

Philosophy's numerical turn: why the Pythagoreans' interest in numbers is truly awesome.

by Catherine Rowett

Philosophers are generally somewhat wary of the hints of number mysticism in the reports about the beliefs and doctrines of the so-called Pythagoreans. It's not clear how much Pythagoras himself (as opposed to his later followers)... more

More Info: Attached file is the uncorrected proof. The full publication can be found on the publisher's web site at the link given.

Publisher: De Gruyter

Publication Date: Oct 2013

Publication Name: Doctrine and Doxography: Studies on Heraclitus and Pythagoras, ed Dirk Obbink and David Sider, pages 3-32

7509

ROWETT, C.

2013

Philosophy's Numerical Turn: Why the Pythagoreans' Interest in Numbers is Truly Awesome.

Catherine Rowett

We do not know a great deal about Pythagoreanism in the early days when Pythagoras was around. However, from what we do know, it seems reasonable, and not hugely controversial, to suggest that a passion for numbers was at least an embryonic part of the early heritage, something that already belongs in the genuine Pythagorean core, and was certainly already central by what I shall call the “second generation” (including Philolaus).¹ It is not just a feature of the crust of lichen and fungus that grew round the name of Pythagoras in later generations (that is, the period we regularly call “Neopythagoreanism”).

¹ As regards Pythagoras himself, this is somewhat controversial. Burkert denies that the stuff about numbers (mainly in Aristotle) has anything to do with the real Pythagoras. He claims that Aristotle is careful to speak of the “so-called” Pythagoreans, and that he means the second generation, the fifth century Pythagoreans, not Pythagoras himself. By contrast, Burkert claims, the authentic material about Pythagoras himself is almost exclusively about mysticism and wonder-working. See *Lore and Science in Ancient Pythagoreanism*, trans. E. L. Minar Jr. Cambridge, Mass. 1972. My claim is not that all the elaborate number theories can be traced to the earliest Pythagoreanism of Pythagoras's time, but that an embryonic interest in harmony, and in the mystical aspects of number, such as the oath by the tetraktys, can safely be traced back to that period, and that a more developed interest in number is evident in the second generation. Burkert's polemic was against F. M. Cornford, “Mysticism and science in the Pythagorean tradition,” in A. P. D. Mourelatos (ed.), *The Presocratics* (Garden City 1974) 135–60 originally published in 1922–3 (Cornford claimed that early Pythagoreans had a “mystical system” that came under criticism from Parmenides, and that later ones had a pluralist system that he called “number atomism” which was the object of Zeno's attack). C. J. de Vogel, *Pythagoras and Early Pythagoreanism: an Interpretation of Neglected Evidence on the Philosopher Pythagoras* (Assen 1966), published around the same time as Burkert's original publication date, and not directly addressed to his finished publication, takes a less austere line than Burkert about science in the first generation period.

But what kind of a passion for numbers do we find in the Pythagoreans? Is it the kind of enthusiasm that philosophers can safely hail as part of their authentic heritage? Can we attribute it to the real Pythagoras (and his early followers) with pride? Or is it an embarrassment, to be hushed up and omitted from the histories of real philosophy? Were some of the Pythagoreans guilty of an unfortunate misjudgement—perhaps even some of the greatest thinkers, perhaps Pythagoras himself, perhaps his followers, even the great Philolaus and Archytas, with the later Neopythagoreans continuing their strange speculations right into late antiquity? Did all or some of them fail to see the difference between respectable mathematics and mystical mumbo-jumbo? Did they get carried away with an inappropriate desire to put numbers where numbers should not be?

Jonathan Barnes gives a dramatic rendering of the dilemma for the historian of philosophy by inviting us first to approve and then to condemn. First he invites us to approve by sketching a story in which Pythagoras figures as a great mathematical hero and astronomer:

These pious offerings portray an impressive figure: Pythagoras, discoverer and eponym of a celebrated theorem, was a brilliant mathematician; by applying his mathematical knowledge, he made progress in astronomy and harmonics, those sister sirens who together compose the music of the spheres; and finally, seeing mathematics and number at the bottom of the master sciences, he concocted an elaborate physical and metaphysical system and propounded a formal, arithmological cosmogony. Pythagoras was a Greek Newton; and if his intellectual bonnet hummed at times with an embarrassing swarm of mystico-religious bees, we might reflect that Sir Isaac Newton devoted the best years of his life to the interpretation of the number symbolism of the book of *Revelations*.

If Greek science began in Miletus, it grew up in Italy under the tutelage of Pythagoras; and it was brought to maturity by Pythagoras' school, whose members, bound in fellowship by custom and ritual, secured the posthumous influence of their master's voice.²

But then he withdraws that story, because (he thinks) it is not true:

What are we to make of this pleasing picture of a Newtonian Pythagoras? It is, alas, mere fantasy: the shears of scholarship soon strip Pythagoras of his philosophical fleece.³

2 J. Barnes, *The Presocratic Philosophers* (London 1982) 100–101.

3 Ibid. 101.

And thirdly he invites us instead to condemn the whole Pythagorean passion for numbers—by implication including the work of Pythagoras himself which is denigrated by association with late works from the Neopythagorean period such as the *Theologoumena arithmeticae* :

What philosophical use did the Pythagoreans make of mathematics? The cynical will speak dismissively of number mysticism, arithmology, and other puerilities. And it is undeniable that a great quantity of Pythagorean ‘number philosophy’ is a ‘number symbolism’ of the most jejune and inane kind. ... ‘Touching on’ arithmetic, the Pythagoreans were impressed by certain properties of the number 10; alas, their impression degenerated into a sort of mysticism: amazement, the nurse of philosophy, soon has her milk soured and turns into silly reverence and superstition. Those with a taste for intellectual folly will have their appetite sated if they go through the *Theologoumena Arithmeticae*. That Pythagorean work is a late compilation; the earliest examples of such symbolism are found in the *acousmata* and probably date from the time of Pythagoras himself: from first to last the Pythagoreans engaged in arithmology.⁴

Barnes offers us two options: we could admire Pythagoras if he was, as suggested, a Newtonian, whose mathematical discoveries were put to fine use in developing a mathematical astronomy and answers to physics that sought confirmation in mathematics. Or we could condemn him (and his followers) if the philosophical use to which they put their number work was mere mysticism and number symbolism. Since the first option seems to be ruled out by the lack of sound historical evidence for the fanciful portrait of the Newtonian Pythagoras, we are left with the second. And so we condemn.

My task in this paper is to persuade the reader that, notwithstanding Barnes’s fine rhetoric on the matter, we should nevertheless admire, and not condemn, the Pythagorean enthusiasm for numbers, tracing its origins to Pythagoras himself and his immediate followers.⁵

4 Ibid. 381.

5 The mathematical interests are only one of the many kinds of wisdom attributed to Pythagoras. C. Huffman, “The Pythagorean tradition,” in A. A. Long (ed.), *The Cambridge Companion to Early Greek Philosophy* (Cambridge 1999), 66–87, 66–7, gives a judicious account of a general difficulty created by the exaggerated reputation of Pythagoras, and the problematic source materials.

Pythagoras and the early Presocratic tradition.

One not particularly original route to defending the Pythagoreans would be to reject Barnes's claim that the Newtonian Pythagoras is mere fantasy. One could try to show that Pythagoras should be reliably credited with certain significant mathematical discoveries, that his work on harmonics was of a serious experimental nature, and that it is not improbable that he applied these studies to astronomy in a spirit of scientific inquiry, so as to show that his work on numbers was not mere speculation or mysticism. That is not the task I have set myself here. For not only is it not a particularly new project, but in any case it is not the case I want to make, for reasons that I shall go on to explain.

A second rather more interesting project does strike me as worthwhile however. In this first part of my paper I shall not so much seek to restore Pythagoras's credibility as a practitioner of the exact sciences as to ask why Pythagoras has been downgraded in the contemporary assessments, dismissed as a mystic and wonder-worker, and his followers dismissed as number mystics, while other Presocratics with rather similar points to make have been exalted as pioneers of embryonic scientific and philosophical thought. Is there really so much difference?

