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CHAPTER 4

Body: M 9.359-440

Gábor Betegh

Introduction

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The chapter in many respects is at odds with Sextus' more usual sceptical strategy. Because of its almost exclusive focus on the mathematicians' conceptions, it is not a systematic consideration of alternative positions of different schools, although, as we shall see, Sextus is well aware of the variety of options, and the motivation behind why one may prefer one conception over another. In particular, he spends very little time and energy on that conception of body which can be ascribed to the most important members of the corporealist camp, namely the Stoics and the Epicureans. He does not discuss in an explicit manner whether, and if so

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how, the arguments against the conceptions of the mathematicians would affect the corporealists' views. In particular, it is not examined whether the question of the conception and existence of bodies can be separated from the question of the existence of fundamental geometrical objects.

The arguments of the chapter are moreover exclusively negative; we are not presented with arguments for the existence of body and fundamental geometrical objects. The conclusion of each series of arguments is that body – and the fundamental geometrical objects – thus conceived cannot exist. We can assume that the reader should provide the positive side on the basis of the common opinion, shared by theoreticians and laymen alike, that bodies exist.¹ The question is, however, more complicated in the case of geometrical objects, for their ontological status was vigorously debated. Yet even in their case, Sextus provides little, if anything, in defence of their existence.

A structural overview may help the orientation in this long and fairly complex chapter and can be found in the first appendix to this chapter.²

I shall follow the structure of Sextus' discussion and speak about each section in turn. Yet, as mentioned above, Sextus' attack on fundamental geometrical notions in the long section E overlaps to a large extent with the corresponding parts of *Against the Geometers*. Now, *Against the Geometers* and its individual arguments have recently been examined in considerable detail by other scholars. After Ian Mueller's pioneering paper, originally read at a previous Symposium Hellenisticum, Luciano Floridi has set Sextus' place in the broader history of 'mathematical scepticism' in a series of studies, whereas Wolfgang Freytag has published a book-length study of Sextus' arguments against the fundamental concepts of the mathematicians concentrating on the twin texts in *M* 3 and *M* 9.³ In an even more recent study Guillaume Dye and Bernard Vitrac have examined the sources and targets of Sextus' attack on geometry.⁴ In view of these studies, I shall concentrate primarily on structural questions, Sextus' argumentative strategy, and the sections before and after Section E. In Appendix II, I shall

¹ For the parallel case in the discussion of time, see Warren 2003: 314–15 with important qualifications by Bobzien, in this volume, pp. 000–000; see also Dye & Vitrac 2009: 163 for the geometrical notions.

² The hierarchical ordering of the sections is not always as clear as it might appear from my numbered list.

³ Mueller 1982; Floridi 1998, 2000 and 2004; Freytag 1995.

⁴ Dye & Vitrac 2009. I received this valuable study shortly before I had to submit this chapter for publication. In some cases Dye and Vitrac and I have arrived at similar conclusions independently of each other; I note these points in footnotes.

present additional remarks on the doxographical material presented in the chapter, including a comparison with parallel texts.

Setting the agenda

In the last sentence of the previous chapter, in 358, Sextus announces the successful completion of the investigation concerning the active principle and indicates that he will now turn to a general, more common treatment of the active and the material principles. It means that we have arrived at the end of the discussion which started at the very beginning of the book, at *M* 9.13. As the reader may remember, Sextus started the treatise with a methodological introduction in which he recommended an attack on what constitutes the principal, most comprehensive and essential (τὰ κυριώτατα καὶ συνεκτικώτατα, *M* 9.1), because, as he argued, an assault on what is common to the individual elements of the doctrine is ‘the more graceful’ (χαριέστερος, *M* 9.3) way of demolishing the dogmatic edifice of the opponent. He put into practice this recommendation by stating, first, that the physicists customarily distinguish between two (kinds of) principles of the universe, active and passive, and by turning, in the next step, to an attack on the notion of god as the active principle. The last sentence of 358 marks the end of the extended discussion of the active principle, the ensuing discussion of cause, and the chapter on parts and wholes.

The reader would expect Sextus now to turn to a discussion of the passive principle. And this is indeed what we find in the introduction of the parallel section in *PH* 3.30: having finished the discussion of the active principle, Sextus announces that he will now continue with an investigation of the material principle (ὕλική ἀρχή), and he first of all provides a doxographical survey of the relevant views. In the articulation of the topics, as well as in formulating the transition, Sextus in *PH* 3.30 is closely following his doxographical source, as the clear parallel with Ps.-Galen’s *De Historia Philosophica* shows (for an analysis of the doxographical survey, as well as the nature of the relationship between Sextus and Ps.-Galen, see Appendix II, section 1). In *PH* 3 after the doxographical survey Sextus immediately points out and exploits the disagreement (*diaphōnia*) among the different views about the material principle(s), and it is only at this point that he turns to a discussion of body. He says that the infinite variety of views about the material principle would in itself suffice to demonstrate the inapprehensibility of the material principle; but let us treat the question more generally, by showing the inapprehensibility of body.

In *M* 9 Sextus presents what is basically the same material as part of a different strategy. Instead of turning to the material or passive principle, Sextus immediately announces the more general discussion, which could provide a common treatment of the two principles, and thus a new beginning. Moreover, if it is true that the discussion he announces here will give a more common (κοινότερον) treatment of the principles and he sticks to his methodology privileging an assault on what is more common, then we should expect that the attack on the physicists reaches its peak in this chapter. Indeed, we may ask why he did not start the whole treatise with what he promises to do now: in so far as this approach is even more general, it may have made *Against the Physicists* even ‘more graceful’.

A general discussion of body could in fact fulfil the role of a more general attack on most of the physicists and also a common investigation of both principles in so far as, at least according to some schools of thought, both the active principle, or god, and the passive principle are bodies. We may think first of all of the Stoics – who must figure prominently in Sextus’ original distinction between the active and the passive principles, as well as in his treatment of god as the active principle – for whom the discussion of body precedes that of the principles (cf. Diogenes Laertius 7.132). Moreover the forthcoming discussion could cover those thinkers as well who do not distinguish between the active and the passive principles. Remember that Sextus said that only the best of the physicists applied this distinction.⁵ The prevalent corporealism of the Hellenistic schools, and of most of the previous philosophers, would then fully justify the importance attached to the attack on the notion of body.

The first sentence of the chapter, moreover, distinguishes between corporealists and incorporealists in a way that seems to confirm our expectation that Sextus now wants to raise the generality of the discussion to the highest level by concentrating on the highest and most primary element (περὶ τῶν ἀνωτάτω καὶ ἀρχικωτάτων στοιχείων), which, in the case of the corporealists, is body. Yet he immediately equates this highest and most primary level with the traditional elements and then starts his doxographical survey: ‘Pherecydes of Syros said that earth is the principle and element of all things, Thales that it is water . . .’⁶ Then comes (almost) the same doxography as the one he gives for the *material principle* in *PH* 3.⁷

⁵ *M* 9.12: ἐπεὶ οὖν τοιαύτη τις ἔστι παρὰ τοῖς ἀρίστοις τῶν φυσικῶν διάταξις . . .

⁶ Note that the reference to elements is missing from the parallel in *PH* 3.30.

⁷ The introductory sentence in Ps.-Galen also marks out the ensuing list as an inventory of views about the passive or material principle.

Identifying the most general level with the element(s) or material principle(s) creates a problem exactly in the cases of those who do make the distinction between active and passive principles; for in the doxographical list we find, for example, that the elements of Empedocles are earth, water, air and fire, whereas the elements of Anaxagoras are the *homoiomeres*. However, in the introductory section, when Sextus was arguing that the best of the physicists distinguish between the active and passive principles, he explicitly included Empedocles' Love and Strife and Anaxagoras' Mind as active principles (*M* 9.4–10). So in the case of these philosophers it is simply not true that their respective (material) elements with which they figure in the list would constitute the most fundamental level common to the active and passive principles; in these cases the elements are merely the passive principles – in conformity with the announced topic of *PH* 3.30 but in contrast with Sextus' proclaimed agenda in *M* 9. Turning to these would be an appropriate sequel to the discussion of the active principle, but this is not what Sextus proposed to do in our chapter.⁸

I have found one addition which may signal that Sextus acknowledges the difference between the two strategies. The lists in Ps.-Galen and *PH* 3.30 end with the Pythagoreans (numbers), the mathematicians (limits of bodies) and, finally, Strato (qualities). The list in *M* 9 omits Strato, but more importantly, adds the Platonists with the Forms. Clearly, the Platonic Forms would be inappropriate for the list of material principles in Ps.-Galen and *PH* 3.30 but are appropriate on the list of highest principles in *M* 9.

Having presented his doxographical survey of both corporealists and incorporealists, Sextus then restates that it will be possible to argue against the members of the two groups in common (ἐνέσται πρὸς πάντας κοινῶς ἀντερεῖν) by going through all the difficulties concerning bodies on the one hand and the incorporeals on the other. By showing that there is no consistent conception of body forthcoming, we can undermine all the

⁸ The situation is actually even more complicated in the case of the Stoics. The Stoics most probably figured in the original doxographical list with their four elements as we can see from Ps.-Galen's text. Yet, even though it is true that a general treatment of body can be prior to the treatment of the active and passive principles, it is not the case that a treatment of the four elements could fulfil that role: in the Stoic ordering of metaphysical topics, the four elements come after the two principles. Remarkably, the Stoics appear with their four elements also in the parallel passage in *PH* 3.30, but there Sextus adds a note – apparently absent from his source that he otherwise follows almost verbatim – which indicates that he is aware of the fact that, technically, he should be speaking about matter as such and not about the elements (περὶ γὰρ τῆς τερατολογουμένης ἀποίου παρά τισιν ὕλης, ἣν οὐδὲ αὐτοὶ καταλαμβάνειν διαβεβαιοῦνται, τί δεῖ καὶ λέγειν). The priority of unqualified matter is also acknowledged in *M* 10.312.) And in our passage he should of course be speaking neither about the four elements, nor about unqualified matter, but simply about body.

corporealists views at once, and we can then proceed in a like manner with the other group and raise puzzles concerning incorporeals. Note that this last promise is never fulfilled in *M* 9–10. In *PH* 3 the general discussion of body is followed by a general discussion of incorporeals, almost as long as the preceding general discussion of body. In *M* 9–10, by contrast, Sextus never actually delivers a general discussion of incorporeals understood as the common principles of the incorporealist group; what we are given instead is a one-by-one treatment of incorporeals. Moreover, the discussions of the individual incorporeals do not focus on those specific incorporeals that figure in *M* 9.364 as the first principles of the respective incorporealist philosophers. The topics covered in the first chapters of *M* 10 involve two of the Stoic incorporeals – place and time – but include motion as well. Number, as the Pythagoreans’ first principles, comes only after these discussions, whereas the Platonists’ Ideas do not receive a separate treatment, but only some remarks embedded in the discussion of the Pythagoreans’ numbers. And, to confuse things even further, the discussion of the limits of bodies, the alleged first principles of the incorporealist mathematicians, is not part of the discussion of the incorporeals but takes up the better part of the discussion of body, the common principle of the corporealists. These oddities in the arrangement of the material, as well as the discrepancies between announced plans and realizations, are characteristic of *M* 9–10 as compared with *M* 3: Sextus apparently does not succeed in integrating his more abundant source material in a large-scale scheme.

The corporealists’ and the mathematicians’ conceptions of body

In accordance with the initial distinction between corporealists and incorporealists, Sextus announces at *M* 9.366 that he will start with the conception (ἐννοία) of body as the ultimate principle of the corporealists. He immediately discards an account, which, he says, some ascribe to Pythagoras and according to which body is ‘what is capable of being acted upon or of acting’ (τὸ οἷόν τε παθεῖν ἢ διαθεῖναι).⁹ He justifies this move by recalling that the previous discussion has already shown the absurdity of the conceptions of cause and effect: if there is no acceptable account of

⁹ The verb διαθεῖναι is not the most common match for παθεῖν in this definition; it is more usual in grammatical contexts and in any case Sextus turns to the more common ποιεῖν language a few lines later. The phrase of course goes back to Plato’s *Sophist*, in which it is suggested as a definition of being that can be acceptable to both corporealists and incorporealists. The attribution to Pythagoras might come from a Pythagoreanizing interpretation of the Platonic material.

cause and effect, the corporealists cannot give an account of body in causal terms. This move, I think, is entirely justified irrespective of the further problem whether or not everybody on the list of corporealists would accept this as a valid conception or definition of body.¹⁰

Sextus' next move is considerably trickier however; for he continues by declaring quite abruptly: 'We must now organize (συντακτέον) the matter at hand according to the conceptions of the mathematicians.' The formulation is not entirely clear,¹¹ but the idea apparently is that in so far as the causal conception of body has been discredited, we should launch the common attack on the corporealist branch of physicists with an examination of the conceptions of the incorporealist mathematicians.¹² As we shall see, this will involve, first of all, the geometers' conception of body, according to which body is that which has three dimensions (τὸ τρεῖς ἔχον διαστάσεις) and, then, their conceptions of dimensions, and, at a later stage, that of point, line and surface. Sextus does not motivate this move, but the wording and the subsequent discussion in the chapter strongly suggest that showing the absurdity of the mathematicians' conception is considered here not just as one among many, but the single most suitable strategy for such a joint attack on the corporealists' principle. Sextus never actually proves this point, so he never shows that once we have discarded the causal account of body, the remaining options will be covered by an attack through the geometers' notions. In so far as this manoeuvre governs

¹⁰ The prime candidates are obviously the Stoics. It has, however, been debated whether they would accept the capacity to act or be acted upon as providing a definition of body, or, being exclusive to bodies, it can merely function as a criterion of corporeality. Reesor 1954 and more recently Falcon 2005: 52 treat it as the Stoic definition of body. Long & Sedley 1987 argue on the other hand: 'It is essential to see that the capacity to act or be acted upon, though peculiar to bodies, is not advanced as a defining characteristic of body *per se*. In confining this capacity to bodies, the Stoics were not redefining body but radically rejecting the thesis, accepted by Plato and Aristotle, that incorporeals can have any causal efficacy' (vol. 1, 273). The question depends on whether we accept that the Stoics in general, and Chrysippus in particular, agreed with Antipater that we obtain a definition by specifying a necessary property of the definiendum which is unique to it. On the Stoic definitions of definition, see the Schol. to Dionysius Thrax 1.107.5–7 (= *SVF* 2.226) and Diogenes Laertius 7.60, with a thorough discussion in Brittain 2005: 186–91. It is equally true on the other hand that the texts most often referred to in this connection, Cicero, *Acad.* 1.39 and S.E. *M* 8.263, do not present it as a Stoic definition of body.

¹¹ There might also be a textual problem. The MSS give συντακτέον, which is accepted by Bekker, whereas Mutschmann, followed by Bury, conjectures συνακτέον. συνάγω is normally used by Sextus in the sense of 'to conclude' as by bringing the premises together. If we accept the emendation, perhaps we should take the verb in a hostile sense, as when warriors engage with each other in battle (cf. LSJ s.v. 3).

¹² This seems to be reinforced also by the fact that the μέν at the beginning of the paragraph dealing with the causal notion of body is picked up by the δέ at the beginning of the section introducing the mathematicians' conception.

most of the subsequent discussion, a general appraisal of the success of the chapter largely depends on the question whether and how far Sextus' move to turn to the mathematicians is legitimate. How far will the corporealists accept an attack on their notion of body through an attack on the mathematical conception of body, and related mathematical notions?

Note first of all that the question is conceptual, and not metaphysical; it is irrelevant – or at least it seems so incipiently – that Sextus' incorporealist mathematicians grant the status of principle to the limits of bodies and thus hold that bodies are derivative of, and thus ontologically dependent on, limit entities, whereas the corporealists denied this. What is at stake at this point is not the ontological relationship between geometrical objects and physical bodies, but the relationship between the mathematicians' and the corporealist physicists' conceptions of body.

Nicomachus' comments on the place of mathematics in the general system of knowledge in his *Introduction to Arithmetic* – possibly the most popular specimen of this flourishing genre in Sextus' time – are more relevant. Before turning to the definitions of the fundamental notions of arithmetic (number, even, odd, etc.), he argues, with frequent references to Plato, for the importance and foundational role of mathematics to science (ἐπιστήμη) and wisdom (σοφία): if we abolish mathematics, we abolish the other sciences as well (*Ar.* 1.6).

