematical thought for philosophical inquiry. For Badiou, mathematics provides a fundamental condition of possibility for philosophy, since mathematics constitutes the first form of secular thinking that, in addressing the problem of infinite multiplicity, is free from a theological concept of both the Infinite and the One while at the same time being indifferent to the implacable historicism that has characterized philosophical thought since Hegel.¹³

And what of the second term in the title of our special issue: “the imagination”? Classical, medieval, and early modern philosophers defined it as the faculty involved in the production of images of things no longer present. Both the Latin term *imaginatio* and the Greek *phantasia* (from *phaos* [light] according to Aristotle, “because it is not possible to see without light”)¹⁴ retain this association with mental imagery, whose “virtual” quality brings the imagination close to the faculty of memory while at the same time distinguishing

Stanford University Press, 1993), and *Mathematics as Sign: Writing, Imagining, Counting* (Stanford, CA: Stanford University Press, 2000); Andrew Pickering, *The Mangle of Practice: Time, Agency, and Science* (Chicago: University of Chicago Press, 1995); Sha Xin Wei, “Differential Geometrical Performance and Poiesis,” *Configurations* 12:1 (2004): 133–160.

12. Imre Lakatos, *Proofs and Refutations: The Logic of Mathematical Discovery*, ed. John Worrall and Elie Zahar (Cambridge: Cambridge University Press, 1977); Reuben Hersh, *What Is Mathematics, Really?* (Oxford: Oxford University Press, 1997).

13. See especially Alain Badiou, *Being and Event*, trans. Oliver Feltham (London: Continuum, 2006), *Number and Numbers*, trans. Robin Mackay (Cambridge, MA: Polity Press, 2008), *Theoretical Writings*, ed. and trans. Ray Brassier and Alberto Toscano (London: Continuum Press, 2004).

14. Aristotle, *De Anima*, in *The Complete Works of Aristotle*, 2 vols., ed. Jonathan Barnes (Princeton, NJ: Princeton University Press, 1984), p. 1:682 (429a4).

it from perception.¹⁵ Throughout classical and medieval accounts, imagination featured as a middle faculty between sense perception and rational intellection; it was also closely associated with “the common sense,” the faculty to which Aristotle attributed the capacity of knowing general sensations and categories of perception: redness in general, for example, rather than the redness of a particular flower. Like the common sense, the imagination was not thought to be concerned either with purely empirical or metaphysical objects. Imagination, according to Aristotle, is not thought, but it is nonetheless required for any kind of thinking to take place: “imagination is different from either perceiving or discursive thinking, though it is not found without sensation, or judgement without it,” Aristotle argues; similarly, “thinking is different from perceiving and is held to be in part imagination, in part judgement.”¹⁶

A persistent philosophical tradition associates the imagination with generating mathematical knowledge. In *De memoria et reminiscentia*, Aristotle argues that the imagination supplies the images that are necessary to all thought and then elaborates by way of analogy with geometrical diagrams: just as thought reasons about magnitude in general by drawing a picture of a triangle with a determinate magnitude, so also “without an image thinking is impossible” (450a1). Proclus’s commentary on Euclid maintains that

the understanding contains the ideas but, being unable to see them when they are wrapped up, unfolds and exposes them and presents them to the imagination sitting in the vestibule; and in imagination . . . it explicates its knowledge of them, happy in their separation from sensible things and finding in the matter of imagination a medium apt for receiving its forms. Thus thinking in geometry occurs with the aid of the imagination. Its syntheses and divisions of the figures are imaginary.¹⁷

Centuries later, Thomas Aquinas argued that geometrical figures are grasped “by means of the imagination alone, which is sometimes

15. On the imagination in classical and medieval thought, see W. D. Ross, *Aristotle* (London: Methuen, 1949), esp. pp. 142–146, 148; Michael V. Wedin, *Mind and Imagination in Aristotle* (New Haven, CT: Yale University Press, 1988); David Summers, *The Judgment of Sense* (Cambridge: Cambridge University Press, 1987); E. Ruth Harvey, *The Inward Wits: Psychological Theory in the Middle Ages and the Renaissance*, Warburg Institute Surveys 6 (London: Warburg Institute, 1975); Dennis L. Sepper, *Descartes’s Imagination: Proportion, Images, and the Activity of Thinking* (Berkeley: University of California Press, 1996); and the comprehensive study by Eva Brann, *The World of the Imagination: Sum and Substance* (Lanham, MD: Rowman & Littlefield, 1991).

