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CHINESE CYCLIC TUNINGS IN LATE ANTIQUITY

Ernest G. McClain

with translations by Ming Shui Hung

Between the first century B.C. and the fifth century A.D. certain aspects of Chinese tuning theory were more than a thousand years ahead of similar developments in the West. The cycle of fifths was studied to the limit of 60 and then 360 tones, and arithmetic approximations to equal temperament were developed. The simplicity and accuracy of Chinese methods, not fully appreciated in the East, have been hidden from the West by the language barrier.

Here is a complete translation of the 60-division tuning of Ching Fang (–78 to –37) together with an analysis of his procedure. From his model, and in accordance with the instructions that survived, I reconstruct the 360-division tuning of Ch'ien Lo-chih (fl. +415 to +455), showing how simply it could have been achieved, and how it may be viewed as the natural culmination of the process Ching Fang set in motion. I then suggest how the emerging interest in temperament, accessible by a formula derived from Ching Fang, may have helped to motivate Ch'ien's extended study of the tone field. In the work of Chinese acousticians we witness a remarkable balance between theory and practice, between the pursuit of absolute perfection on the one hand, and the acceptance of common-sense simplification on the other.

HISTORICAL BACKGROUND

Throughout history the Chinese scale has remained basically pentatonic, on the model C D E G A, with its modal variations, tuned as a sequence of falling fourths and rising fifths. The first 12 tones of the extended series gave their names to the months. In some historical periods various auxiliary tones were also recognized, and ritual transpositions coordinated the scale with the calendar (Nakaseko 1957:147-172). The sequence of fifths 2:3 and fourths 3:4 was sometimes extended to as many as $5 \times 12 = 60$ tones, or 64 tones (providing for 12 pentatonic transposi-

tions), or 84 tones (12 diatonic transpositions), and eventually, theoretically, to 360 tones. Successive dynasties established their own pitch standards and freely revised or discarded the calendrical ritual, yet maintained a strongly conservative bias. Successive tones were generated by the perfect fifths and fourths of "Pythagorean" tuning, and new pitch standards varied over the range of a musical sixth (Chuang 1963:54-55). Ritual transposition coupled with a fondness for huge orchestras of winds, strings, and tuned bells gave Chinese acoustical theory the same impetus toward a 12-tone simplification that chromatic modulations and triadic harmonies of the 17th century fostered in the West. Chinese theorists were far ahead of us in grappling with the problems of temperament, but Chinese conservatism consistently doomed every innovation, however brilliant, that threatened the purity of perfect fourths and fifths.

The oldest surviving Chinese tuning, that by Kuan-tzu for the first 5 tones (recorded ca. 3rd c. B.C.), establishes the pattern followed by later tunings. He alternately adds and subtracts $\frac{1}{3}$ of successive tone-values (pipe and/or string lengths) to produce the falling fourths and rising fifths of Pythagorean tuning (Nakaseko 1957:147-150):

schema	C	G	D	A	E
numbers	81	108	72	96	64
Chinese "solmization"	<i>kung</i>	<i>chih</i>	<i>shang</i>	<i>yü</i>	<i>chiao</i>

In the Confucian ritual tradition, the tone now known as *Huang-chung* and translated as "Yellow Bell" was tonic (*kung*) at the winter solstice in the 11th month. When *Huang-chung* is assigned the numerical value of 81, conceived as a pitch-pipe length of 8.1 inches (Chinese subdivisions normally being decimal), or as 9 inches divided into 9ths, as during the Chou dynasty, only 5 tones can be designated accurately by successive additions and subtractions of $\frac{1}{3}$ without the further use of fractions. The Kuan-tzu tuning is thus in "smallest integers." Odd-numbered tones in the tuning sequence are *yang* (male) and even-numbered tones are *yin* (female) irrespective of whether generation begins "downward," as this one does, or "upward," as later tunings do. The 3 *yang* tones in this first set (C D E) and the 2 *yin* tones (G A) show a double implication for the "male" prime number 3 and "female" prime number 2, which generate the tuning.

The complete cycle of 12 tones (*lü*) is first given in the *Huai Nan Tzu*, the book composed for Liu An, Prince of Huai Nan, ca. -122, both in 6-digit precise values and in 2-digit approximations whose derivation remains controversial (see Table I). The accurate designation for the cycle of 12 tones must obviously begin on $3^{11} = 177,147$ to avoid fractions.

TABLE 1
Pitch Numbers in the *Huai Nan Tzu*

1.	177,147 = 81	C	Huang-chung	11th month
2.	$-\frac{1}{3} = 118,098 = 54$	G	Lin-chung	6th month
3.	$+\frac{1}{3} = 157,464 = 72$	D	T'ai-ts'ou	1st month
4.	$-\frac{1}{3} = 104,976 = 48$	A	Nan-lü	8th month
5.	$+\frac{1}{3} = 139,968 = 64$	E	Ku-hsi	3rd month
6.	$-\frac{1}{3} = 93,312 \approx 43^*$	B	Ying-chung	10th month
7.	$+\frac{1}{3} = 124,413 \approx 57$	F#	Jui-pin	5th month
8.	$+\frac{1}{3} = 165,888 \approx 76$	C#	Ta-lü	12th month
9.	$-\frac{1}{3} = 110,592 \approx 51$	G#	I-tse	7th month
10.	$+\frac{1}{3} = 147,456 \approx 68$	D#	Chia-chung	2nd month
11.	$-\frac{1}{3} = 98,304 \approx 45$	A#	Wu-yi	9th month
12.	$+\frac{1}{3} = 131,072 \approx 60$	E#	Chung-lü	4th month

*The original *Huai Nan Tzu* value of 42 was corrected to 43 by Prince Chu Tsai-yü in the + 16th c.

