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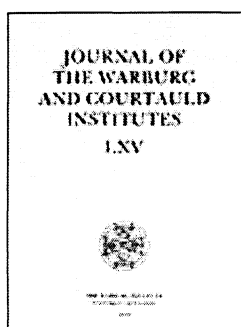


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R. Wittkower and B. A. R. Carter

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THE PERSPECTIVE OF PIERO DELLA FRANCESCA'S 'FLAGELLATION'

By R. Wittkower and B. A. R. Carter

I

By R. Wittkower

The present investigation was originally undertaken in connexion with the paper on the foregoing pages. There, the thesis has been submitted that architecture in Quattrocento paintings contains useful clues to the problem of 'proportion in perspective' in real architecture. If this is true, it appeared pertinent to gather more information from detailed space analyses of Quattrocento paintings. Instead of many, only one can be offered here; and since it developed further than was at first envisaged and in a somewhat different direction, it has been separated from its original context. Little serious work in this respect has been done in the past; and more often than not, manipulating a pair of dividers has tempted scholars to exchange the thorny path of exact analysis for the easy road to amateurish exercises. Instances are too frequent to be quoted, but reference may be made to a case not without interest in connexion with the present inquiry—namely, Winterberg's¹ fantastic diagrammatical 'reconstructions' of Piero della Francesca's paintings.

In looking for a test-case to analyse, one is naturally led to Piero della Francesca and, in particular, to his 'Flagellation' (Pl. 43),² which has always been regarded as a key-piece—in Sir Kenneth Clark's words³—of "the *mystique* of measurement." The reluctance of art historians to undertake such analyses is easy to understand, for not only do they require infinite patience, but also professional knowledge in the technique of perspective. Myself lacking these qualifications, I joined forces for this complicated inquiry with Mr. B. A. R. Carter, who had already independently devoted considerable time to an investigation of the perspective of this work. While the plans and elevation offered on plates 44 and 45 are the result of continuous consultation between us, the credit for them rests entirely with him. The reader must also be referred to his own words for a detailed statement concerning his procedure.

For the purpose of the following analysis⁴ the hall of Piero's 'Flagellation' should be imagined as a three-dimensional model corresponding to the actual size of the hall shown in the picture. We may draw this hall in plan and elevation (Pl. 44), and measure and discuss it in terms of a real building. If it has the properties of a building constructed, one should expect to find a unit of measurement governing its various parts. Such a unit is, in fact, traceable.⁵

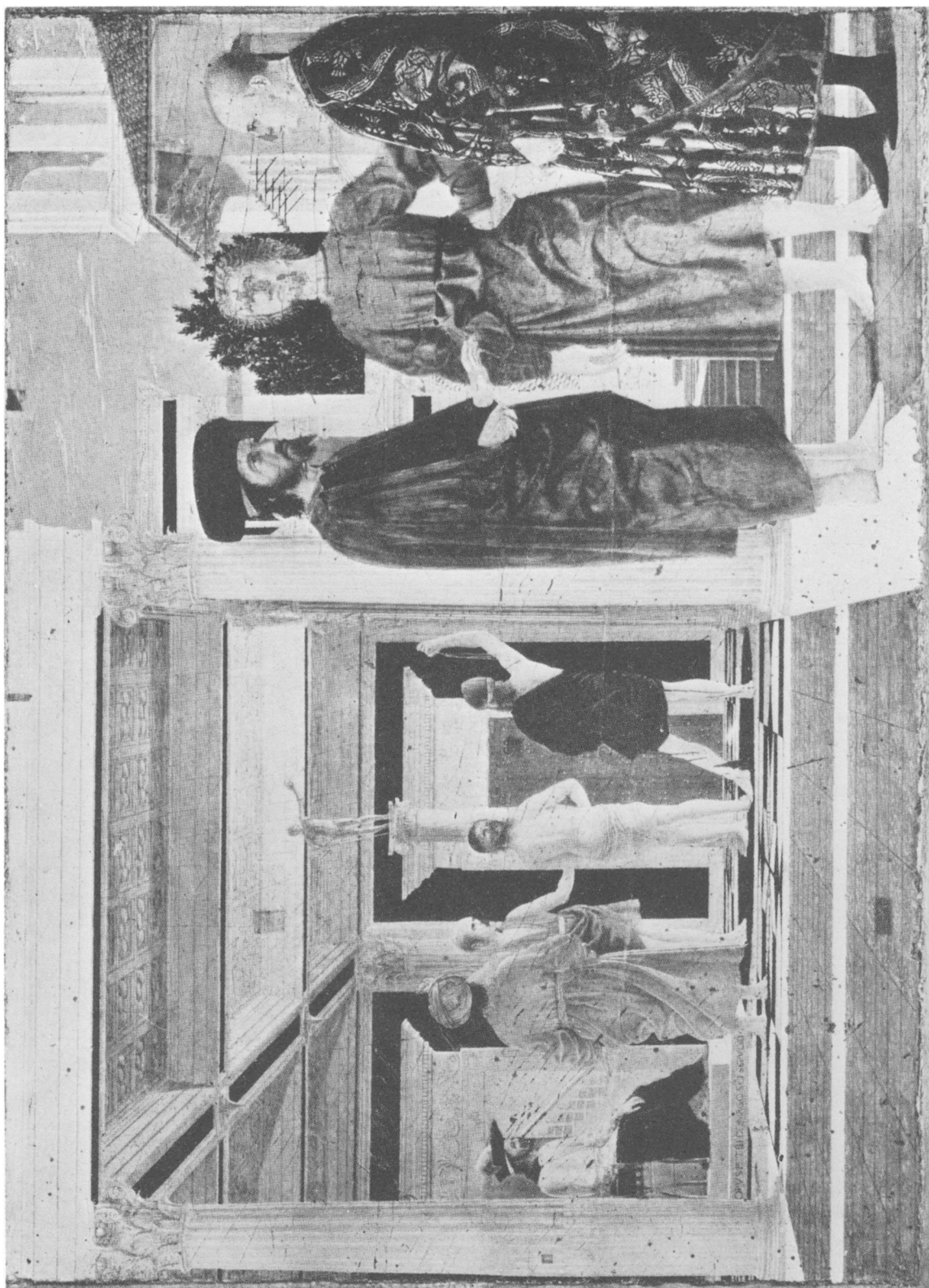
¹ *Petrus Pictor Burgensis De prospectiva pingendi*, Strasbourg, 1899, p. 21 ff.