16. Aristotle, *De Anima* (above, n. 14), p. 1:680 (427b14–28).

17. Cited by Brann, *World of the Imagination* (above, n. 15), p. 594.

referred to as an intellect.”¹⁸ Even Descartes, who sought to reduce geometrical relationships to algebraic symbols and equations, argued that the imagination was the place where sensory images could encounter intellectual thought and that both arithmetic and geometry provided the best models for how this was the case.¹⁹

Mathematician Barry Mazur has written engagingly on the use of the imagination in mathematical calculations of all kinds: it requires a special stretch, an extra effort that can be practiced and gradually mastered, to imagine solutions to new mathematical problems. “Imagination is held to be a movement,” Aristotle argues in *De Anima*, “a movement resulting from an actual exercise of a power of sense.”²⁰ And one key to imagining number, Mazur argues, is to imagine it, too, as a movement: number is not a thing, but a “verb,” an “act” of doubling or tripling. But imagining number as a transformational movement can be quite different from creating a mental picture, as we usually do when we imagine in a literary or artistic way, as Mazur shows (137–139). Many mathematical objects are quite difficult to imagine in the usual sense, including even the familiar natural numbers. Gottlob Frege has proposed that, if asked to imagine the number 4, we may picture the written numeral (Arabic or Roman), or four dots on a single die, or four letters in the word “gold.” In each case, however, it is not number itself that we picture, but rather a token or example of a particular grouping: we could say that the “four” we imagine is an attribute of the object or concept that we picture, when that object is considered as a multiplicity.²¹ But this is still not a direct imagination of number per se, for which, in the accounts of both Plato and Descartes, for instance, we would have to abandon the imagination (albeit temporarily) in favor of

18. Ibid.

19. See “Rules for the Direction of the Mind,” in *The Philosophical Writings of Descartes*, 2 vols., trans. John Cottingham, Robert Stoothoof, and Dugald Murdoch (Cambridge: Cambridge University Press, 1985), rule 14, esp. 1:58–62, and rule 12, esp. 1:42–43; discussed by Brann, *World of the Imagination* (above, n. 15), pp. 75–77; Sepper, *Descartes’s Imagination* (above, n. 15); Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra*, trans. Eva Brann (Cambridge, MA: MIT Press, 1968), pp. 197–211.

20. Aristotle, *De Anima* (above, n. 14), p. 1:681 (428b10–429a2).

21. Gottlob Frege, “The Concept of Number,” in Benacerraf and Putnam, *Philosophy of Mathematics* (above, n. 10), p. 132. Bertrand Russell would call it a “plurality”: “Number is what is characteristic of numbers, as *man* is what is characteristic of men. A plurality is not an instance of number, but of some particular number. A trio of men, for example, is an instance of the number 3, and the number 3 is an instance of number; but the trio is not an instance of number.” See Russell, *Introduction to Mathematical Philosophy*, 2nd ed. (London: George Allen & Unwin, 1920), p. 11.

another kind of reasoning, distinguishing the concept of “number” in general from “numbers,” and “numbers” in turn from the notion of an undifferentiated unit that allows any counting to take place.²² “Thought often leads us far beyond the imaginable without thereby depriving us of the basis for our conclusions,” Frege observes (133), while the philosopher Eva Brann points out that the objects of non-Euclidean geometry are “flawlessly thinkable yet quite unimaginable” (596).

In its relation to mathematical objects, therefore, and especially to difficult concepts such as irrational numbers, sets, or Riemannian manifolds, the imagination is perhaps best approached in several different ways. The broadest is to regard the imagination as proximate to, even continuous with, a notion of “intuition” in general: that kind of knowledge or way of knowing, common in philosophical discussions of mathematical thought, in which we seem to grasp abstract insights immediately and without conscious reasoning. This position could be said to be broadly Kantian,²³ and it has obvious affinities with the intuitionist position in the philosophy of mathematics as described by Arend Heyting.²⁴ Heyting has written an engaging dialogue to illustrate the positions held by the intuitionist (characterized as “Int”), who is at one point addressed by the character “Letter,” who objects specifically to the intuitionist treatment of infinity: it introduces “obscurity and confusion” (71) and reveals

22. See the discussion of *arithmos* and *logistic* in classical Greek mathematics by Klein, *Greek Mathematical Thought* (above, n. 19), pp. 10–25, 46–60: “Only that can be ‘counted’ which is *not one*, which is before us in a certain number: neither an object of sense nor *one* ‘pure’ unit is a *number* of things or units” (49); see also Klein’s discussion of the problem in Descartes (*ibid.*, pp. 199–202).