Ching Fang will later generate his 60 tones from this same base, his last 48 necessarily being approximations, for he also avoids fractions in his basic calculations. From our equal temperament perspective, this tuning by perfect fourths and fifths produces a spiral rather than a cycle, for there is a cumulative excess of 2 cents per interval. *Chung-lü* E#, the 12th tone in the sequence, is thus 22 cents sharp, and cannot lead to a cyclic return to C. The mean deviation for the set of 12 tones, shown below in Table V, is almost 12 cents. A remarkable feature of the accompanying 2-digit approximations, when 42 is corrected to 43 as Prince Chu Tsai-yü suggests, is that their mean deviation from equal temperament is only 7 cents (Kuttner 1975:176). The familiar Pythagorean comma of 531441:524288 between the 13th tone of such a series and the original reference tone here acquires a Chinese simplification to 81:80 (notice that 60 plus $\frac{1}{3}$ of itself gives 80), a value familiar in the West in quite a different tonal context as the "syntonic comma" (approximately 22 cents instead of 24). Two successive additions for *Jui-pin* F# and *Ta-lü* C# compress operations within an octave. Among the 2-digit numbers, the rising fifths E-B, C#-G#, and D#-A# and the falling fourth B-F# are no longer "perfect" ratios of 3:2 and 4:3 respectively. Notice that when divided by 3, the numbers 64, 43, and 76 give remainders of $\frac{1}{3}$, which are apparently dropped, while 68 gives a remainder of $\frac{2}{3}$, which is rounded off to the full unit so that $\frac{2}{3} \times 68 = 45$. What purpose these 2-digit numbers served is not clear, and there is no agreement as to how they were achieved. They prove, however, that for at least a century before Ching Fang the Chinese were accustomed to working both with precise values and with approximations. His achievement can be understood, then, as a virtuoso variation on an ancient theme.

CHING FANG

The 60-division tuning of Ching Fang (–78 to –37) and his own extensive commentary on it were recorded by Ssu-ma Piao three centuries later in the *Hou Han Shu*, the official history of the late Han dynasty (+25 to +219). Ssu-ma Piao informs us that an imperial Bureau of Music had been established by Emperor Wu (–140 to –88), who “wanted to correct the music.” Ching Fang received his appointment to the Bureau from Emperor Yüan (–48 to –33), after an examination by Hsuang Cheng, “lecturer to the heir-apparent, and a certain official Chang, who was a censor” (Fan 1965:3000). The Early Han dynastic history contains a biographical entry for Ching Fang written by Pan Ku, from whom we learn that Ching Fang was also an expert at divination, that from the hexagrams of the *I Ching* he could predict the weather—“always very accurately.” His original name, Li Fang, he changed also according to a rule of divination. At the age of 41 he was executed by order of the emperor who had appointed him—his head was chopped off in the market place—for “accusations against a high official” (Pan 1962:3160-67).

Ching Fang has an honored place in the history of acoustical theory as the first man to compute the greatly reduced comma between the 1st and 54th tones in the spiral of fifths (Barbour 1953:124). The simplicity and accuracy of his approximation technique commands attention here. His tuning is given in full in Table II, in scale order; modern tone names and the tuning order, or generation order, have been added to clarify his procedure. We do not translate his tone names; beyond the first 12 in the tuning order, associated with the months, names appear to be merely convenient labels for the operations. That the whole construction has survived in nearly perfect condition is a tribute to the quality of Chinese scholarship and to the care Ching Fang took to express results in three complementary ways: 6-digit *shih* (“substance”) numbers are accompanied by 3-digit *lǚ* (pitch-pipe) lengths, including symbols for fractional remainders, and by 2-digit *chun* (long zither) string lengths, with *exact* fractions that serve as a cross-check against copyists’ errors in the *shih* and *lǚ* numbers (Fan 1965:3003-14).

Ching Fang’s calculation is first carried out in 6 digits, as an extension of the earlier series in the *Huai Nan Tzu*—corrected to keep the approximations in tones 13 to 60 within a fraction of a cent of their true values in the spiral of fifths. These 6-digit *shih* numbers are then divided by $3^9 = 19,683$ to give pipe lengths for the *lǚ* (whose *Huang-chung* fundamental is 9 inches) and string lengths for the *chun* (whose fundamental is 9 feet), all subdivisions being decimal. The 3-digit *lǚ* numbers are practical for *both* pipes and strings. They are accurate to within 2 to 4 cents,

TABLE II
The 60-Division Tuning of Ching Fang

Scale order	Tuning order	Name	Shih (number)	Lü (pipe)	Chun (string)
1	C 1	黃鍾 Huang-chung	177,147	9 inches	9 feet
2	54	色育 Se-yü	176,776	8.98+	8.9+15,973/x*
3	13	執始 Chih-shih	174,762	8.87+++	8.8+15,516/x
4	25	丙盛 Ping-sheng	172,410	8.76-	8.7+11,679/x
5	37	分動 Fen-tung	170,089	8.64++	8.6+8,152/x
6	49	質末 Chih-mo	167,800	8.52½++	8.5+4,945/x
7	C# 8	大呂 Ta-lü	165,888	8.43--	8.4+5,508/x
8	20	分否 Fen-fou	163,654	8.31++	8.3+2,851/x
9	32	凌陰 Ling-yin	161,452	8.21--	8.2+514/x
10	44	少出 Shao-ch'u	159,280	8.09++	8+18,160/x
11	D 3	太簇 T'ai-ts'ou	157,464	8	8
12	56	未知 Wei-chih	157,134	7.98++	7.9+16,383/x
13	15	時息 Shih-hsi	155,344	7.89(+)	7.8+18,166/x
14	27	屈齊 Ch'ü-ch'i	153,253	7.79--	7.7+16,939/x
15	39	隨期 Sui-ch'i	151,190	7.68++	7.6+15,992/x
16	51	形晉 Hsing-chin	149,156	7.58--	7.5+5,335/x
17	D# 10	夾鍾 Chia-chung	147,456	7.49++	7.4+18,018/x
18	22	開時 K'ai-shih	145,470	7.39+	7.3+17,841/x
19	34	族嘉 Tsu-chia	143,513	7.29+	7.2+17,954/x
20	46	爭南 Cheng-nan	141,582	7.19++	7.1+18,327/x

Remainders: 微強 +; 強 ++; 半強 ½++; 大強 +++; 弱 --; 微弱 -
少強 (+).

$$*x = 19,683 = 3^9$$

TABLE II (continued)