² Dr. Giuseppe Marchini, Soprintendente delle Belle Arti at Urbino, was kindness itself in giving me every help and facility in studying the picture. Thanks are also due to Professor Cesare Brandi who supplied photographs taken after the cleaning of the picture.

³ *Piero della Francesca*, London, 1951, p. 20.

⁴ No attempt is made here to do justice to the picture as a whole. We are only concerned with the important but limited problem of the rationalization and harmonization of space and figures.

⁵ The unit of measurement referred to by Sir K. Clark (p. 20) appears to be *ca.* half the height of the eye.



By courtesy of the Istituto Centrale del Restauro, Rome

Piero della Francesca, 'Flagellation,' Palazzo Ducale, Urbino (p. 292)

There is good reason to believe that Piero worked with the unit later published in Pacioli's *De Divina Proportione*.¹ But since we cannot be certain, the following calculations have been made in inches. It appears that the basic unit informing the space development of the architecture is 1.85"; we may call this the module. Each little square subdividing the large dark squares of the pavement has sides of two modules (3.7") and the white divisions between the large squares are 3 modules wide (5.55"). The large squares of the pavement, corresponding to the squares of the ceiling, have sides of 16 modules. But the first large square nearest to the picture plane is not complete: it shows six and a half instead of eight small squares in depth. One and a half squares would lie in front of the picture plane and, as the ground plan shows, must have played a part in Piero's deliberations.²

For the organization of depth 19 modules are of particular importance. This is the grand unit from centre of column to centre of column. It was also used for the placing of the figures in space. From the group of figures close to the picture plane³ to the turbaned figure seen from the back is 38 modules, i.e. 2 grand units of 19, and from that figure to the figure of Christ is again 19 modules. The salient points of the picture, namely the foreground figures, the first column, the column of Christ and the back-wall are equally distant, namely $28\frac{1}{2}$ modules; i.e., 19 plus half of 19.⁴ So architecture and figures are integrated into one system of spatial relationships. Like a canon or fugue, the same 'theme' binds the architectural parts, architecture and figures, and the figures themselves together.

However, the distance from the eye to the picture plane is not simply co-ordinated with the larger units inside the picture. It is approximately $31\frac{1}{2}$ modules, and the only obviously intentional relationship to it is that of the column of Christ, which is placed twice that distance—63 modules—behind the picture plane. It is also worth mentioning that the distance from the eye to the foreground figures corresponds almost precisely to that from the latter to the turbaned figure seen from the back ($37\frac{1}{2}$ and 38 modules). Since the distances from the eye to the picture plane ($31\frac{1}{2}$), from there to the first column ($34\frac{1}{2}$) and on to the following columns (19) are not simple multiples of the module, it follows that the diminishing sizes of the columns only roughly correspond to the diminishing arithmetical progression discussed in Piero's treatise.⁵ The same applies to other elements of the composition and the figures. It must be borne in mind that more than 25 years lay between this picture and the conception of the treatise, and no close relationships can be expected. But it appears that Piero's later contention that the ratio of diminishing progression is difficult to express in figures is supported by his own practice.

¹ Cf. below, p. 298.

² In the plan, Pl. 44, only one row of small squares (and not one and a half) are shown to lie in front of the intersection, i.e. the picture plane. The reason for this discrepancy between picture and reconstruction will be given by B. A. R. Carter.

³ To be precise, from the figure on the

right-hand border, which is the one nearest to the spectator.

⁴ The grand unit of 19 modules is also significant in the elevation, for it is repeated in Christ's column with the figure above it.

⁵ See above, p. 283 f.

It may be asked whether one is justified in discussing space relationships in such a picture in terms of modules, as if one were talking of real architecture; and it may be argued that the whole picture can be constructed on the surface by normal perspective procedure. On the other hand, it cannot be doubted that Piero mapped out the whole composition in plan, similar or even corresponding to our Plate 44, as if it were real architecture. This is supported not only by the stringent logic of the development in depth,¹ but also by Piero's own method, as demonstrated in the later part of his treatise, of working out perspective from plan and elevation. This method must have been much more common than is generally realized.² A reflection of it is still to be found in Vasari's life of Baccio d'Agnolo,³ where he maintains that painters, in order to succeed with their perspective of architecture, have to draw it in plan. Moreover, further support for Piero's 'architectural' planning of the composition is supplied by the complex pattern of the marble floor in front of and behind the compartment in which Christ has been placed. It is at once obvious that this pattern must first have been designed in plan; and its genesis doubly confirms this.