I shall consider two comparisons here: first between Anaximander's notion of the *apeiron* and Pythagorean discussions of the limited and unlimited, and secondly between Heraclitus' notion of *logos* and harmony, and the Pythagoreans' theories relating to proportion and harmony. Other examples could also be used, but these will be sufficient for this task.⁶

Let us start with the infinite. First, we should note the appearance of '*apeiron*' as a technical term in Anaximander's cosmology:

Anaximander, son of Praxiades, of Miletus... said that the origin and element of things is τὸ ἄπειρον, being the first to use this term of the origin.
Hippolytus *Refutatio* 1.6

This term is also prominent in certain Pythagorean documents, especially in Philolaus (chief among the second generation of Pythagoreans). When this term '*apeiron*' occurs in Pythagorean documents (and in Ar-

6 For instance astronomy, in which Philolaus was arguably far more advanced than his contemporaries such as Democritus, who appears to have rejected or ignored the advances made by the Pythagoreans. See D. R. Dicks, *Early Greek Astronomy to Aristotle* (London 1970) 80–81. Thanks to Carl Huffman for suggesting this additional example.

istotle's reports about them), it invokes a contrast between limit and unlimited, and leads to a classification of numbers that spills out into reality more widely.⁷ For the second generation Pythagoreans, the unlimited is evidently part of what makes number the basis of the whole of reality. This then gets taken up into some of the more peculiar bits of Platonic theory, in particular the so-called "unwritten doctrines" and the idea of the one and the indefinite dyad.⁸

In our modern accounts of Pythagoreanism, we take the Pythagoreans to be talking about funny stuff, some kind of weird mathematical nonsense, when they mention the unlimited or indefinite, and we dismiss it as so much garbage, despite the fact that some respected interpretations take Philolaus's *apeira* to be kinds of material stuff or qualities rather similar to those that appear in other cosmologies of the time.⁹ Yet when we meet the *apeiron* in Anaximander we are not so quick to diagnose something weird or mathematical. We treat it very differently. In Anaximander's case we do not see a primarily mathematical notion being used to create a metaphysical basis for reality, but we read Anaximander as speaking of an unlimited material: physical stuff. We read him as a materialist, and for that reason (it seems) the *apeiron* is defused. It doesn't smack of funny stuff, as it does in the hands of the Pythagoreans. It looks instead like a primitive kind of prime matter, something that Aristotle could look back to, and in which he could trace the origins of the material cause:

For all things are either an origin or derivative from an origin, but of the *apeiron* there is no origin..., but it seems to be the origin of the rest and to encompass all things and control all things, as all those say who do not make any other causes apart from the *apeiron* such as mind or friendship. And this is what is divine, for it is deathless and indestructible, as Anax-

7 The terminology is widespread, particularly in material from Philolaus. See Philolaus B 1, 2, 3, 6 etc, and Aristot. *Meta.* A ch. 5.

8 Aristot. *Meta.* 987b18–988a1. Details discussed in J. N. Findlay, *Plato; The Written and Unwritten Doctrines* (London 1974) ch. 2.

9 Nothing in the extant fragments succeeds in making clear exactly what Philolaus has in mind when he refers to limiters and unlimiteds. Jonathan Barnes (above, n. 2), is among the readers who take the unlimiteds to be material stuffs. Carl Huffman, *Philolaus of Croton*, Cambridge 1993, discusses these interpretations, and mines the fragments for hints (pp. 37–53), and suggests (on the basis of a fragment from Aristotle's lost work on the Pythagoreans) that Philolaus's unlimiteds included time, void, fire and breath; but Aristotle's wording hardly makes that reading secure.

imander says, and most of the natural philosophers. Aristotle *Physics* 203b6–7, 10–15.

But what really is the difference between the notion of *apeiron* in Anaximander and the notion of *apeiron* in the second generation of Pythagorean thought? Why do we take the one as a great advance in science, and the other as a kind of metaphysical mistake?

Secondly, we should notice that Anaximander has a universe structured with concentric circles, which is invoked to explain the positions and apparent movements of the heavenly bodies relative to the earth. We do not hear the music of the spheres in Anaximander's universe, but it is surely improbable that they do not utter sounds, for the circles that carry the stars have "flute-like" pipes with breathing holes through which the fire bursts forth when they are not blocked up.¹⁰ It seems certain that the pressurised fiery vapour escaping from these ἐκπνοιαί must make sounds or notes that reflect the size and diameter of the pipe, rather like the sound of huge pan pipes played across the dark and misty heavens. How are their notes related? Harmonically or out of tune? Who can say? But we do know—roughly speaking—that Anaximander posited some numbers which claim to be the sizes (i. e. diameter) or distances (i. e. radius) of the circles of the sun and the moon and the stars.¹¹ The numbers seem to form a pattern, a sequence of a geometrical kind, probably 9, 18, 27, with the earth too having the numerical proportion 3:1 between its height and its diameter.¹² In fact, it seems that Anaximander was applying a sort of geometrical thinking to his speculations about the shape and the movements of the heavens.¹³ Given that all

10 Hippol. *Ref* 1.6.

11 Hippol. *Ref* 1.6.

12 The evidence for these numbers in the doxography, and the reconstruction of the mutilated texts, are discussed by D. O'Brien, "Anaximander's measurements," *CQ* 17 (1967) 423–32, and G. Naddaf, "Anaximander's measurements revisited," in Anthony Preus (ed.), *Before Plato* (Albany 2001) 5–23. See also C. Kahn, *Anaximander and the origins of Greek Cosmology* (New York 1960) 61–3, KRS 133–137, "The Discovery of space: Anaximander's astronomy," in id., Robert Hahn, and Gerard Naddaf (eds.), *Anaximander in Context* (Albany, 2003) 165–254.

13 This is widely agreed. Kahn suggests that the inspiration for Anaximander's numbers was mathematical rather than mystical; cf. Kahn (above, n. 12) 96–97). A possible link with the geometrical calculations used in architecture has been explored in detail by; and id., "Proportions and numbers in Anaximander and early Greek thought," in D. L. Couprie, Robert Hahn, and Gerard Naddaf, *Anaximander in Context* (Albany 2003) 73–163. Much effort has gone into try-

must be aware that larger and longer flutes make deeper notes, it would be surprising if these numerical ratios did not also yield a sense that lower pitch sounds would issue from the vents of the larger wheels at the outer spheres of the cosmos.

When Pythagoras, or some Pythagoreans, posit harmonic proportions, based on ratios of numbers, as the explanatory principles in determining the structures of the heavens, and do this on the basis of discovering that physical sounds can indeed be analysed mathematically, as systematically related to the size and shape of the physical object that produces them, we generally dismiss their suggestions as idle speculation.¹⁴ When Anaximander engages in some similar, though less well grounded, fantasy about the relative sizes of the heavenly rings, he is

ing to show that, although the work was speculative rather than experimental, Anaximander was not just engaged in arithmology. See also "The Pythagoreans and Greek mathematics," in David Furley and R. E. Allen (eds.), *Studies in Presocratic Philosophy* (London 1970) 379. The alternative line that this is a rationalisation of some mythic or poetic materials is proposed by M. L. West, *Early Greek Philosophy and the Orient* (Oxford 1971), 94; D. L. Couprie, "Anaximander's discovery of space," in A. Preus (ed.), *Before Plato* (Albany 2001) 23–48, 40–41.

- 14 It is hard to find references that explicitly present this attitude as strongly as Barnes does in the passage cited above. Yet I think that the judgement is evident in the general approach to the study of the Presocratic philosophers and in the extent to which Pythagorean number theory is marginalized in main-stream collections of work on Presocratic philosophy. See the evidence presented by P. Kingsley, *Ancient Philosophy, Mystery and Magic: Empedocles and Pythagorean Tradition* (Oxford 1995) 317–320, showing that how a tradition in intellectual history has tried to cleanse the early Pythagoreans of the mystical, by implying that it was a feature of a decadent, late, pseudo-Pythagoreanism, not the true philosophical period of Pythagoras and his early followers. That is not quite the pattern I am seeking to highlight here, but is related. See also G. E. R. Lloyd, *Early Greek Science: Thales to Aristotle* (London 1970) 26–7, "Secondly many of the resemblances that the Pythagoreans claimed to find between things and numbers were quite fantastic and arbitrary.... Obviously while the search for numerical ratios proved fruitful in such fields as the analysis of musical harmonies, and mathematics itself, it also and more often led to mumbo-jumbo and crude number mysticism"; and D. Furley, *The Greek Cosmologists* (Cambridge 1987) 58, who dismisses some parts of Pythagorean astronomy as fantasy, though he does this in the service of a more positive assessment of their particular emphasis on form and structure. See Huffin (above, n. 9) 271–2, for a survey of the bizarre ideas about the inhabitants of the moon, attributed to supposedly more "rational philosophers", that are quietly ignored in modern histories (while those attributed to Philolaus are taken to discredit the whole of his astronomy as mere fantasy).

hailed as a pioneer and an impressive forerunner of mathematical astronomy.¹⁵ Of course, Anaximander lived a little earlier than Pythagoras, in the early sixth century rather than the second half of it. But really there is not a lot of difference in the dates, and we might suppose that by appealing to harmonic structures, Pythagoras was attempting to put more plausibility and accuracy into the speculative arithmetic, on the basis of his discoveries that related audible harmony to numerical proportions and the sizes of physical objects,¹⁶ in place of whatever obscure patterns lay behind Anaximander's guesswork.

So it seems that when Anaximander indulges in *pure* numerical fantasy, plucking multiples of 3 and 9 apparently out of the air, he is hailed as the pioneer who saw for the first time that the plausibility of one's cosmological theory can be enhanced by showing that it makes mathematical sense. When Pythagoras, or early Pythagoreans, do a more sophisticated version of the same thing, selecting harmonic and geometrical sequences in preference to arbitrary patterns, they are accused of superstitious and fanciful numerological speculation. And yet the reason for positing harmonic ratios in nature is that harmonic ratios are found in nature, and are perceptible by us because we are naturally attuned, so that we find such ratios beautiful, when they occur.