Scale order	Tuning order	Name	Shih (number)	Lü (pipe)	Chun (string)
21	E 5	姑洗 Ku-hsi	139,968	7.11+	7.1+2,187/x
22	58	南授 Nan-shou	139,674	7.09+++	7+18,930/x
23	17	變虞 Pien-yü	138,084	7.01½++	7+3,030/x
24	29	路時 Lu-shih	136,225	6.92+	6.9+4,123/x
25	41	形始 Hsing-shih	134,392	6.83--	6.8+5,476/x
26	53	依行 I-hsing	132,582	6.73½++	6.7+7,059/x
27	E# 12	中呂 Chung-lü	131,072	6.66--	6.6+11,642/x
28	24	南中 Nan-chung	129,308	6.57-	6.5+13,685/x
29	36	內負 Nei-fu	127,567	6.48+	6.4+15,958/x
30	48	物應 Wu-ying	125,850	6.39++	6.3+18,471/x
31	F# 7	蕤賓 Jui-pin	124,416	6.32+	6.3+4,131/x
32	60	南事 Nan-shih	124,154	6.31--	6.3+1,511/x
33	19	盛變 Sheng-pien	122,741	6.23½++	6.2+7,064/x
34	31	離宮 Li-kung	121,089	6.15+	6.1+10,227/x
35	43	制時 Chih-shih	119,460	6.07--	6+13,620/x
36	G 2	林鍾 Lin-chung	118,098	6	6
37	55	謙待 Ch'ien-tai	117,851	5.99--	5.9+17,213/x
38	14	去滅 Ch'ü-mieh	116,508	5.92--	5.9+3,783/x
39	26	安度 An-tu	114,940	5.84-	5.8+7,786/x
40	38	歸嘉 Kuei-chia	113,393	5.76+	5.7+11,999/x
41	50	否與 Fou-yü	111,867	5.68++	5.6+16,422/x

TABLE II (continued)

Scale order	Tuning order	Name	Shih (number)	Lü (pipe)	Chun (string)
42	G# 9	夷則 I-tse	110,592	5.62--	5.6+3,672/x
43	21	解形 Chieh-hsing	109,103	5.54++	5.5+8,465/x
44	33	去南 Ch'ü-nan	107,635	5.46+++	5.4+13,468/x
45	45	分積 Fen-chi	106,187	5.39½++	5.3+8,671/x
46	A 4	南呂 Nan-lü	104,976	5.33++	5.3+6,561/x
47	57	白呂 Po-lü	104,756	5.32++	5.3+4,361/x
48	16	結躬 Chieh-kung	103,563	5.26++	5.2+12,114/x
49	28	歸期 Kuei-ch'i	102,169	5.19+	5.1+17,857/x
50	40	未卯 Wei-mao	100,794	5.12+	5.1+4,107/x
51	52	夷汗 I-han	99,437	5.05++	5+10,220/x
52	A# 11	無射 Wu-yi	98,304	4.99++	5.9+1,857/x
53	23	閉掩 Pi-yen	96,980	4.93--	4.9+5,333/x
54	35	鄰齊 Lin-ch'i	95,675	4.86+	4.8+11,966/x
55	47	期保 Ch'i-pao	94,388	4.79½++	4.7+18,779/x
56	B 6	應鍾 Ying-chung	93,312	4.74+	4.7+8,019/x
57	59	分鳥 Fen-wu	93,116	4.73+	4.7+6,059/x
58	18	遲內 Ch'ih-nei	92,056	4.68--	4.6+15,142/x
59	30	未育 Wei-yü	90,817	4.61(+)	4.6+2,752/x
60	42	遲時 Ch'ih-shih	89,595	4.55++	4.5+10,215/x

meaning to within .01 inches on the pipes, and to within .1 inches on the 10 times longer strings. These *lü* numbers, however, are accompanied by 7 symbols that increase accuracy by almost another decimal place, hence to within a fraction of a cent. Five symbols, translated here as +, (+), ++, ½++, and +++, indicate the relative sizes of remainders that have been dropped. Two other symbols, translated as – and --, indicate that remainders have been raised to the next larger digit. The only errors in the table are trivial ones concerning these symbols for the remainders.¹ The large fractional remainders for the 2-digit *chun* numbers are too awkward

for use on pipes or strings, but they are the essential bibliographic tool for verifying the *shih* numbers.

The main significance of the construction can be found in the first three tones of Ching Fang's scale order. *Chih-shih*, third tone in the scale but 13th in the spiral of fifths, should differ from *Huang-chung* by the Pythagorean comma of 531,441:524,288, almost 24 cents. Ching Fang's *shih* approximation is accurate to within .01 cents, and his *lŭ* reduction shows this to be worth about 12 parts in 900 (9.00–8.87+++). *Se-yŭ*, second tone in the scale but 54th in the spiral of fifths, subdivides this comma, missing *Huang-chung* by only about 1/6 as much. To express accurately this new ratio of the so-called "54-comma" requires numbers with 26 digits; we are now involved with 3^{53} and the nearest power of 2. It is here on *Se-yŭ* that we appreciate the power in Ching Fang's computational technique:

$$\frac{\text{Huang-chung}}{\text{Se-yŭ}} = \frac{3^{53}}{2^{84}} \approx \frac{\text{shih } 177,147}{176,776} = \frac{\text{lŭ } 9.00}{8.98+}$$

The *shih* approximation is accurate to within .14 cents, phenomenal by any standard, and the *lŭ* reduction allows us to see this as less than 2 parts in 900. The "54-comma" formed first by *Se-yŭ* is duplicated between the 2nd and 55th tones and by all subsequent pairs. The complete set is rounded off to 60 elements probably because of that number's hoary prestige as a basic calendrical unit. Ching Fang's table thus represents sequences of Pythagorean commas generated by successive 12th intervals in the spiral of fifths (see 1-13-25-37-49 starting from *Huang-chung*, or 8-20-32-44 from *Ta-lŭ*), with the last 7 tones producing "54-commas" within the first 7 Pythagorean commas. Note that each diatonic semitone (C#-D, D#-E, etc.), or Greek *leimma*, contains 4 Pythagorean commas, while each chromatic semitone (C-C#, D-D#, etc.), or Greek *apotome*, contains 5 such commas. Philolaus taught the Greeks that the Pythagorean wholetone of 9:8 spanned about 9 commas; Chinese theorists could read the same lesson in Ching Fang's table. We see at a glance exactly how the whole spiral of fifths unfolds through wholetones, semitones, and Pythagorean commas to reach near agreement with the reference tone after 53 consecutive operations—as Mersenne, Kircher, and Mercator were to learn in the 17th century (Barbour 1953:124-25).