The pattern, one of the most elaborate to be found in any Renaissance picture, is not developed from the module. It can be taken for granted that for a man of Piero's mathematical turn of mind this is not just a decorative design found by trial and error, but that it was the result of a distinct process of mathematical reasoning. Now a relation exists between the compartment in which Christ stands and the two framing ones, if only in the sense that those with the geometrical design serve to enhance the importance of Christ's position. The column of Christ is placed in the centre of a circle, which has, of course, particular significance;⁴ it must be understood as a symbolic reference to Christ and, even more than that, as a symbol of Christ.⁵ It would therefore appear logical to look for a relationship between this circle and the "misterioso gioco pavimentale" (Longhi) in front of and behind it.

Wherein would such a relationship consist? One is tempted to look in one particular direction. Amongst the great problems of classical geometry, that of the quadrature of the circle—that is, the determination of the area of the circle by way of calculating its circumference (i.e. the calculation of π)—holds a place of special importance. For its solution, the inscribing of regular polygons into the circle was a method already well tried by Greek geometers.⁶ New impetus was given to the study of this problem after Jacopo da Cremona's Latin translation of Archimedes for Nicholas V.⁷ Nicholas of Cusa devoted years of work to it,⁸ and possibly under his influence Alberti turned his

¹ E.g., fragments of the pattern of the pavement appear in the line of vision, left and right of the left-hand column.

² G. J. Kern demonstrated it for the construction of a circle in Botticelli's Berlin 'Virgin with the Seven Angels'; see *Jahrbuch der Preuss. Kunstslg.*, 1905, p. 137 ff. See also the same author's "Der Mazzocchio des Paolo Uccello," *ibid.*, 1915, p. 13 ff.

³ Vasari-Milanesi, *Vite*, V, p. 349.

⁴ R. Longhi, *Piero della Francesca*, 1942,

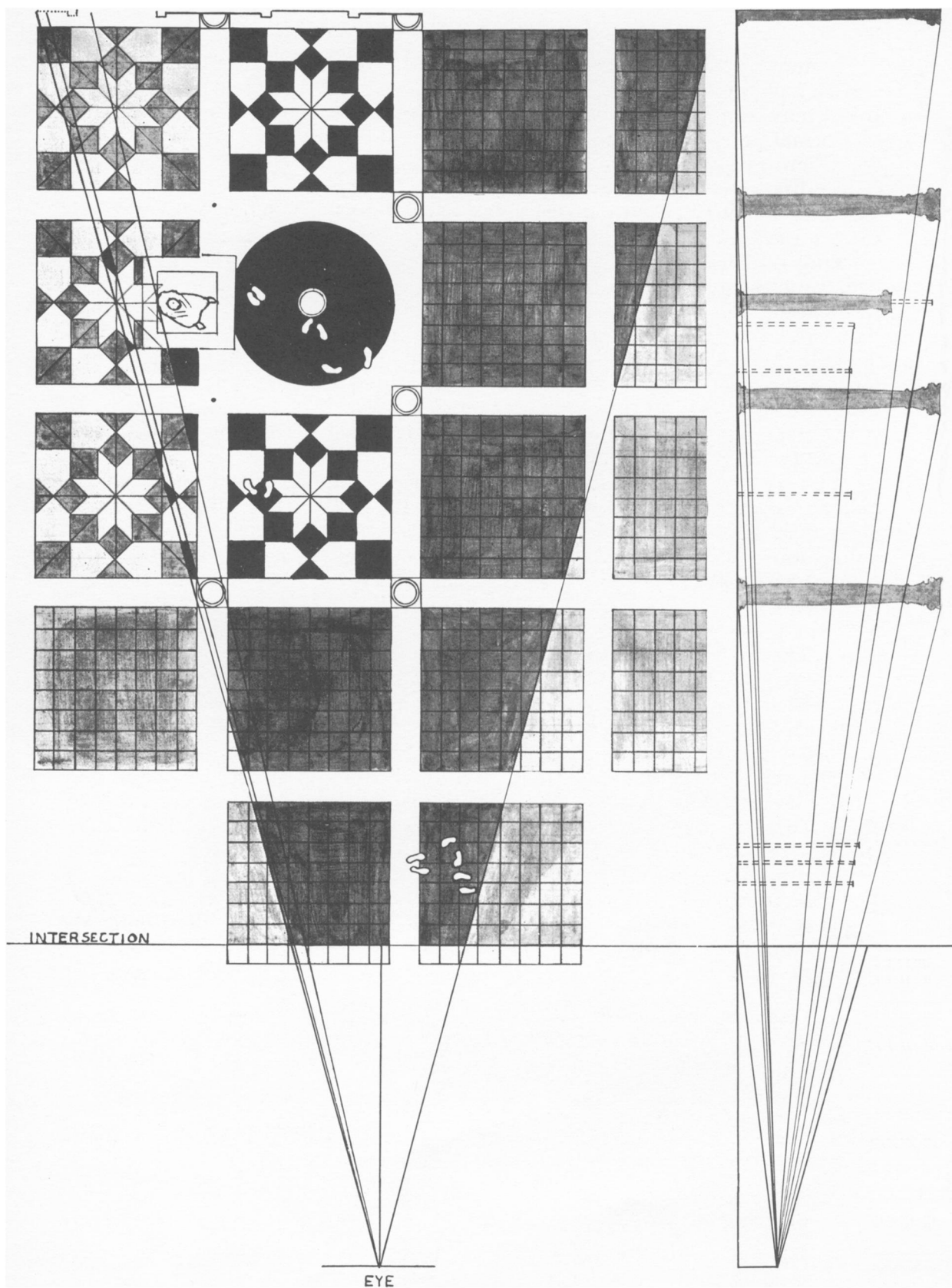
p. 41, talks of the "ellisse nero che sta al centro del misterioso gioco pavimentale." Objectively, "ellisse" is not a correct statement.