Of course, the view that geometry is indispensable to the description of the physical world finds its most illustrious expression in the *Timaeus*. Yet, one does not need to be a Platonist to accept some role of mathematics, and the use of mathematical notions, in the description of the physical world; one can think also of the way in which Aristotle specifies the use of mathematics in the scientific understanding of certain physical objects and phenomena (cf. *Ph.* 2.2). So, irrespective of one's position about the ontological status of geometrical objects, or the ontological relationship between geometrical and physical bodies, one may hold the view that fundamental geometrical notions are necessary for thinking about, and having a conception of, certain aspects of the physical world. Sextus' strategy seems to assume exactly this with specific regard to the corporealists' common first principle: once the causal account of body has been eliminated, the corporealists' conception of physical body will, at some level of analysis, necessarily involve fundamental geometrical notions. If those geometrical notions turn out to be incoherent and untenable, the corporealists find themselves without a plausible conception of their own principle. The corporealists' principle can thus be attacked through a rejection of the relevant geometrical conceptions. Sextus appears to accept

the view expressed also by Nicomachus – ‘if we abolish mathematics, we abolish the other sciences as well’ – and turn it against the physicists: let us abolish geometry so that we abolish physics as well.

The validity of this strategy can, however, be contested by repudiating the entire discipline of geometry, or, more specifically, by denying the relevance of the allegedly requisite geometrical notions. First of all, there were schools, such as the Cyrenaics, the Cynics, and of course the sceptics themselves, who rejected geometry with other branches of mathematics as part of their wholesale dismissal of the sciences. This kind of indiscriminate rejection will, however, be of little significance for Sextus’ present purposes in so far as these schools will be unlikely to develop a dogmatist corporealist physics.¹³ Yet there were others, namely the Epicureans, who rejected geometry in a targeted way and denied any truth to it exactly because of its incompatibility with their physical theory involving theoretical minima.¹⁴ Such a comprehensive dismissal of the discipline involves, in all likelihood, a refusal to accept the validity of the fundamental notions of geometry and hence their relevance in the understanding of physical reality in general, and of body in particular. Moreover, the Epicureans’ specific reason for rejecting conventional geometry is precisely that the geometrical conception of spatial magnitudes, including limit entities, are fundamentally misconceived. As Sextus himself states explicitly, the proper object of geometry is continuous spatial magnitude (*M* 4.1), whereas the Epicureans emphatically deny that magnitudes are continuous. If so, they will not be prepared to accept the relevance of the geometrical notions at any level of the analysis of their conception of body. Indeed, there are reasons to think that one major source of Sextus’ arguments against the fundamental geometrical notions is the Epicurean polemics against the geometers. Furthermore, even if a corporealist physicist does not reject the entire (traditional) discipline of geometry as misconceived, he may still object that specific geometrical concepts, most importantly those of limit entities, are not applicable in the analysis and description of physical bodies; as we shall see, this might turn out to be the (early) Stoic position. In general, the pivotal point will be whether a given physical theory accepts the geometrical analysis of spatial magnitudes applied to physical spatial magnitudes.

The picture is complicated even further because some of the arguments in the chapter against fundamental geometrical notions aim to show

¹³ On the Cyrenaic rejection of physics, see e.g. *M* 7.11 and 13.

¹⁴ On the Epicurean approach to geometry, see Sedley 1976; White 1992: 230–9.

precisely that certain physical phenomena, such as the juxtaposition of limits of bodies, cannot be coherently described in geometrical terms (414–18; 431–3); and these arguments, once again, are likely to go back to Epicurean and Stoic sources.¹⁵ So Sextus, on the one hand, seems to assume that the corporealists are bound to use geometrical notions in formulating their conceptions of physical body but, on the other hand, uses arguments coming from the corporealists to show that geometrical notions are inappropriate to describe physical bodies. In view of these considerations, it will be important to check at each major juncture of Sextus' argument how the most important representatives of the corporealists would react to that specific move.

It is worth noting, first of all, that, as other texts evince, Sextus is well aware that the mathematicians' conception of body formally differs from the corporealist physicists' conceptions. In *M* 1.21 he provides the following inventory:

Now they [i.e. bodies] are not perceptible as is clear from the conception of them. For body is either (i) a conjunction by aggregation of magnitude, shape, and resistance (ἀντίτυπία), as Epicurus says, or (ii) that which is extended in three dimensions (i.e. that consisting of length, width, and depth), as the mathematicians say, or (iii) that which is extended in three dimensions and has resistance (ἀντίτυπία), again as Epicurus says so that he can also distinguish it by this from the void, or (iv) a resistant mass (ὄγκος ἀντίτυπος) as others say (trans. Blank, modified).

In line with what we read in *M* 9, the conception referring exclusively to three-dimensional extension is attributed to the mathematicians – and to the mathematicians only. The causal conception of body ('what is capable of being acted upon or of acting') that has been briefly discarded at the beginning of our section in *M* 9 is not mentioned. On the other hand, three further conceptions are listed, two of which are explicitly ascribed to Epicurus, and all three of which make reference to resistance (ἀντίτυπία).¹⁶ From this fourfold list Sextus in *M* 9 focuses almost exclusively on the one attributed to the mathematicians.¹⁷

¹⁵ Cf. Dye & Vitrac 2009: 181–2. One of Dye and Vitrac's principal theses is that, in *M* 3, Sextus primarily attacks geometry as a means of 'modelling' physical reality.

¹⁶ Blank 1998 excises the last definition and argues in his commentary (96, n. 39) that it must be a later interpolation, because it is not attributable to anyone in particular and does not advance Sextus' argument.

¹⁷ At the end of the chapter, in 437, he briefly considers a version of (i) in relation to the question whether bodies are perceptible.

At this point, it will once again be instructive to compare Sextus' strategy with the way in which he proceeds in the parallel passage in *PH* 3. The starting point is the same as in *M* 9: the account according to which body is that which can act or be acted upon is summarily discarded with reference to the previous discussion of cause and effect. Yet, Sextus then turns to what he takes to be the general conception of body: body is that which is three-dimensional *and has resistance* (τὸ τριχῇ διαστατὸν μετὰ ἀντιτυπίας, *PH* 3.39). In the subsequent discussion in *PH* 3 Sextus discusses and attacks ἀντιτυπία alongside the dimensions.¹⁸ As the presence of ἀντιτυπία indicates, what is treated here as the general conception of body is not that of the mathematicians,¹⁹ but the one that is ascribed to Epicurus in *M* 1 (cf. also *M* 11.226). It is worth noting that the author of the treatise *Are Qualities Incorporeal* transmitted under the name of Galen attributes the exact same definition to the Stoics.²⁰ Thus, Sextus in *PH* 3.39 turns to the conception of body that he explicitly ascribes to Epicurus, but that might have been accepted also by the Stoics; if so, the conception targeted by Sextus could be common ground between the two most prominent contemporary representatives of the corporealist group. In *M* 9.367, by contrast, he turns to the conception of body that he attributes exclusively to the mathematicians.

Some in the corporealist camp, most prominently Aristotle, should accept the relevance of the mathematicians' conception; indeed he also defines body in similar terms at *Cael.* 1.1.268a7, *Ph.* 3.5.204b20 and 4.1.209a4–6. Yet, the Epicureans and Stoics might object at this point that three-dimensionality is not sufficient to define body.²¹ As Sextus himself states in *M* 1.21 (quoted above), Epicurus needs to include resistance (ἀντιτυπία) 'so that he can also distinguish it [i.e. body] by this from the void'. In a similar vein, Sextus makes Epicurus say in *M* 10.222

¹⁸ Incidentally, a successful dismissal of ἀντιτυπία could disqualify also the first conception of body attributed to Epicurus in *M* 1, even if Sextus does not mention that conception in the *PH* 3.

¹⁹ That the two conceptions are different is also emphasized by Annas & Barnes 2000 in their note ad loc., 153, n. 50.

²⁰ Ps.-Galen, *Qual. Inc.* 19.483.13–16 = *SVF* 2.381: ... τοῦ σώματος τοῦτον ὅρον εἶναι φασιν τὸ τριχῇ διαστατὸν μετὰ ἀντιτυπίας ... Cf. also Plotinus 6.1.26. Reesor 1954: 57 denies that the definition was accepted by the Stoics and maintains that the author of *Qual. Inc.* (whom, following Orth, she takes to be Albinus) and Plotinus apply their own definition of body in their polemics against the Stoics.

²¹ We see here the historical origins of the long and exciting debate whether three-dimensional extension is sufficient to define body, a debate that will be taken up by Philoponus (cf. De Haas 1997 and Sorabji 1988) to flare up again in the early modern period with Descartes on the one side and people like Newton, Boyle, Locke and Leibniz on the other.

that body can never be conceived without ἀντισυρία; in that context, ἀντισυρία is presented as the differentia, while three-dimensionality provides the genus of per se existents comprising bodies and the void. And the Stoics should agree that three-dimensionality cannot in itself deliver a defining characteristic of body, because there is also the three-dimensionally extended extra-cosmic void. Indeed, the author of *Are Qualities Incorporeal* gives the very same reason why the Stoics also insisted that ἀντισυρία has to be included in the definition of body.²² That ἀντισυρία is the distinguishing attribute (ἴδιον) of body is also stated at M 10.12 in the thought experiment that functions as an argument for the existence of place: if in thought we abolish everything, three-dimensional extension and hence place will still remain.²³

Sextus, however, can have ready answers to these worries. He could point out, first of all, that resistance (ἀντισυρία) is a property that is primarily, if not exclusively, related to the causal characterization of a body; so we have effectively disposed of it with the destruction of the conceptions of cause and effect. Much more importantly, he could argue that even if the Epicureans and the Stoics do not accept that three-dimensional extension is a uniquely defining characteristic of body, their own conception also includes reference to three-dimensional extension; therefore they, too, must give an account of dimensions in order to make their conception of body intelligible – and this remains so, irrespective of their insistence on ἀντισυρία. At this point what the corporealists think about the validity of geometry is irrelevant. Indeed, it is of no immediate consequence whether the mathematicians and the physicists speak about the same thing, or the mathematicians' conception is of geometrical solids whereas the physicists focus on physical bodies, or how clear at all the

²² Ps.-Galen, *Qual. Inc.* 19.483.10–14 = *SVF* 2.502. It may be objected how the inclusion of ἀντισυρία in the definition of matter could be made compatible with the view that matter is 'unqualified being'. One possibility, I think, is that ἀντισυρία is not conceived as a tangible quality but rather as that feature of body which constitutes its causal efficacy; which, in the case of matter, is that it is capable of being acted upon.

²³ Falcon 2005: 53–4 argues that the Epicureans and the Stoics had both a general and a specific notion of body, such that the first, expressed in terms of three-dimensional extension only, included both geometrical and physical bodies, whereas the addition of ἀντισυρία in the second served to delimit physical bodies. This might be true in the case of the Stoics (cf. Diogenes Laertius 7.135 with the definition provided by the Stoic Apollodorus: 'A body is what is extended in three ways, in length, in breadth, and in depth', apparently coming from a work called *Physics*). But it seems to me questionable whether the Epicureans were interested in such a generic concept of body enveloping both geometrical and physical bodies. On the whole, I think that the ancient sources correctly identify the primary motivation for the inclusion of ἀντισυρία that is true for both schools: to distinguish body from non-corporeal spatial entities such as the void.

parties were on this distinction;²⁴ what matters is that both conceptions involve three-dimensionality, and thus all parties need to be able to explain what they mean by that. As Sextus puts it in *M* 1.25: ‘Besides, anyone who conceives the body compounded of these dimensions must first know the dimensions themselves, in order to be able to know the body in addition’ (trans. Blank).²⁵

To sum up: Sextus never shows that once we get rid of the causal account of body on the basis of the previous discussion of cause and effect, all other conceptions of body will include reference to extension in three dimensions.²⁶ But, if that point is granted, Sextus’ strategy turns out to be legitimate; actually, it is more economical than the strategy he uses in *PH* 3. Yet, he should have helped his reader by making his reasons more explicit instead of simply stating: ‘We must now organize the matter at hand according to the conceptions of the mathematicians.’

Excursus: an oddity in the mathematicians’ definition of dimensions

The definition of body Sextus attributes to the mathematicians is completed by an account of the dimensions:

For they say that body is that which has three dimensions, length, breadth, depth, from which length is that which is from above to below, breadth is that which is from left to right, and the third dimension, that is depth, is that which is from front to back (367).²⁷

In fact, Sextus characterizes the dimensions, and length in particular, in two different ways in the outset of the two main argumentative parts. When he starts the second series of arguments focusing on the different conceptions of line at 376, he characterizes length as ‘the greatest dimension of the body’ (τὸ μέγιστον ἢν τοῦτο τοῦ σώματος διάστημα), and not as ‘that which is from above to below’ as he does here. Sextus does not mention that he is using two different conceptions in the two sections; but

²⁴ Cf. Mueller 1982: 77: ‘I am inclined to think that the Stoics did not distinguish clearly between mathematical and physical body, but I doubt that anyone outside the Platonic tradition did so.’

²⁵ Sextus could even say that Epicurus’ alternative conception, listed in *M* 1, also involves magnitude and shape, so it is incumbent on him to say something about dimensions also in view of that account of body.

²⁶ At this point it becomes significant whether the fourth conception of body, ‘a resistant mass’, in the *M* 1 list was part of Sextus’ text or is a later addition as Blank 1998 argues (see n. 16 above). To show that it is necessary to give an account of dimensions in view of this conception, too, would certainly need further arguments.

²⁷ Bury puts into quotation marks only the first part of the sentence; I think it is clear that the second part is also meant to be part of the quotation, verbatim or not.

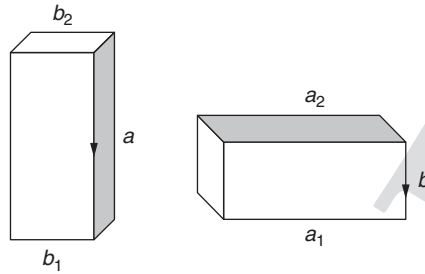


Figure 4.1

Stobaeus lists exactly these two in substantiating the claim that ‘length is said in many ways’ in the context of the definition of body as that which has three dimensions.²⁸ The characterization of dimensions in terms of directions is repeated almost verbatim in the parallel passage in *M* 3.19, with the difference that there, in accordance with what we find in Stobaeus—and as we shall see, in all other parallel texts – breadth is from right to left and not from left to right. Moreover, Sextus specifies in *M* 3.19 that length is the *first* dimension.²⁹

The outcome of this characterization is the seemingly curious idea that the ‘length’ (μήκος) of an object will depend on the position of the object and not on its intrinsic geometrical properties. Thus, on Figure 4.1 ‘length’ will be *a* in the case of the left-hand object, and *b* in the case of the right-hand object. It is worth noting in this respect that in some non-technical contexts μήκος could also designate the height of an object even when its horizontal dimension was larger.³⁰

Even more peculiar, on the definition quoted by Sextus, the dimension (διάστασις) of length is not simply the vertical dimension of the object: it has a fixed directionality as well, from above to below. It differs in

²⁸ Stobaeus, *Ecl.* 1.143.24 W: σῶμα ἐστὶ τὸ τριχῇ διαστατόν, πλάτει, βάθει, μήκει ταῦτα δὲ πλεοναχῶς λέγεσθαι. ὅτε μὲν γὰρ μήκος [εἶναι] λέγεσθαι τὸ μέγιστον διάστημα τοῦ σώματος, ὅτε δὲ μόνον τὸ κάτωθεν ἄνω καὶ πλάτος ὅτε μὲν τὸ δεύτερον διάστημα, ὅτε δὲ τὸ ἐκ δεξιᾶς καὶ ἐξ εὐωνυμοῦ καὶ βάθος ὅτε μὲν τὸ εἰς αὐτὸ διάστημα, ὅτε δὲ τὸ πρόσω καὶ ὀπίσω. In Stobaeus’ text the definition is not assigned to anybody. Diels thought that it comes from Arius Didymus (fr. 19 Diels) and that it is a report of the Stoic view. Hence it is also included in *SVF* (*SVF* 2.357). I found no good reason to think that the definition as a whole, including the two characterizations of the dimensions, should be Stoic.