Before analyzing Ching Fang's approximation technique, and in view of his historical importance, we translate in full the exposition that Ssuma Piao attributes to him at the time of his appointment to the Bureau of Music. Notice that Ching Fang does not claim originality. He explains the

generation process, links it to traditional *yin-yang* dualism, the calendar, the *I Ching*, and, rather vaguely, to his weather forecasting, and then concludes with a description of the instrument he built for acoustical demonstrations. Our explanations are in brackets [].

I study, under Chiao Yen-shou, a magistrate of Hsiao-huang, the method of 60 *lǚ*. For *shang* [generation upwards] to give birth to *hsia* [generation downwards], 3 gives birth to 2 [subtract $\frac{1}{3}$], and for *hsia* to give birth to *shang*, 3 gives birth to 4 [add $\frac{1}{3}$]. *Yang* generating from below produces *yin*, and *yin* generating from above produces *yang*, ending at *Chung-lǚ* to complete the 12 *lǚ*. *Chung-lǚ* [E #] generating from above produces *Chih-shih* [the first Pythagorean comma], and *Chih-shih* produces *Ch'ü-mieh* [the subsequent comma, a fifth higher]. "Up" and "down" give birth to each other and end with *Nan-shih* [a comma above *Jui-pin* F #] to complete the 60 *lǚ*. 12 *lǚ* develop into 60 *lǚ* just like 8 hexagrams develop into 64 hexagrams. Fu Hsi created the [*Book of*] *Changes* to record the beginning of the *yang-ch'i* ["male force"] and made it into a rule [*lǚ-fa*] to establish a date for the sound of the arrival of winter and of *Huang-chung* as *kung* ["do"], *T'ai-ts'ou* as *shang* ["re"], *Ku-hsi* as *chiao* ["mi"], *Lin-chung* as *chih* ["sol"], *Nan-lǚ* as *yü* ["la"], *Ying-chung* [B] as *pien-kung* [transformed *kung*, an auxiliary tone], and *Jui-pin* [F #] as *pien-chih* [transformed *chih*, a second auxiliary tone]. This is the origin of the sound and *ch'i* ["force"] and the correct doctrine of the 5 tones [2 of the 7 named being merely auxiliary tones]. Therefore each occupies a day; the rest of them follow in succession. Each in succession is adopted as *kung* ["tonic"] for a particular date. *Shang* and *chiao* follow the rule. *The Book of Rites* states: "5 sounds, 6 *lǚ*, and 12 pipes each in succession become *kung*." That is what I have just said. The period is divided by 60 *lǚ*. *Huang-chung* starts from the arrival of winter and appears again the following winter. This is how divination can predict *yin* and *yang* and cold or warm, wind or rain. Use this to examine different sounds and to investigate whether they are high or low. Except for the sound of trees or grass [i.e., except for the "white noise" of the wind?], everything else will be matched. This is what it means in *Yü Shu*, which says, "*lǚ* harmonizes with sound."

The bamboo pipe cannot be used for tuning [evidence of awareness of the problem of "end-correction"?] and therefore I make the *chun* to determine the number. The shape of the *chun* is like a *se* [zither], with a length of one *chang* [ten feet], and with 13 strings. Divide [the sounding length] into 9 feet to correspond with the 9 inches of the *lǚ* of *Huang-chung*. Underneath the central string mark with *ts'un* [inches] and *fen* [tenths of inches] to measure the 60 *lǚ*.

Ssu-ma Piao concludes his introduction to the Ching Fang tuning by observing that "his writing is not recorded in its entirety, therefore I sum up the outline to supplement the records in the formal dynasty history" (Fan 1965:3000-01).

How did Ching Fang achieve his remarkable accuracy? Since he began with $3^{11} = 177,147$, as in the *Huai Nan Tzu*, his first 12 tones are untroubled by fractional remainders. In the alternating addition and subtraction of thirds that follows, however, only a few numbers are evenly divisible by 3, hence there are several remainders of $\frac{1}{3}$ and $\frac{2}{3}$. Reconstruction of the arithmetic shows that if Ching Fang had simply dropped *all* remainders while computing the 6-digit *shih* numbers, 32 of his 6-digit numbers would be 1 or 2 units smaller than they are—and his calculations

would still be accurate to within about .1 cents. His corrections, however they were made, improve accuracy to within about .01 cents, far beyond the limit of perception. How did he achieve such near-perfection?

I suggest that Ching Fang trusted the oscillation of addition and subtraction to balance the effects of positive and negative remainders and allow most of them to be dropped. But he closely watched the unpredictable rhythm of remainders of $\frac{2}{3}$ and made corrections at exactly 4 points, on tones numbered 23, 34, 35 and 51 in the *tuning* (not *scale*!) order. The arithmetic suggests the following rationale:

- a) 3 successive remainders of $+\frac{2}{3}$ on tones 19, 21, and 23 of the tuning order justify raising the 23rd number by 2 digits ($3 \times \frac{2}{3} = 2$), in the absence nearby of compensating subtractions.
- b) Tone 35 in the tuning order is rounded to a full digit, from a $+\frac{2}{3}$ remainder, partly because there is also a $+\frac{1}{3}$ on tone 36.
- c) At tones 34 and 51, a $-\frac{2}{3}$ is rounded to a full digit during the subtractions, again in the absence of nearby compensating additions. (The only other $-\frac{2}{3}$ in the entire set is at tone 39, balanced by $+\frac{2}{3}$ on tone 38, hence both remainders are dropped.)

The effect of Ching Fang's corrections is to let his set match the uncorrected set (with *all* fractions dropped) on tones 1 through 22, rise slightly above it until the momentary agreement at tone 34, then rise slightly above it again until—aided by a subtraction on tone 51—there is a natural convergence on tone 56. Ching Fang could have improved his accuracy very slightly by other choices of corrections, but to no purpose.² His calculations as they stand display that harmony of precision and simplicity properly called elegance.