⁵ R. Wittkower, *Architectural Principles in the Age of Humanism*, 1952, p. 24 ff.

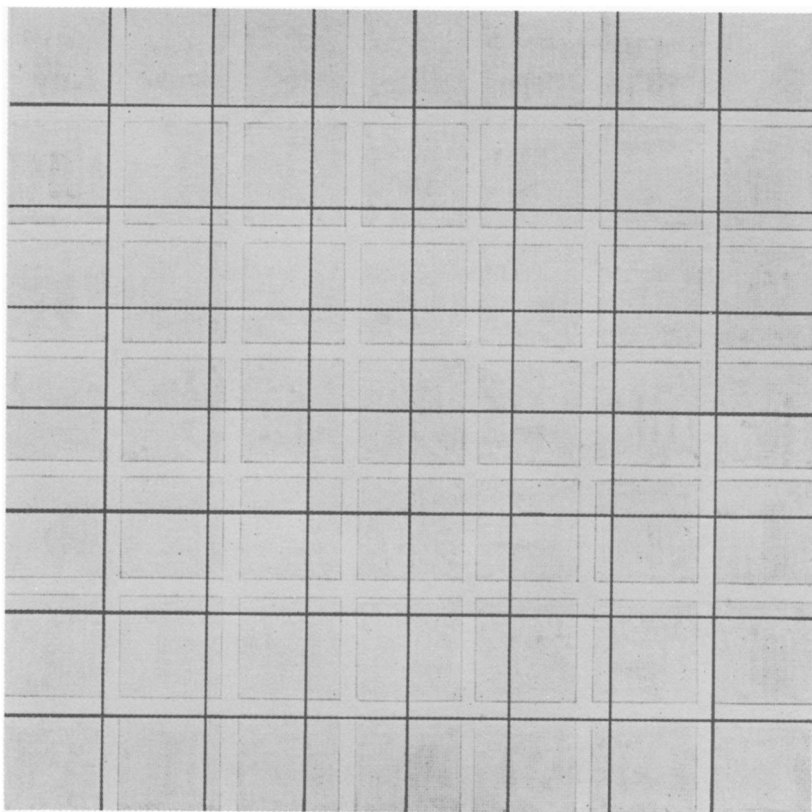
⁶ David Eugene Smith, *History of Mathematics*, 1925, II, p. 303.

⁷ Cantor, *Vorlesungen über die Geschichte der Mathematik*, 1913, II, p. 193.

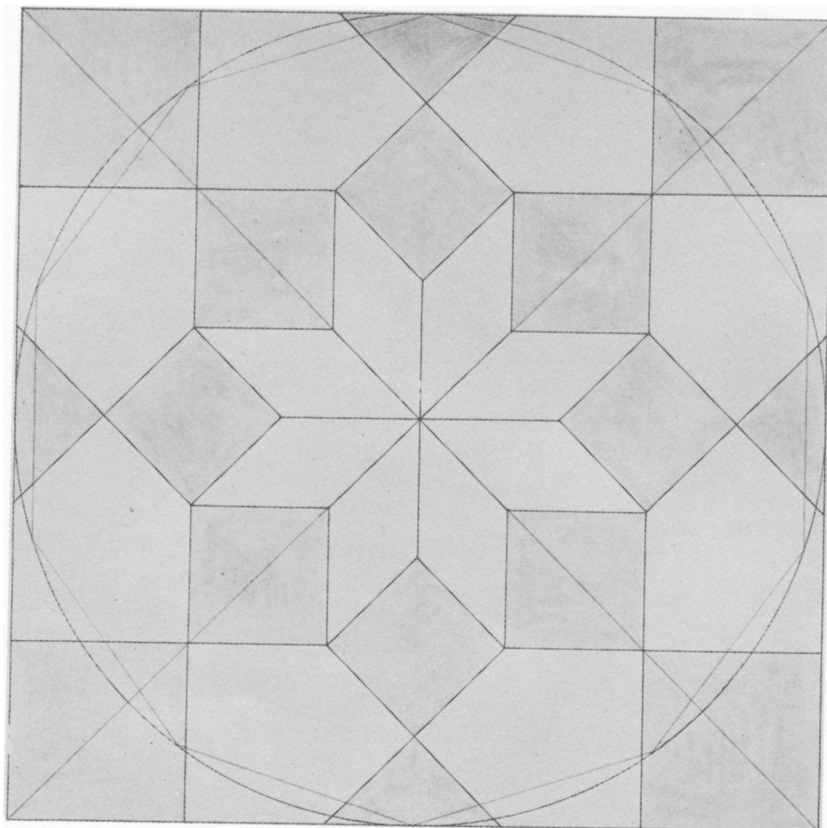
⁸ *Ibid.*, p. 194 ff.