²⁹ *M* 3.19: ... σῶμα μὲν ἐστὶ τὸ τὰς τρεῖς ἔχον διαστάσεις, μήκος πλάτος βάθος, ὧν πρώτη μὲν διάστασις ἐστὶν ἢ κατὰ μήκος ἄνωθεν κάτω, δευτέρα δὲ ἢ κατὰ πλάτος ἀπὸ δεξιῶν ἐπ’ ἀριστερά, τρίτη δὲ ἢ κατὰ βάθος ἀπὸ τῶν πρόσω εἰς τοῦπίσω. Curiously, Bury here translates ἄνωθεν κάτω by ‘up and down’.

³⁰ Cf. e.g. Aristophanes, *Aves* 1130, listed in LSJ, speaking about the height of a wall.

this respect from the relevant directions, which, as Sextus specifies in *M* 9.367, can be both from the top down or from the bottom up. Length as a dimension (διάστασις) appears to be identical with one of the two vertical directions (παράτασεις).

As far as I am aware, it is not at all common in the technical mathematical literature to make an immediate connection between the three dimensions and the six directions.³¹ Moreover, the directionality of length (and the other dimensions), not surprisingly, is not a common view in the mathematical literature. It is, however, closely paralleled in the *Definitions* transmitted under the name of Hero of Alexandria.³² This is how that text defines line:

Line is length without breadth and without depth or what first takes existence in magnitude or what has one dimension and is divisible as well; it originates when a point flows from up downwards according to the notion of continuum, and is surrounded and limited by points, itself being the limit of surface (trans. Cuomo).³³

And when defining surface, the author says that it is generated as the line flows from right to left along breadth,³⁴ and that solids come into being when the surface flows from before to behind.³⁵ The text of the *Definitions* clearly shows that the directionality of the dimensions is based on the genetic view of dimensions. This is reinforced by Sextus in *M* 3.19 where he claims that length is the *first* dimension, just as Hero tells us in the text

³¹ The term I have translated as ‘direction’ is παράστασις in N, which has been corrected to παράτασις by Bekker, followed by Mutschman and Hicks. Neither the transmitted nor the emended term is part of the technical mathematical vocabulary. Neither is included in Mugler 1958. LSJ list only our passage where παράτασις would have the sense ‘direction of extension’, ‘dimension’. The closest we get in both respects is Nicomachus, who, however, uses the word περίστασις: ‘By these [i.e. depth, breadth and length] are defined the six directions (περιστάσεις) which are said to exist in connection with every body and by which motions in space are distinguished; forward, backward, up, down, right and left; for of necessity two directions opposite to each other follow upon each dimension, up and down on one, forward and backward upon the second, and right and left upon the third’ (*Ar.* 2.6.4, trans. D’Ooge). Note that Nicomachus clearly connects directions to motions. If we want to correct the received παράστασις in Sextus, I wonder if we are not better off making it uniform with Nicomachus’ text and read περίστασις.

³² The parallel is also noted by Freytag 1995: 164 and Dye & Vitrac 2009: 176–7. Heiberg has argued that the work is a Byzantine collection of which 1–132 were derived from Hero. See, approvingly, Mansfeld 1998: 56.

³³ *Def.* 2: Γραμμή δὲ ἐστὶ μῆκος ἀπλατὲς καὶ ἀβαθὲς ἢ τὸ πρῶτον ἐν μεγέθει τὴν ὑπόστασιν λαμβάνον ἢ τὸ ἐφ’ ἑν διαστατὸν τε καὶ διαιρετὸν γίνεται δὲ σημείου ρυέντος ἄνωθεν κάτω ἐννοία τῇ κατὰ τὴν συνέχειαν, περιέχεται τε καὶ περατοῦται σημείοις πέρας ἐπιφανείας αὐτῇ γενομένη.

³⁴ *Def.* 8 [Επιφανεία] γίνεταί δὲ ῥύσει ὑπὸ γραμμῆς κατὰ πλάτος ἀπὸ δεξιῶν ἐπ’ ἀριστερά ῥύεισης.

³⁵ *Def.* 11 περατοῦται δὲ πᾶν στερεὸν ὑπὸ ἐπιφανειῶν καὶ γίνεται ἐπιφανείας ἀπὸ τῶν πρόσω [ἐμπροσθεν] ἐπὶ τὰ ὀπίσω ἐνεχθείσης.

quoted above that the line is ‘what first takes existence in magnitude’. The priority of line in generation is of course current among mathematicians in the Platonist-Pythagorean tradition. Nicomachus, for example, says that ‘The first dimension is said to be the line, for the line is that which is one-dimensional’ (*Ar.* 2.6.4), explaining a little later how the successive dimensions are generated. What the *Definitions* adds to this view is that the line comes into being when the point flows *downwards*.³⁶ The remaining difference is that both Hero and Nicomachus speak about line whereas Sextus and Stobaeus speak about length. As we shall see, the identification of length and line will be central to Sextus’ second set of arguments, and we shall turn back to this issue in that context.³⁷

Now, the theory of the successive generation of dimensions, and especially the view that an n dimensional entity is generated from the $n-1$ dimensional entity by ‘flowing’ (a theory that will come to the fore at a later point in Sextus’ discussion), can easily lead to the view that the entity thus generated has a directionality. Nonetheless, this idea in itself does not determine the specific direction assigned to the entity in question. The view that the line is the point flowed *from above* may at first seem to build too much on the image of ‘flowing’ – the point behaves as some kind of liquid and so it flows downwards – but of course this will not work with regard to the other dimensions. It seems much more likely that the specific directionality of the dimensions expresses the view that there is a hierarchy of directions: up, right and front are prior to down, left and behind. I wish to suggest that the source of this hierarchical systematization of dimensions and directions may be found in Aristotle’s startling discussion of the directions in the cosmos in *De Caelo* 2.2 together with a Neo-Pythagorean response to Aristotle’s criticism of the Pythagoreans in the same context.³⁸

Building on the results of the no less curious treatment of the six directions in *De Incessu Animalium* 2–7, in which he connects the different directions with the functions of living beings, Aristotle says in *Cael.* 2.2.284b24–5 that: ‘Above is the starting point (ἀρχή) of length, right of breadth, before of depth.’ He adds a little later, at 285a19, that length is prior in the sense of generation to breadth, and – although he does not make it explicit here – presumably also to depth. The outcome matches exactly what we have found in the group of texts discussed above both in

³⁶ Sextus will speak about the generation of dimensions by ‘flowing’ at a later point. See below, pp. 000–000.

³⁷ See below, pp. 000–000.

³⁸ The relevance of the *De Caelo* is noted also by Dye & Vitrac 2009: 177.

the order of dimensions and in the respective starting points of the dimensions. Moreover, like Sextus, Aristotle speaks about length, breadth and depth, and not about line, surface and solid as the *Definitions* (and Nicomachus). Yet, clearly, Aristotle's results come not so much from a theory about the generation of dimensions (which seems to be the immediate background of the view in the *Definitions* and Sextus), but rather from the assumed connection between the directions and the functions of living beings. According to the analysis of *De Incessu Animalium* 4 – which is surely connected to the *Timaeus*' discussion of the directions in the organism – growth and the distribution of nutriment is from above to below (with the awkward consequence that the roots are the superior part of plants), locomotion is from the right to left (supported by the 'facts' that all men carry burdens on the left shoulder and that they hop easier on the left leg) and the sense organs, directing movements, are positioned in the front. The outcome is that although all bodies are extended in three dimensions, the six directions properly speaking characterize only animals and that the directions in an animal are relative neither to absolute directions nor to the perspective of the observer, but to the functions of the animal.³⁹ Aristotle adds, however, that we can assign directions to inanimate objects analogously and relative to ourselves.

It is remarkable that Aristotle couches the whole discussion of the directions of the cosmos in a polemic against the Pythagoreans. Ultimately his two points of criticism are (i) that the Pythagoreans speak only about right and left, omitting above and below, which are prior to them in so far as length is prior to breadth and (ii) that they assign right and left to inanimate things as well.⁴⁰ What seems to justify Aristotle's criticism is that right and left are included in the Table of Opposites, but the other two pairs of directions are not. I would suggest that some later Pythagoreans accepted the force of (i), but rejected (ii). The ensuing view is that in

³⁹ The question may actually be even more complicated. For Aristotle says: 'The distinctions are three, namely, above and below, front and its opposite, right and left – all these three oppositions we expect to find in the perfect [or: complete] body [cf. *Cael.* 1.1] – and each may be called a principle. Above is the principle of length, right of breadth, front in depth. Or again we may connect them with the various movements (Ἐτι δ' ἄλλως κατὰ τὰς κινήσεις), taking principle to mean that part, in a thing capable of movement, from which movement first begins . . . Hence we must not look for above and below, right and left, front and back, in every kind of body, but only in those which, being animate, have a principle of movement within themselves' (trans. Stocks). He thus clearly mentions the directions also before introducing the perspective of movements. It is not entirely clear whether the restriction expressed in the last sentence quoted is limited to approaching the directions from the perspective of motions.

⁴⁰ *Cael.* 2.2.285a25–7: Διὰ τε δὴ τὸ παραλείπειν τὰς κυριώτερας ἀρχὰς δίκαιον αὐτοῖς ἐπιτιμᾶν, καὶ διότι ταύτας ἐν ἅπασιν ὁμοίως ἐνόμιζον ὑπάρχειν.

speaking about the derivation sequence of the dimensions, one must also include the priority relations among pairs of opposite directions and say that the derivations are directional. The account of line, surface and body in the *Definitions* and the definitions of the dimensions Sextus attributes to the mathematicians register this development.⁴¹

The relationship between body and dimensions

Having presented the definition of the mathematicians, Sextus next turns to the possible ways to account for the relationship between the dimensions and the body (367–75). At this point the difference between the incorporealist and corporealist standpoints is already of consequence. For the incorporealists, Sextus' problem will amount to the following question: what is the relationship between first principles and derivative entities? For the corporealists, by contrast, the question will concern the relationship between the ontologically basic primary entities and those items that figure in their conceptions. In so far as Sextus' primary targets should remain the incorporealists, the success or failure of this section will depend on whether his arguments will effectively be applicable to the way in which the corporealists would conceive the issue. In particular, Sextus' argument uses disjunctions as its premises at several levels: 'the relationship between body and dimensions is either A or B; if A, then it is either Ai or Aii'. Now it is *prima facie* conceivable that different options are available to express the relationship between primary and derivative entities on the one hand, and the relationship between primary entities and entities that are included in their conceptions, on the other. If so, it may well be possible that what is an exclusive and exhaustive disjunction in view of the first question is not so in view of the second, and thus the argument is valid in the first, but not in the second case. Moreover, as we shall see in a moment, Sextus' main candidate is that body is, in some way or other, a compound (ἄθροισμα) of the dimensions, and this is how the dimensions 'constitute' the body; in this respect the pivotal question will be what ἄθροισμα means and, furthermore, whether 'constitution' would have the same sense in the two contexts.

According to Sextus' initial dilemma, we can either conceive of the body (A) independently of its dimensions or (B) as an aggregate or

⁴¹ This suggestion may add further substance to Isnardi Parente's conclusion (1992: 151) that Sextus' source for these chapters of the *Against the Physicists* should be granted the status of an important source of information on Hellenistic Neo-Pythagorean views.

compound (ἄθροισμα) of the three dimensions (*M* 9.368). We may readily grant to Sextus that (A) is a non-starter in view of the conception of body we are considering. The remaining discussion considers different alternatives for conceiving of body as an aggregate of the dimensions. In the first step we get a dichotomy in terms of the corporeal–incorporeal distinction: the dimensions are either (Bi) corporeal or (Bii) incorporeal. In the next step Sextus adds two further options for the ἄθροισμα view: either (Biii) the dimensions contained the logos of body in themselves before they formed a body, or (Biv) body supervenes on (or emerges from, ἐπισυνεβή) the complex of three dimensions. The whole discussion is cursory, and the presentation of the alternative views, never assigned to individual authors or schools, is rather crude. Moreover, there is no attempt to show either that the (A)–(B) disjunction is exhaustive, or that the four versions presented cover all the possible variants of (B), the ἄθροισμα view.

All the arguments against the different versions of the ἄθροισμα view turn on the corporeal–incorporeal dichotomy. (Bi), that the dimensions are corporeal, can quickly be disposed of because it immediately results in an infinite regress: if the dimensions taken individually are bodies, then each of them will have three dimensions as well.⁴² (Bii), that the dimensions are incorporeal, may initially appear considerably more plausible – also because it is fair to assume that not only the incorporealists, but also most, if not all, the corporealists would consider the dimensions incorporeal.⁴³ Yet, Sextus summarily points out that the mere addition of incorporeals will never result in anything bodily. More precisely, he says that the conjunction or ‘coming together’ (συνέλευσις) of lines, which are incorporeal, and the compounding of points will never result in a solid and *resistant* body (στερεόν . . . σῶμα καὶ ἀντίτυπον), so also length, depths and breadth will not produce body (370). The addition of ἀντίτυπον, which makes it unambiguous that Sextus is speaking here about physical bodies, renders the argument problematic for both camps. The mathematicians may formulate an objection on the basis of the distinction between physical

⁴² Note, however, that the parallel argument in *PH* 3 does not stop at establishing the regress but goes one step further: the body will then be composed of infinitely many bodies and must be of infinite size. Sextus applies the infinitely many-bodies argument specifically to surface at the very end of the chapter in *M* 9.435. Note that by the application of the doctrine of blending through and through, the Stoic can accept that a body is constituted by entities that are themselves bodies in such a way that the constituent bodies are spatially coextensive with each other and with the body they constitute: more bodies does not mean larger extension.

⁴³ Long & Sedley 1987: 301, followed tentatively by White 1992: ch. 7, develops the interpretation according to which limit entities fall completely outside the corporeal–incorporeal distinction and are members of the class of pure mental constructs.

bodies and geometrical solids.⁴⁴ The Stoics and the Epicureans may insist, on the other hand, that even if they grant that they need to give an account of the relationship between body and the dimensions in so far as their conception also includes reference to dimensions, they have never agreed that the dimensions are sufficient to provide a resistant physical body; their point was exactly that we need something further, namely resistance, to obtain physical body. The real targets of the argument could, however, be Sextus' 'mathematicians', that is those in the Neo-Pythagorean-Platonist tradition who, in the manner of the *Timaean*, seek to generate also physical bodies from geometrical entities.

(Biii), that the dimensions contained the logos of body in themselves before they formed a body, and (Biv), that body supervenes on (or emerges from, ἐπισυνεβή) the complex of three dimensions, first appear to be more refined alternatives to the rigid corporeal-incorporeal dichotomy of (Bi) and (Bii), yet they are soon collapsed into them. Thus, it never becomes clear in exactly what way (Biii) differs from (Bi). And even though (Biv) sounds promising in so far as the term 'emerged' (ἐπισυνεβή) is indeed sometimes used to describe the way in which physical objects are derived from mathematical entities,⁴⁵ it also becomes reduced to the corporeal-incorporeal dichotomy in a rather mechanical way. In this case, Sextus first assumes, reasonably, that the 'supervenience' view posits incorporeal dimensions but then asks what 'happens' to the dimensions when they come together to form a body. If they remain incorporeal, we are back to (Bii): incorporeal entities that remain incorporeal cannot deliver a (physical) body. If, on the other hand, one wants to claim that the dimensions *become* corporeal in their conjunction, one needs to accept that they have already been corporeal from the start, because only bodies can undergo qualitative change (μεταβολή), and becoming corporeal is assumed to be such a change. Now, I am not suggesting that those who formulated, or could find attractive, the ἐπισυνεβή view had a full story about supervenience or emergence, but it is fairly clear that Sextus does not block all the routes that could be available to these thinkers. Sextus' approach in formulating the dilemma is acceptable in so far as it may legitimately be asked whether anything 'happens' to the lower-level constituents when the higher-level entity emerges.⁴⁶ Yet, he does not seem to allow for instance

⁴⁴ Cf. n. 24 above, with reference to Mueller 1982: 77.

⁴⁵ Cf. e.g. Alexander of Aphrodisias, in *Metaph.* 75.2 commenting on the way in which numbers, for the Pythagoreans, are supposed to be causes of the physical cosmos.

⁴⁶ For a contemporary formulation in which the lower-level entities undergo a change, cf. Paul Humphreys' conception of emergence as 'fusion' Humphreys 1997.

that the first prong of the ἐπισυνεβή view may differ from (Bii): that there can be other, more sophisticated, ways to conceive the relationship between the static constituents and the composite entity. He does not even employ puzzles he raises elsewhere about the generation of a composite entity: how can a further, additional thing come to be from a combination of components (cf. e.g. *M* 10.338).

