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SOME EXAMPLES OF GREEK ARITHMETIC.

PAPYRUS No. 186 of the *Papiri della Società Italiana* contains two arithmetical problems which have not been adequately dealt with by the editors. They say: “Poichè non sono conservati tutti i dati, non è possibile controllare la soluzione, secondo cui si avrebbero 8400 spettatori al teatro, e 4860 artabe nel vaso.” It is, however, easy, in both cases, to reproduce the complete mathematical calculation, although it may be impossible to restore with certainty the exact words. The text of the papyrus, which is assigned to the fourth century A.D., and is written in extremely bad Greek, is as follows:—

Recto.

δέαδρον τὸ μὲν ἄνω [
 π, τὸ δὲ κάτω βάθρον [— βα-]
 θρων ρκ, εὐρεῖν πόσ[ους?
 χωρήσει τὸ θέαδρον]ν
 5 σύνθεσ τῷ ἄνω βάθ[ρω
 κάτω, τουτέστιν π [
 ἐπὶ τὰς τῶν βάθρων ρ[κ?
 χωρήσι τὸ θέαδρον τῷ [
 ην ἐπὶ τῶν ὁμοί[ων
 10 σχῆμα ὑπόκειται.

(*Figure.*)

Verso.

|γυλοῦν, οὗ ἡ διάμετρος
]βάθος πηχ' λ, εὐρεῖν
]* χωρήσει τὰ η τῆς δια-

15 [μέτρον —] α γί(νονται) ζδ τούτων οί φενό-
 [μενοι —] μη ἐπὶ τὸ βάθος πηχ λ''
]* ον γ δ'η' διὰ τί ἐπὶ τὸν
 σ]τερεὸς πηχ' χωρήσει ξηροῦ
]χωρήσει ξηροῦ ὃ δωξ'.

(Figure.)

These problems resemble very closely some of those in the mathematical papyrus of Akhmîm (J. Baillet, *Mém. de la Mission Arch. Franc. au Caire*, ix, 1892), from which we may make some inferences as to the way in which arithmetic was taught in Egyptian schools. It is, I think, quite certain that the Akhmîm papyrus is the exercise-book of a schoolboy; it is a collection of disconnected problems, with no method in their arrangement, except that the earlier are somewhat easier than the later problems, as would be expected in a schoolboy's book. The writer has also made mistakes in his work, which are not mere errors of calculation, but due to the application of wrong methods. A remarkable feature of this papyrus is the use of the word *ὁμοίως* to introduce the solution and often some of the subsidiary steps. The writer was clearly working to a pattern, and we must suppose that, instead of learning general principles, he was taught a separate rule for each class of problems, and given a model example to copy: a method which, unfortunately, is only too common at the present day. Behind the solution of each of these Greek problems there lies a general formula, which we should now represent by algebraical symbols: by the Greeks it was probably expressed in words, accompanied by a particular example. A student, having such an example before his eyes, when working out a similar problem with different numbers, might naturally introduce the several steps of his calculation by the word *ὁμοίως*.

The first problem is a good example of his procedure,

and is also instructive, because the result is wrong ; if we can discover the source of the error, we shall learn a good deal about the methods of teaching Greek logistic.

λάκκος στογύλουν ἦν ἄνω περιμετρος πυχῶν κ
 ἡ κάτω περιμετρος πυχῶν ιβ βάθος πυχῶν σL
 ὁμοίως κ και ιβ γί(νονται) λβ τὸ L τῶν λβ γί(νεται) ις ὁμοίως
 ις ἐπὶ ις γί(νεται) σνς ὁμοίως σνς ἐπὶ σL
 γί(νεται) ,αχξδ μέρισον λς ὅς εἶναι
 μς σ' ιη'

“ There was a circular pit. Its upper perimeter was 20 cubits, its lower perimeter was 12 cubits, its depth $6\frac{1}{2}$ cubits (find its contents). Similarly $20 + 12 = 32$; $\frac{1}{2}$ of $32 = 16$; similarly $16 \times 16 = 256$; similarly $256 \times 6\frac{1}{2} = 1664$; divide by 36, so that the result is $46\frac{1}{8} \frac{1}{16}$.”