Reconstruction of Plan and Elevation of Piero della Francesca's 'Flagellation,' Palazzo Ducale, Urbino (pp. 292, 293, 294)



a—Pattern of Floor and Ceiling Squares, Superimposed; Piero della Francesca's 'Flagellation,' Palazzo Ducale, Urbino (*pp.* 292, 298)



b—Diagram showing Construction of Floor Pattern in Piero della Francesca's 'Flagellation,' Palazzo Ducale, Urbino (*pp.* 292, 301)

attention to it.¹ But none was more closely identified with these investigations than Piero della Francesca who, late in life, wrote his *Libellus de quinque corporibus regularibus*,² which was incorporated in Italian translation into Pacioli's *Divina Proportione*. It was therefore a reasonable line of approach to inquire whether the floor patterns derived from a polygonal division of the circle. Mr. Carter found that essential points in the construction of the pattern were won by using as a unit the length of a side of the decagon inscribed into the circle of Christ, and that this unit supplied altogether the means of evolving the pattern. The patterned squares contain therefore suggestions of polygon and circle, and reconcile geometrically Christ's circle with the module which determines the over-all size of the squares—indeed a “misterioso gioco pavimentale.”

Once it has been recognized that squaring the circle played an important part in the geometry of the picture, one may even go a step further. A close approximation to the true value of π was known to Piero. Multiplications of the module 1.85" with π coincide with the distances of the eye from the picture plane, with the foreground figures, the first column, the turbaned figure, the near executioner and the column of Christ. This seems to be more than chance: Piero may have chosen this curious relation between the module scale and the ‘mystic’ scale to symbolize the interweaving of this-worldly space with that belonging to the Kingdom of Christ.

II

By B. A. R. Carter

It has been shown above that at some early stage in the evolution of the ‘Flagellation’ Piero must have drawn out a ground plan and, it may be added, probably also a side elevation of the scene, which he then put into perspective. Given the three essential data for making a perspective analysis, one can re-discover his original plan and elevation. These data are:

- (1) The centre of vision (through which the horizon line is drawn).
- (2) The distance of the artist's eye measured perpendicularly from the picture plane.
- (3) The line of intersection of the plane of the floor with the picture plane (the so-called ground line), which is placed by measuring the height of the eye down from the centre of vision.

The first two can be deduced from clues in the picture. The centre of vision is to be found by producing lines in the picture, which represent lines perpendicular to the picture plane, to their common point of convergence, situated somewhat below the centre of the painting.

The distance of the artist's eye from the centre of vision can only be measured if the representation of a horizontal square having one side parallel to the ground line be found in the picture. The diagonals of such a representa-

¹ Mancini, *Leonis Baptistae Alberti Opera inedita*, Florence, 1890, p. 305 ff. Leonardo

Olschki, *Geschichte der neu-sprachlichen wissenschaftlichen Literatur*, 1919, I, p. 81 ff.

² A critical assessment of Piero's performance in Olschki, *op. cit.*, p. 216 ff.

tion, if produced to meet the horizon line to the right and to the left of the centre of vision, provide two points on the horizon line which are the same distance as the eye from the centre of vision. These are usually called distance points, and are used in conjunction with the ground line for measuring distances behind the picture plane. Since it is most probable that the dark elliptical shape in which Christ stands represents a circle, the circumscribed rectangle may be taken for a true square, the diagonals of which, if produced to meet the horizon line, will provide the distance points and the knowledge of the distance of the artist's eye from the picture plane.¹ Due to lack of exact horizontals or verticals in the painting² some difficulty was at first experienced in drawing a true horizontal through the centre of vision for the horizon line, and consequently in determining the correct position of the distance points. They were at first computed to be at distances varying approximately between 59" and 56" from the centre of vision. It was only in the gradual process of completing the plan and elevation, and by checking the construction by the results, that the optimum distance of 58.2" was achieved. Only this length gave entirely consistent results. Once the distance points are placed, the whole floor and ceiling can be found to be laid out in squares (Figs. 1 and 4),³ since all the diagonals produced meet in these two points.