23. This is the approach that Eva Brann takes, arguing that intuition provides the “mental place or medium which the imagination activates and in which it inscribes its figures”; see Brann, *World of the Imagination* (above, n. 15), pp. 584–587. Barry Mazur also describes a version when he distinguishes two acts of imagination: “the generation of a single visual image in our minds as we read lines of verse,” and “the cultivation of a comprehensive inner intuition for *imaginary numbers*, the fruit of collective imaginative labors over time”; see Mazur, *Imagining Numbers* (above, n. 4), p. 157. On Kant, see Brann, pp. 585, 594–595, and Bernard Freytag, *Imagination in Kant’s Critique of Practical Reason* (Bloomington: Indiana University Press, 2005).

24. For the intuitionist, statements that presume the existence of a mathematical object are illegitimate, because they introduce a metaphysical, rather than a strictly mathematical (i.e., calculable) concept. Mathematical objects can only be calculated according to rules defined by mathematicians; questions and beliefs about the putative existence of these objects must simply be set aside. See Arend Heyting, in Benacerraf and Putnam, *Philosophy of Mathematics* (above, n. 10), p. 53, and “Disputation,” pp. 67, 70: “A mathematical construction ought to be so immediate to the mind and its result so clear that it needs no foundation whatsoever.”

the intuitionist as “not only dogmatic, but even theological” in his metaphysical assumptions about “objective and eternal truths.” Int calmly responds with an account that we could regard as being eminently imaginary in the sense given above and defends the intuitionist position by aligning it with philosophy, history, the social sciences, and even “arts, sports, and light entertainment” (74)—the last activities in which usefulness of application is not the criterion of success. The very problem of mathematical “reality” must never be presumed but always be at issue, Int argues: “It is too often forgotten that the truth of . . . constructions depends upon the present state of science and that the words ‘in reality’ ought to be translated into ‘according to the contemporary view of scientists’” (75).

As noted above, this problem of the “realness” or “unrealness” of *all* mathematical objects—and not simply specific objects such as imaginary numbers—represents an important domain of discussion in the philosophy of mathematics and presents yet another way to discuss a relationship between mathematics and the imagination. Are mathematical objects, such as integers and sets, or mathematical relations, such as one–one correlation among sets, “real”? Brann has pointed out that one of the imagination’s most familiar qualities is its capacity for being “twice unreal”: the imagination produces images that are not real in the sense of existing independently outside the mind, and these images can be of things that do not exist in a real sense (425), such as unicorns and chimeras. From Plato onward, fiction has provided philosophy with its most potent examples of such “unreal” entities. But mathematics also can be said to provide equally complex objects of an unreal or, more controversially, of a fictional type. The sixteenth-century Italian mathematician Gerolamo Cardano referred to all negative numbers, as well as imaginary numbers, as *fictae*.²⁵ Jeremy Bentham offered a systematic account of the “fictions” of thought and language, which encompassed primary fictional entities such as quantity, quality, motion, and relation and included numbers and mathematical operations of simple and complex types. In Bentham’s view, real entities are sensible entities, but they are also the ideas we form from sense impressions (for Bentham, the idea of a cat is as real as the furry, whiskered creature so many of us have seen and pet). Fictional entities, in turn, are linguistic artifacts, invented by the imagination, which we take *as if* they were real; they are *necessary* entities without which discourse on many subjects would be impossible.²⁶

25. Mazur, *Imagining Numbers* (above, n. 4), p. 40.

26. Jeremy Bentham, *Bentham’s Theory of Fictions*, ed. C. K. Ogden (New York: Harcourt, Brace and Co., 1932), pp. xlii–xliii.