The difficulty is the final division by 36. Baillet says : “ Au lieu d’une division par 12, nous trouvons une division par 36, soit par un nombre 3 fois plus fort. Ou bien il y a là une erreur grossière, ou bien il y a un changement d’unité non annoncé, de même que cela semble avoir lieu également dans le problème 2.” The theory of a change of unit is ingenious, but, I think, untenable. It is true that not only in problem 2, where the result is multiplied by $3\frac{3}{8}$, but also in problem 5, where the result is divided by 27, there is a change of unit, a most important change, which will be discussed later ; but for the present it is sufficient to remark that both $3\frac{3}{8}$ and 27 are perfect cubes, and therefore the change is in the linear unit. In this case we should have two units of volume in the ratio 3 : 1, and the corresponding linear units would be incommensurable. We must have recourse to the alternative that there is a gross error ; but I think that the source of this error can be determined.

The general problem, of which this is a particular case,

may be stated in more modern language thus :—Find the volume of a frustum of a circular cone, given the perimeter (P) of the upper circle, the perimeter (p) of the lower circle, and the distance (d) between them. It is known that sometimes, as in P. Ox. III. 470, the ancients erroneously substituted for this volume that of a cylinder of the same height standing on a base with a radius equal to half the sum of the given radii. The volume of such a cylinder is $\left(\frac{P+p}{2}\right)^2 \frac{d}{4\pi}$. If we take $\pi = 3$, the rule as stated to the Greek student would be :—

Add the perimeters of the upper and lower circles; divide by 2; square the quotient; multiply by the height, and divide by 12. This is exactly what the writer of the papyrus has done, except that in the last step he has divided by 36 instead of 12.

The true value of the volume of a cone was well known to the Greeks. Eudoxus is said to have been the first to discover the proof, but we now know that Archimedes in the *Ἑφῶδος* (see *Hermes*, vol. xlii, 1907, p. 245) ascribed much of the merit to Democritus, who first stated the theorem without proof. An accurate formula for the volume of a truncated cone, a *κῶνος*, *κόλουρος*, such as is considered in this problem, is given by Heron of Alexandria (*Metrika*, II. 9); but it is expressed in terms of the diameters instead of the perimeters of the bounding circles, and the value of π is taken as $3\frac{1}{7}$. In terms of the perimeters the volume is $(P^2 + p^2 + Pp) \frac{d}{12\pi}$; and the rule for calculating it would have been—Add together the squares of the perimeters and their product. Multiply by the height and divide by 36 ($= 12\pi$).

I think it is almost certain that the writer of the papyrus was given both rules, as alternatives, for problems of this kind, and that he in this case adopted the first rule, but in

his last step employed the divisor belonging to the second rule.

The second problem of the Akhmîn papyrus is very simple, but of great importance, because a change of unit is introduced. It is to find the contents of a *θησαυρὸς τετράγωνος* measuring $10 \times 10 \times 8$ cubits. The three dimensions are multiplied together, giving 800 cubic cubits, and this product is then multiplied by $3\frac{3}{8}$. Baillet did not recognize that the symbol here used was that for artabae, but thought it stood for "l'unité," probably because he wrongly interpreted the symbol for aroura supposing it to stand for artaba. But he guessed that the artaba was the measure intended, for he says, "Cette mesure serait l'artabe, assimilée au pied cube."

In the second of the problems in the Italian papyrus the same multiplier is employed to reduce cubic cubits to artabae.

Wilcken (*Ostr.* I, p. 752) has pointed out that we can deduce from this the important equation:—

$$1 \text{ cubic ell} = 3\frac{3}{8} \text{ artabae.}$$

We can go a step further than this, and assert that, at least theoretically, an artaba was the amount of dry produce contained in one cubic foot. Since $3\frac{3}{8}$ is the cube of $1\frac{1}{2}$, the linear unit must be $\frac{2}{3}$ of a cubit; and, since a cubit is $1\frac{1}{2}$ feet, the linear unit is 1 foot.

It is known that artabae of different sizes were used in Ptolemaic times. Wilcken (*Ostr.* I, p. 744) enumerates artabae of 40, 30, 29, 26, and 24 choenices. The existence of the first is proved by P. P. II, 25, of the second by Rev. Pap. 39, 2. The third (P. Gren. I. 18, 19) was a private measure, which may have been intended to hold 30, but was found, when tested, to contain only 29 choenices: but the reading is doubtful, and Wilcken suggests (*Archiv.* II, p. 123) that *πρὸς τὸ βα(σιλικόν) χ(αλκοῦν)* should be read for