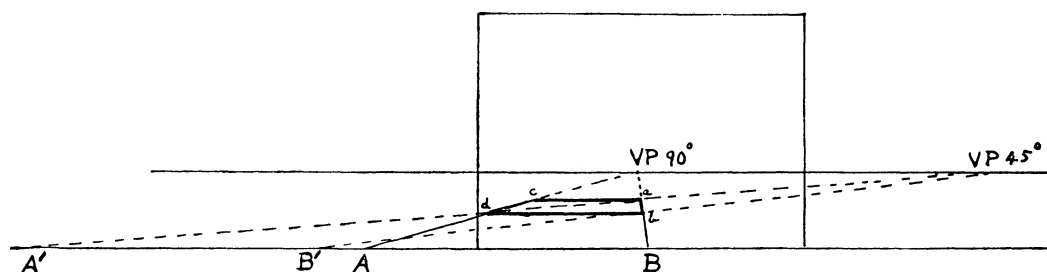


Fig. 1

The third essential datum, the position of the ground line, given by the height of the eye above the floor, is much more difficult to discover; yet only at this line may direct contact be made with the floor in the picture and objective measurements be taken. At the ground line the real floor passes from the tangible world into the illusory and inaccessible space of the picture surface. However, by using an arbitrary ground line drawn at a convenient level, one can reconstruct the layout of the floor. Its dimensions will only lack absolute scale. Note 4 and the diagram Fig. 2 will help in grasping this point.⁴

¹ Although it is improbable that Piero actually used distance points, they are implicit in any rigorous perspective drawing. The perspective diagrams of *De Prospectiva Pingendi* contain only one indication of a distance point construction, and that a doubtful one, to which no reference is made in the text (Ed. 1942, fig. 23).

² See p. 298, note 1.

³ Figure 1 shows how the length and breadth of the square abcd (e.g. the second

large square of the picture) are measured and found to be equal at the ground line ($A'B' = AB$). The receding parallels of the square (ab and cd) converge to VP 90° (the centre of vision). The diagonal ad and its parallel going through b converge in VP 45° (a distance point).

⁴ The image on the picture plane (shaded in, see Fig. 2), may represent l, m, n, o, or r, s, t, u, or any other figure that intersects the visual pyramid, either in front of, or be-

The floor in the picture is assumed here to be continuous with the floor on which the artist is supposed to have stood.¹ It intersects the picture plane at the ground line. But as a picture can provide no direct clue about this intersection, the decision to draw the ground line in one position rather than another must rest upon conjecture, the validity of which will depend upon the consistency and the probability of the resulting conclusions. The small rectangles seen in the foreground of the picture measure approximately 3.6" in width at a ground line drawn along the bottom edge of the painting; but if they are measured at a line drawn above or below this edge their apparent width is diminished or increased accordingly. Hence it is clear that their objective width (i.e. the actual width that the image *represents*), cannot be found without knowledge of the height of the eye. In the process of trying to reach a decision in this equivocal problem, a ground line was at first

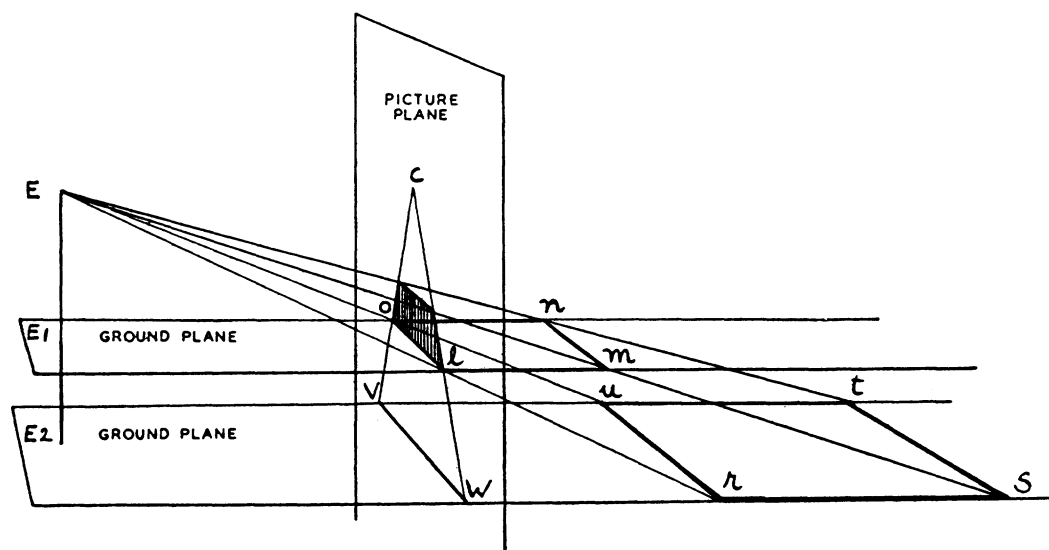


Fig. 2

drawn in what appeared to be the most obvious position, i.e. along the bottom edge of the painting; but after further experiments, a ground line drawn slightly below this edge was found to give results more consistent with a systematic spatial construction. The processes of deduction leading to this decision will now be described. Although the actual height of the eye cannot be found, yet the *image* of the height of the eye can be measured on the picture plane (Fig. 3). The central visual ray may be imagined to pass from the eye of the artist, through the centre of vision marked as a point on the picture plane, and thence to continue on into the picture space, where the first object it meets is the wall seen behind the nearer executioner. The distance from the point where the ray meets the wall (this point is covered by the centre of vision), to the floor beneath, as measured on the picture, is the image of the

hind, the picture plane. The height of the eye may be respectively as E, E1, or E, E2, etc., and the ground line may be at o l or at v w, etc., etc.

¹ According to the scale of the picture (see end of note 1, p. 300), the artist would have made his drawing seated on the floor, the height of his eyes being 23.4" above the ground.

of the plane containing the ceiling with the picture plane has been represented in the diagram Fig. 4.