If we reflect that 1 cent is approximately the ratio 1730:1731, we can see that Ching Fang's 6-digit numbers, approximately 100 times larger, will safeguard distinctions of about .01 cents—with optimal corrections. That is more than enough accuracy for acoustical theory. In dividing his 6-digit *shih* numbers by $3^9 = 19,683$ to produce the 3-digit *lü* lengths, Ching Fang displayed his eminently practical concerns. In denying that the pitch-pipes were accurate enough for acoustical theory, and in making demonstrations on a 9-foot string, Ching Fang proved his competence in the acoustics laboratory. But the significance of Ching Fang's work lies mainly in the *insight* later theorists could gain from it. He lays the foundation for the first Chinese effort towards equal temperament.

THE TEMPERAMENT OF HO CH'ENG-T' IEN (+370 TO +477)

The earliest numerical approximation to equal temperament is that by Ho Ch'eng-t'ien, ca. +400, possibly inspired by Ching Fang's presen-

tation of the comma as about 12 parts in 900 (see *Huang-chung* 9.00 versus *Chih-shih* 8.87+++ in Table II). By the clever device of adding one digit (meaning .01 inches) to each successive *lŭ* length, Ho Ch'eng-t'ien avoided the comma and achieved the first truly cyclic 12-tone tuning. The result is an "unequal" temperament, for his *linear* corrections ignore their own *logarithmic* implications, but the mean deviation from equal temperament of only 2.2 cents makes this "a remarkable temperament for the time when it was constructed" (Barbour 1953:55-56 and Kuttner 1975:173-6). Ho Ch'eng-t'ien's procedure is described in the *Sui Shu* (History of the Sui Dynasty), compiled in the +7th century and edited in +1024 during the T'ang dynasty (Wei 1973:389):

Ho Ch'eng-t'ien started to establish a new rule and from *Chung-lŭ* [E # , the 12th tone in the tuning order], went back to *Huang-chung*, completing the cycle of 12 *kung* [instead of arriving on *Chih-shih*, a comma higher], The length for *Huang-chung* was 9 inches, *T'ai-ts'ou* 8.2 [not 8.00], *Lin-chung* 6.1 [not 6.00], and *Ying-chung* 4.79+ [not 4.74+]. *Chung-lŭ* gives birth from above to 177,147 [the 6-digit *shih* number for *Huang-chung*], completing the cycle of 12.

Neither Ho Ch'eng-t'ien's temperament nor those of later theorists found favor in China. The perfection of the fifth 3:2 legitimized any dynasty's link with China's glorious past when 1 was the number of heaven, 2 of earth, and 3 of man, and the musical scale was the model for an harmonious society (Needham 1956:II, 271). In such a context it was natural to search for a cyclic return in a more extended tone-field of fifths 3:2 and fourths 4:3, and Ho Ch'eng-t'ien himself pointed the way in his book *Li-fa Chih-i*, ending with an interesting accusation against Ching Fang (Wei 1973:389):

Shang [generation upwards] and *hsia* [generation downwards] give birth to each other. To subtract or add $\frac{1}{3}$ is the simple and easy method of the ancients. It is like the ancient calendar, which has $365\frac{1}{4}$ degrees for the cycle of the sky [on oracle bones from the 2nd millenium B.C.]. Later people changed the system in different ways. But Ching Fang did not realize that and by this absurd mistake made it into 60.

THE 360 DIVISION TUNING

The *Sui Shu* contains two accounts of the extension of Ching Fang's tuning from 60 to 360 tones, and we shall translate both accounts in full. The extension was first accomplished by Ch'ien Lo-chih (Wei 1973:389):

Ch'ien Lo-chih, a historian during the reign of Yüan-chia [+424 to 453] of the Sung [in Southern Dynasties], followed the *Nan-shih* of Ching Fang with 300 more tones ending with *An-yün* of 4.4+ inches. Including the old tones, there are a total of 360, one for each day. Each follows the succession *kung chih* [meaning the standard pentatonic order].

The calculation was repeated in the following century by Shen Chung, described as a "scholar of the 5 [Confucian] classics," tutor for the heir-apparent, whose lectures at the court about the 3 religions (Confucianism, Taoism, and Buddhism) were attended by 2000 Confucian-Taoist officials (Ling-hu 1974:808-11). (The texts are silent on the relationship between these two efforts.)

Ch'ien Lo-chih of the Sung followed Ching Fang's *Nan-shih* with 300 more tones. During the Liang Dynasty [+502 to 556], a doctorate, Shen Chung, in his *Chung Lū I* [Discussion on musical rules] says:

The *I Ching* has 360 *ts'e* [divining straws] to meet the number of days of a cycle [year]. This is the number for music and the calendar. *Huai Nan Tzu* says: "each *lū* gives birth to 5 sounds, 12 *lū* give birth to 60 sounds, and 6 repetitions give birth to 360 sounds to meet the number of days in a year. This number for music and the calendar reflects the way of heaven and earth." This has been the case since antiquity.

[Shen] Chung calculated the 360 *lū* with the original number in the *Huai Nan Tzu* and the method of Ching Fang.

The *Sui Shu* then names all 360 tones in scale order, in columns headed by the original 12 tones, and adds a few important interpretative comments, but omits the numbers (Wei 1973:397-401).