Certainly, to suggest that the heavens manifest a harmonic structure which we find beautiful is not to engage in *empirical* science of quite the sort we are used to. But if we are in the business of *speculative* astronomy, rather than empirical astronomy, then Pythagoras (or whatever Pythagorean invented this idea) has at least as good a grounding for his approach as Anaximander seems to have. And if empirical support is a virtue, at least Pythagoras can point to his work on harmony, which evidently does have some empirical support.¹⁷

15 "The importance of this theory is that it is the first attempt at what we may term a mechanical model of the heavenly bodies in Greek astronomy"—Lloyd (above, n. 9) 17; "His theory of equilibrium was a brilliant leap into the realms of the mathematical *a priori*"—KRS 134. Recent fashion has been rather more low key in its estimate of Anaximander (e.g., "The beginnings of cosmology," in A. A. Long (ed.), *The Cambridge Companion to Early Greek Philosophy* (Cambridge 1999) 45–65, 55).

16 See Xenocrates fr. 9 Heinze, apud Porphyry *Commentary on Ptolemy's Harmonics* 30 1–6 Düring.

17 Two key texts are Xenocrates fr. 9 Heinze, apud Porphyry *Commentary on Ptolemy's Harmonics* 30 1–6 Düring; Aristoxenus fr. 77 Müller, in a scholiast on Plato *Phaedo* 108d, Greene p.15.

Why, then, do we cite Pythagorean harmony theory as a type of worthless superstition, but cite Milesian science as an impressive forerunner of modern mathematical techniques in empirical science? Could it be that the idea that something is beautiful, or that there should be *music* in it, seems not to be a good reason for supposing it to be true? But if that is so, we need to examine our preconceptions. For it seems, first, that we are bringing to the enquiry a prejudice in favour of the idea that nature is random, disordered or arbitrary, rather than systematic, displaying patterns and orders at more than one level. Why should it not be more likely that harmonic patterns figure in the structure of the heavens? If so, the Pythagoreans have the better approach to the task than, say, materialists such as the atomists.

Secondly, it seems plausible to suppose, as I have suggested, that Anaximander too thought that the heavenly bodies uttered a flute-like whistle. Perhaps he, too, was moved by that thought in composing his theory about the sizes and shapes of the hoops that circle the earth. So that even if we do, sadly, start from a post-enlightenment prejudice in favour of seeing the world as random, meaningless and lacking in beauty, still there seems to be some inconsistency in our preference for the speculations of Anaximander over those of Pythagoras. Is that just because we don't happen to have any texts on the music of the spheres—or rather wheels—in Anaximander?

Moving on from Anaximander to Heraclitus, let us ask a different question, this time about *logos*. It has become customary in writing about Heraclitus to leave the word *logos* untranslated even when writing for a Greekless readership.¹⁸ Alternatively translators look for a standard formulaic or non-committal translation (such as “account” or “principle”)¹⁹ in order to avoid giving any specific meaning to the term in any particular occurrence.²⁰ These high-minded practices have an unfore-

18 R. Waterfield in the commentary in *The First Philosopher*, Oxford 2000; KRS; R. McKirahan, *Philosophy before Socrates*, Indianapolis 1994).

19 “Account” in Barnes, *Early Greek Philosophy*, Harmondsworth 1987; T. M. Robinson, *Heraclitus; Fragments*, Toronto 1987; and Kahn, *H.*; “principle” in R. Waterfield in *The First Philosophers*, Oxford 2000. See also Long at n. 19 in this volume.

20 The motives for both practices are, of course, admirable in their way, in so far as the translator tries to avoid imposing an interpretation by rendering the term one way rather than another, or concealing the same term under unrecognisable variant translations. I am not suggesting that there is a better solution, but rather that translation is inherently unsatisfactory.

seen consequence, it seems to me. First they make the term *logos* stand out as something like a key concept, inviting us to think that Heraclitus has a theory of “the *Logos*”. This is reinforced by the habit of adding the definite article in the English translation. The definite article is there in some, but not all, of Heraclitus’ references to *logos*,²¹ but it would not always be there, typically, in English even where it is normal in Greek (for instance if ὁ λόγος were translated by a term such as “discourse,” “language,” “reason,” “proportion,” “rationality” or “logic,” we would not put in a definite article at all, on any occasion of use, even if it were there in the Greek). But because we do not offer one of these translations, but leave it as *logos*, and because we therefore add “the” on every occasion even where it is missing in the Greek, we produce an unanticipated effect: the *Logos* becomes almost imperceptibly hypostasised, until before we have even observed it happening, we find it occupying, for us, a place that looks plausibly god-like, and at that point we are sorely tempted to identify it with the ‘god’ of B 67. What might well have looked like an immanent pattern in the behaviour of the world, if it had been properly translated, now takes on a metaphysical role instead, as a divine entity that explains or dictates the reciprocal patterns in the world. This seems to happen, at least in part, because our increasingly entrenched translation practices, and exegetical practices, irresistibly privilege the term *logos* and exalt it to become a term of art.

Aside from the damage that this does to our understanding of Heraclitus, it also has a strange effect on our understanding of the relation between Heraclitus and Pythagoras. Since we now think of *Logos* as a kind of god in Heraclitus’ system, we fail to notice how close is the resemblance between *Heraclitus*’ interest in proportion and ratios and the same topics in *Pythagoreanism*. The failure to translate *logos*, the adoption of a special systematic pseudo-translation, and the addition of the definite article and capital letter as though the *logos* were an hypostasis or divinity, mask the links between the notion of *logos* and the notion of “harmony” in Heraclitus’ thought. Harmony (as in the harmony of opposites) is also seen as a key concept in Heraclitus’ thought, as it is in Pythagorean thought. So both Pythagorean thought and Heraclitean thought are constantly playing with the twin notions of ratio and harmony, and using these as their main explanatory concepts in natural philosophy. The Pythagorean resonances in Heraclitus would, of course, be

21 *Logos* occurs with the definite article in B 1, 2, 31b, 50; without in B 39, 45, 72, 87, 108, 115.

much more apparent if Heraclitus' *logos* were read as "ratio" or "proportion," and the numerical significance of harmony were allowed to emerge in the context of both opposition (in Heraclitus B 8), of ratio (in B 49, 79, 82–3), and of measure, and limit, and "the unlimited" (in B 120, 94, 30,31 and 45). It is true that Heraclitus uses a wider range of terms for "limit" and "boundary"—including οὐρος (120), τέρμματα (120), μέτρον (94), as well as the πείρατα of 45—so that even in the Greek we are not alerted very strongly to the link between these various fragments that are in several ways obsessed with measure and limits. By contrast, in Pythagorean sources we tend to find a kind of technical terminology—the ἄπειρα καὶ περαίνοντα of Philolaus, for instance, and the πέρας καὶ ἄπειρον of the first pair in Aristotle's table of Pythagorean opposites. This draws attention to the theme in the Pythagoreans more prominently than the *variatio* in the Heraclitean vocabulary does.²² Still it remains undeniable that Heraclitus uses the idea of *logos* or proportion to bring order to the measured processes of the world, and that he draws connections between patterns of opposition and the idea of a cosmic harmony (hidden or otherwise).

So while the Pythagoreans' attempts to put numbers on the patterns and proportions that they saw in the cosmos, and to link those numbers to geometrical and harmonic proportions, are easily dismissed as puerile fantasies by Barnes (and not just Barnes)²³, Heraclitus' mysterious *logos* is given a definite article, a capital letter, and hailed as an attempt to bring reason and order into a world of opposition and strife.²⁴ What is the difference? Is it that Heraclitus does not give us the *numbers* but only hints tantalisingly at the existence of measures and ratios (the sea returns "to the same measure as was there before it became earth", B 31)? And is it

22 The impression that the Pythagoreans have a systematic technical terminology may be exaggerated because of the prominence of Philolaus as a prime source for Presocratic Pythagoreanism.

23 I have used Barnes because he represents an extreme end of a certain kind of Anglo-Saxon tradition. Not all historians of Presocratic philosophy display such antagonism towards the Pythagorean school, so that my generalisations should be taken to have distinguished exceptions. I am happy for the reader to identify with me or with my opponents as he/she feels most at home.

24 Barnes (above, n. 2) 59, is careful to warn us against taking the *logos* as a technical term and the key to Heraclitus's secrets, but he goes (80–81) on to allow that there may be a metaphysical *logos doctrine*, and to recommend that Heraclitus be placed with the Milesian rational tradition, and sheltered from the pejorative term "mystic" (which is doubtless reserved for those who fall into the Pythagorean mire).

because the Pythagoreans tell us the numbers (that the number of the heavenly bodies is ten, for example), but we suspect that their choice of numbers is motivated by the desire for perfection, not the desire to save the phenomena? And of course, we ourselves would *never* countenance a practice of adjusting the observational results to comply with the predictions of mathematics and theory....