The most problematic part of Sextus' argument is however his initial dichotomy: if body is conceived in terms of dimensions, it will be conceived as an ἄθροισμα, aggregate, of the dimensions. What is common to the different variants of this view is that, in accordance with 'the mathematicians' incorporealism', but in contrast with the fundamental tenet of the corporealists, these conceptions treat the dimensions as ontologically prior to bodies. The underlying assumption seems to be that body is generated from the 'coming together' or conjunction of the independently existing dimensions.

This general assumption will certainly be inadmissible for the corporealists. Epicurus, for one, is at pains to work out an alternative to this picture by explicitly refusing to conceive the relevant relationship in terms of ἄθροισμα. The key text is a difficult passage in the *Letter to Herodotus* in which Epicurus discusses the relationship between bodies and their permanent attributes (*Ep. Hdt.* 68–70). However, Epicurus does not refer to dimensions in this context but mentions shape, colour, size and weight (keeping the list open). Now, the role he assigns to permanent attributes is the same as the role of dimensions in the alternative conception of body under scrutiny in Sextus' text: they are necessary ingredients of the conception of body (ὧν ἄνευ σῶμα οὐ δύνατον νοεῖσθαι) and the body is in a way a complex of these. That Sextus, too, recognizes the parallel is shown by the fact that in describing the relationship Epicurus posits between body and size, shape and resistance in his list of conceptions of body in *M* 1.21 he uses the same language that he uses now for describing the relationship between body and dimensions: he says that according to the first conception of Epicurus, body is a conjunction by aggregation (σύνοδος . . . κατὰ ἄθροισμὸν) of size, shape and resistance.⁴⁷

Now, Epicurus explicitly discards some of the assumptions used in Sextus' argument. He makes it clear, first, that the items listed in the conception of body are not some incorporeals that would exist on their

⁴⁷ At the very end of the chapter on body, Sextus will consider a version of the Epicurean definition in terms of the permanent attributes and there he phrases it in terms of the ἀθρόος that figures in Epicurus' original text.

own, independently of the body. He then argues that the nature of the body is a complex (ἄθροον) of the permanent properties, but not in the way that a larger body is an aggregate (ἄθροισμα) of its bodily constituents, atoms or smaller component parts.⁴⁸ We can on, the other hand, distinguish these constitutive properties by a special way of the mental operation of focusing (ἐπιβολή), which does not mean that what we can focus on in these ways can be separated from the complex conception of the body.⁴⁹ A body can be conceptually analysed into these items, the complex of which provides us with a grasp of the nature of body. Body, however, is not constituted by them in the sense that body is not generated by somehow putting together these items, as they were material parts. The way in which Epicurus speaks about the relationship between body and permanent attributes is clearly applicable also to the relationship between body and dimensions. Moreover, there are good reasons to think that, notwithstanding all the doctrinal differences about the ontology and nature of limit entities, both Aristotle and the Stoics⁵⁰ would follow roughly the same route. They would agree that although body can be analysed into dimensions by a mental operation, it does not mean either that the dimensions arrived at in these ways can be conceived independently of body, or that they would exist independently of body so that the dimensions would be generative of body in the way that Sextus' presentation appears to assume. Thus they could actually all agree with Sextus that no version of the ἄθροισμα view will describe the relationship between body and dimensions, yet they will object that this is not the only available way in which to conceive this relationship. Indeed, Sextus' arguments could well serve the corporealists in their polemics against the incorporealists and may ultimately go back to such contexts.

⁴⁸ The emphasis here is on the fact that the physical constituents also have magnitude – and are hence bodies – but are smaller than the aggregate. No such relationship holds between the body and its permanent properties.

⁴⁹ οὐθ' ὡς ἕτερον ἄττα προσυπάρχοντα τούτῳ ἄσωματα οὐθ' ὡς μόρια τούτων, ἀλλ' ὡς τὸ ὅλον σῶμα καθόλου μὲν <ἐκ> τούτων πάντων τὴν ἑαυτοῦ φύσιν ἔχον αἰδίον οὐχ οἶόν τε εἶναι, συμπεφερημένων ὥσπερ ὅταν ἐξ αὐτῶν τῶν ὄγκων μείζον ἄθροισμα συστήῃ ἥτοι τῶν πρώτων ἢ τῶν τοῦ ὅλου μεγεθῶν τοῦδε τινος ἐλαττόνων, ἀλλὰ μόνον ὡς λέγω ἐκ τούτων ἀπάντων τὴν ἑαυτοῦ φύσιν ἔχον αἰδίον. καὶ ἐπιβολὰς μὲν ἔχοντα ἰδίας πάντα ταῦτα ἐστὶ καὶ διαλήψεις, συμπαρακολουθοῦντος δὲ τοῦ ἀθροῦ καὶ οὐθαμῇ ἀποσχιζομένου, ἀλλὰ κατὰ τὴν ἀθρόαν ἔννοιαν τοῦ σώματος κατηγορίαν εἰληφότος. The text is exceedingly difficult and I do not claim to understand it in every detail. In the main lines I follow the interpretation and construal suggested in Long & Sedley 1987: fr. 7b (with interpretation in vol. I, 36–7 and additional notes in vol. II, 28), possibly with the exception of the last clause quoted.

⁵⁰ Cf. Proclus, in *Enc.* 89.15–18, according to which the limit entities exist only κατ' ἐπίνοιαν, contradicted by Posidonius (Diogenes Laertius 7.135), according to whom they exist also κατ' ὑπόστασιν.

I think it is instructive to consider at this point how Descartes treats the relationship between body and dimensions. He comes back to this question time and again, but a brief passage from Rule Twelve could function as a direct response to Sextus' puzzles:

If, for example, we consider some body which has extension and shape, we shall indeed admit that, with respect to the thing itself, it is one single and simple entity. For, viewed in that way, it cannot be said to be a composite made up of corporeal nature, extension and shape, since these constituents have never existed in isolation from each other. Yet with respect to our intellect, we call it a composite made up of these three natures, because we understood each of them separately before we were in a position to judge that the three of them are encountered at the same time in one and the same subject. That is why, since we are concerned here with things only in so far as they are perceived by the intellect, we term 'simple' only those things which we know so clearly and distinctly that they cannot be divided by the mind into others which are more distinctly known. Shape, extension and motion, etc. are of this sort; all the rest we conceive to be in a sense composed out of these. (AT 10.418 = CSM 1.44)

Descartes, like Epicurus, starts by emphasizing the different ways in which something can be considered a compound. He also stresses that the conceptual analysis by which we decompose body into items that constitute its nature – extensions, shape – does not deliver ontologically more basic and separable entities. The distinction between body and extension is achieved by a mental operation of the intellect which 'alone has the ability to separate out abstract entities of this sort' (AT 10.444 = CSM 1.60). But in the next move he also has to stress that the items that are the outcome of the conceptual analysis, are 'simple natures' that we cannot analyse further. Decomposition even in this sense has to stop here. As he says also in his letter to Princess Elizabeth (21 May 1643, AT 3.665 = CSM 3.218), extension is a 'primitive notion'. Sextus, however, moves on and seeks to bring the analysis further by asking what length, taken to be the primary extension, is. This move, in itself, is unobjectionable in so far as neither Greek mathematical thinking, nor the relevant philosophical theories block such a step by introducing the conception of primitive notions.

The existence of dimensions: length and line

At *M* 9.375 Sextus turns to his second main set of arguments, which extends to 433 and thus takes up the larger part of the chapter. The proclaimed general aim of this long section is to present arguments for

the non-existence of body through arguments for the non-existence of dimensions: if we conceive of body as that which is constituted by length, breadth and depth, and it turns out that length, breadth and depth do not exist because no conception of them is forthcoming, then body – so conceived – cannot exist either. The argument thus combines conceptual and ontological considerations at more than one level.⁵¹

The fact that these arguments concentrating on the existence of dimensions are separated from the previous ones that focus on the way in which dimensions can constitute the body is an advantage of *M* 9 compared with *PH* 3, where these questions are treated together in a somewhat jumbled way. In *M* 3 these two arguments, formulated in the very same terms as in our text, are separated and follow each other in the order we have in *M* 9. It is important to emphasize, however, that in *M* 3 these two arguments come after the discussion of point, line and surface, the joint discussion of line and surface, the discussion of straight line, and so forth. The argument about the existence of dimensions is presented as a brief addition, taking up only six lines, before Sextus concludes in *M* 3.92 that the *archai* of the geometers are unfounded. As we shall see, in *M* 9 all the arguments about the fundamental geometrical notions not only follow the argument which makes the existence of body dependent on the existence of dimensions but are subordinated to it. The reorganization of the material is clearly motivated by the fact that in *M* 9 Sextus approaches the fundamental geometrical notions in the perspective of his general examination of conceptions of body.

Even if the argument about the existence of dimensions is distinguished from the argument that discusses the different ways in which the three dimensions may constitute body, it takes as its tacit premise that there is a part-whole relationship between body and the dimensions. The parallel argument in *PH* 3 (which includes resistance (ἀντισυπτία)) makes the point explicit: ‘Now without length and breadth and depth and resistance, nothing will be a body; but if a body is these items, then anyone who shows that they are unreal will do away with bodies too (for wholes are done away together with all their parts)’ (*PH* 3.40 trans. Annas & Barnes 1994). Thus, the task now is ‘to do away’ with the dimensions that constitute body.

Sextus immediately translates the question of the existence of dimensions into the question of the existence of fundamental geometrical

⁵¹ Cf. the discussion by Bobzien, in this volume (pp. 000–000), about the problematic nature of such moves.

objects. The entire later development between 375 and 433 is ultimately dependent on this move. The crucial sentences run as follows:

... nothing is length, breadth and depths, as we shall establish; therefore body does not exist. *For length does not exist, since this is the greatest dimension of the body which is called 'line' by the mathematicians*, and the line is a point which has flowed, and the point is a partless and unextended sign (σημεῖον). Hence, if nothing is a partless and unextended sign, there will not be line either, and since there is no line, there is no length either, and since there is no length, body will not subsist (ὑποστήσεται) either. (375–6; my emphasis)

The key step comes in the phrase I have emphasized; apparently it is the identification of length and line that provides the basis for the elimination of dimensions through the elimination of limit entities. That the non-existence of length follows from the non-existence of line – which in turn follows from the non-existence of point – is reiterated a little later at 379 as a coda to the short arguments for the inconceivability of point. And when Sextus next turns to the arguments for the inconceivability of line, he says once again that one can argue for the non-existence of length by establishing that line does not exist, because length is line (ἤν γὰρ τὸ μήκος γραμμή, 380). The same conclusion is repeated at the end of the section dealing with the derivation of line from point (389).

Remarkably, this explicit and emphatic identification of length and line is absent from *M* 3. We may perhaps interpret it as a clear recognition from Sextus that in the entire long subsequent section he is still focusing on the same conception of body, expressed in terms of dimensions, and he must therefore first establish that the dimensions are dependent on, or indeed identical with, the fundamental geometrical entities. This move, on the other hand, may seriously threaten the efficacy of Sextus' strategy. If the dogmatist opponent refuses to accept this identification, the arguments for the inconceivability of point, line and surface will not threaten his conception of body in terms of dimensions. Disagreeing with the Platonists, a corporealist dogmatist may accept that body is to be conceived of in terms of three-dimensional extension (with or without resistance) without nonetheless accepting that body is in any way constituted by points and lines or that the dimensions should be conceived of in terms of fundamental geometrical notions.

It is thus highly significant that Sextus immediately translates the dimensions into limit entities as conceived of by the mathematicians, and apparently does not allow for possible alternatives formulated by the corporealists. The consequence of this identification is that once Sextus

disposes of the various mathematical conceptions of limit entities to his own satisfaction, he considers the job of eliminating body – also as conceived of by the corporealists – done. Yet the physicists could, and did, come up with other conceptions of limit entities; for instance, the Stoics could claim that limits have depth, albeit infinitesimal depth, whereas the Epicureans taught that limit entities will have a definite three-dimensional magnitude. Thus they can readily agree with Sextus that the mathematical conception of limit entities is flawed, or that limits as conceived of by the mathematicians are not applicable to the description of physical reality, without conceding that all this threatens the conception of physical body as three-dimensionally extended (with or without resistance).⁵² Indeed, it is highly probable that many of the subsequent arguments against the fundamental geometrical notions originate in Stoic and especially Epicurean texts.⁵³ At this point the distinction between geometrical and physical bodies also becomes crucial; for it may be much easier to argue that geometrical body stands or falls together with other fundamental geometrical entities, point, line, and surface, so that the conceivability and ontology of these geometrical objects comes in a package, than to accept that physical body is also part of the package.

It is important to note, moreover, that *PH* 3 offers a further possibility, namely that surfaces and lines are ‘observed only in connection with so-called bodies’ (μόνον περὶ τοῖς λεγομένοις σώμασι θεωρεῖσθαι, *PH* 3.41, trans. Annas & Barnes), which would reverse the priority relationship between line and body. This option, corresponding to the focus on the incorporealist mathematicians, is not entertained here or elsewhere in our chapter; an omission that creates yet another momentous lacuna in the argument of *M* 9.

Let us see then how Sextus justifies the identification of length and line. In a first step he defines length as the greatest dimension of the body. This, in itself, does not seem to be problematic. If we take a three-dimensional object the actual position of which is not fixed, we may designate the longest dimension of it as its length. It is plausible to say that we measure the length of a pencil or a couch always along its largest extension irrespective of the position of the object. This conception is, however, clearly in contrast with the position-based definition of length

⁵² I cannot enter here into the discussion of the ontological status of limits, point, line and surface according to the different parties concerned. For an instructive treatment of the relevant Aristotelian, Epicurean and Stoic views, see White 1992.

⁵³ Cf. Dye & Vitrac 2009.

(‘length is that which is from above to below’) that Sextus used in his initial presentation that we considered earlier. We have also seen, on the other hand, that Stobaeus lists exactly these two as alternative conceptions of length.⁵⁴ Stobaeus, however, hastens to point out that that conception of dimensions which defines length as the largest dimension is problematic in the case of bodies such as a sphere: as a body it must have length together with breadth and depth, yet none of these is larger than the other two. Nonetheless, Simplicius in his commentary on Aristotle’s *De Caelo* 2.2 emphasizes that ‘length is said to be the greatest dimension in *every* body’ to supply a premise for his explanation of Aristotle’s argument for the claim that the length of the spherical cosmos is the axis joining the two poles.⁵⁵

Yet even if we accept the somewhat counterintuitive claim that in the case of every body one of the dimensions is greater than the others, and that this is the length of the body (or in the case of a sphere, we can designate in a non-arbitrary way one of the diameters as the length of the sphere), how can this motivate the identification of length and line as Sextus’ formulation would require it? The background of the identification of length and line, I would suggest, lies elsewhere and is closely connected to the developments I have tried to reconstruct above in discussing the directionality of dimensions. If it is true that length is prior to the other dimensions (as Aristotle also argues in *De Caelo* 2.2 and *IA* 2–7) and, on the other hand, line is the first dimension which is generated from the point, we can say that when the point has flowed, we get the first dimension of the future body, namely its length. And in so far as in the next phase of the generation of dimensions we get surface from line, and breadth is the second dimension of the body, surface can be treated as the breadth of the body, or, alternatively, surface is length and breadth. Yet this conception of length is not the same as the one which identifies it with the *greatest* dimension of the body. If you take the generative view of the body, there is nothing, as far as I can see, that could guarantee that what is generated first is also its quantitatively greatest dimension. Moreover, the claim that length is the greatest dimension indicates that we are not dealing with indeterminate extensions. Thus, I think, Sextus’ move would be much better motivated if he had said ‘For length does not exist, since this is the *first* dimension of the body which is called “line” by the

⁵⁴ Cf. above, pp. 000–000.

⁵⁵ Simplicius, in *Cael.* 390.2–6: μήκος γὰρ ἐν πᾶσι τοῖς σώμασι λέγεται τὸ μέγιστον ἐν αὐτοῖς διάστημα.

mathematicians.’ There seems to be a confusion here between the two conceptions of dimension that Stobaeus duly distinguishes.