Since, for Bentham, mathematics deals with “physical existences, i.e. bodies and portions of space . . . in respect of their quantities and nothing else,” and since quantity is itself a category of fiction, mathematical ideas are also a species of fiction.²⁷ Number, whether taken as a pure abstraction (as a unit of quantity) or as a notation (a word or symbol), is also a fiction, as are the various “contrivances” (as Bentham calls them) that are developed in mathematics to find the unknown solution to a problem (we would call them “operations”).²⁸ These fictions include the four basic operations (addition, subtraction, multiplication, division), as well as higher-level operations such as the finding of tangents, the translation of algebraic equations into geometrical figures, and the use of calculus. Insofar as mathematical notation fixes conventional signs to ideas that are thinkable only through the system of notation that allows us to manipulate them, for Bentham, mathematics is like a language, and the ideas of relation that it designates are fictions.²⁹

One reason Bentham’s theory of fiction could be so extensive was because he had categorically distinguished “fabulous” objects from fictional ones, assigning to the category of the fabulous fictions of the literary or mythical type.³⁰ The usefulness of this distinction in thinking about mathematical objects is visible in Bertrand Russell’s similar notion of “logical” or “symbolic fiction” in his *Introduction to Mathematical Philosophy* (1919), in which he uses the notion of “fiction” to characterize both numbers and sets (or “classes”—numbers being defined in terms of classes by Russell). Throughout the *Introduction*, Russell evinces an ambivalent, even contradictory attitude toward fiction and the imagination. In his view, we accept mathematical fictions such as number and “class” on the grounds of logic, and indeed they have a foundational place in mathematical reasoning. But we should remain “agnostic” toward the question of their existence or reality, brushing the dust of the imagination off our hands as we advance the argument.³¹ And we must certainly distinguish logical fictions from distinctly *unreal* literary fictions such as *Hamlet*, as real as Shakespeare’s own imagination must have been

27. *Ibid.*, p. xcv.

28. *Ibid.*, pp. 25, 52.

29. *Ibid.*, pp. xcvi–cvi. On the relationship between fiction and the philosophy of mathematics, see Jean-Pierre Cléro, *Les raisons de la fiction* (Paris: Armand Colin, 2004).

30. Bentham, *Bentham’s Theory of Fictions* (above, n. 26), pp. xxxv–xxxvi.

31. Russell, *Introduction* (above, n. 21), pp. 14, 45, 137, 138, 140, 182, 184.

when he wrote the play.³² In this sense, Russell effects a kind of “settlement” (in Latour’s terms) between science and literature by asserting a distinction between them, and then fortifying that distinction, as necessary, with disdain and dismissiveness.³³

Admitting the question of mathematical reality into direct inquiry, however, in turn raises intriguing questions about the nature of the imagination. If one accepts that the imagination is necessary to mathematical thought, one is led to the paradoxical statement that mathematical entities are *both* real and imaginary, or real because imaginary, or real insofar as the imagination allows us to think of them as real—which is why the concept of fiction is so useful, and we might borrow a term of classification from literary study and call such mathematical entities *realist*. Rudolf Carnap has sought to clarify the question of mathematical realism by distinguishing claims that pertain within a given system of explanation (“internal” claims), from claims made about facts outside the system (“external” claims).³⁴ In Carnap’s view, asking whether mathematical objects such as numbers “really exist” confuses an external problem with an internal one, since it would be trivial to pose the question of reality to any mathematical system of explanation that presumes that reality (prompting the answer, “Of course they are!”). Serious questions about the reality of numbers could only belong to a different domain—philosophical speculation, say, or certain kinds of reflection prompted by naïve experience, or musings by literary critics or other nonmathematicians—that is conducted according to different systems of explanation. Thus the pressing question—“But are numbers *real*?”—should be understood, in Carnap’s view, as a disingenuous version of the more proper question: “Should I accept this system of explanation about the realness of numbers as useful or relevant to me or not”? It is not a theoretical or metaphysical question that presents itself, but a practical one—namely, a question of whether or not one system of explanation is to be preferred over another in a given circumstance or for a given problem.

32. *Ibid.*, pp. 168–169.

33. Hartry H. Field has proposed a controversial theory of “mathematical fictionalism,” which aims to demonstrate how science could be done without mathematics if one were to accept that mathematics is based on notions that are not truths or actual objects, but rather on constructed conventions. According to Field, mathematics is, in essence, akin to narrative fiction with its own internal logic (“nominalism” is a similar theory). See Field, *Science Without Numbers* (Princeton, NJ: Princeton University Press, 1980), and *Realism, Mathematics, and Modality* (Oxford: Blackwell, 1989).

34. Rudolf Carnap, “Empiricism, Semantics, and Ontology,” in Benacerraf and Putnam, *Philosophy of Mathematics* (above, n. 10), pp. 241–257.