Or is it that (by tradition) we systematically translate out, or interpret out, the references to harmony and ratio in Heraclitus, so that we don't see them as number mysticism, because instead we interpret Heraclitus' interest in *logos* as a kind of monotheism—something in the tradition of Xenophanes in which a Zeus-like divinity is rationalised and demythologised to become an immanent physical regulatory principle in the world, a guiding principle that sees to it that we don't need to appeal to weird metaphysical structures or mystical number patterns. It seems that we try to see in Heraclitus a step on the route from Milesian materialism to post-enlightenment physics, and we therefore let him have his *logos* in the guise of personified reason—exalted, but not transcendent, much as Cartesian theism accounts for the existence and orderly functioning of a mathematically regular cosmos by assigning certain roles to a kind of “god” figure that is really not an object of cult or mystery at all.

It is salutary to take a look at what Sextus Empiricus has to say. He provides a lengthy analysis of the role of reason as a criterion of knowledge in the Presocratic philosophers, an analysis heavily coloured by the interests and concerns of Hellenistic epistemology. Having dealt with Anaxagoras (“the most physical” of the Presocratics)²⁵ he introduces the passage on the Pythagoreans thus:

ὥστε ὁ μὲν Ἀναξαγόρας κοινῶς τὸν λόγον ἔφη κριτήριον εἶναι· οἱ δὲ Πυθαγορικοὶ τὸν λόγον μὲν φασιν, οὐ κοινῶς δέ, τὸν δὲ ἀπὸ τῶν μαθημάτων περιγινόμενον, καθάπερ ἔλεγε καὶ ὁ Φιλόλαος, θεωρητικόν τε ὄντα τῆς τῶν ὄλων φύσεως ἔχειν τινὰ συγγένειαν πρὸς ταύτην, ἐπεὶ περ ὑπὸ τοῦ ὁμοίου τὸ ὁμοιον καταλαμβάνεσθαι πέφυκεν· ... ἦν δὲ ἀρχὴ τῆς τῶν ὄλων ὑποστάσεως ἀριθμός· διὸ καὶ ὁ κριτὴς τῶν πάντων λόγος οὐκ ἀμέτοχος ὦν τῆς τούτου δυνάμεως καλοῖτο ἂν ἀριθμός.²⁶

Sextus Empiricus *Adv Math* 7.92–3 (= DK 44 A29)²⁷

25 Sextus Empiricus *Adv. Math.* 7.90.

26 So that Anaxagoras said that the *logos* generally was the criterion. The Pythagoreans also say that it is the *logos*—but this time not the *logos* generally but the *logos* that is acquired from studies (mathematics?)—just as Philolaus also said—and that given that it contemplates the nature of the universe, it has a certain affinity

Sextus goes on to provide six or seven further paragraphs of evidence in support of the claim that mathematical reason is the criterion for the Pythagoreans before proceeding to apply the same kind of treatment to Xenophanes, Parmenides, Empedocles, Democritus and Heraclitus.²⁸ It is no surprise (to us, at least) that Heraclitus too is said to make the *logos* a criterion of truth.²⁹ In Heraclitus' case it is not cashed out as mathematical reason—naturally enough, since that is supposed to be distinctively Pythagorean. Instead Sextus cites the sequence of fragments that we standardly use as the prime evidence for “the *Logos* doctrine” in Heraclitus.³⁰ So what do we do when we use this material? We accept, from this passage, Sextus's account of Heraclitus and his notion of the *logos*—this survives almost unscathed into our interpretation of several major and crucial moves in Presocratic thinking; but we completely ignore the bits about *logos* in Pythagoreanism, and its basis in number and the idea that like is known by like that leads to the idea that if the universe is numerically ordered, then our understanding of it will be similarly ordered as a mathematical kind of science.³¹

That bit—the appeal to a specifically mathematical type of calculation—is not *exactly* what we find in Heraclitus, although there are (as we observed) hints of a kind of thinking that invokes proportions and ratios in Heraclitus too.³² But there is no reason to think that the addition of mathematics as a criterion of sound understanding of the world is a development that we should dismiss as mystical mumbo-jumbo, or despise as a fairy story, by comparison with the rather vaguer and more general notion of *logos* in Heraclitus. On the contrary, we should probably agree, nowadays, that cosmology requires not just a generic brand of

with that nature, if like is by nature grasped by like... But number was the principle of the structure of the universe; hence the *logos* that is the judge of all things is not devoid of this power, and would therefore be called number.

27 Text as in Huffinan (above, n. 9) 199.

28 S.E. *AM* 7 92–140.

29 S.E. *AM* 7.126.

30 That is, B 1 and 2.

31 In defence of this selective use of Sextus's material, one might appeal to the fact that we do have genuine fragments of Heraclitus in which he *uses* the term *logos*, whereas there is no textual support for attributing that term in that sense to Philolaus. But we should notice that Sextus is talking in terms that are alien to the Presocratic discourse throughout, and this applies as much to his search for a criterion of truth in Heraclitus as it does in Philolaus. On the anachronism of the terms of the enquiry see Huffinan (above, n. 9) 199–201.

32 E.g., B 30, 31, 79, 82, 83.

“rational enquiry” but more specifically a *mathematically trained investigator*, whose criterion for whether he has reached something worthy to count as knowledge will be *whether the mathematics works*. So why should we think of Heraclitus as a model of Presocratic philosophy at its best, while dismissing Pythagorean theory as an embarrassing disfigurement of an otherwise pure stream of increasingly rational investigation? Surely it should be the other way around?

Let me offer a proposal for what lies behind this widespread preference for the Heraclitean over the Pythagorean harmony theory. It is this. It seems to me that Barnes (as well as others who share his judgments) is following a tradition that prefers signs of materialism and reductionism over any kind of metaphysical or teleological picture. It is a tradition that sees the pure materialist reductionism of the atomists as the culmination and high point of the Presocratic achievement, and it assesses the contribution of earlier thinkers by how closely they approximate to that ideal—an ideal that is seen as a kind of no-nonsense physics, even if it has little ambition to provide genuine empirical support for its speculations.³³

It is true that I have suggested that Heraclitus' *logos* gets hypostasised as “The *Logos*” with a capital letter, and in the process takes on a quasi-god-like role as the governor of cosmic processes. That might suggest that our admiration for Heraclitus is not because we see him as eliminating metaphysical and religious entities. But, as I suggested above, despite the theistic terminology, we tend to conceive of that move as somewhat reductionist, like Xenophanes's theological endeavours. On that reading of Heraclitus, “God” or “The Logos,” just *is* the world, when all's said and done:

God is day, night, winter, summer, war, peace, hunger satiety...³⁴

God *is* the processes that once seemed mysterious; but really (according to this version) they are not so much mysterious as regular, not unpredictable but reasonable. Reason, not religion, is the way to get control

33 Barnes himself (above, n. 2) 343–4, 76–7, is careful to warn us against over-enthusiastically assimilating ancient atomism to modern science, and he points out many ways in which the atomism of Leucippus and Democritus raise problems that they cannot answer. But these warnings leave untouched the general sense that the atomists' theory is virtuous just in so far as it approximates to the ideal of modern science—so that ancient atomism is *not quite admirable* because it *does not quite live up to that ideal* in all respects.

34 Heraclitus B 67.

of the physical events, we understand. That is what we take Heraclitus to be saying, in his rather obscure and difficult way. So God turns out to be no more than our picture of how the world works to a predictable pattern. Harmony theory is then read not as a metaphysical thesis but as a materialist one.

Conceivably this is correct as an interpretation of Heraclitus. In support, one might notice the way in which Heraclitus appeals to the notion of measure in connection with specific physical processes that manifest this kind of proportionality—the notion that we find this *logos* in the quantity of sea that you get back when earth has been turned back into water (B 31), and that there are observable limits to the passage of the sun across the sky.³⁵ It seems that, no matter how little evidence Heraclitus gives for observing such regularities in nature, he does mean that the regularities in question are part of *nature*.³⁶ Similarly, to return to Anaximander's picture of the cosmos, those heavenly pipes seem to be substantial and very concrete material chariot wheels revolving in the sky, invisible only because they can't be seen for the mist. Anaximander too is giving us *material* explanations with numbers on them, not numerical explanations with no matter to do the work.

Pythagorean mathematics, by contrast, tries to make numbers *do the work*. If you're looking for an account of material and efficient causes in the cosmos, it's odd to point to numbers *as such*, as opposed to applying numbers to quantities of other things, quantities of some material stuffs or physical forces that could do the work. So *if* you conceive the Presocratic project as a project to suggest and improve explanatory factors that are to be invoked in the interests of a reductionist thesis about how the world works, numbers as such seem to be the wrong kind of thing.³⁷

35 This is one traditional interpretation of the claim in B 94 that “the sun will not overstep its measures.” The Derveni papyrus (which appears to combine what we used to know as B 3 and 94) opens the possibility that the measures are the sun's size (a foot across, B 3) rather than its tropics. See G. Betegh, *The Derveni Papyrus: Cosmology, Theology and Interpretation* (Cambridge 2004) 10–11. This alternative is also compatible with the traditional reductionist interpretation that I am sketching.