When formulated in that way, the identification of line and length may moreover reveal a further confusion between ‘having length’ and ‘being length’. Length appears together with line because the line already has length. But this does not mean that line is length or length is line. The identification nonetheless seems to lurk also in the Euclidean definition of line. For the Euclidean definition states not merely that ‘line is that which is extended in one dimension’, as Nicomachus will later say (*Ar.* 2.6.4), or that ‘line is what has length only’ but that it is ‘breadthless length’ (*Def.* 1.2). Compare the definition of surface: ‘A surface is that which *has* length and breadth only’ (*Def.* 1.5). Note also the priority relations among dimensions seem to be presupposed in these definitions as well.⁵⁶

Point, line and surface

As we have seen, Sextus in the programmatic sentences of 375–6 makes the existence of body ultimately dependent on the existence of point. If the conditional in the last sentence in the passage quoted above holds (‘if nothing is a partless and unextended sign, there will not be line either, and since there is no line, there is no length either, and since there is no length, body will not subsist either’), it should in theory be sufficient to argue merely for the non-existence of point. However, presenting all the arguments included also in *Against the Geometers*, Sextus systematically goes through the available conceptions of point, line and surface. Surely, those conceptions of line that derive it from point (‘line is a point which has flowed’ and ‘line is a row of points’) are also relevant in so far as they substantiate the claim that line depends on point. Yet Sextus does not stop there but examines further conceptions of line, and then surface, and examines whether they are consistent with the mathematicians’ ‘theorems’.⁵⁷ Now, this overall strategy is clearly relevant in the context of *Against the Geometers*. The notions in question form the basis of geometry as a *technē*, which builds its whole edifice on these; this is why it is

⁵⁶ Hero, *Def.* 2.1 lists both definitions. Note that μήκος in specific geometrical contexts, moreover, can be used synonymously with εὐθεία. See Mugler 1958: 293 s.v. μήκος. It is also remarkable that Aristotle in some key passages uses the word μήκος where one would rather expect γράμμη. See e.g. *Metaph.* 13.3.1078a8 and *Ph.* 2.2.193b24–6.

⁵⁷ The examination of surface (430–6) may actually be taken as independent from the project announced in 375–6 in so far as it primarily concentrates on the question of what happens to limits when two bodies touch.

informative to relate these conceptions to further, more complex statements, operations and notions used by the same *technē*. A *technē* can effectively be destroyed by showing that its fundamental notions, as well as its more complex operations, turn out to be incoherent. In such a context, this kind of overkill can also be a powerful rhetorical tool. Indeed, for destroying geometry as a *technē*, one does not need to show that its assumed fundamental objects do not exist; it is sufficient to show that a *technē* is incapable of construing a coherent conception of them.

Prima facie, the whole exercise is much less relevant in the context of *Against the Physicists*, where the primary target should still be the first principles of the dogmatic physicists. There are cases, however, where this tactic furnishes considerations that turn out to be relevant in this context as well. Discussing various alternative conceptions of the different limit entities may deliver such formulations as could be accepted also by the dogmatist physicists. So along with the two definitions of surface that would surely be rejected by Epicureans and Stoics alike – that ‘surface is the line which has flowed’ and ‘surface is breadth without depth’ – we are given a third definition, ‘surface is the limit of body’, which could be accepted by them. Similarly, the examination of the question of what happens to limit entities when two bodies touch (430–6) is relevant in the context of *Against the Physicists* as well. The impression we get, especially in view of the close correspondence with *Against the Geometers*, is that Sextus keeps concentrating on the geometers, casts his net far and wide, and then some of his catch turns out to be material to his present agenda, without his explicitly discriminating between what is and what is not relevant.

The overall structure of the examination of the basic geometrical objects is as follows. First comes a relatively brief discussion of point conceived as a partless and unextended sign (377–8). In the next step Sextus turns to line, starting with those conceptions of it that derive line from point (380–8). He wraps up this section with an interim conclusion stating that line cannot be conceived in relation to point, therefore body does not exist (389). Next, he turns to doing away with line directly (προηγούμενως), according to its own conception (390). The direct attack on line concentrates primarily on the definition that we know from Euclid (‘Line is breadthless length’, Eucl. *Def.* 1.2)⁵⁸ but also includes a short section on line conceived as the limit of surface (cf. Eucl. *Def.* 1.6). At 418

⁵⁸ Dye & Vitrac 2009: 174 point out very reasonably that from the presence of definitions identical to those in Euclid one should not infer that Sextus consulted Euclid’s *Elements*, or for that matter any other more technical geometrical treatises, or that his primary targets were these works and authors.

Sextus formally announces the end of the direct examination of limit entities and body, but he appends a longish section in which he discusses the *aporiai* that result when one relates the conceptions of the fundamental geometrical entities to geometrical hypotheses and theorems about more complex, derivative geometrical objects (circle, straight line, parallel lines, square, cylinder, etc., 419–30). For once, Sextus makes explicit that this section has no direct bearing on the conception of body under investigation,⁵⁹ but he claims that it provides further evidence to show the absurdity and inconsistency of the geometers' conceptions. Finally, he completes the programme with a section on surface (430–6). It is formally connected to the previous discussion of line and starts with the thesis that the surface is generated from line, but it concentrates rather on the puzzles stemming from treating surface, understood as a two-dimensional entity, as the limit of body. Sextus proclaims the end of the examinations concerning the conceptions of basic geometric objects and the related theorems at 437. As almost all the arguments contained in this section are identical with the ones in *Against the Geometers* and hence were discussed by Ian Mueller, and more extensively by Wolfgang Freytag, I shall treat this section briefly, primarily concentrating on those points that reveal the articulation of the arguments and their place in the overall argumentative strategy of the chapter.

Collecting the different conceptions of the fundamental geometrical entities, there appear to be three ways to give an account of them:⁶⁰

- (i) by derivation: the $n+1$ -dimensional object is the n -dimensional object which has flowed: 'line is the point which has flowed' and 'surface is the line which has flowed'.
- (ii) as a limit: the n -dimensional object is the limit of the $n+1$ -dimensional object 'point is the limit of line', 'line is the limit of surface' and 'surface is the limit of body'.
- (iii) by privation: the n -dimensional object is an extension that does not have the extension characteristic of $n+1$ -dimensional objects: 'point is a partless and unextended sign', 'line is breadthless length' and 'surface is breadth without depth'.

An important difference between (i) and (ii) is whether we take the lower- or the higher-dimensional object as primary: (ii) assumes that n -dimensional entities belong to $n+1$ -dimensional objects, whereas (i) takes it that $n+1$ -dimensional entities can be generated from n -dimensional entities;

⁵⁹ For this reason I shall not discuss this section.

⁶⁰ For a somewhat different formulation of the three types of definition, see Freytag 1995: 162.

obviously, (ii) will be more palatable to the corporealists. Sextus in *M* 3.19 seems to treat the genetic account as primary from which the other accounts somehow follow (παρό).⁶¹ We shall also see that Sextus apparently takes the view according to which the $n+1$ -dimensional entity is a multitude of n -dimensional entities as a possible interpretation of the derivation by ‘flowing’. However, he does not add to the list a fourth type of account, according to which we define an entity as the origin or beginning of the next entity in the derivation sequence (although, as we shall see in a moment, he considers that it is essential to point that it is generative of the line). There is evidence that such definitions were also current in the Pythagorean tradition. Aristotle mentions as an example the definition according to which point is the *archē* of line (*Top.* 108b31–2) and it has been plausibly argued that he takes the example from Archytas.⁶² Nicomachus also uses this definition (*Ar.* 2.7).

Point

Even though the way in which Sextus introduces his overall strategy suggests that the examination of point will carry much of the weight of the argument, the point is dealt with very briefly (377–8). The section on point in *Against the Geometers* is considerably longer (*M* 3.22–8). According to Sextus’ initial dilemma, the point is either corporeal or incorporeal. As it does not satisfy the criteria of three-dimensionality, the point cannot be a body. Sextus is thus using the conception under investigation as a premise in the argument—this is fine of course in so far as he wants to show the internal inconsistency of the conception. The argument to the effect that the point cannot be incorporeal uses as its premise that the point is generative of the line. Yet, if the point is incorporeal, it cannot function as a source of generation for further entities, because generation presupposes contact (θίξις) and only something corporeal can be in contact.⁶³

⁶¹ The list of definitions considered by Sextus in Mueller 1982: 73–4 is not complete.

⁶² Huffman 2005: 499–503.

⁶³ It is notable that Sextus does not base his case on the Aristotelian argument from *Ph.* 6.1 that what is partless cannot be in contact (ἀπτεται) but states that incorporeals cannot be in contact. From the same assumption we can deduce that lines and surfaces, which have parts but are incorporeal, cannot be in contact either. For Stoic parallels, cf. e.g. Nemesius 81.8 = *SVF* 2.790 (part): οὐδὲ γὰρ ἐφάπτεται σώματος ἀσώματος. For the view that all causal interaction presupposes contact, see *M* 9.258. In *M* 10.325 Sextus says more specifically that generation and perishing presupposes contact. See also e.g. Plutarch, *Comm. Not.* 1080E about Chrysippus’ insistence on this point. For a more detailed discussion of why incorporeals cannot be in contact, see Freytag 1995: 183–202; 4.2.2 and 4.2.3 on why contact is needed, according to Sextus, for generation.

The whole discussion turns on the expectation that the point is that from which the other dimensions, and ultimately the body will be built up. Sextus thus does not consider whether the notion of the point as a partless and unextended entity is coherent in itself, and he does not exploit the numerous puzzles, mentioned also by Aristotle, that may arise from such a conception. Similarly, taking the point to be incorporeal may be coherent in itself, but it will not do for our purposes, because then it will not be able to produce line. A discussion of that conception which takes point to be a limit entity – one which receives a treatment also in *M* 3 – is missing too.

Line

Derivation from point

Sextus next turns to line, and the critical examination of the different conceptions of line will take up the larger part of the remainder of the chapter. Remember that line is crucial since this is what Sextus has identified with length, and length is supposed to be essential for the constitution of body, being its primary dimension. First come those conceptions that derive line from point. The discussion of the relationship between point and line had had of course a long history by that time, starting at least with Archytas, and becoming an especially important issue among the disciples of Plato. Part of the problem is the old Zenonian one: how can something that has magnitude be constituted by things that have no magnitude? The other part of the problem is the topological relationship between points that are supposed to be constitutive of a continuous magnitude: how can we imagine the relationship between two neighbouring points? It is also important to see, as has often been emphasized, that these questions cannot be adequately treated without a fairly advanced set-theoretical apparatus developed in the second half of the nineteenth century.⁶⁴

Because the problems with the different conceptions of line are so apparent, Sextus can be generous and employ the customary sceptic strategy: let us ignore that we have already done away with point, and hypothetically grant that it exists; even so the line will not exist because it is impossible to derive it from point. According to the old tag that Sextus constantly ascribes to the mathematicians, we obtain the line when ‘the point has flowed’ (στικμὴ ἐρρηκυῖα, 376); alternatively the line is the ‘flux of sign’ (ῥύσις

⁶⁴ Cf. e.g. White 1992: chs. 1 and 4, esp. 179–85; Freytag 1995: chs. 2 and 4.

σημείου, 380).⁶⁵ The ῥύσις doctrine is actually the only mathematical view to which Sextus attaches a name, if not in *M*9, at least in *M*3. At *M*3.28 Sextus reports that Eratosthenes, in order to ward off attacks, used to say that the point does not have a place, neither does it measure the interval of the line, but it produces the line by flowing (ῥυέν δὲ ποιεῖ τὴν γραμμὴν). The attribution to Eratosthenes is confirmed by a parallel report by Theon of Smyrna in a context where he explicitly discusses Eratosthenes' views: 'As to the point, it produces the line neither by multiplication, nor by addition, but by a continuous movement, just as the line produces the surface, and the surface the solid' (*Exp.* 31, 83.21–4 Hiller).⁶⁶

Somewhat curiously, Sextus seems to consider that all those views that derive the line from the point can be described by the terms of 'flux' (ῥύσις) and 'flowing' (ἔρρηκυῖα), so that these terms cover both those views (A) that obtain the line from a single point and those (B) that conceive the line as a set of points in a row. Now (B) not only is an unintuitive interpretation of the labels but is the group of views in contradistinction to which Eratosthenes apparently put forward the ῥύσις doctrine in the first place. Indeed, other specifics of Eratosthenes' conception, most notably that the point does not have a place, are also ignored in Sextus' arguments which are based on the assumption that 'flowing' can only be conceived in physical terms.

The distinction between (A) and (B) constitutes the first dilemma, which then leads to several further levels of dilemmas. Sextus first deals with the less probable option at each level and then constructs a dilemma from the other horn. (A) is thus first divided into: (A1) the point remains at the same place; and (A2) the point moves from one place to another. (A1) is implausible both because it does not deliver a line – a stationary point will remain a point – and because why would we then say that the point flows? But (A2), the *prima facie* more plausible candidate, needs interpretation. When the point moves, does it (A2i) occupy a new place by leaving its previous place behind, or does it (A2ii) lay hold of the new place without giving up its first place? Again, (A2i) will leave us with a single moving point – Sextus does not even need to indulge in the difficulties that may be raised about the motion of a partless entity⁶⁷ and by whether something which is not a body and has no extension can have a place.⁶⁸

⁶⁵ On the origins of this conception, see e.g. Isnardi Parente 1992: 120–68. She defends the view according to which the conception may ultimately go back to Archytas.

⁶⁶ Cf. Freytag 1995 and Dye & Vitrac 2009: 185. ⁶⁷ Cf. Aristotle, *Ph.* 6.10.240b8–241a6.

⁶⁸ This problem arises from Aristotle's discussion of place in *Ph.* 4.1–5, according to which only a moving *body* can be in a place. Eratosthenes' insistence that the point does not have a place might be

If one assumes that the point can flow and that it has a place, (A2ii) might be the most intuitive interpretation of the definition; and it is at this point that we can expect the argument to become more interesting. Sextus, however, formulates this option in a rather perplexing way. According to the next level of dilemmas, when the point lays hold of a new place it becomes ‘coextensive by stretching out’ (ἀντιπαρεκτείνομαι).⁶⁹ As far as I am aware, this notion is not used in technical mathematical contexts and evinces once again Sextus’ fundamentally physicalist interpretation that treats point as a body. The rare term used here belongs to the Stoic technical vocabulary to designate the way in which two bodies interpenetrate each other in *krasis* and thus take up the same extension; in Sextus’ presentation the same relationship would hold between point and its place.

According to Sextus’ dilemma the point ‘stretches out’ either (A2iia) to an indivisible place or (A2iib) to a divisible place. Once again, the first option is implausible: what is coextensive with an indivisible place is still a point. Now what about (A2iib)? For, presumably, this is what the derivation is supposed to mean: the indivisible point has become the first divisible magnitude, the line. Sextus’ objection is double. On the one hand, he points out, reasonably enough, that the resulting entity can no longer be a point in so far as it is divisible. The basis of this objection must be that the conception under investigation assumes that the resulting line is not a qualitatively different entity, but still a point – a point which has flowed.⁷⁰ Besides, the change described by the verb ‘to flow’ does not seem to refer to a process of generation, but rather to locomotion or growth.

Sextus nonetheless adds another objection, according to which, in so far as it is divisible, it must have parts, whereas that which has parts is a body (τὸ δὲ ἔχον μέρη σῶμα ἐστίν, 385) – so the point must be a body, which is unacceptable for those who hold this conception: it is not extended in three dimensions. So Sextus does not merely point out that the conclusion according to which the point has parts is incompatible with the definition according to which the point is partless but introduces the further premise stating that that which has parts is a body. Perhaps we can

a recognition of Aristotle’s point. On how the ancient commentators tried to make this claim compatible with the discussion of contact in *Ph.* 5.3, which presupposes that limit entities also have place, see Furley 1982 and White 1992: 24–8.

⁶⁹ Cf. *SVF* 2.471; 472 and 473 from Alex. *Mixt.* and Arius Didymus. This otherwise rare word occurs in Sextus ten times, twice in the chapter on part and whole (*M* 9.262–3), three times in our passage, which is exactly paralleled in *M* 3, and once in the chapter on time (*M* 10.225).