In Table III we convert the 360 *Sui Shu* names into numbers, arranged into similar columns (except for the reversal of right and left)—by following clues gleaned from the material translated above. Numbers are computed by alternate subtractions and additions of $\frac{1}{3}$ —"the simple and easy method of the ancients"—beginning with Ching Fang's *Nan-shih* = 124,154, 32nd in his list but 60th in his generative order. Since these 6-digit numbers are already generally accurate to within about .1 cents, we have left them uncorrected, simply dropping all fractional remainders instead of trying to compete with Ching Fang's even greater accuracy. "Step numbers" added for the first column only will reveal at a glance that the table essentially consists of Ching Fang's original sequences of Pythagorean commas (note underlined numbers), easily recognized by their tone names, each one subdivided now by consecutive 54-commas.³ Note the long column for each chromatic semitone (*apotome*), now containing 34 tone-numbers, and the shorter column for each diatonic semitone (*leimma*), containing only 27 numbers. These totals are given in the *Sui Shu*, probably to guard against copyists' errors. One additional number, the last tone-number of the set, 88,472, has been placed in the last column, giving it a total of 28 tones. This 360th number—named *An-yün*, meaning "peaceful circuit"—is the only one allowed to exceed the range of an octave from *Huang-chung*; when divided by $3^9 = 19,683$, in Ching Fang's manner, it gives the *lū* value of 4.4+ inches (more precisely 4.494), exactly as we were told that Ch'ien Lo-chih discovered. *An-yün* is sharper

TABLE III
The 360-Division Octave of Ch'ien Lo-chih (reconstruction)

Step	C	C#	D	D#	E	E#	F#	G	G#	A	A#	B
	Huang- chung (34)	Ta-lü (27)	T'ai- ts'ou (34)	Chia- chung (27)	Ku-hsi (34)	Chung- lü (27)	Jui- pin (27)	Lin- chung (34)	I-tse (27)	Nan-lü (34)	Wu-yi (27)	Ying- chung (28)
1	177147	165888	157464	147456	139968	131072	124416	118098	110592	104976	98304	93312
54	176776*	165538	157134	147145	139674	130796	124154	117851	110359	104756	98097	93116
107	176400	165189	156800	146834	139378	130520	123892	117600	110126	104534	97890	92919
160	176028	164840	156469	146525	139084	130245	123630	117352	109894	104313	97684	92723
213	175665	164501	156146	146224	138797	129977	123376	117110	109668	104098	97483	92532
266	175298	164157	155821	145917	138508	129704	123118	116866	109438	103881	97278	92339
319	174929	163810	155493	145609	138216	129430	122858	116620	109207	103662	97073	92144
13	174762	163654	155344	145470	138084	129308	122741	116508	109103	103563	96980	92056
66	174394	163309	155017	145164	137793	129034	122482	116263	108873	103345	96776	91862
119	174026	162965	154690	144858	137502	128762	122224	116018	108644	103127	96572	91668
172	173660	162624	154365	144554	137213	128493	121968	115774	108416	102910	96370	91476
225	173302	162288	154046	144256	136930	128228	121716	115535	108192	102698	96171	91287
278	172938	161946	153722	143952	136642	127957	121460	115292	107964	102482	95968	91095
331	172573	161605	153398	143649	136354	127688	121204	115049	107737	102266	95766	90903
25	172410	161452	153253	143513	136225	127567	121089	114940	107635	102169	95675	90817
78	172045	161110	152929	143209	135937	127297	120833	114697	107407	101953	95473	90625
131	171682	160770	152606	142906	135650	127028	120578	114455	107180	101738	95271	90434
184	171324	160437	152288	142610	135368	126765	120328	114216	106958	101526	95074	90246
237	170970	160104	151973	142314	135088	126501	120078	113980	106736	101316	94876	90059
290	170609	159765	151653	142013	134802	126234	119824	113740	106510	101102	94676	89868
343	170250	159429	151333	141714	134518	125968	119572	113500	106286	100889	94476	89679
37	170089	159280	151190	141582	134392	125850	119460	113393	106187	100794	94388	89595
90	169729	158940	150870	141280	134106	125582	119205	113153	105960	100580	94187	89404
143	169370	158605	150552	140982	133824	125317	118954	112914	105737	100368	93988	89216
196	169020	158277	150240	140690	133546	125058	118708	112680	105518	100160	93794	89031
249	168668	157948	149928	140398	133269	124798	118461	112446	105299	99952	93599	88846
302	168312	157613	149610	140101	132986	124534	118210	112208	105076	99740	93401	88658
355	167957		149296		132708**			111972				88472**
49	167800		149156		132582*			111867				
102	167442		148837		132300			111628				
155	167089		148524		132021			111393				
208	166744		148217		131749			111163				
261	166397		147909		131474			110932				
314	166045		147596		131197			110697				

*132582 generates the 53-comma at 176776

**132708 generates the 360-comma at 88472

than the octave of *Huang-chung* C = 4.5 inches by less than 2 cents, that is, by only $1/12$ of the Pythagorean comma formed by *Chih-shih*, and by about half of the 54-comma formed by *Se-yü*.

Two parenthetical comments in the fifth column, for *Ku-hsi* E, call attention to the consecutive numbers that generate *Se-yü*, the first of the 54-commas, and *An-yün*, the almost perfect octave. The whole table is thus carefully presented to make clear the internal logic of the endlessly spiraling fifths, and the successive reductions of the Pythagorean comma through the 54-comma to the infinitesimal 360-comma. Since the Chinese had used a basic calendrical unit of 360 days for at least a millenium, it is easy to sympathize with Shen Chung's enthusiasm—in the +7th century—for linking this 360-division tuning to the revered *I Ching*, causing embarrassment to modern Chinese scholars less sympathetic with numerical coincidences. The whole point of the construction is missed, however, by those who only notice the coincidence with the number of days in a schematic year. From internal evidence the table was clearly intended to complete the process that the Ching Fang tuning sets in motion. Generation stops at the 360th tone not because that makes enough tones for the year, or enough for the weeks in a 5-year cycle, but because on *An-yün*, as its own literal meaning emphasizes and as the *Sui Shu* points out, we can see the cycle of fifths reaching a "near-conjunction" with the octave. The point is driven home by the reminder that *An-yün* has the *lū* value of 4.4+ inches, just beyond the octave at 4.5. The table solves a legitimate question in musical number theory: Where does the spiral of fifths come closest to the octave? The last two numbers in the last column, that for *Ying-chung* B, tell the tale: the number 88,658, generated as the 307th tone, falls just short of the octave *Huang-chung* C' = $177,147/2$, while *An-yün* at 88,472 slightly exceeds the octave, reducing the 54-comma.