36 Here perhaps we assume too readily that the image of the Erinyes in B 3 (“otherwise the Furies, ministers of Justice, will find it out”) is just a picturesque metaphor to convey what we take to be a natural constraint on the behaviour of the sun. Again the Derveni papyrus, with its obsession with the daimonic, tells us that Heraclitus was not always read so rationalistically in antiquity (see previous note).

37 See Furley (above, n. 14) 52–3 for a brief discussion of this thought.

This post-enlightenment complaint is closely related to an objection made long since by Aristotle in *Metaphysics* N:

οἱ δὲ Πυθαγόρειοι διὰ τὸ ὄραν πολλὰ τῶν ἀριθμῶν πάθη ὑπάρχοντα τοῖς αἰσθητοῖς σώμασιν, εἶναι μὲν ἀριθμοὺς ἐποίησαν τὰ ὄντα, οὐ χωριστοὺς δέ, ἀλλ' ἐξ ἀριθμῶν τὰ ὄντα· διὰ τί δέ; ὅτι τὰ πάθη τὰ τῶν ἀριθμῶν ἐν ἁρμονίᾳ ὑπάρχει καὶ ἐν τῷ οὐρανῷ καὶ ἐν πολλοῖς ἄλλοις. ... 1090a20–25

οἱ μὲν οὖν Πυθαγόρειοι κατὰ μὲν τὸ τοιοῦτον οὐθενὶ ἐνοχοί εἰσιν, κατὰ μέντοι τὸ ποιεῖν ἐξ ἀριθμῶν τὰ φυσικὰ σώματα, ἐκ μὴ ἐχόντων βάρους μηδὲ κουφότητα ἔχοντα κουφότητα καὶ βάρος, ἰοίκασι περὶ ἄλλου οὐρανοῦ λέγειν καὶ σωμάτων ἀλλ' οὐ τῶν αἰσθητῶν.³⁸ 1090a30–35

Aristotle exonerates the Pythagoreans from any charges to the effect that they treat numbers as separate from sensible things or as intermediate between sensible things and forms.³⁹ But this is only because he takes it that they think that sensible things *are numbers*, which he thinks is, if anything, an even more peculiar idea, albeit one that avoids the problem of duplication of Platonic entities. Aristotle suggests that the Pythagoreans adopted this theory because they perceived that there were *πάθη* of numbers, such as harmonies, ratios and the like, which can be seen as numerical characteristics of things, and that these show up all over the place in astronomy and the other aspects of nature. Seeing those mathematical phenomena, he supposes, made the Pythagoreans go for the idea that things actually *are* numbers.

Although the tendency to exclude Pythagorean speculations from the serious history of Presocratic philosophy comes from a tradition that is heavily indebted to Aristotle, and to Aristotle's reconstruction of early cosmology, it does not seem to me that the modern objections to Pythagorean numerology reflect quite the same motivation as Aristotle's bemused comments in *Metaphysics* N. Aristotle suggests that numbers aren't the right kind of thing to be the constitutive substance of things

38 But the Pythagoreans made things be numbers (because they saw many numerical effects existing in perceptible bodies) but not separate numbers, but things being constituted of numbers. But why so? Because the numerical effects exist in harmony in the heavens and in many other things. ... On the one hand the Pythagoreans seem not to be liable to any charge on this matter (sc. separating the mathematical). But in respect of making physical bodies out of numbers, making things that have lightness and heaviness out of things that have no heaviness or lightness, they seem to be talking about a different heaven and different bodies, but not the perceptible ones.

39 The Platonists and Speusippus are under attack for a variety of modifications of the idea of numbers that are separate from *aistheta*.

that have mass. Numbers don't have heaviness and lightness, so—he complains—you can't make things out of them. But Aristotle does not simply dismiss the Pythagoreans as stupid or muddled. Rather, he concludes that in some way they would have to be talking about something else, if their thoughts are to make any sense. They can't really be talking about things that have weight and so on at all. "They seem to be talking about a different heaven and different bodies, but not the perceptible ones."⁴⁰

Aristotle's observations about whether people 'separate' numbers are pertinent here. In the case of Plato, he says, the theory allows that there *are* things that have physical mass, on the one hand, and then, on the other hand, there are ἀριθμοί that are χωριστοί – a separate realm of things that are not in the same category as the physical things that manifest the numerical effects. It is a two-world view. But among the Pythagoreans he does not find a two-world view. They neither separate numbers, nor make them intermediates.⁴¹ Instead he finds that they have just one world, and since it is a world made of numbers, it should be the incorporeal one, although it is also the world of perceptible things. That is why Aristotle says that they seem not to be liable to any charge of separating numbers from the perceptible things.⁴² For there is just one set of things, namely the numbers (which nevertheless in some way *are* the perceptible things).⁴³

So the Pythagoreans have just the one world, but it is apparently composed of what we (at least) regard as incorporeals. The result is that (as Aristotle observes) the Pythagoreans seem to be talking about

40 1090a34–5. Aristotle is bemused because they talk as though they are explaining physical bodies, yet the elements they cite suggest that this can't be so. The word "seem" in this sentence suggests that he thinks that to make sense of what they are saying we need to adopt some hypothesis such as this.

41 Cf. Aristotl. *Meta.* A, 987b27 καὶ ἔτι ὁ μὲν τοὺς ἀριθμοὺς παρὰ τὰ αἰσθητά, οἱ δ' ἀριθμοὺς εἶναι φασιν αὐτὰ τὰ πράγματα, καὶ τὰ μαθηματικά μεταξὺ τούτων οὐ τιθέασιν.

42 1090a30.

43 Aristotle's comment at 1090a30 seems to conflict with the passage in *Metaphysics* A (987b11) where he tells us that the Pythagoreans have a notion comparable to Plato's notion of μέεξις. It seems that at 987b11 Aristotle is assimilating Plato and Pythagoreanism—a tendency that was to have a long subsequent history—by contrast with the careful attempt to draw distinctions between different kinds of Platonism about numbers in the passages in *Metaphysics* N.

a different world and different bodies, 1090a34. If things are made of numbers, then they are a very funny kind of “things”.⁴⁴

So Aristotle resists making the sort of objection that one might expect from a scientist, complaining that it is naïve and stupid to try to create bodies out of numbers, or that this is the wrong kind of matter to do the job. Instead, Aristotle seems to be saying that if their suggestion that the material is numbers is to make any sense, their question must have been different. They must have taken themselves to be explaining a world of incorporeal perceptibles, not the world of corporeal perceptible things that Plato was talking about, which he regarded as something quite separate from numbers.

Nevertheless, Aristotle is evidently assuming from the start that the Pythagorean project (including the appeal to numbers) was the same project as the Ionian scientists' project. He thinks that the question was how to explain the perceptible world in terms of its constitutive matter. Once the constitutive matter has been specified as non-separated numbers, Aristotle concludes that the Pythagoreans had some funny idea of the world. They were evidently explaining a rather incorporeal world, with all its entities composed of numbers; but he does not drop the idea that the Pythagoreans are to be assessed for their competence at reducing perceptible reality to explanatory components that are immanent and not transcendent.⁴⁵

44 I have not questioned Aristotle's claim that the Pythagoreans made things “out of numbers” at face value, because I am interested in trying to explain what his objection to that thought is, and how it differs from our objections to the Pythagoreans' use of numbers in physics. It may well be that he was wrong to think that they meant that numbers were a material cause, despite the fact that he surely had access to the works of Philolaus and Archytas and had written extensively on them. For a diagnosis of Aristotle's mistake, and the evidence in Philolaus for what he really meant, see Huffman (above, n. 9) 57–64.

45 The problem is surely only that Aristotle takes the reduction to be materialist in its outlook. Without that assumption the project to reduce ontology to a system of numbers is not self-evidently flawed. W. V. Quine entertains precisely this project and investigates what it lacks, if anything, as a serious candidate in a number of works. See Quine, “Ontological reduction and the world of numbers,” in id., *The Ways of Paradox and other essays* (New York 1966), 199–207; “Ontological relativity,” in id., *Ontological Relativity and other Essays* (New York: Columbia University Press, 1969) 26–68; “Propositional objects,” in id., *Ontological Relativity and other Essays* (New York 1969) 137–60. I am grateful to Nick Denyer for pointing me to these references.

From Incomprehension to Admiration

I have suggested that there are striking similarities between the Pythagorean concerns with number, ratio and harmony, and some of the material that has been regarded as pioneering and profound in Anaximander and Heraclitus. In asking why these moves should be considered important and profound in the latter cases, I have suggested that they meet with approval among modern scholars because the modern scholars are assessing the Presocratic thinkers for their progress in a sequence of developments in the direction of reductive materialist physics. Because they see the numerical patterns in the universe as immanent $\pi\acute{\alpha}\theta\eta$ of things, and do not presuppose a set of theoretical entities that are 'numbers' with properties of their own, the numerical fantasies of these early Ionian thinkers are seen as an acceptable—or even progressive—part of that generally materialist project.