If we adopt Carnap's position, we may argue that the imagination allows us to cross over from one domain of explanation (philosophy, or psychology, or literature) to another (mathematics); we could go further and say that *the imagination is that faculty of thinking that facilitates movement across systems of explanation that seem irreconcilable*, and that, as a consequence, allows for new thoughts, new arguments, and new explanations to occur. The question then becomes one of recognition: Do we acknowledge that we are employing our imagination or not? A more technical way to make the same point would be to turn to a fairly restricted definition of the imagination typical of recent work in the philosophy of mind, which regards the imagination as fundamentally "propositional": specifically, the imagination allows us to formulate propositions about hypothetical conditions, possible worlds, counter-factual arguments, and statements about what might be the case.³⁵ Understood in this way, the imagination would be fundamental to many different kinds of mathematical arguments; indeed, it is difficult to conceive of any discussion of a mathematical problem that does not presume it, at least implicitly, from the word problems of children's textbooks to the most sophisticated equations. But we may generalize from this definition and describe the imagination as that which facilitates the invention of new propositions about the world by drawing on propositions that we already know, a definition of the imagination that both recalls classical discussion and that seems especially congenial to our neo-Romantic moment, which continues to prize notions of creativity and discovery. Instead of individual or personal creativity and discovery, however, the imagination would, in this view, facilitate a kind of *collective* thinking across disciplines and within institutions; it would mark a certain adventurousness or experimentation in intellectual work that results when one borrows from many systems of explanation at the same time.³⁶

This special issue contemplates the relationship between mathematics and the imagination; that is, how/what mathematicians

35. Shaun Nichols, ed., *The Architecture of the Imagination: New Essays on Pretence, Possibility, and Fiction* (Oxford: Oxford University Press, 2006), pp. 1–10.

36. Mazur has proposed a similar notion of collective imagination over time in the history of mathematics and compares it to the development of epic poetry (*Imagining Numbers* [above, n. 4], pp. 156–157). Nor would this mode of thinking necessarily have to be "human," however, if we presume artificial conditions that facilitate juxtaposition, selection, or pattern-recognition from a very wide array of data: in this view, a computer could be said to be working "imaginatively" once it crosses a given threshold of diversity in the data that it synthesizes (which may account for the certain aura of "imaginativeness" that surrounds search engines such as Google).

imagine when they do math, and how mathematics is imagined by mathematicians and nonmathematicians alike. While the essays range across disciplines and time periods, there are numerous threads connecting them, such as the persistent question of whether mathematics is transcendent/external or imminent/internal; the quest for understanding the nature of abstract thought and how to represent it; the reality of mathematical entities and concepts; explorations of topological theory; and the impact of social relationships and politics on the development of mathematical thought and practice.

The six essays are arranged chronologically by topic. The first essay, "Imagination and Layered Ontology in Greek Mathematics," is by classicist Reviel Netz, who examines the use of the imagination in Greek mathematics when envisioning the virtual presence of geometrical objects, such as a point or circle. Netz performs a philological analysis of the term *noein*, and how for Greek mathematicians, especially Archimedes, it primarily implied a kind of "seeing-as" operator, offering a layered ontology of both reality and the imaginary. Drawing on a study of *noein* by von Fritz and on the kinds of seeing that Wittgenstein describes in his *Philosophical Investigations*, Netz dissects the "philosophical grammar" of the verb and shows how it is *not* about imagining a mathematical object in such a way as to bring it into existence. To the Greeks, mathematics only pointed to the really real and ideal forms; it could never completely present or represent it. What was to be imagined was a "trace" of the object—hence the layered ontology. In Greek mathematics, this sort of envisioning was an act of "make-believe seeing as," which is why he translates *noein* as "to imagine," instead of the equally viable "to understand" or "to construe." In observing how Greek mathematicians used this term, Netz also discusses the low status of the diagram in classical mathematics, and how Greek mathematical writing required the reader to imagine in the mind's eye what the equations and proofs are describing: "As long as the object is possible, it does not really matter [to the Greek mathematician] that it is purely imaginary; mathematics is the art of the possible." And this "art of the possible" was also conceived by the Greeks as hypothetical, which is not what we generally think of today when we think of the "certainty" that mathematical reasoning offers. One of the consequences of Netz's essay is a clear demonstration of how much perceptions of reality and of mathematical truth have changed since antiquity.