⁷⁰ Freytag 1995: 175–6 argues that Sextus’ fundamental objection is that one cannot identify the moment when the generation of line from point could take place.

derive this premise from the discussion of part and whole: that which has parts is a whole, but only a body is a whole (cf. *M* 9.332 and 334 for the Stoic and Peripatetic acceptance of this statement). We have seen above that the final stage of the discussion about the conception of point depends not on the partlessness of point, but on the corporeal–incorporeal distinction and can therefore be extended to line and surface: it implies that not only point but line and surface cannot be in contact either. Although Sextus does not draw this conclusion, in the present case the last premise implies that – in so far as they are not bodies – the line and the surface cannot have parts either. Partlessness ceases to be a distinctive feature of point.

Sextus now turns to (B): line is a multitude of points in a row (πολλὰ στοιχηδὸν κείμενα, 386). The argument turns on the question how the neighbouring points are spatially related. They are either (B₁) divided by intervening spaces or (B₂) touch one another. Again, (B₁) is less plausible, because we have lost the continuity of the line. (B₂), in turn, is dealt with by a variant of Aristotle's dilemmas in *Ph.* 6.1.231a–24b6: when points touch, do they touch whole by whole, part by part, or part by whole. As we know from Plutarch (*Comm. Not.* 1080E), Chrysippus' chief objection against the advocates of indivisible magnitudes went along similar lines. One difference is that Aristotle simply points out that points, being partless, cannot touch part with part, whereas Sextus dwells on this issue at some length. He says that points should, on this conception, have distinguishable parts not only along the line – one by which they touch the previous point and one by which they touch the next point in the row – but also in the other directions as well. The line is imagined to lie on a surface so that each point constituting the line touches the underlying surface with a further part; and to have something above, so that the point has to have a fourth part by which it can touch the corresponding part of that thing. One may be struck by the literal-mindedness of the visual imagery: the points are supposed to touch the underlying surface as the pearls of a necklace touch the dressing table. Yet this elaboration reveals the denial that 'naked' lines can exist just on their own in something like abstract space: the line must have some environment and, if we suppose that the line is constituted of points, these points must have some topological relation not only to one another but also to the corresponding parts of the environment. Indeed, the whole section is characterized, once again, by strongly physicalist, corporealist assumptions. Point, line and surface are systematically treated as bodies, and then it is shown that they cannot be bodies: their parts are conceived as physical parts, whereas 'touch', as we have just seen, is portrayed as a physical contact between bodies.

The section is wrapped up by the conclusion that since it has been shown that there is no such thing as line, and we need line to conceive length, and length to conceive body, no such thing as body exists (389).

Line is breadthless length

This is the point where Sextus turns to abolishing the line directly (προηγούμενως) according to its own conception. The expression may simply refer to a distinction between different conceptions: he is now turning to that conception of line which does not relate it either to point or to surface⁷¹ – so neither (i) nor (ii) but (iii) on the list above (see p. 000). We get a slightly stronger interpretation if we take the phrase to mean that Sextus will now attack line directly in the sense that the conception of line to be discussed is independent from the more specific doctrines of the mathematicians about the ontology and derivation sequence of geometrical entities. The conception in question – line is breadthless length – is thus not only a different one, but one that is less theory-laden and could be more widely accepted. The parallel use of προηγούμενως in M 10.189 offers some support for this construal.⁷² Notably, this is also where Sextus starts to speak about ‘geometers’; up to this point he has spoken only about the ‘mathematicians’. The text does not make the relationship between the two designations explicit, but I find it tempting to think that, if the distinction is to carry any weight, then the geometers are those who do not need to be committed to the derivation sequence of geometrical objects. Indeed, none of the arguments of the subsequent section – where geometers are mentioned, and the coherence of their definitions and theorems is under attack – assumes the more robust metaphysics of the ‘mathematicians’.⁷³

The conception at issue is of course the Euclidean definition of line (Def. 1.2). Yet, as Sextus will also remind us, Aristotle had already formulated a defence of it; the definition and the controversies around it therefore must go back to pre-Euclidean times.

Sextus’ strategy consists in applying an empiricist epistemology to show that what is defined in this way is inconceivable. It is notable that Sextus concentrates the empiricist artillery on this definition of line and

⁷¹ So Mueller 1982: 72. ⁷² Cf. Warren 2003: 314 and Bobzien, in this volume, pp. 000–000.

⁷³ When he turns to the difficulties surrounding surface in M 9.430, Sextus refers to the derivation of surface from line by flowing, but he does not exploit any of the puzzles that may derive from this conception. Instead, he immediately turns to consider those difficulties which arise when we define surface as that which has two dimensions, having length and breadth only, and which takes it to be the limit of body. These latter are of course acceptable also for those who otherwise disagree with the derivation sequence view.

does not apply it to the point (as he does in *M* 3.22–8). Sextus first presents a general typology of different ways in which we conceive (νοεῖται *M* 9.393; ἐπινοεῖται *M* 3.40) a concept (τὸ ἐπινοούμενον *M* 9.393; τὸ νοούμενον *M* 3.40).⁷⁴ According to the first division, we form concepts either by direct encounter with things that are manifest, or by transference (μετάβασις) from them; the experience of manifest things is thus a precondition of concept formation. Putting to work his usual method, Sextus contends that line can be grasped in neither way.

One might think that it is not a central issue for the geometer whether or not a length without breadth is directly perceptible. Yet there is some evidence that there were authors also in the mathematical tradition who tried to claim that one-dimensional entities are available to direct perceptual experience by referring to, for example, ‘what divides illuminated regions from those in shadow.’⁷⁵ The purported direct perceptibility of such entities not only should ward off sceptical attacks, like the one we find in Sextus, but could also be directed against those empiricist physicists who, for different reasons, deny that one-dimensional limit entities have any reality in the physical world. Sextus, however, ignores such examples and finds it sufficient to assert that length is never perceived in itself, but always together with some measure of breadth.

Sextus then divides the other option, transference, into three ways of concept formation: according to resemblance (κατὰ ὁμοιότητα), composition (κατὰ ἐπισύνθεσιν) and proportionality (κατὰ ἀναλογίαν), this last one comprising diminution and increase. Once again, Sextus proposes that we cannot conceive length without breadth by any of these modes. The ultimate origin of this specific typology is unknown, but it closely corresponds to somewhat different lists attributed to the Epicureans and the Stoics respectively. First, it differs from the theory of concept formation that Diogenes Laertius ascribes to Epicurus only in that Epicurus does not subsume the three non-direct forms under ‘transference’ (μετάβασις).⁷⁶ What differentiates the non-direct modes is that in those cases there is some measure of contribution from reasoning. Diogenes attributes a similar, but more extensive, list to the Stoics (7.52–3), where we also find the examples of the Cyclops (increase) and the pygmy (diminution),

⁷⁴ Cf. also *M* 8.59–60. On this epistemological passage, see the fuller analysis in Mueller 1982: 78–81 and Freytag 1995: 1.3. See also Ierodiakonou, in this volume, pp. 000–000.

⁷⁵ Proclus, in *Euc.* 100.14–16. The claim is attributed to Apollonius by Heiberg; cf. also Hero, *Def.* 16.5–11.

⁷⁶ Diogenes Laertius 10.32: καὶ γὰρ καὶ ἐπίνοιοι πᾶσαι ἀπὸ τῶν αἰσθήσεων γεγόνασι κατὰ τε περίπτωσιν καὶ ἀναλογίαν καὶ ὁμοιότητα καὶ σύνθεσιν.

mentioned by Sextus. The term *μετάβασις* does figure on that list, but not as a more general concept covering resemblance, composition and proportionality. It is briefly mentioned as the specific way in which things such as *lekta* and place, and so presumably also void and time, are conceived. So if one, apparently like Plutarch (*Comm. Not.* 1080E), thinks that limit entities belong in the category of incorporeals, one could speculate that line would be conceived by *μετάβασις*, but still it would not be identical with any of the modes examined by Sextus.⁷⁷

Sextus formulates his objections to two of the three forms of *μετάβασις* analogously. He points out that all three require that there is some entity such that it is directly available to perceptual experience and itself shows the relevant kind of property that the entity to be conceived by transference also possesses. Thus, we can conceive of a giant by increase or a pygmy by diminution, because human beings that are directly perceptible also have size. Yet we always perceive length with some quantity of breadth (as Sextus has already proclaimed in blocking the first horn of his original dilemma), so by increase or diminution we could conceive of lengths with larger or smaller breadths, but not without breadth. Similarly, there is no resemblance in the relevant way between a perceivable length with *some* breadth and a length *without* breadth. The remaining way, composition, is even easier to dispose of: for what manifest thing should be added to what other manifest thing – as we add horse and man to arrive at the conception of a centaur – to obtain the conception of a breadthless length?

Although Sextus does not mention them in his original classification, he adds and discusses a little later two further possible ways of concept formation: intensification or ‘stretching’ (*κατὰ ἐπίτασιν*, 403–6) and privation (*κατὰ στέρησιν*, 407). Such later additions are always worrying. The original typology had the air of an exhaustive list – Sextus even added that ‘there being this many ways of conceiving, if a length without breadth is conceived, it should be conceived in one of these ways’ (396) – but then the reader has to learn that other options are also available and may wonder whether even further possibilities, not considered by Sextus at all, would be available, so that they would make Sextus’ argument inconclusive.

Now, the formulation in *M* 3 suggests that ‘stretching’ (*ἐπίτασις*) was the way (some) geometers actually tried to answer the empiricist challenge against the conception of line as breadthless length. The idea seems to be

⁷⁷ Mueller 1982: 78 notes that in so far as the Stoic *μετάβασις* appears to be a ‘quasi-scientific inference to an explanatory concept’, but the line does not seem to have such a role, *μετάβασις* may not be applicable to line.

that we can conceive of line as the limit towards which an infinite series of successive reductions of the other dimension of a surface converge. It would be exciting to see how exactly this idea was expressed by the geometers. In any case, Sextus does not accept such a notion of a limit of an infinite series and he states, not unlike a Stoic would do, that there will be some breadth remaining after any finite number of steps; the length that has however small a breadth is still not breadthless. Or, there is a least breadth, and once we have removed that, there will be no length left either.

One cannot but agree with Mueller that Sextus' treatment of privation (στέρησις) is 'rather unsatisfactory'.⁷⁸ Sextus' notion of στέρησις is very narrow and in this sense remains close to what, judging from the example in Diogenes, the Stoics may have meant by that term ('by στέρησις, for instance man without hands' Diogenes Laertius 7.53). His principal point is that privation cannot operate by negating an essential attribute of the subject. The examples he gives to substantiate this point are particularly noteworthy: flesh cannot be conceived of without vulnerability just as *body cannot be conceived of without resistance*. This last one may have been entirely appropriate in its original context but comes quite abruptly after the complete neglect of this question all through the chapter. And of course, Sextus has not yet established that the possession of breadth is such an attribute without which length cannot be conceived; it is exactly what he is supposed to show now.

Sextus' argument against Aristotle is similarly unrefined. Aristotle, in his defence of this particular conception of line, pointed out that 'when we grasp the length of the wall, we apprehend it without the breadth of it' (*M* 9.412; fr. 29 Rose). Sextus' response consists in stating that even if we apprehend the wall's length without the actual breadth of the wall, we never do it without any breadth.

Much more could be said about this epistemological section, but this last point is a good reminder of the larger structural problems that the whole discussion of line as breadthless length raises. Remember that Sextus has undertaken the discussion of point and line because he assumed that the (the concept of) length, and hence (that of) body, is dependent on these. We need line to conceive of length: line is primary, so if there is no line, there will be no length either. Yet the treatment of line in this section takes the opposite approach. It takes for granted that we have an object which has at least length and breadth, and it argues that from this we cannot arrive at the conception of something which has length only (or is

⁷⁸ Mueller 1982: 80.

breadthless length). But even if the argument went through, from the inconceivability of line thus defined, it does not follow that we cannot conceive something that has length, breadth and depth. Thus, this long section does not seem to have any real effect on the question whether the conception of body as that which has length, breadth and depth (with or without resistance) is coherent in itself. It is clearly an argument that the empiricist corporealists can use against fundamental geometrical notions; more specifically, this argument can also be used against those geometers, such as Euclid, who do not base their case on the derivation sequence of these entities. We find, once again, an argument which is entirely appropriate in a general attack on the geometers but is at odds with what is supposed to be on Sextus' agenda in this chapter.

Touching limits

The remaining part of the section on line (414–18), as well as the better part of the section on surface (431–3), exploit *aporiai* arising from touch: when two entities are juxtaposed, or touch one another, what happens to their corresponding limit entities? Do they become fused into one or do they remain distinct, reiterating thus the contact problem? Sextus in this section does not develop the corresponding separation problem, which appears to require the division of a point into two, or the cutting of a line along its length, or the reduplication of a surface by slicing. This is a genuine, and much discussed, problem for which ancient mathematical theory does not seem to furnish an entirely satisfactory answer.⁷⁹

It is noteworthy that Sextus, in both sections, formulates the touching problem in such a way that the limit entities in each case belong to bodies. So when he speaks about the problems concerning the touching of two lines, he does not discuss what 'happens' to the points that are the limits of the lines, whether they melt or remain distinct. Instead, he construes the issue thus that the touching lines are themselves the limits of surfaces of bodies (415). The question, formulated in this way, will apparently be as follows: imagine two rectangular objects touching each other with two respective surfaces, so that the surfaces perpendicular to the touching surfaces (surfaces *A* and *B* on Figure 4.2) are in line with one another; now the lines that are the limits of these surfaces, *a* and *b*, are parallel to each other and are in touch.

⁷⁹ Cf. White 1992, esp. Part One, and the relevant parts of the Cone Problem 293–306 and Mueller 1982: 82–6.

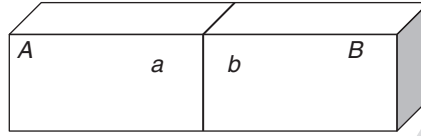


Figure 4.2

The question then is whether these two touching parallel lines become one or remain two; Sextus obviously aims at showing that both options result in absurdities. The first horn of the dilemma, namely that the two lines become coincident and unified, applies the geometrical description that would be acceptable also to Aristotle, and possibly to the Stoics,⁸⁰ whereas the counter-arguments combine geometrical and physical considerations. Assuming that two entities (surfaces or bodies) cannot share, or be separated by, one limit, and ignoring the possibility that one of the entities is an open segment (which would be the contemporary solution), Sextus claims that the unification of the two lines would result in the unification (ἔνωσις) of the two surfaces, and hence of the two bodies. The first objection to this scenario is that this is not what we observe in the case of physical bodies: liquids might, but stones and other solid bodies do not become unified when juxtaposed. Then, apparently forgetting that he explicitly stated at the outset of this set of arguments that, *ex hypothesi*, we are still considering the line to be breadthless, he objects that by the unification of the two lines we would thus lose one edge, and therefore the resulting unified object would be smaller than the sum of the two original bodies. The treatment of the other horn of the dilemma – that the two lines remain distinct – is, however, supposed to be contradicted by precisely that assumption.

The problem of touching bodies recurs in a different, and even less refined, form in the discussion of surfaces (431). In this case, the purported difficulty arises from the distinction between the limit and the limited. If limits are distinct from the body they are the limits of, and if they cover the body from the outside and contain it as the jar contains the liquid, then we must say, absurdly, that the two bodies are not in touch, but only their limits are, or that the bodies somehow reach beyond (ἐκτός) their own limits. What might give force to these arguments is that at least some Stoics

⁸⁰ Cf. Plutarch, *Comm. Not.* 1080E–1081A. Plutarch objects, however, that on the Stoics' own view limit entities, being incorporeals, cannot undergo such changes, i.e. come into being or pass out of being, as would be required by this view.

apparently claimed that limits are not parts of the body in so far as the parts of bodies must be bodies themselves (Stobaeus, *Ecl.* 1.167.9–14 W).

The final argument

At the end of the chapter Sextus presents a further, independent argument for the inconceivability of body (437–9). This is the point where he finally leaves the mathematicians behind and presents a more general consideration. The argument is closely paralleled in *PH* 3 – with some differences that we shall briefly consider below – where it is also the concluding section of the treatment of body. In *PH* 3, however, this argument takes up almost one third of the discussion of body, whereas in our chapter it is dwarfed by the disproportionately long section on the mathematicians' conceptions. Sextus, however, introduces the argument in *M* 9 as the more powerful way to engage with the matter at hand (σθεναρῶς συνάγοντα τὸ προκείμενον). Note that if one accepts Mutschmann's conjecture in *M* 9.367, then Sextus uses here the same construction as the one by which he announced that he would next turn to the mathematicians' conceptions (κατὰ δὲ τὰς τῶν μαθηματικῶν ἐννοίας νῦν συντακτέον τὸ προκείμενον).