It seems to me that this evaluation of the early Greek philosophers displays an agenda that is built into our heritage of modern Presocratic scholarship. The agenda is very evident in Barnes, but that is only because he is particularly blatant about expressing his prejudices in outspoken terms. In practice he is following an existing tradition. One would say that the tradition was Aristotelian in origin—the rejection of fancy metaphysical entities, the down-to-earth preference for specifying that the world of particulars is what is real, the analysis of the Presocratics as engaged in diagnosing the material cause, all these seem to be Aristotle's prejudices—except that I think that it is a modern version of Aristotelianism that has acquired a great deal of baggage from the Enlightenment, from logical positivism and from a more recent scientism that equates truth with what can be proved by empirical methods. Aristotle is certainly opposed to some of the things that he finds in Platonism, such as the separation of Forms, but his opposition is not for quite the same reasons as the reasons that modern Presocratic scholarship would offer for why it doubts that Pythagorean numerology was a valuable contribution to Western Philosophy's overall development.

In this second part of the paper I shall move beyond my initial thought, that Pythagorean speculations are *no worse* than the comparable bits of Heraclitus or Anaximander, if one is looking for empirically verifiable reasons in favour of a particular theory. My second thought is more ambitious. I want to propose that there is something in the Pythagorean enthusiasm for numbers that is far more significant philosophically, and has had far greater ramifications in the story of Western phi-

losophy than anything Anaximander or Heraclitus ever did, despite their superior credibility in the contemporary western system of values. That is, I am suggesting that we should not *apologise* for the Pythagoreans' tendency to idolise numbers, or certain particular numbers, nor should we try to *discard* those bits and look for some cleaner bits of respectable doctrine instead. Rather we should celebrate them.

A number of thinkers from Plutarch to Leibniz have anticipated my point. We should start however with a well known passage attributed to the fifth century Pythagorean Philolaus, which is quoted by Stobaeus.⁴⁶ The point that is relevant to my topic is where Philolaus says that without number it is not possible for anything to be thought or known:

καὶ πάντα γὰρ μὲν τὰ γινωσκόμενα ἀριθμὸν ἔχοντι· οὐ γὰρ οἶόν τε οὐδὲν οὔτε νοηθῆμεν οὔτε γνωσθῆμεν ἄνευ τούτου.⁴⁷

Philolaus B 4

The idea that “having a number” is a criterion of knowability is not one that is often proposed or given prominence in epistemological discussions, but it is worth comparing it to the point made by Parmenides about the relation between being and knowability:

ταῦτόν δ' ἐστὶ νοεῖν τε καὶ οὔνεκεν ἔστι νόημα.
οὐ γὰρ ἄνευ τοῦ ἑόντος, ἐν ᾧ πεφρατισμένον ἔστιν,
εὐρήσεις τὸ νοεῖν.⁴⁸

Parmenides B8.34–6

It seems that Philolaus is giving to numbers a role very similar to the role that is served by being and truth in Parmenides. There is no true thinking without being in Parmenides. There is no true thinking without numbers in Philolaus. It is a very severe epistemology, in which nothing counts as knowing unless it is knowledge of numbers. There is only one set of knowable objects, namely mathematical, or at least numbered items. So study of mathematics is not just one of the sciences, alongside physics, but mathematics is the only science that relates effectively to knowable objects.⁴⁹

46 Philolaus B 4–5; Stobaeus 1.21.7b–c.

47 And indeed all the things that are known have number. For without this it is not possible for anything to be thought or known. Text from Huffman (above, n. 9).

48 It's the same thing—thinking and that whose thought it is. For you won't find thinking without the reality, in which it is an utterance.

49 See Huffman (above, n. 9) 173–7, responding to M. C Nussbaum, “Eleatic conventionalism and Philolaus on the conditions of thought,” *HSCP*, 83 (1979) 63–108). I do not deny that Philolaus would probably allow that we can perceive

Secondly we may notice a quotation from Archytas (B 1) preserved by Porphyry in his commentary on Ptolemy's *Harmonics*, and the echo of the same thought that is known from Plato's *Republic*:

παρακείσθω δὲ καὶ νῦν τὰ Ἀρχύτα τοῦ Πυθαγορείου, οὗ μάλιστα καὶ γνήσια λέγεται εἶναι τὰ συγγράμματα. λέγει δ' ἐν τῷ Περὶ μαθηματικῆς εὐθύς ἐναρχόμενος τοῦ λόγου τάδε· Καλῶς μοι δοκοῦντι τοὶ περὶ τὰ μαθήματα διαγνώμεν, καὶ οὐθὲν ἄτοπον ὀρθῶς αὐτοὺς οἶα ἐντι περὶ ἐκάστου φρονέν. περὶ γὰρ τὰς τῶν ὄλων φύσιος καλῶς διαγνόντες ἐμελλον καὶ περὶ τῶν κατὰ μέρος, οἶα ἐντι, ὄψεσθαι. περὶ τε δὴ τὰς τῶν ἄστρων ταχυτάτος καὶ ἐπιτολάν καὶ δυσίων παρέδωκαν ἡμῖν σαφῇ διάγνωσιν καὶ περὶ γεμετρίας καὶ ἀριθμῶν καὶ οὐχ ἥκιστα περὶ μωσικᾶς. ταῦτα γὰρ τὰ μαθήματα δοκοῦντι εἶμεν ἀδελφεά.⁵⁰ Porphyry *In Ptolem. Harm.* 1.3, p. 56

Düring (quoting Archytas B 1)⁵¹

This passage confirms Plato's claim, at *Republic* 530d, that the Pythagoreans called arithmetic, geometry, harmonics and astronomy 'sister' sciences," and it suggests that Archytas was the one who coined the phrase. But my interest is not in that point, but in Archytas's suggestion that one would expect the experts in these sciences – which are grouped together because they work by theoretical manipulation of abstracted mathematical, not empirical data from physical bodies – one would expect *these* experts to be the ones who correctly discern the nature of things, and of the universe as a whole. It is οὐθὲν ἄτοπον, says Archytas, that these people think correctly about things. But notice also the idea that they do this καλῶς – they discern the workings of the universe *beautifully*.

musical intervals and other relations without knowing the formulae, but I am suggesting that when we *know* the formulae, the knowable things are the numbers, so that other things (ratios, harmonies etc) are knowable just in virtue of being numerical, or having numbers (and their numbers are what we know about them). This is to go somewhat beyond what is strictly justified by the text.

- 50 And now let us set alongside the words of Archytas the Pythagorean, to whom the writings are most reliably attributed. He says in the work on mathematics, right at the beginning, the following: "The people who are versed in learned subjects (mathematics?) seem to me to discern beautifully, and there is nothing absurd in their thinking correctly about each of the things just what it is like; for, since they discern beautifully with regard to the nature of the universe as a whole, it is to be expected that they will observe beautifully about the particular things, just what they are like. They have handed down to us clear knowledge concerning the speed of the stars and their risings and settings, and about geometry and numbers, and not least about music. For these subjects seem to be sister-subjects."

- 51 Text as in C. Huffman. *Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King*, Cambridge 2005.

And this surely links in to the idea that the workings of the universe are themselves a fine object of attention.

That point is made more explicit by Plutarch in a passage in his *Quaestiones convivales*:

πάσι μὲν οὖν τοῖς καλουμένοις μαθήμασιν, ὥσπερ ἀστραβέσι καὶ λείοις κατόπτροις, ἐμφαίνεται τῆς τῶν νοητῶν ἀληθείας ἵχνη καὶ εἶδωλα· μάλιστα δὲ γεωμετρία κατὰ τὸν Φιλόλαον ἀρχὴ καὶ μητρόπολις οὕσα τῶν ἄλλων ἐπανάγει καὶ στρέφει τὴν διάνοιαν, οἷον ἐκκαθαίρομένην καὶ ἀπολυομένην ἀτρέμα τῆς αἰσθήσεως. διὸ καὶ Πλάτων αὐτὸς ἐμέμψατο τοὺς περὶ Εὐδόξου καὶ Ἀρχύταν καὶ Μέναιχμον εἰς ὀργανικὰς καὶ μηχανικὰς κατασκευὰς τὸν τοῦ στερεοῦ διπλασιασμὸν ἀπάγειν ἐπιχειροῦντας, ὥσπερ πειρωμένους δίχα λόγου δύο μέσας ἀνὰ λόγον, ἧ παρεῖκοι, λαβεῖν· ἀπόλλυσθαι γὰρ οὕτω καὶ διαφθεῖρεσθαι τὸ γεωμετρίας ἀγαθὸν αὐθις ἐπὶ τὰ αἰσθητὰ παλινδρομούσης καὶ μὴ φερομένης ἄνω μὴδ' ἀντιλαμβανομένης τῶν αἰδίων καὶ ἀσωμάτων εἰκόνων, πρὸς αἵσπερ ὧν ὁ θεὸς αἰεὶ θεὸς ἐστίν.