$\pi\rho\acute{o}\varsigma \tau\acute{o} \kappa\theta\bar{\chi}$; the fourth is also doubtful, depending, as it does, upon an uncertain reading of a single ostrakon. We are thus left with three artabae of 40, 30, and 24 choenices. It is certain (see P. P. III. App. and P. Lille 1) that *naubia* and *aoilia* were generally measured by the double royal cubit; but, in one instance (P. P. III, p. 125) it was specially directed that the *aoilia* were to be measured $\tau\omega\iota \tau\rho\iota\sigma\kappa\alpha\iota\delta\epsilon\kappa\alpha\pi\alpha\lambda\alpha\sigma\tau\omega\iota \mu\epsilon\tau\rho\omega\iota$. Hence we have three different linear measures, proportional to 14, 13, and 12, employed for calculating volumes. If the variation in the size of the artaba depended upon this variation in the linear unit, we should expect to find three artabae proportional to the cubes of these numbers: that is, in the proportion 2744 : 2197 : 1728, which is equivalent to 38·1 : 30·5 : 24. This is so close to the ratios of the known artabae 40 : 30 : 24, that there can be little doubt that the variation in the size of the artaba was due to the difference in the linear feet by which the vessels were measured.

The fifth problem is interesting because it is another example of the application of a wrong method, and because it contains another and different unit of capacity. The problem is:—Given a rectangular 'channel ($\delta\iota\acute{o}\rho\nu\xi \tau\epsilon\tau\rho\acute{\alpha}\gamma\omicron\nu\omicron\varsigma$), length 20 cubits, breadth 8 cubits, and depth $3\frac{3}{4}$ cubits, to find its contents. He should have multiplied together the three linear dimensions; but, instead of doing so, says:— $\acute{o}\mu\omicron\iota\omega\varsigma \kappa \kappa\alpha\iota \eta \gamma\acute{\iota}\gamma\nu\epsilon\tau\alpha\iota \kappa\eta, \tau\acute{o} L \tau\acute{\omega}\nu \kappa\eta \gamma\acute{\iota}\gamma\nu\epsilon\tau\alpha\iota \iota\delta, \iota\delta \acute{\epsilon}\pi\iota \iota\delta \gamma\acute{\iota}\gamma\nu\epsilon\tau\alpha\iota \rho\theta\varsigma, \rho\theta\varsigma \acute{\epsilon}\pi\iota \gamma L \delta' \gamma\acute{\iota}\gamma\nu\epsilon\tau\alpha\iota \psi\lambda\epsilon \mu\epsilon\rho. \kappa\zeta, \acute{o}\varsigma \acute{\epsilon}\iota\nu\alpha\iota \kappa\zeta\varsigma' \iota\acute{\eta}$. “Similarly $20 + 8 = 28$; $\frac{1}{2}$ of 28 = 14; $14^2 = 196$; $196 \times 3\frac{3}{4} = 735$; divide by 27, so that the result is $27\frac{1}{8} + \frac{1}{18}$.”

Thus he uses the formula $\left(\frac{P + p}{2}\right)^2 d$, as in the first problem following a model intended for a different class of question. But the most important point is that the result in cubic cubits is divided by 27. This implies a unit of volume measured by a cube with a side of 3 cubits: can it be

identified? It should be observed that the problem relates to a διῶρυξ, and the answer is the amount of earth or sand excavated. In Ptolemaic times the unit of volume used in measuring such excavations was an *aoilion*, and the same volume of material used in building or constructing embankments was called *naubion*. In the appendix to Petrie Papyri III I proved that an *aoilion* is a volume equal to the cube whose side is a royal double cubit. There was at that time no evidence to determine the size of a *naubion*; but subsequently Lille Pap. I furnished proof that its volume was equal to that of the *aoilion*. *Naubia* and *aoilia* were in Ptolemaic times of the same volume, containing 8 cubic royal cubits. But in Roman times the standard was changed, and the *naubion* was a cube whose side measured a ξύλον (Ox. P. iv. 669). There were, according to the same papyrus, two ξύλα, a βασιλικόν of 3 cubits, and another of $2\frac{2}{3}$ cubits: the volume of a *naubion* measured by the former would be 27 cubic cubits. Hence, having obtained the volume in cubits, the number of *naubia* (or *aoilia*) is determined by division by 27.

We may now return to the Italian problems, and discuss the general formulae of which they are particular examples. The first is to find the number of spectators in a theatre, given the number of seats in the top and bottom rows, and the number of rows. It is assumed that the increase in the number of seats in the rows is uniform from bottom to top. It is easy to construct the general solution: let x be the number of seats in the top row, y the number in the bottom row, n the number of rows. Then if a be the average increase in the number of seats in each row $x = y + (n - 1) a$, and the total number of seats will be

$$N = \frac{n}{2} \{ 2y + (n - 1) a \} :$$

eliminating α , we get

$$N = \frac{n}{2} (x + y).$$

Hence the general rule: add together the number of seats in the top and bottom rows; multiply by the number of rows, and divide the result by 2.