Following Netz's piece is Robert Goulding's analysis of the intellectual battle between Petrus Ramus and Jacques Charpentier over

how classical mathematics should be conceived. As Goulding shows in “Pythagoras in Paris: Petrus Ramus Imagines the Prehistory of Mathematics,” one of Ramus’s main tactics was to rewrite elements in the history of mathematics so as to defend the notion of mathematics as applicable and linked to the physical world, taught to all, and not arcane, metaphysical, or spiritual, as the scholastic Charpentier would have it. The conflict between Ramus and Charpentier was highly public and politicized, especially given the latter’s links to the elite oligarchy of France’s old regime. Politics, religion, academic freedom, pedagogy, self-fashioning, and ideologies on the nature and value of progress all played their part in the debates. Although Charpentier, with his old-school thinking and old-regime ties won the battle, Ramus’s vision for the teaching and doing of mathematics would ultimately win the war, the scientific revolution already beginning to be born. Goulding’s essay brings to the fore Ramus’s impulse to author a history of mathematics, the *Prooemium mathematicum* (1567)—something that had yet to be done in any formal, extensive, or, most importantly, continuous (as opposed to fragmentary) way. The imagination thus serves in a multiple capacity in Goulding’s essay: as part of what Ramus saw as central to the doing of mathematics; as central to Ramus’s construction of the past, especially his refashioning of Pythagoras as the ideal university professor and originator of the reforms he wanted to see at the University of Paris; and as fundamental to all the liberal arts.

Just postdating the Ramus–Charpentier debate and also located within French borders is Michel de Montaigne’s 1580 *Essais*. In “A Devil in Diversion: Number and Line in the *Essais*,” Tom Conley offers a study of the geometry, topography, and typography at play (literally) in the language of Montaigne’s famous collection of autobiographical anecdotes and philosophical contemplations. The *Essais*, in particular two essays of the third volume (“Of Three Kinds of Associations” and “Of Diversion”), Conley shows, utilize a sort of ludic mathematics: ciphering numbers, proportions, and measures in curious discursive and indexical ways. The reader must thus take on the role of the decipherer, armed with knowledge of quantitative functions, especially those found in fields such as combinatorics and mapmaking. Writer and reader enter into games of *gematria* and plotting that make manifest not only what Montaigne wishes to convey, but an indication of prose’s—and language’s in general—natural and powerful links to the computational, spatial, and visual. Conley’s “topographic reading” of the *Essais* embraces what could be called a “tropological mathematics.” Conley cites Michel de Certeau’s discussions on the aesthetics of space (and evokes that

of Bachelard), as well as Derrida's and Deleuze's considerations of the formal nature of language, but focuses primarily on the typographic tricks and epistemological paradoxes of rhetorical figures such as double entendre, mise en abyme, and triadic formulations, as well as on transpositional tropes such as metathesis. Montaigne's imagining of how numeric relations reflect and even inspire human relations compels us to look at the numerations and calculations we make—and those we reject—when we read, write, think, and live.

Considering topology as a mathematical field from the point of view of one of its early creators, the next essay, "Bernhard Riemann's Conceptual Mathematics and the Idea of Space," moves us into nineteenth-century Germany and into the remarkable imagination of mathematician G. F. Bernhard Riemann. Arkady Plotnitsky chooses as his focus Riemann's work on manifolds—an analysis of space in terms of subspaces and the relations among them—and investigates how Riemann develops this new "spatiality" by engaging with the concept of "concept" in mathematical thinking. Plotnitsky builds on three thinkers for whom Riemann's work was of great inspiration: the mathematician Hermann Weyl, for his theories on the interaction between mathematical thinking and phenomenal intuition; and Gilles Deleuze and Félix Guattari, for their writing on how the mind confronts *doxa* and chaos. What Plotnitsky reveals is that Riemann, when doing mathematics, thought in terms of "concepts" (as many mathematicians do, and discuss doing), rather than in terms of formula or manipulations of sets (as Georg Cantor would later do) or in other possible thought-modes (there are many), and that his theory of manifolds emerged from a broad "sociological" and "material" understanding of space, responding to both *doxa* and chaos. Plotnitsky sees Riemann's mathematical and philosophical thought and imagination as deeply interlinked. Riemann's work displays an extraordinary imaginative power for encountering and translating many sorts of phenomena, so much so as to contribute revolutionary notions and advances in nearly all fields of mathematics and elsewhere, most especially in physics, where Riemann's concept of manifold was used by Einstein in his general relativity theory—a non-Newtonian theory of gravitation.