(Plat. *Phaedr.* 249c).⁵²

Plutarch *Quaestiones convivales* 8.2.1, 718E (= DK 44 A 7a)

The thought developed by Plutarch in this passage is supposed to go back to Philolaus in some sense,⁵³ and indeed it is faintly reminiscent of the passage from Sextus Empiricus which we noticed above,⁵⁴ where Philolaus was said to have claimed that one gets an affinity with the harmony of the universe from assimilation to mathematical knowledge. Here too, in Plutarch's passage, the thought attributed to Philolaus—or built upon Philolaus's foundations by Plutarch—is that handling numbers does something splendid for you.⁵⁵ And for Plutarch,

52 In all the so called (mathematical?) studies, the traces and images of the truth of intelligible objects are reflected, as in even and polished mirrors; and most of all geometry, according to Philolaus, being the source and mother-city of the other studies, leads the mind up and converts it, like a mind purified and released effortlessly from perception. Hence Plato himself criticised the followers of Eudoxus and Archytas and Menaechmus, who tried to divert the doubling of the cube to instrumental and mechanical devices, as though they were trying to obtain the two mean proportionals, however practicable, aside from rationality. For this is to destroy and corrupt the good of geometry, when it is dragged back to perceptible things and not carried up and not grasping eternal and bodiless icons instead—"those things closeness to which makes god always be god."

53 The name Philolaus is not in the manuscript, but it is obtained by a plausible emendation of a corrupt reading φίλαον in the manuscripts.

54 S.E. *AM* 7.92 (= DK 44 A 29).

55 Huffman (above, n. 9) 193–194, takes the material from Philolaus to be very brief, only the reference to geometry as the source and mother city, while all the reflections on that are from Plutarch's Platonist context. On the other

the less empirical the science is, the better it is at this task. Plutarch supports this Platonist thought by reference to a legend according to which Plato is supposed to have raised objections to the method for duplicating the cube attributed (here) to Eudoxus, Archytas, and Menaichmus. The details of the mathematics are not immediately relevant for now (except perhaps to note that Archytas should be exempt from the criticism).⁵⁶ Plutarch's point is simply this: that we should not be attending to material examples. We should be handling numbers. That's how we get to think of incorporeals, he thinks... This is the Platonist thought: that we need to get close to God by escaping from perceptible things.

Of course, Plutarch is feeding us Platonism served up on a bed of Philolaus. I'm not meaning to pretend that the importance or the significance of the Pythagoreans' devotion to the elegance of numbers was explicitly appreciated by them, at the time that they first developed their fascination. On the contrary, I want to suggest that it is with hindsight that we can see that this was one of the most profound and lasting legacies of Presocratic thought—the discovery of what we would call incorporeals. It is a point made by Plutarch, and by Plato too of course,⁵⁷ that learning mathematics and geometry helps us to turn our gaze “upwards” (meaning towards intellectual objects rather than corporeal or perceptible ones), or to abstract the intelligible objects and intelligible truths from material things. They are making a point about the usefulness of a particular kind of abstract discipline for intellectual exercise and training. But in addition to that point, I would also want to add that

hand it is not clear why Plutarch would be prompted to cite Philolaus at this point if there were not *some* invitation to this line of thinking in the text to which he is alluding. I think that Huffman is keen to exclude any hints of proto-Platonism from Philolaus, and as a result he may be skimming the material too hard.

56 There is something wrong with the story, though how exactly it has ended up in this form in Plutarch is not entirely clear. In fact, it would appear that Archytas's solution to the problem of obtaining the two mean proportionals was more theoretical and did not resort to practical methods as implied here. It makes no sense to suggest that Archytas was one of the offenders against whom Plato would have laid such a charge, therefore. The allusion does not seem to be to any existing text of Plato, although the issue of the need for two mean proportionals between cubic numbers figures in Plato's *Timaeus* 31c–32b, and there is a passage in the *Republic* 7.528a–d, which criticises stereometry for some failings that commentators have tried to link to the dispute mentioned by Plutarch. See Huffman (above, n. 9) 344–401.

57 Plato *Republic* 527b.

there is something important, even revolutionary, about the idea that the most important reality might be the one that obeys the mathematical rules, not the one that observably falls short, and approximates only rather rarely to mathematical accuracy. In other words, Plato saw that when mathematics talks about the $\pi\acute{\alpha}\theta\eta$ that are numbers, it does not treat them as $\pi\acute{\alpha}\theta\eta$ of things (that is, of things that are more real than their $\pi\acute{\alpha}\theta\eta$) but it treats the numbers as the most perfectly real things (or at least the more perfectly real, as compared with bodily things).

This was an approach with which Aristotle was only partly out of sympathy. It is true that he does not think that the Platonic separation of mathematical forms from perceptible things is a good philosophical move. He prefers to think of the numbers as $\pi\acute{\alpha}\theta\eta$ of bodies, and he is pretty sceptical of the weird results when (as he supposes) the Pythagoreans imagine that things just are numbers or made out of numbers. But the preference for the perfection of mathematics, and the location of genuine truth and beauty in the intelligible world, is less alien to Aristotle's way of thinking than it is to the agenda with which modern scholarship has approached the Pythagoreans—those prominent parts of modern scholarship which have been responsible for relegating them to the status of superstitious mystics.⁵⁸

What do I mean? I mean that there is a seamless continuity between Pythagorean awe at the perfect patterns in number, Parmenides's awe at the eternity of Being, Plato's awe at the Form of the Good, and Aristotle's awe at the Unmoved Mover. All these are objects of love and admiration, but their power is derived entirely from their beauty and perfection, not from any efficient or material causal efficacy.⁵⁹ At the end of

58 There are, of course, notable exceptions in modern scholarship on the Presocratics. Most importantly perhaps, Carl Huffman, who has done much to bring the contributions of neglected Pythagorean thinkers such as Philolaus and Archytas to our attention in their own right, and show that they have serious philosophical and theoretical meat to offer. Others, including most prominently Peter Kingsley, have sought to show why the religious and mystical side of Pythagorean traditions needs to be taken seriously.

59 Some sources credit the Pythagoreans (Hippasus in particular) with the discovery of irrational numbers, or particularly the incommensurability of the side and diagonal in a square, and suggest that this was a challenge to their belief in the mathematical perfection of the universe. (Porphyry *Life of Pythagoras* 246–7; Clement *Stromateis* 5.58). But one might equally suppose that the fact that the side and diagonal are commensurable when squared (effectively Pythagoras's theorem) would reveal a hidden rationality, a virtual rationality, in numbers that

the day, Aristotle too would locate the best and most perfect causal power in the teleological cause, a cause discovered by abstract reasoning, not by experimental science. The reason why Aristotle found the Pythagoreans puzzling was because he thought that they must be looking for the material cause. He could not fit into his system a weird attempt to explain bodies that have mass by appeal to entities that have no mass; but his bemusement is created by the story that he has to tell about the development of Presocratic philosophy. Presocratic *phusikoi* are (he thinks) trying to answer the question “Out of what, ἐκ τίνος, did the world come?” (or “Out of what, ἐκ τίνος, is the world made?”).⁶⁰ Aristotle glosses the “stuff,” out of which they suppose the world comes, as their *arche*, and his account of his predecessors tends to be couched as an analysis of their various attempts to cope with the logical complexity of the notion of coming to be *out of something*, while supposing that the something in question was logically playing the role of the material substrate. So it makes sense that if he found the Pythagoreans saying that the ἀρχή is number (or saying what he took to be that claim), he would suppose that something very weird was going on. But it was only weird because he was looking at their ἀρχή as a material substrate. It is not at all weird if you place it alongside the formal causes of the Platonists, or the final cause that is so powerfully there in Aristotle. If, instead, you start by wondering whether the Pythagoreans are talking about the explanatory power of beauty, structure, form, and indeed teleology, in the universe, the idea of appealing to patterns of numbers makes much more sense.⁶¹ If that was what they were doing, then their appeal to numbers as explanatory principles of the universe is not just methodologically sound, but also extremely perceptive.

So I suggest that Aristotle’s incomprehension might be created by his own agenda, which informs his investigation and presentation of the pre-Aristotelian history of ideas,—in other words, by that story that he tries to tell, of an investigation solely into material causes in the Presocratic period— but it does not reflect any ideological antipathy

were apparently irrational when treated as lines, and restore one’s faith in the idea of a mathematically coherent universe.

60 Aristot. *Meta.* 983b6–11; cf. *Phys.* 187a12–26.

61 Indeed there is evidence that Aristotle was partially aware of this alternative construal, as for instance in his comments at *Metaphysics* N 1092b8, where he admits that it is not clear in what sense numbers are explanatory of being, and suggests (as the second alternative) that it is because harmony is (explained as) a ratio of numbers and that this idea extends to explanations of other things.

on his part to the possibility that there might be other worthy causes to investigate besides the material cause.⁶² In modern thought the lack of appreciation for this pioneering work on numbers, which paved the way for all the most influential thinking from Plato to the Renaissance,⁶³ is due to something worse. It is due, I think, to a fundamental prejudice in favour of reductionism, materialism, and mechanistic conceptions of the physical world. It seems to me that, unlike Aristotle who thought that the Presocratics were *primitive* in trying to explain things by reference to matter alone, contemporary scholars assume instead that they are to be *congratulated* for exactly that. They think that *progress* was manifested in the Presocratics' move towards the more and more mechanistic theories of Anaxagoras and Democritus. They are *embarrassed* by Empedocles, and they often try to rescue him, by giving him one physical poem in which mechanistic forces explain everything and there are no appeals to immaterial values, so that they can conveniently ignore the other bits which don't fit that ideal. And they find themselves quite unable to stomach the Pythagoreans, when they discover that even abstract maths is imbued with ethical and religious value. "Mathematicians and philosophers just *shouldn't be talking about numbers with that kind of language*", I hear them say. "It's *superstitious*. It worries us."