According to the opening dichotomy of the argument, body is the object of perception or the object of thought. The first horn seems very promising, especially in view of the fact that the Epicureans argued for the existence of bodies by simply referring to perception: 'That bodies exist is universally witnessed by sensation itself' (σώματα μὲν γὰρ ὡς ἔστιν, αὐτὴ ἡ αἴσθησις ἐπὶ πάντων μαρτυρεῖ, *Ep. Hdt.* 39, trans. Long 8 Sedley; cf. Lucretius 422–3). Acknowledging that he has primarily Epicurus in view, Sextus immediately provides the relevant Epicurean definition of body:

For it [sc. body] is a complex quality grasped according to composition of shape, size and resistance.⁸¹

The formulation strongly resembles the one in *Ep. Hdt.* 68–70 that we considered above, and the closeness to the Epicurean original is further indicated by the use of the word 'complex' (ἄθροος). We have seen that this is Epicurus' technical term for the complex of permanent attributes.⁸² Epicurus may nonetheless object to calling body a complex *quality* (ἄθροα ποιότης): he speaks about a complex conception of body (ἄθροα ἐννοία

⁸¹ ἄθροα γὰρ ἦν ποιότης κατ' ἐπισύνθεσιν σχήματος καὶ μεγέθους καὶ ἀντιτυπίας λαμβανομένη. The list is different in *PH* 3.47: 'length and breadth and depth and resistance and colour and various other items together with which they are observed'.

⁸² Note that *PH* 3.47 speaks about συναθροισμός at this point.

τοῦ σώματος, *Ep. Hdt.* 68). Sextus' argument, however, turns exactly on this point. He declares that such a composite quality is not the object of perception. *PH* 3 is a little more explicit: 'But they say that the senses are simply affected (ἀπλοπαθεῖς).' According to a full corpus *TLG* search ἀπλοπαθής occurs only in Sextus, and only in *PH*,⁸³ yet the wording suggests that Sextus is not introducing an external premise but is referring to an epistemological doctrine advocated, or accepted, by those who hold this view about body. The problem he raises may be construed as a version of the 'wooden horse' problem of the *Theaetetus* (184c–d): the complex is not perceived directly by the senses, and we need something further to bring together the information provided by the senses. Yet Sextus seems to refer directly back to the epistemological interlude in the discussion of line: the complex quality is grasped according to composition (ἐπισύνθεσις), and composition was there listed as one form of transference (μετάβασις), which was in turn contrasted with immediate perceptual encounter with manifest things. The recurrence of this term – missing from *PH* 3 – is notable because it indicates that the argument present also in *PH* 3 was rephrased in *M* 9 in terms of the preceding epistemological material, which is part of the mathematical section, and hence absent from *PH* 3. We have moreover seen that there is some evidence coming from Diogenes Laertius to indicate that ἐπισύνθεσις was part of Epicurean epistemology; a further signal that Sextus' criticism is internal. Although this line of reasoning is not exploited by Sextus, we may add that from an Epicurean point of view, when we form the conception of something by composition, we do so 'with the help of a measure of reasoning' (Diogenes Laertius 10.32), and thus the existence of these objects is not directly guaranteed by perception.

The problem raised by Sextus seems to be explicitly discussed by the Epicureans. First, it is crucial that, just as for Aristotle, the proper objects of the different senses do not overlap; the Epicurean argument for this claim is that if this were not so, different senses might provide contrasting evidence on the same thing. *Prima facie*, it would be tempting to think that at least some of the properties, such as shape, are the objects of vision. Yet as a number of texts make clear, the Epicureans follow Aristotle in thinking that colour, and only colour, is the proper object of vision. We can focus on shape on the basis of the stream of images coming from a body, but the shape perceived thus is not the shape of the body, but the

⁸³ In *PH* 3.108 it is used in an argument to show that change is not perceptible, because we should be able to perceive both from what and into what the object changes.

shape of the colour.⁸⁴ This is emphasized also in a papyrus text which appears crucial in finding an answer to the problem Sextus raises. Its Epicurean author, perhaps Philodemus, stresses that body as such is the proper object of touch.⁸⁵ On this view, then, we do not grasp body as a complex of properties each of which is the proper object of different senses; body is presented directly in a single perception by touch.

Assigning body to touch as its proper object is a notable divergence from the Aristotelian view according to which the proper objects of touch are the ranges of qualities defined by the contraries of hot and cold, dry and wet, and, possibly other qualities such as hard and soft (*de An.* 2.11.422b27). It is true that Aristotle later specifies that what unites these qualities, and hence guarantees that touch is a single-sense modality with a unified proper object, is that these are the distinctive qualities of body *qua* body, in so far as these are the primary qualities that characterize the elements (*de An.* 2.11.423b27–31); this, however, still does not make body the proper object of touch as the Epicureans apparently held. Indeed, it is tempting to think that the Epicurean doctrine is motivated precisely by its being able to ward off the objection that, in so far as it is not the proper object of any of the senses, body is not immediately perceived. On the basis of this modification, the Epicureans could still agree that the senses are ἀπλοπαθής but would refuse that we need composition, and hence ‘some measure of reasoning’, to grasp body. Within body we can then distinguish, by the special mental act of focusing, the different items of the ἄθρόον (shape, size and resistance), as Epicurus in *Ep. Hdt.* 68–70 also claims, just as we can focus on the shape of the colour that we are presented with in vision.

Unfortunately, this may not be the end of the story, for at this juncture it becomes important which properties are included in the ἄθρόον. The list of properties given in our chapter (shape, size and resistance) may be taken care of in the way just suggested. If, however, we take the list given in the parallel text in *PH* 3.47, which includes also colour (and the inclusion of colour may find support in Epicurus’ own formulation in *Ep. Hdt.* 68), then Sextus’ unification problem re-emerges.

⁸⁴ *M* 7.207 (reporting the Epicurean view): οὐ γὰρ ὅλον ὁρᾶται τὸ στερεμνιον, ἵνα ἐπὶ τῶν ὁρατῶν ποιῶμεθα τὸν λόγον, ἀλλὰ τὸ χρῶμα τοῦ στερεμνίου.

⁸⁵ P.Herc. 19/698, cols. 17–18: τὴν μέ[ν] γ[ὰ]ρ [δ]ψιν ὁρατὰ κατα[λ]αμβ[ά]νειν ἡγοῦμεθ[α], τεῖνε δὲ ἀφὴν ἀπτεῖα, κα[ὶ] τὴν μὲν χρώματο[ι]ς, τὴν δὲ σώματος ... ὥστε κατ’ αὐτὴν ἀναλογίαν κοινὰ κρίματ’ εἶναι τῶνε αἰσθήσεων τοῦ[των] τὸ σχῆμα καὶ τὸ μέ[γε]θος, ὃν λόγον ἔχει τ[ὰ το]ῦ χρώματος π[ρ]ὸς τὸ χρῶμα, τοῦτον ἔχόντων [τ]οῦ σώματος πρὸς τὸ σῶμα, καὶ ὃν λόγον ἔχει τὸ χρῶμα πρὸς τὴν διὰ τῆς ὁράσεως [κατ]άληψιν, τοῦτον το[ῦ σ]ώματος π[ρ]ὸς τὴν διὰ τῆς ἀεφῆς ... See Monet 1996; see also Sedley 1989.

The other horn of the initial dilemma, according to which body is the object of thought, appears less interesting. It is based on the strict empiricist principles evidenced also in the epistemological interlude: A can be the object of thought only if there exists some B such that B is the object of perception, and the conception of A can be derived, presumably by the different modes of transference (μετάβασις), from the direct perception of B. If body is not an object of perception but is an object of thought, we are left with no candidate to take the role of B, for incorporeals are clearly inadequate for the task.

Concluding remarks on relative chronology

Can the preceding analysis offer any clues regarding the relationship between *PH* and *Against the Physicists* on the one hand, and *Against the Geometers* and *Against the Physicists* on the other? As to the first question, our analysis may give some slight support to the received view that *Against the Physicists* comes after *PH* 3. Or, to put it in an even more qualified way, there are indications that the chapter on body in *M* 9 as we have it is later than the corresponding part of *PH* 3.

I would tentatively suggest the following scenario. In *PH* 3 Sextus remains closer to the original plan of discussing the material principle after the discussion of god. In a second phase he turns to the discussion of the conception of body in which resistance (ἀντισυπία) is never lost from sight. Finally, the discussion of Epicurus' alternative conception occupies a proportionately large part of the chapter. True, the series of arguments is not particularly well structured, for example questions concerning the relationship between the constituents and body and the existence of the dimensions are not clearly separated. But on the whole, the chapter is relatively well balanced.

Then comes *Against the Physicists*. The chapter on body confuses the articulation between the discussion of the material principle and body. The separate discussion of the material principle is skipped; Sextus introduces the corporealist–incorporealist distinction instead and immediately jumps to the discussion of body. I find it easier to think that this is an unhappy modification of the original plan occasioned by the more extensive material than that the awkwardness of this ordering presents the original version, which was then cleared up in *PH*. Consider, for instance, the use of the doxographical material. We have every reason to believe that it was presented in Sextus' source as an inventory of the different views on the material principles; this is confirmed also by the introductory words in

Ps.-Galen. Sextus nevertheless presents it in our chapter as part of a different strategy. I find it unlikely that Sextus first used it in *M* 9 removed from its original context and in a confusing way and then reused it in *PH*, restoring it to its proper context, and going back to his original source, adding also the introductory words that he first left out in *M* 9.

Now, what about the main structural difference between the relevant section of *PH* 3 and our chapter, namely the prominence of the discussion of the mathematicians' conceptions, that creates a considerable imbalance in our text? Roughly, there are two possibilities. We can think that Sextus turned away from the agenda of *PH* 3, perhaps because he thought that the discussion of the geometrical notions might pull the rug from under all conceptions of body, introduced the mathematicians' conception of body, and then got carried away – sometimes quite far away – and included considerations that have no immediate bearing on the primary issue on his agenda. Point, line and surface, to be sure, were already mentioned in the relevant section of *PH* 3. Yet I find it difficult to believe that all the arguments presented in this long section, including the ones about the geometers theorems, and so on, were developed or collected in view of the proclaimed objective of our chapter. It is a much more economical hypothesis, I believe, that this series of arguments originally formed part of a systematic attack on the geometers and then were integrated in our chapter with some reshuffling and little shortening, with the result that the discussion of body in *Against the Physicists* became so strikingly imbalanced. This hypothesis implies that, whether or not *Against the Geometers* had by that time received its final form, the main bulk of the material presented in it was already available in a fairly organized form when Sextus was composing this part of *Against the Physicists*.⁸⁶

⁸⁶ I am of course aware of the fact that the received view holds that *M* 7–11 is earlier than *M* 1–6. Some earlier scholars, e.g. Zeller and Brochard, however, argued for the sequence *PH*; *M* 1–6; *M* 7–11 (cf. Zeller 1876–1909: vol. III.2, 51, n. 2; Brochard 1923: 318–19). Without undertaking a full examination of this issue now, let me merely mention that the principal argument for taking *M* 7–11 as earlier than *M* 1–6 is that Sextus in the course of *M* 1–6 seems to refer back to *Against the Physicists* twice (see e.g. Floridi 2002: 10). First in *M* 1.35: 'One must bring over the puzzles from those we have already brought forward in our controversy against the physicists concerning change and going through generation and perishing.' Yet, clearly, this may just as well be a reference to the relevant parts of *PH* 3; indeed the distinction between μεταβολή on the one hand and generation and perishing on the other seems to point to *PH* 3 rather than to *M* 10. The other cross-reference comes from the concluding sentence of *Against the Geometers* and refers back to the arguments that have established the impossibility of subtraction in ἐν τῷ πρὸς τοὺς γραμματικούς καὶ ἐν τῷ πρὸς τοὺς φυσικούς ὑπομνήματι: part from whole or from part, equal from equal, less from greater, greater from less. Once again, I find no guarantee that this reference is to *M* 9.297–307 and not to *PH* 3.85–96.

Appendix I: Structural overview of M 9.359–440

- A. The doxography of primary elements (359–64)
- B. Transition to the investigation of body (365)
- C. The capacity to act or to be acted upon (366)
- D. The relation between body and dimensions (367–75)
 - 1. Body is conceptually separate from the three dimensions (368)
 - 2. Body is the complex (*athroisma*) of the three dimensions (369–74)
 - 2.1 Dimensions are incorporeals (369–70)
 - 2.2 Each dimension contains corporeality and the *logos* of body (371–2)
 - 2.3 Body supervenes on the conjunction of the three dimensions (373–5)
 - 2.3.1 After conjunction they retain their incorporeality (373)
 - 2.3.2 After conjunction they become corporeal (374)
- E. The non-existence of dimensions and the conceivability of fundamental geometrical objects (375–433)
 - 1. Introduction: the connection between length and line
 - 2. The inconceivability of point (377–8)
 - 2.1 The point is corporeal (377)
 - 2.2 The point is incorporeal (378)
 - 3. The inconceivability of line (380–429)
 - 3.1 Line is a point which has flowed (380–5)
 - 3.1.1 The point occupies the same places (381)
 - 3.1.2 The point changes its place (382–5)
 - 3.1.2.1 Leaving one place and taking up another (383)
 - 3.1.2.2 Occupying one place and extending to another (384–5)
 - 3.1.2.2.1 Extending to an indivisible place (384)
 - 3.1.2.2.2 Extending to a divisible place (385)
 - 3.2 Line is a row of points (386–8)
 - 3.2.1 With intervening places (386)
 - 3.2.2 The points touch each other (387)
 - 3.2.2.1 They touch parts with parts (387)
 - 3.2.2.2 They touch wholes with wholes (388)

- 3.3 Interim conclusion: in so far as line cannot be conceived in relation to point, body does not exist (389)
- 3.4 Line is breadthless length (390–413)
 - 3.4.1 Length without breadth is not perceptible (391)
 - 3.4.2 Length without breadth is not intelligible (392)
 - 3.4.3 General typology of ways of concept formation (393–5)
 - 3.4.3.1 According to encounter with manifest things (394)
 - 3.4.3.2 According to transformation of manifest things (394–5)
 - 3.4.3.2.1 Resemblance
 - 3.4.3.2.2 Composition
 - 3.4.3.2.3 Analogy: increase and decrease
 - 3.4.4 Application of the above to the conception of line (397–402)
 - 3.4.4.1 According to encounter with manifest things (397)
 - 3.4.4.2 According to transformation of manifest things (398–401)
 - 3.4.4.2.1 Resemblance (398)
 - 3.4.4.2.2 Composition (399)
 - 3.4.4.2.3 Analogy: increase and decrease (400–1)
 - 3.4.5 Interim conclusion: if these are the ways of concept formation, the line is inconceivable (402)
 - 3.4.6 Further ways of concept formation: intensification and abstraction (403–13)
 - 3.4.6.1 Intensification (403–6)
 - 3.4.6.2 Abstraction (407–13)
 - 3.4.6.2.1 Privatives do not exist (407–11)
 - 3.4.6.2.2 The criticism of Aristotle (412–13)
- 3.5 Line is the limit of surface (414–18)
 - 3.5.1 When two lines are juxtaposed, they become one (415–16)
 - 3.5.2 When two lines are juxtaposed, two parallel lines remain (417)
- 4. *Aporiai* about geometrical hypotheses/theorems (419–30)
 - 4.1 Revolving line and the surface of the circle (420–5)
 - 4.1.1 Concentric circles are not continuous (422)
 - 4.1.2 Concentric circles are continuous (423–4)
 - 4.2 The revolving line measures out the surface of the circle (426–7)
 - 4.2.1 The line does not move over all parts of the surface (427)
 - 4.2.2 The line moves over the entire surface (427)
 - 4.2 The line which is the side of the square measures out the square (428)
 - 4.3 The revolving cylinder touching the surface at a line (429)

- 5. The inconceivability of surface (430–6)
 - 5.1 Conceptions of surface (430)
 - 5.2 What happens when the limits of bodies are juxtaposed? (431–3)
 - 5.2.1 Limit touches limit (432)
 - 5.2.2 Limited touches limited (432)
 - 5.2.3 Limited touches limited and limit touches limit (433)
 - 5.3 Surface is:
 - 5.3.1 A body (434)
 - 5.3.2 Incorporeal (435)
 - 5.4 Conclusion: The absurdities following from the conception of surface as limit of the body leads to suspension of judgement
- F. Is body perceptible or intelligible (437–9)
 - 1. Body as such is not perceptible (437)
 - 2. Body is not intelligible (438–9)
- G. Conclusion and transition to the examination of incorporeals (440)

Appendix II: The doxographical section

I. Pseudo-Galen and Sextus

The treatise called *Philosophical History* (Φιλόσοφος ἱστορία) transmitted under the name of Galen is one of the ‘low-end’ products of the doxographical tradition. It draws heavily on the material provided by Ps.-Plutarch but incorporates material from a different tradition as well. According to his introductory paragraph (ch. 2), Ps.-Galen tried to make a collection for the profit of those who are eager to know. The expected advantage is not the realization of the forlornness of the enterprise of the earlier philosophers, and sceptical suspension of judgment, but rather gain in knowledge which has ethical significance as well. Ps.-Galen starts his presentation with a summary of the succession of philosophical schools according to the traditional divisions (ch. 3). Next come sections on the various definitions of philosophy, then a chapter on the different views on the parts of philosophy and their ordering. In organizing his work Ps.-Galen apparently wants to follow what he presents as the majority view: logic–physics–ethics, without, however, ever really reaching ethics. Accordingly, after further brief preparatory sections on the notion of a philosophical school (*hairesis*) and the *archē* of philosophy, comes a set of logical sections (chs. 9–15: on sign, syllogism, definition, the criterion of truth, truth, diairesis, proof), followed by physical topics. The physical section falls into two parts. The first series (chs. 16–24) is a very coarse selection of the most important physical topics: the cosmos, the material principle, motion, body, and soul, whereas the second, much longer series (chs. 25–133) offers a considerably more fine-grained presentation of general physical topics, followed by sections on cosmology, astronomy, psychology and physiology. On a number of occasions items in the two sets overlap, and the author does not make any attempt at coordinating or harmonizing them. For the last longer section the author epitomized Ps.-Plutarch, reducing it to half, occasionally adding some short remarks.