Western scholars, generally unaware of Ching Fang's computational method, have mistakenly supposed that he must have kept track of *exact* fractional remainders, a moderately heroic task involving up to 27 digits, and have therefore been misled into supposing that an extension to 360 tones would have required a super-human patience with grotesque fractions. Nothing could be further from the truth. Using Ching Fang's 6-digit approximations (*without* corrections), and at the casual pace of 2 or 3 subtractions and additions per minute, the additional 300 tone-values can be computed in a couple of hours. *An-yün* at the end of this process is actually correct to within .14 cents of its absolutely perfect value—which would require calculations running to 3^{359} , a 172-digit number. If Ch'ien Lo-chih or Shen Chung used, say, two clerks doing the computations (to forestall errors) and one scribe to enter results in appropriate columns as

they were read off to him (positions are known in advance from Ching Fang's table), they could have completed the 360-division tuning before the morning tea break.

We are unwilling to stop here with this recital of facts, for further reflection on Ching Fang's tuning suggests the possibility of still another level of meaning in the extension to 360 tones.

APPROXIMATIONS TO EQUAL TEMPERAMENT

The work of Ho Ch'eng-t'ien proves that the 360-division tunings were developed within the context of a concern with temperament and with perfectly cyclic tunings—and awareness that pure fifths 3:2 and pure fourths 4:3 produce only spirals. Kuttner has shown that among the first 60 tones in the spiral of fifths there are 12 with a mean deviation from equal temperament of only 4.7 cents, and that among 360 there are 12 with a mean deviation of only .76 cents, “so close to maximum tuning accuracy by the finest craftsmen that, from a practical point of view, no improvement is possible or necessary” (1975:173). Both of Kuttner's tempered sets can be identified merely by inspecting Ching Fang's table, and the second one provides a test of the accuracy of any 12-tone approximation computed directly, like that of Ho Ch'eng-t'ien. I suggest that the following lines of thought were accessible to any of the theorists we have discussed—once their curiosity was aroused by the question of temperament.

The most important fact in Ching Fang's table is the near coincidence between *Huang-chung* and *Se-yü*, 1st and 54th respectively, in the tuning sequence. Since there is an accumulative excess of 2 cents per interval over equal temperament norms and near agreement on the 54th tone, it follows logically that the first and last 6 tones of the tuning sequence, numbers 1-6 and 48-53, will lie closest to equal temperament. None can be further than about $2 \times 6 = 12$ cents from the desired values. To locate Kuttner's 12 best choices from among Ching Fang's 60 candidates, take the first 6 (C G D A E B) as given, then take the last 6 (F# C# G# D# A# E#) *one step earlier* in his *scale* order. Note how this slightly lengthens string and/or pipe measures, offsetting the sharpness/shortness of the Pythagorean values (see Table II).

Now let us take an even closer look at Ching Fang's ratios. *Se-yü* at 176,776, or 8.98+ inches, the 54th tone, is *almost* 1/6 of the distance between *Huang-chung* and *Chih-shih*, the Pythagorean comma. Now the Pythagorean wholetone of 9:8 has an excess of *exactly* 1/6 of this comma; hence—it follows logically—two tuning operations before *Se-yü*, at *I-han*

= 99,437, the 52nd tone, we possess a rather good equal tempered $A^\sharp = B^\flat$, a minor-seventh above *Huang-chung* C and a wholetone (200.29 cents) below its octave C'. (It is very slightly flat because *Se-yü* is not quite $1/6$ of the comma sharp.) We can now see that successive 51st intervals in the spiral of fifths will generate the 6 descending tones of an equal tempered wholetone scale, $A^\sharp G^\sharp F^\sharp E D C$, the 6 *yang* tones. If the remaining 6 *yin* tones are generated from these in the usual manner (by adding or subtracting $1/3$ of the *yang* values), no tone can possibly be more than 2 cents sharper than in equal temperament. The slight cumulative flatness in the *yang* sequence can actually be used to offset this characteristic sharpness: for tones 2 and 53, early in the *yin* series, substitute the later tones 308 and 359, slightly flatter, hence closer to equal temperament (see Table IV). By this reasoning an acoustical theorist can identify the 12 best approximations to equal temperament from Ching Fang's table *before* any of them are computed. Their values, when computed, are accurate enough to serve as a standard of reference (see Table V). Since the 359th tone is $E^\sharp = F$, we know that the 360th is sharper than the octave by only the trivial excess of the pure fifth over a tempered fifth. The location of *An-yün*, then, can be understood perfectly without actually performing the last 300 calculations attributed to Ch'ien Lo-chih and Shen Chung.

CONCLUSION

Were Chinese theorists aware of all the meanings we have read from Ching Fang's table? There is no evidence either way, but he was studied intensely for centuries by competent men. At our distance in time, Ching Fang seems brilliant, but he claimed no originality ("I study, under Chiao Yen-shou, the method of 60 *Lü*"), his computational devices were foreshadowed in the *Huai Nan Tzu*, and Ho Ch'eng-t'ien did not hesitate to

TABLE IV
Equal Temperament Approximations Within the 360-Division Tuning

<i>yang</i>	tone	operation	tone	<i>yin</i>
C	1	$[-1/3]$	2	$G]^\ast$
$A^\sharp$	52	$[+1/3]$	53	$E^\sharp = F]^\ast\ast$
$G^\sharp$	103	$+1/3$	104	$D^\sharp$
$F^\sharp$	154	$+1/3$	155	$C^\sharp$
E	205	$-1/3$	206	B
D	256	$-1/3$	257	A
[C	307]	$-1/3$	308	$G^\ast$ (better)
			359	$E^\sharp = F^\ast\ast$ (better)