The reason why we have come to think like that is because we have been brought up with an agenda that is more ideologically blinkered than Aristotle's. A deep set ideological preference for mechanistic theory prevents us from seeing that one might—one should—want to explain what is beautiful and awe-inspiring about the world; and that the explanation we give for beauty and wonder should *not explain it away*, in such a way that there is no awe and no beauty to move us after all.

62 My diagnosis of the source of Aristotle's incomprehension differs somewhat from Huffman (above, n. 9) 57–64. This is largely because Huffman thinks that Philolaus *was* investigating the corporeal world in the terms that Aristotle envisages (so Aristotle's mistake was in confusing the claim that things come from limiters and unlimiteds with the claim that they come from numbers). I am suggesting instead that Philolaus might indeed—nay surely did—assign a role to numbers in explanation, but not as a material explanation.

63 It would be good to say something about Plato, and particularly about those bits of Plato that have been taken to be somewhat Pythagorean in inspiration (including parts of the *Phaedo*), but that will have to wait for a more substantial opportunity to treat it in its own right.

To encapsulate the point I want to make, and to remind ourselves that the prejudice against the Pythagoreans is not universal, we should look at a passage from Leibniz.

It is rightly said in the paper given to the Princess of Wales, and which Her Royal Highness was kind enough to send to me, that next to vicious passions, *the principles of the Materialists* contribute much to support impiety. But I do not think that there were grounds to add that *the Mathematical Principles of Philosophy are opposed to those of the Materialists*. On the contrary, they are the same except that the Materialists, following in the footsteps of Democritus, Epicurus, and Hobbes, restrict themselves to mathematical principles alone, and admit nothing but bodies, whereas the Christian Mathematicians also admit immaterial substances. Thus it is not Mathematical Principles (in the ordinary sense of the term) but *Metaphysical Principles* which ought to be opposed to those of the Materialists. Pythagoras, Plato, and to some extent Aristotle, had some knowledge of these, but I claim to have established them demonstratively, although in my *Theodicy* this was done in a popular manner.⁶⁴

There is a distinction to be made between the materialist way of doing numbers, which uses numbers to give exact accounts of the behaviour of bodies, and the metaphysical move, which admits immaterial substances. What the Pythagoreans give us, according to Leibniz, is the metaphysical, which is, I would claim, the origin of real philosophy. It wasn't Anaximander who started us moving towards real philosophy, nor Democritus, nor even perhaps Heraclitus. It was Pythagoras, and he did it when he told us to take our oaths by the *tetraktys*, and that justice is the number 4 and *kairos* is the number 7.⁶⁵

Don't get me wrong. I am not trying to say that metaphysical theories are right, or that a metaphysical theory is better (as a theory) than a materialist one. I am just saying that we would not have had a history of western philosophy if there hadn't been Pythagoreans and Platonists: that the metaphysical turn (initiated or preceded by the numerical turn) is what is distinctive, and that is what first sets the debate about the nature of reality and the status of the perceptible world going. Parmenides does this, and the Pythagoreans do it, but the materialists

64 Gottfried Wilhelm Leibniz: *Correspondence with Clarke*: Leibniz's second paper, in G. W. Leibniz, ed. C. I. Gerhardt *Die Philosophischen Schriften von Gottfried Wilhelm Leibniz* (Berlin: Weidmann, 1890) vol. 7, p. 355. I am grateful to Lloyd Strickland who kindly supplied this new translation for me.

65 Alexander *In Meta*. 38.10).

don't.⁶⁶ And that is one reason why one might want to say that philosophy started in southern Italy, not on the coast of Turkey, and that Parmenides was, after all, an honorary Pythagorean.⁶⁷

Bibliography

- Algra, Keimpe, "The beginnings of cosmology," in A. A. Long (ed.), *The Cambridge Companion to Early Greek Philosophy* (Cambridge 1999) 45–65.
- Barnes, Jonathan, *The Presocratic Philosophers* (2nd edn.; London: Routledge and Kegan Paul, 1982).
- *Early Greek Philosophy*. Harmondsworth 1987.
- Betegh, Gábor. *The Derveni Papyrus: Cosmology, Theology and Interpretation*. Cambridge 2004.
- Burkert, Walter. *Lore and Science in Ancient Pythagoreanism*, trans. E.L. Minar Jr. Cambridge, Mass. 1972.
- Cornford, Francis MacDonald, "Mysticism and science in the Pythagorean Tradition," in Alexander P. D. Mourelatos (ed.), *The Presocratics*. (Garden City 1974) 135–60.
- Couprrie, Dirk L. "Anaximander's discovery of space," in Anthony Preus (ed.), *Before Plato* (Albany 2001) 23–48.
- "The Discovery of space: Anaximander's astronomy," in id., Robert Hahn, and Gerard Naddaf (eds.), *Anaximander in Context* (Albany, 2003) 165–254.
- Dicks, D. R., *Early Greek Astronomy to Aristotle*. London 1970.
- Findlay, John N. *Plato; The Written and Unwritten Doctrines*. London 1974.
- Furley, David. *The Greek Cosmologists*. Vol. 1. Cambridge 1987.
- Hahn, Robert. *Anaximander and the Architects*. Albany 2001.

66 One feature of the prevalent view that presocratic philosophers were seeking mechanistic and materialist explanations is the assumption that all or most of the Presocratics were materialists in their metaphysics, and indeed that they had no notion of the incorporeal at all. This is not the place to develop a full defence of my claim that having a notion of the incorporeal is older and more ordinary than the idea that there are no incorporeal entities. I see no reason to suppose that it was *hard* for primitive thinkers to come up with an idea of non-bodily powers and causes: assuming that there are such powers and causes seems to me to be the norm in pre-scientific societies. I will develop this idea in relation to the Presocratics in two further papers, as yet unpublished.

67 This paper was originally composed in 2005. The current version has not been heavily revised to address more recent work, but it has benefited from discussion with various audiences, (in Samos and the B club in Cambridge in 2005, and the pure mathematics seminar at UEA in 2006). Extensive written comments from Carl Huffman on my first draft have been of immense value.

- “Proportions and numbers in Anaximander and early Greek tThought,” in Dirk L Couprie, Robert Hahn, and Gerard Naddaf, *Anaximander in Context*. (Albany 2003) 73–163.
- Heidel, William A., “The Pythagoreans and Greek mathematics,” in David Furley and R.E. Allen (eds.), *Studies in Presocratic Philosophy* (London 1970) 350–81.
- Huffman, Carl. *Philolaus of Croton*. Cambridge 1993.
- “The Pythagorean tradition,” in A. A Long (ed.), *The Cambridge Companion to Early Greek Philosophy* (Cambridge 1999) 66–87.
- *Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King*. Cambridge 2005.
- Kahn, Charles. *Anaximander and the Origins of Greek Cosmology*. New York 1960.
- Kingsley, Peter. *Ancient Philosophy, Mystery and Magic: Empedocles and Pythagorean Tradition*. Oxford 1995.
- Leibniz, Gottfried Wilhelm. “Correspondence with Clarke,” in C. I. Gerhardt (ed.), *Die Philosophischen Schriften von Gottfried Wilhelm Leibniz* (Berlin 1890) 7. 345–440.
- Lloyd, Geoffrey E. R. *Early Greek Science Thales to Aristotle*. London 1970.
- McKirahan, Richard. *Philosophy before Socrates*. Indianapolis 1994.
- Naddaf, Gérard, “Anaximander’s measurements revisited,” in Anthony Preus (ed.), *Before Plato* (Albany 2001) 5–23.
- Nussbaum, Martha C. “Eleatic conventionalism and Philolaus on the conditions of thought,” *HSCP*, 83 (1979) 63–108.
- O’Brien, Denis. “Anaximander’s measurements,” *CQ* 17 (1967) 423–32.
- Quine, Willard V. “Ontological reduction and the world of numbers,” in id., *The Ways of Paradox and other essays* (New York 1966), 199–207.
- “Ontological relativity,” in id., *Ontological Relativity and other Essays* (New York: Columbia University Press, 1969) 26–68.
- “Propositional objects,” in id., *Ontological Relativity and other Essays* (New York 1969) 137–60.
- Robinson, Thomas M. *Heraclitus: Fragments*. Toronto 1987.
- Vogel, Cornelia J. de. *Pythagoras and Early Pythagoreanism: an Interpretation of Neglected Evidence on the Philosopher Pythagoras*. Assen 1966.
- Waterfield, Robin. *The First Philosophers*. Oxford 2000.
- West, Martin L., *Early Greek Philosophy and the Orient*. Oxford 1971.