If we know any three of the quantities in the equation

$$N = \frac{n}{2} (x + y),$$

we can determine the fourth. Here $x = 80$, $n = 120$, $N = 8,400$, and we find that $y = 60$.

Hence we may restore the text with certainty as to the meaning, but hesitation as to the exact words.

δέαδρον. τὸ μὲν ἄνω [βάθρον χωρεῖ
 π, τὸ δὲ κάτω βάθρον [ξ, ὄντων βά-
 θρων ρκ εὐρεῖν πόσ[ους ἀνθρώπους
 χωρήσει τὸ θέατρο ν.
 5 σύνθες τῷ ἄνω βάθ[ρον καὶ τὸ
 κάτω, τουτέστιν π [καὶ ξ / ρμ.
 ἐπὶ τὰς τῶν βάθρων ρ[κ / Ᾱ, σω. τούτων
 χωρήσει τὸ θέατρον τω [ῆμισυ, τουτέστιν
 ην. ἐπὶ τῶν ὁμοί[ων προβλημάτων τὸ
 10 σχῆμα ὑπόκειται.

“A theatre. The top row has 80 seats, the lowest has 60; if there are 120 rows, find how many spectators the theatre will contain. Add the top row and the lowest row, that is 80 and 60; the sum is 140; multiply by 120, the number of rows; result 16,800. The theatre will contain half of this, that is 8,400. The figure for similar problems is given below.”

I suppose the τῷ of l. 5 and the τῷ of l. 8 to be mistakes for τὸ. In l. 7 ἐπὶ τὰς would be ἐπὶ τὰ in ordinary Greek.

The second is an example of the general problem to

find the number of artabae contained in a cylindrical receptacle of given diameter and depth. The volume of a cylinder of diameter d and depth h is $\frac{\pi}{4}d^2h$. If we take $\pi = 3$, we get the rule: Square the diameter, take $\frac{3}{4}$ of the result, then multiply by the depth. This is the volume in cubic cubits. To find the contents in artabae, multiply by $3\frac{3}{8}$, the number of artabae in a cubic cubit.

We can now reconstruct the text. The first problem in the Akhmîm papyrus begins with the words *λακκος στονγυλουν*, and here we have]*γυλουν*; we may presume, the Greek being equally bad in both papyri, that the beginning was [*λακκος στρογ*]*γυλουν*.

[*λάκκος στρογ*]*γυλοῦν*, οὗ ἡ διάμετρος
 [*πηχ(ῶν) ἡ τὸ δὲ*] *βάθος πηχ(ῶν) λ*, εὐρεῖν
 [*πόσας ἀρτάβας*] *χωρήσει. τὰ ἡ τῆς δια-*
 [*μέτρον ἐφ' ἑαυτ'*] *ἀ γί(νεται) ξδ, τούτων*
 [] *μη, ἐπὶ τὸ βάθος πηχ(ῶν) λ*
 [*γί(νεται) ,αυμ . . .*] *. ον γδῆ, διατι ἐπὶ τον*
 [] *ὁ σ*] *τερεὸς πῆχ(υς) χωρήσι ξηροῦ*
 [*ὁ δὲ λάκκος*] *χωρήσι ξηροῦ ὁ δωξ.*

“A circular pit, of which the diameter is 8 cubits and the depth 30 cubits: to find how many artabae it will contain. Multiply the 8 of the diameter by itself; result 64. Take $\frac{3}{4}$ of this; result 48. Multiply by the depth 30 cubits; result 1,440. Multiply by $3\frac{3}{8}$, because a cubic cubit contains so many artabae. The pit will contain 4,860 artabae.”

In l. 14 the editors read *τούτων οἱ φενό | μενοι*, and say = *οἱ φαινόμενοι*. This cannot be right; such a word has no place in this context. What was written must have been one of the many ways in which “three-quarters of this is 48” could have been expressed in Greek, such as *τούτων*

τὸ *L* καὶ δ', or τούτων τὰ τρία τέταρτα, or τούτων ἄφειλε τὸ τέταρτον.

In l. 16 I suppose διὰ τί to be a mistake for διότι.

The word ξηροῦ in ll. 17 and 18 is not superfluous, because the sentence means "the amount of dry stuff a cubic cubit, or the receptacle, contains is so many artabae." Such a vessel might contain wine or water, in which case the writer would have said "a cubic cubit contains so many metretae of fluid."

J. G. SMYLY.