TABLE V
Equal Temperament Approximations In Chinese Cyclic Tunings

Chinese	Western	NORM Cents	Huai Nan Tzu ca. -122 12-Division		Step	Value	Cents	Ching Fang -78 to -37 60-Division <i>Hou Han Shu</i>		Step	Value	Cents	Ch'ien Lo-chih ca. +450 360-Division Reconstruction	
			2-digit Value	Cents										
Huang-chung	C	0	81	0	1	177147	0	177147	1	177147	0	0	177147	0
Ta-lü	C#	113.69	76	110.31	49	167800	93.85	167800	155	167089	101.20	93.85	167089	101.20
T'ai-ts'ou	D	203.91	72	203.91	3	157464	203.91	157464	256	157948	198.60	203.91	157948	198.60
Chia-chung	D#	317.60	68	302.86	51	149156	297.75	149156	104	148837	301.46	297.75	148837	301.46
Ku-hsi	E	407.82	64	407.82	5	139968	407.82	139968	205	140690	398.91	407.82	140690	398.91
Chung-lü	E#	521.51	60	519.55	53	132582	501.67	132582	359	132708	500.03	501.67	132708	500.03
Jui-pin	F#	611.77	57	608.35	48	125850	591.89	125850	154	125317	599.24	591.89	125317	599.24
Lin-chung	G	701.96	54	701.96	2	118098	701.96	118098	308	118210	700.31	701.96	118210	700.31
I-tse	G#	815.64	51	800.91	50	111867	795.80	111867	103	111628	799.50	795.80	111628	799.50
Nan-lü	A	905.87	48	905.87	4	104976	905.87	104976	257	105299	900.55	905.87	105299	900.55
Wu-yi	A#	1019.55	45	1017.60	52	99437	999.71	99437	52	99437	999.71	999.71	99437	999.71
Ying-chung	B	1109.78	43	1096.30	6	93312	1109.78	93312	206	93794	1100.86	1109.78	93794	1100.86
Maximum deviation		21.51		19.55			9.79					9.79		1.46
Total deviation		129.10		82.84			52.01					52.01		8.45
Mean deviation ($\div 11$)		11.74		7.53			4.73					4.73		.77

call him "absurd" for failing to follow tradition more closely. The rising fifths and falling fourths associated with subtractions and additions of $\frac{1}{3}$, "the simple and easy method of the ancients," have been documented as a basic Babylonian lyre tuning in the 2nd millenium B.C. (Kilmer 1976). In that very early period the Chinese were already casting bronze bells and cutting sonorous stones in the same tuning, and with great accuracy (Kuttner 1964). We must apparently concede the possibility that serious investigations of the extended spiral of fifths may have been carried out far earlier in history than surviving data can prove. Whatever the early historical facts, which remain elusive, our study of Chinese acoustical theory reveals the elegance of simple methods, and the great strength of a continuous tradition.⁴

APPENDIX

Further study of the 2-digit numbers in the *Huai Nan Tzu* suggests the following rationale:

$42 = 14 \times 3$
 $45 = 15 \times 3$
 $48 = 16 \times 3$
 $51 = 17 \times 3$
 $54 = 18 \times 3$
 $57 = 19 \times 3$
 $60 = 20 \times 3$ or 15×4
 $64 = 16 \times 4$
 $68 = 17 \times 4$
 $72 = 18 \times 4$
 $76 = 19 \times 4$
 $[80 = 20 \times 4 \text{ suppressed}]$
 $81 = 9 \times 9 \text{ (traditional base)}$

With all respect to Chu Tsai-yü's intention, he was probably historically wrong to correct 42 to 43. The virtue of the *Huai Nan Tzu* series is partly that it succeeds in approximating an equal-tempered chromatic scale by simple arithmetic progressions, semitones of 14:15:16:17 being oversize, and 17:18:19:20 being under-size. The Prince's improvement mars the elegant and simple pattern of the original, for the number 43 does not fit this scheme.

NOTES

1. It is easy to discover the exact value of the remainders of the 3-digit *lü* numbers by carrying out the division by 19,683 two more decimal places, but it is not clear what values Ching Fang's 7 symbols for the remainders are intended to represent because errors have intruded, thus making categories overlap. I suggest the following:

Symbol	Tones	Meaning	Comments
+	14	.0006 to .0020	Should possibly stop at .0015, with tone 34 changed to (+).

(+)	2	.0023 to .0040	Should possibly be .0019 to .0026, with tone 59 changed to ++
++	16	.0012 to .0045	Should probably start at .0030, with tones 5, 15, 17, and 48 changed to + and tones 10, 47, 51, and 60 changed to (+). This set is most erratic.
½++	6	.0049 to .0059	Probably correct.
+++	3	.0062 to .0088	Overlaps next category.
--	13	.0062 to .0092	Tone 9, with a remainder of .0026, should be changed to (+).
-	3	.0093 to .0096	Probably correct

There is probably little point in actually making these hair-splitting corrections. The modern editor of the *Hou Han Shu* reports many variants in the *lǚ* modifiers and *chun* remainders in his various sources, and makes many corrections. He also reports the following variants among the *shih* numbers (Fan 1965:3017-24):

22	<i>Nan-shou</i>	139,670	45	<i>Fen-chi</i>	106,188
43	<i>Chieh-hsing</i>	119,103	57	<i>Fen-wu</i>	93,117

2. Courant suggests that there are 8 errors among the 6-digit *shih* numbers in Ching Fang's table, but Courant himself computed these numbers by carrying exact fractions to the very end, although these ran to 21 digits (1924:88). Such a grotesque procedure misses the whole point of Ching Fang's elegance, burying Chinese common sense and affection for simplicity under our modern passion for absolute perfection, however irrelevant that may be. Courant succeeded in making the 360 division tuning seem impossibly difficult, supposing that exact fractions would similarly be carried to the end. My own study was inspired by Kuttner's personal conviction that the Chinese "devised some efficient method of cutting decimals without sacrificing accuracy," and owes much to his data and personal advice (1975:173). If Ching Fang had made his first correction earlier and also had made another near the end he could have improved a few values *very* slightly—but to no purpose.

3. The 360-comma at *An-yün* = 88,472 has the value of 1.8453 cents *above* the octave of *Huang-chung*, while the 307-comma has the value of 1.77 cents below it. Whereas the 307-comma falls short of a cyclic repetition, the 360-comma reduces the 54-comma. (Proof: $An-yün\ 88,472 \times 2 = 176,944$, slightly larger than $Se-yü = 176,776$, but smaller than *Huang-chung*.) Modern Chinese studies of the 360-division tuning project *An-yün* into the wrong octave, thus missing its significance, and also ignore the interesting question as to how the calculations were made (Liu 1948:332-366).

4. Edith Borroff, Fred Fisher and F. Joseph Smith directed my attention to various problematic aspects of the 360-division tuning, and my wife, Augusta, helped with translations and calculations.

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