The ground line drawn at the edge of the completed row of fragmentary small squares makes the height of the artist's eye $7.15''$. This number is not derived from the module, but it may be significant that the height of Christ measured on the picture is approximately $7.15''$, and that this is also a unit of measurement which appears to have played an important part in the surface organization of the whole rectangle.¹

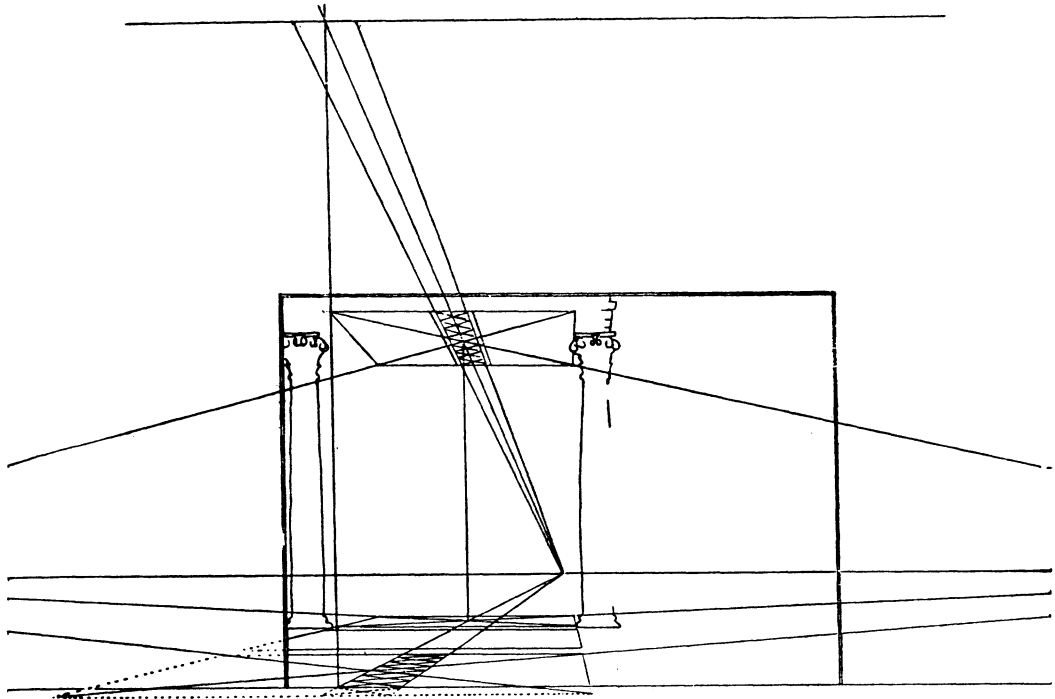


Fig. 4

¹ (See also page 301, note 2.) The rectangle in which the picture is composed can be resolved into 12 squares of $7.15''$, and these fill the centre space, leaving an equal margin at the top and at the sides. Measuring upwards from the horizon line, the top of the black wall comes at $7.15''$, and again from here to the highest black inlay is $7.15''$. Proceeding horizontally from the centre of vision towards the left, $2 \times 7.15''$ brings one approximately to the inside edge of the nearest left-hand column.

At a first approach there appeared to be indications that the picture might have been at some stage designed for a rectangle having the proportions of the side of a square to its diagonal, though the actual rectangle deviated by about $\frac{1}{4}$ inch from these proportions. Sir K. Clark gives 815×590 mm., i.e.

$32.1'' \times 23.23''$ as the size of the picture; our measurements, made on the painting after recent cleaning, were 815×584 mm., i.e. $32.1'' \times 23''$. A hypothetical reconstruction of the rectangle is given below which agrees more closely with the actual measurements.

If 2 squares of $21.45''$ are drawn (composed of 3×3 smaller squares of $7.15''$), and they overlap by two-thirds of their widths, a rectangle of $28.6'' \times 21.45''$ is formed. A diagonal of one of the squares of $21.45''$, measuring $30.33''$, exceeds the width of this rectangle by $1.73''$. Now $1.73''$ may be taken as the width of a border to be added to the top and to the sides of the original rectangle, increasing its dimensions to $32.06'' \times 23.18''$. This is very close to the given size of the painting. When the rectangle is constructed geometrically, it is found that the two diagonals drawn from

With these basic measurements a ground plan was now drawn to the scale of one-tenth, and the positions of the figures represented in the painting were marked in.¹ A half-scale side elevation of approximately $9\frac{1}{2}'$ in length was also drawn in order to test further the consistency of the reconstruction. This was particularly valuable in making a final adjustment to the distance of the eye from the picture plane. The objective heights of the figures and columns were marked in at their appropriate distances, and the correspondence between the objective heights and the heights measured on the picture plane was tested by stretching a thread from the head and foot of each figure and column to the point representing the eye in elevation. The height between the threads at the intersection corresponded very exactly in each case with the height measured on the picture.²

The geometrical pattern before and behind the figure of Christ consists of squares and half squares. This can be proved by testing the diagonals with a thread stretched from a distance point. The approximate dimensions of the component parts of the pattern are easily measured on the picture. It has been mentioned in Part I that the pattern does not show an obvious link with the 1.85" scale as the ceiling did, and that the connexion is to be sought via the circle in which Christ stands. In trying to develop the geometry of the pattern from a regular figure inscribed in the circle,³ it appeared that the

the outside top corners of the overlapping squares, intersect in the centre of vision and touch the head of Christ below their point of intersection. that the diagonals of the left-hand square