Art historian Linda Dalrymple Henderson contributes the fifth essay in the issue, "The Image and Imagination of the Fourth Dimension in Twentieth-Century Art and Culture," by extending her ground-breaking work on early twentieth-century mathematics, physics, and art with a discussion of how the fourth dimension was imagined among mathematicians and artists from cubism to the

age of the computer. Henderson shows how popular publications on mathematics by E. A. Abbott, Claude Bragdon, David Hilbert, and Henri Poincaré, among others, drew on the visual and iconic aspects of the imagination to explain new geometrical models of space, and how these books in turn influenced visual artists who were seeking graphic conventions for representing space in new ways. But the fourth dimension was more than a mathematical hypothesis, becoming the basis for liberatory and utopian worldviews such as Charles Howard Hinton's "hyperspace philosophy," which argued that human minds loosened from their conventional categories and released into the new limitless space of free thought would revolutionize mental and physical reality. Henderson examines the place of the imagination and the fourth dimension in cubism, suprematist abstraction, and in the work of Marcel Duchamp, before turning to the art of Tony Robbin, "the most serious artist-scholar in four-dimensional geometry of the twentieth century." She shows how artists experimented with a range of techniques for representing the invisible and intellectual formations of mathematical equations, whether by experimenting with color and modeling complex sensory perception (as in the work of Juan Gris), by pushing toward greater abstraction (that of Kazimir Malevich), by creating layers of allegory and visual play (Duchamp's *The Large Glass*), or by collaborating with mathematicians and painting from computer-generated models of geometrical relationship (Robbin).

The final essay, "A Hyperspace Poetics, or, Words in Space: Digital Poetry Through Ezra Pound's Vorticism," by literature scholar Lori Emerson concludes the issue by establishing conceptual links between early twentieth-century Vorticism and contemporary digital poetry, showing how non-Euclidean geometries and new philosophies of mathematics provided the ground for experiments in poetic form by Ezra Pound, experiments that in turn allow us to understand the "kinetic" verse of contemporary digital poets such as David Knoebel, Eduardo Kac, and Jim Rosenberg as fundamentally continuous, in both formal and intellectual terms, with the earlier modernist moment. Emerson interleaves her discussion of Pound's poetics and of the larger shifts in mathematical thought about space that characterized the early twentieth century with italicized reflections on contemporary digital poetry, particularly its interactivity, its complex play between materiality and virtuality, and its self-conscious exploitation of its screened mode of appearance, in this way suggesting how digital poetry, like new media more broadly, requires reading practices that reorient in turn how we approach the traditional poetry of high modernism.

The essays presented here offer examples of the new questions that become possible once we begin to think imaginatively about mathematics, and, at the same time, to think mathematically about the imagination. Scholars in neuroscience, psychology, and cognitive science have been examining how the brain does math,³⁷ how the brain perceives and knows what it knows,³⁸ and how to model thought through geometrical representation.³⁹ Ethnomathematics, linked to both anthropology and the history of mathematics, considers the cultural relativism of mathematical understanding, formation, and education.⁴⁰ Women's studies and feminist studies have looked at the role of gender in the doing of mathematics.⁴¹ Mathematical models have been subjects of and have informed a great amount of literary theory.⁴² The role of mathematics in digital arts is a large and expanding topic, especially when discussing computer-based interactive literature, gesture and performance, and electronic music.⁴³ There have been annual international conferences

37. Stanislas Dehaene, *The Number Sense: How the Mind Creates Mathematics* (Oxford: Oxford University Press, 1997); Brian Butterworth, *What Counts: How Every Brain Is Hardwired for Math* (New York: Free Press, 1999).

38. Jean-Pierre Changeux, *The Physiology of Truth: Neuroscience and Human Knowledge*, trans. M. B. De Bevoise (Cambridge, MA: Harvard University Press, 2004).

39. Peter Gärdenfors, *Conceptual Spaces: The Geometry of Thought* (Cambridge, MA: MIT Press, 2000).

40. See, for example, the classic works of Florian Cajori, *A History of Mathematical Notation* (1928–29; reprint, New York: Dover Publications, 1993); Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers* trans. P. Broneer (1924; reprint, Cambridge: MIT Press, 1969); Claudia Zaslavsky, *Africa Counts: Number and Pattern in African Culture* (Boston: Prindle, Weber & Schmidt, 1973); and the more recent, Bill Barton, *The Language of Mathematics: Telling Mathematical Tales* (New York: Springer-Verlag, 2007), and John Allen Paulos, *Mathematics and Humor: A Study of the Logic of Humor* (Chicago: University of Chicago Press, 1982).