The logical part (chs. 9–15) and ch. 18 on the material principle find close, in some case verbatim, parallels in Sextus; these parallel texts, moreover, are not paralleled in other texts of the Aetian doxographical tradition. The nature of the relationship between the *Philosophical History* and Sextus has been a matter of debate, and, as Mansfeld and Runia have shown,⁸⁷ this debate constitutes a

⁸⁷ Mansfeld & Runia 1997: 60–1 and 141–52.

notable episode in the historiography of the doxographical tradition. Diels devoted his doctoral dissertation (1870) to Ps.-Galen's text and assumed that the relevant parts of the *Philosophical History* are copied from Sextus. This contention was forcefully criticized by Volkmann 1873, who focused on the parallels in the treatment of logical topics and argued that there was no direct relationship between the two authors, but both drew on a common source, which is, however, not Ps.-Plutarch. Diels returned to the question in *Doxographi Graeci* and discussed it in considerable detail (233–58). Somewhat grudgingly, he accepted Volkmann's hypothesis.

2. Comparison of the parallel passages

Ps.-Galen, <i>De Historia Philosophica</i> 18	PH 3.30–2	M 9.360–4
Καὶ περὶ μὲν τῆς δραστηκῆς αἰτίας ἐπὶ τοσοῦτον. διεξελεθῆν δ' ἂν εἴη καιρὸς καὶ περὶ τῆς ὑλικῆς. οἱ φυσικοὶ περὶ ταύτης εἰπόντες εἶναι μὲν ἀρχὴν ὑλικὴν ἅπαντες ὁμοίως δεδώκασιν, οὐ μὴν ἅπαντες εἶναι τὴν αὐτήν.	Περὶ μὲν οὖν τῆς δραστηκῆς τοσαῦτα νῦν ἀρκέσει λελέχθαι συντόμως δὲ καὶ περὶ τῶν ὑλικῶν καλουμένων ἀρχῶν λεκτέον. ὅτι τοῖνυν αὐταὶ εἰσιν ἀκατάληπτοι, ῥᾶδιον συνιδεῖν ἐκ τῆς περὶ αὐτῶν γεγενημένης διαφωνίας παρὰ τοῖς δογματικοῖς.	
ἀλλὰ Φερεκύδης μὲν ὁ Ἀσσύριος γῆν εἶναι ταύτην ἐνόμισε,	Φερεκύδης μὲν γὰρ ὁ Σύριος γῆν εἶπε τὴν πάντων εἶναι ἀρχήν,	Φερεκύδης μὲν ὁ Σύριος (Ἀσσυαρρο– N) γῆν ἔλεξε πάντων εἶναι ἀρχήν καὶ στοιχεῖον,
Θαλῆς δὲ ὕδωρ, Ἀναξίμανδρος δὲ τὸ ἄπειρον, Ἀναξίμενης δὲ καὶ	Θαλῆς δὲ ὁ Μιλήσιος ὕδωρ, Ἀναξίμανδρος δὲ ὁ ἄκουστής τούτου τὸ ἄπειρον, Ἀναξίμενης δὲ καὶ	Θαλῆς δὲ ὁ Μιλήσιος ὕδωρ, Ἀναξίμανδρος δὲ ὁ ἄκουστής τούτου τὸ ἄπειρον, Ἀναξίμενης δὲ καὶ Ἰδαῖος ὁ Ἰμεραῖος καὶ
Διογένης ὁ Ἀπολλωνιάτης ἀέρα,	Διογένης ὁ Ἀπολλωνιάτης ἀέρα,	Διογένης ὁ Ἀπολλωνιάτης καὶ Ἀρχέλαος ὁ Ἀθηναῖος, Σωκράτους δὲ καθηγητής, καὶ κατα; ἐνίου; Ἡράκλειτος ἀέρα,
πῦρ δὲ Ἴππασος ὁ Μεταποντῖν	Ἴππασος δὲ ὁ Μεταποντῖνος πῦρ,	Ἴππασος δὲ ὁ Μεταποντῖνος καὶ κατ' ἐνίου; Ἡράκλειτος πῦρ,
Ξενοφάνης δ' ὁ Κολοφώνιος γῆν καὶ ὕδωρ.	Ξενοφάνης δὲ ὁ Κολοφώνιος γῆν καὶ ὕδωρ,	Ξενοφάνης δὲ ὕδωρ καὶ γῆν (πάντες γὰρ γαίης τε καὶ ὕδατος ἐκγενόμεσθα),

Ἴππων δὲ ὁ Ῥηγίνος πῦρ καὶ ὕδωρ, Οἰνοπίδης δὲ ὁ Χῖος πῦρ καὶ ἀέρα, Ὀνομάκριτος δὲ ἐν τοῖς Ὀρφικοῖς γῆν καὶ πῦρ καὶ ὕδωρ, οἱ Στωικοὶ δὲ τέσσαρα στοιχεῖα γῆν καὶ πῦρ καὶ ὕδωρ καὶ ἀέρα,	Οἰνοπίδης δὲ ὁ Χῖος πῦρ καὶ ἀέρα, Ἴππων δὲ ὁ Ῥηγίνος πῦρ καὶ ὕδωρ, Ὀνομάκριτος δὲ ἐν τοῖς Ὀρφικοῖς πῦρ καὶ ὕδωρ καὶ γῆν, οἱ δὲ περὶ τὸν Ἐμπεδοκλέα καὶ τοὺς Στωικοὺς πῦρ ἀέρα ὕδωρ γῆν—περὶ γὰρ τῆς τερατολογουμένης ἀποίου παρά τισιν ὕλης, ἣν οὐδὲ αὐτοὶ καταλαμβάνειν διαβεβαιοῦνται, τί δεῖ καὶ λέγειν; οἱ δὲ περὶ Ἀριστοτέλην τὸν Περιπατητικὸν πῦρ ἀέρα ὕδωρ γῆν τὸ κυκλοφορητικὸν σῶμα,	Ἴππων δὲ ὁ Ῥηγίνος πῦρ καὶ ὕδωρ, Οἰνοπίδης δὲ ὁ Χῖος πῦρ καὶ ἀέρα, Ὀνομάκριτος δὲ ἐν τοῖς Ὀρφικοῖς πῦρ καὶ ὕδωρ καὶ γῆν, οἱ δὲ περὶ τὸν Ἐμπεδοκλέα καὶ οἱ ἀπὸ τῆς Στοᾶς γῆν καὶ ὕδωρ καὶ ἀέρα καὶ πῦρ (τέσσαρα γὰρ πάντων ριζώματα πρῶτον ἀκούει Ζεὺς ἀργῆς Ἥρη τε φερέσβιος ἡδ' Ἀιδωνεύς Νηστὴς ἢ δακρύοις τέγγει κρούνωμα βρότειον),
Ἀριστοτέλης δὲ τούτοις προσέθηκε καὶ τὸ κυκλοφορητικὸν σῶμα. Ἐμπεδοκλῆς δὲ τέτταρσι στοιχείοις προσήγαγε φιλίαν καὶ νεῖκος, τῶν μὲν τεσσάρων στοιχείων ὑλικῶν ὄντων καὶ τῆς φιλίας ταῦτα συγκρινούσης, τοῦ δὲ νείκους διαλύοντος καὶ διακρίνοντος. Δημόκριτος δὲ καὶ Ἐπίκουρος τὰς ἀτόμους ἀρχὰς πάντων νομίζουσιν, Ἡρακλείδης δὲ ὁ Ποντικὸς καὶ Ἀσκληπιάδης ὁ Βιθυνὸς ἀνάρμους ὄγκους τὰς ἀρχὰς ὑποτίθενται τῶν ὅλων, Ἀναξαγόρας δὲ ὁ Κλαζομένιος τὰς ὁμοιομερείας, Διόδωρος δὲ ὁ Κρόνος ἐπικεκλημένος ἀμερῇ καὶ ἐλάχιστα σώματα,	Δημόκριτος δὲ καὶ Ἐπίκουρος ἀτόμους, Δημόκριτος δὲ καὶ Ἐπίκουρος ἀτόμους, Ἀναξαγόρας δὲ ὁ Κλαζομένιος ὁμοιομερείας, Διόδωρος δὲ ὁ ἐπικληθεὶς Κρόνος ἐλάχιστα καὶ ἀμερῇ σώματα, Ἡρακλείδης δὲ ὁ Ποντικὸς καὶ Ἀσκληπιάδης ὁ Βιθυνὸς ἀνάρμους ὄγκους,	Δημόκριτος δὲ καὶ Ἐπίκουρος ἀτόμους, εἰ μὴ τι ἀρχαιοτέραν ταύτην θετέον τὴν δόξαν καί, ὥς ἔλεγεν ὁ Στωικός Ποσειδώνιος, ἀπὸ Μώχου τινὸς ἀνδρὸς Φοῖνικος καταγομένην, Ἀναξαγόρας δὲ ὁ Κλαζομένιος ὁμοιομερείας, Διόδωρος δὲ ὁ ἐπικληθεὶς Κρόνος ἐλάχιστα καὶ ἀμερῇ σώματα,
Πυθαγόρας δὲ τοὺς ἀριθμούς, οἱ μαθηματικοὶ δὲ τὰ πέρατα τῶν σωμάτων,	οἱ δὲ περὶ Πυθαγόραν τοὺς ἀριθμούς, οἱ δὲ μαθηματικοὶ τὰ πέρατα τῶν σωμάτων, Στράτων δὲ ὁ φυσικὸς τὰς ποιότητας.	Ἀσκληπιάδης δὲ ὁ Βιθυνὸς ἀνάρμους ὄγκους. τῶν δὲ ἀσώματα δογματίζοντων οἱ μὲν περὶ Πυθαγόραν τοὺς ἀριθμούς ἔλεξαν πάντων ἀρχεῖν, οἱ δὲ μαθηματικοὶ τὰ πέρατα τῶν σωμάτων, οἱ δὲ περὶ τὸν Πλάτωνα τὰς ἰδέας.

3. Overview of the inventory

The list observes strict structuring principles. First, the different views are grouped systematically, according to the number of elements. Then, wherever it is applicable, the ordering follows the ordering of elements according to heaviness–lightness: earth, water, air, fire. This principle is observed even in the less conspicuous cases, as for example in the two-elements group.

Corporealists

One element

Pherecydes: earth
 Thales: water
 Anaximander: apeiron
 Anaximenes: air
Idaeus of Himera: air
 Diogenes of Apollonia: air
Archelaus: air
Heracritus (according to some): air
 Hippasus of Metapontum: fire

Two elements

Xenophanes: water and earth
 Hippo of Rhegium: fire and water
 Oenopides of Chios: fire and air

Three elements

Onomacritus in *Orphica*: fire and water and earth

Four elements

Empedocles: earth, water, air, fire
 Stoics: earth, water, air, fire

[[Five elements]]

[[Aristotle]]

Unlimited number of elements

Democritus: atoms
 Epicurus: atoms
Mochus (according to Posidonius): atoms
 Anaxagoras: homeomeres
 Diodorus Cronus: minimal and indivisible bodies
 [[Heraclides: *anarmoi onkoi*]]
 Asclepiades: *anarmoi onkoi*

Incorporealist

Pythagoreans: numbers
 Mathematicians: limits of bodies
 [[Strato: qualities]]
Platonists: ideas

4. Additional remarks on the list in *M* 9

Much of the list is standard and must come directly from Sextus' source. From those present on the lists in Ps.-Galen and *PH* 3, Aristotle, Strato and Heraclides of Pontus are missing from our chapter (I have indicated these

with double square brackets in the list above). For these omissions we are, however, compensated by some curious additions which figure only in *Against the Physicists* (indicated by bold typeface). The mention of Idaeus of Himera in *M* 9.360 appears to be the one and only reference to this person in antiquity. This is not the place to discuss the historiography in detail, but it is fascinating to see the wild speculations that this single mention could lead to. Even though Sextus clearly says that Idaeus' primary element was air – and this is all that we know about him – Zeller and Diels maintained that when Aristotle says that some people think that the *archē* is an intermediate substance between air and fire (*Metaph.* 1.7.988a23; *Ph.* 1.4.187a14; cf. *Cael.* 3.5.303b10), he must be referring to Ideaus. On this basis, Idaeus received an entry in Diels–Kranz, where all the relevant Aristotelian passages, plus Simplicius' comments on those passages – none of them mentioning the name of Idaeus – are listed as testimonia. As a proud possessor of an entry in Diels–Kranz, Idaeus became an official Presocratic, to whom one can also assign a date. Guthrie, for example, dates him to the second half of the fifth century.⁸⁸ This is of course not based on anything, and as far as Sextus' evidence goes, Idaeus could just as well have been a Hellenistic figure. Or he could have been born before the Trojan war as Mochus of Sidon, another curious addition to the list, supposedly was. The context of Posidonius' testimony on Mochus, as well as the reason for including this reference here (missing from both Pseudo-Galen and *PH* 3), is mysterious.⁸⁹

That Heraclitus figures on such a list would not be remarkable in itself. It is noteworthy because Heraclitus is conspicuously missing from both Ps.-Galen and *PH* 3. But Sextus makes up for this omission by mentioning Heraclitus twice here: once making fire his principle and then, with reference to an alternative tradition (which must have reached Sextus through Aenesidemus, cf. *M* 10.233), aligning him with those who took air to be the principle.

⁸⁸ Guthrie 1965: 354. Some people are even more precise. Bernard Pullman, in his *The Atom in the History of Human Thought* (Oxford, 1998) p. 18, puts Idaeus around 450 BCE.

⁸⁹ The testimony is closely paralleled by Strabo 16.2.24: εἰ δὲ δεῖ Ποσειδωνίῳ πιστεῦσαι, καὶ τὸ περὶ τῶν ἀτόμων δόγμα παλαιὸν ἐστὶν ἀνδρὸς Σιδονίου Μώχου πρὸ τῶν Τρωικῶν χρόνων γεγονότος. Strabo's text does not bring us any closer to seeing the basis of the assertion in Posidonius, or to Sextus' reason to include it here. (Strabo has just turned to the description of the achievements of the Phoenicians, and this is the only bit of information he quotes from Posidonius. See Kidd's notes ad loc. 1988: 972–5.) We find a more detailed doxography of Mochus' principles in Damascius, *Pr.* 1.323.6 (Ruelle), which speaks about aither and air, and a series of gods and a cosmic egg. I have no idea how a (proto-)atomist theory can be read into this.