41. See, for example, Bonnie Schulman, "What If We Change Our Axioms? A Feminist Inquiry into the Foundations of Mathematics," *Configurations* 4:3 (1996): 427–451, and "Implications of Feminist Critiques of Science for the Teaching of Mathematics and Science," in *The Gender and Science Reader*, ed. M. Lederman and I. Bartsch (London: Routledge, 2001).

42. See, for example, N. Katherine Hayles, *Chaos and Order: Complex Dynamics in Literature and Science* (Chicago: University of Chicago Press, 1991), and *Electronic Literature* (South Bend, IN: University of Notre Dame Press, 2008); Franco Moretti, *Graphs, Maps, and Trees: Abstract Models for a Literary History* (New York: Verso, 2005).

43. See, for example, Nick Montfort, *Twisty Little Passages: An Approach to Interactive Fiction* (Cambridge, MA: MIT Press, 2005); Sha, "Differential Geometrical Performance" (above, n. 11); Aden Evens, *Sound Ideas: Music, Machines, and Experience* (Minneapolis: University of Minnesota Press, 2005); David Benson, *Music: A Mathematical Offering* (Cambridge: Cambridge University Press, 2006).

on mathematics and the arts, and a high-profile conference on "Mathematics and Narrative" in Greece in 2005.⁴⁴ Interdisciplinary programs, such as Dartmouth's "Mathematics across the Curriculum," are being established. And it is difficult not to notice the last decade's flurry of films, biographies, plays, and television shows on mathematicians and/or involving mathematics.⁴⁵

Some of these new questions have found provisional answers, while others will prompt future inquiry: How have advances in mathematical knowledge affected the ways we experience and depict the world? Do mathematicians imagine differently than poets, painters, philosophers, or novelists? Are the cognitive operations necessary for solving problems, writing equations, and engaging in abstract mathematical thought the same as those used in other sorts of creative endeavors? Are the semiotic differences among words, numbers, and diagrams as distinct as they seem? How have accounts of the imagination (philosophical, psychological, physiological, neurological, literary, aesthetic) positioned it in relation to the kind of knowledge that mathematics is thought to provide? Can understanding mathematical concepts such as object, model, structure, relation, function, derivation, abstraction, transformation, property, identity, infinity, and so on enrich our experience and use of these concepts in other fields? What is beauty in mathematics? This special issue of *Configurations* invites us to take a few steps toward these questions, and toward others we have not *yet* imagined—toward what mathematician Hermann Weyl called "thinking the possible."⁴⁶

44. Since 1998, the Bridges Organization, which focuses on "mathematical connects in art, music, and science," has been holding an annual conference; see <http://www.bridgesmathart.org/>. The "Mathematics and Narrative" meeting organized by Thales + Friends, "Bridging the Chasm between Mathematics and Culture," was held in Mykonos, Greece, on July 12–15, 2005; see http://thalesandfriends.org/en/index.php?option=com_content&task=view&id=41&Itemid=76.

45. Examples of films include Gus Van Sant's *Good Will Hunting* (1997), Darren Aronofsky's *Pi* (1998), Ron Howard's adaptation of *A Beautiful Mind* (2001), and John Madden's *Proof* (2005); television shows include *Numb3rs* and *Lost*; plays include Tom Stoppard's *Arcadia* (1993) and Michael Frayn's *Copenhagen* (2000).

46. Hermann Weyl, "The Unity of Knowledge," a lecture given at the bicentennial of Columbia University in 1954; reprinted in Raymond G. Ayoub, ed., *Musings of the Masters: An Anthology of Mathematical Reflections* (Washington, DC: Mathematical Association of America, 2004): "Summarizing our discussion I come to this conclusion. At the basis of all knowledge there lies: (1) *Intuition*, mind's originary act of "seeing" what is given to him . . . (2) *Understanding and expression* . . . (3) *Thinking the possible*. In science a very stringent form of it is exercise when, by thinking out the possibilities of the mathematical game, we try to make sure that the game will never lead to a contradiction; a much freer form is the imagination by which theories are conceived. Here, of course, lies a source of subjectivity for the direction in which science develops" (76).