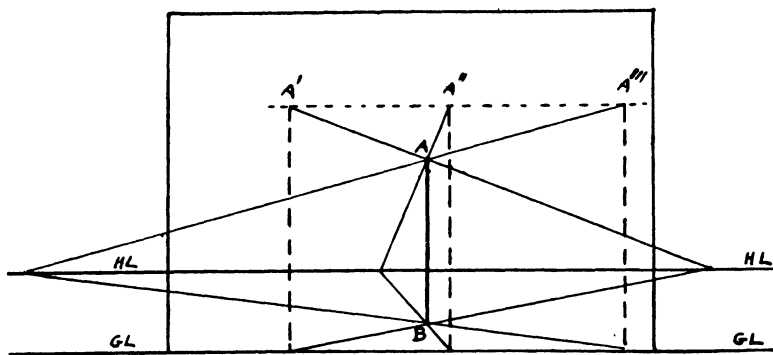


Fig. 5

¹ The correspondence between the plan and the picture may be checked by stretching a thread from the "eye" in plan, e.g. past the inside of the heel of the nearer executioner's right foot to the corner of the distant dark square of the floor pattern which is partly screened by the leg.

The approximate scale of the picture to life size can be calculated. For example, the height of the central figure of the group in the foreground is 22" at the intersection. If the real height of this figure is estimated at 6', 22" may be divided into 72", to give the scale of the picture to life size as 1 to 3.273.

² In doing this, care was taken to select points, as far as possible, in *spatially* vertical relationship. The method of measuring heights within the picture space is represented in the diagram (Fig. 5). The diagram shows how a height AB in the picture space may be transferred to the picture plane by means of any pair of parallels which contain it. Its correct objective height is revealed equally at A', A'' and A''', etc., etc.

³ A relationship may also be found to exist between the visual pyramid and the regular figures. For the visual pyramid contains an angle of exactly 30° in plan, which links it

decagon gave the governing points. For if a circle is inscribed in a square of 29.6", a side of the inscribed decagon¹ provides a diagonal of a dark corner square (Pl. 45b), a side of which in turn provides a diagonal of the $\frac{1}{2}$ square at the centre of each side of the pattern. The intervening space is filled by the second square along the diagonal.²

The distance of the eye and the positions of the figures relative to the picture plane may conceivably have been measured with a second scale derived from the basic module of 1.85" via the circle. This scale may be made by multiplying the multiples of 1.85" by π (π may be valued here as $3\frac{1}{7}$, 3.14286), giving a new unit of measure of 5.8143". It is seen that the distance of the eye arrived at empirically exceeds 10 of these units by as little as 0.057". If, however, Piero had used a value for π of $3\frac{2}{3}$ (3.15), the distance of the eye from the picture plane might be reckoned as $10 \times 1.85" \times 3.15$, i.e. 58.275". Twice this figure would give exactly the distance measured on the ground plan of the centre of Christ's circle to the picture plane, i.e. 116.55". Furthermore, an appropriate emphasis would be given to these two key points of the construction by bringing them into conjunction with both scales. For 116.55 is 63×1.85 and is also 20×5.8275 . The distances of the figures from the picture plane coincide with multiples of the π unit.³

It is possible that Piero may have used a value of π as $3\frac{2}{3}$, for in the fifteenth century, following the translation of Archimedes into Latin, the value of π was not constant. Cantor points out⁴ that the whole of the Middle Ages regarded $3\frac{1}{7}$ as being the final and accurate value of π ; but when, in the first half of the fifteenth century, the Archimedean upper and lower limits of π were made known ($3\frac{1}{7}$ and $3\frac{10}{71}$), mathematicians saw that the problem of measuring the circle offered scope for further research. Pacioli, in his *Summa de Arithmetica Geometria Proportione e Proportionalita*, 1497, writes of the great difficulty of the "quadratura," which, "finora per nulla sia trovata se non quanto per Archimede," and he gives the Archimedean $3\frac{1}{7}$, but explains that

with the equilateral triangle and with the regular hexagon, while its angle of elevation being exactly $22\frac{1}{2}^\circ$ relates it to the regular octagon.

¹ The side of the decagon is related to the radius of the escribed circle as 1 to 1.618, namely in the ratio of the *Divina Proportione*.

² The diagonal of this second square, 7.212", is within 0.06" of the estimated height of the eye. If multiplied by 1.618 it makes 11.669", i.e. one-fifth of 58.275", the revised distance of the eye, to within 0.014".

Certain intersections of the diagonals of the decagon with those of the main square may appear to give certain governing points of the pattern, but a close examination reveals these coincidences to be only approximate.

³ The centre of the group of three figures in the foreground is at approximately $2\frac{1}{2} \pi$ modules from the picture plane (14.5"). The figure standing to the left of the centre of the nearer floor pattern is at 14π modules (81.4").

The nearer executioner is at 18π modules (104.6"). The figure of Christ and the other figures at approximately the same distance from the picture plane as the centre of the circle may be taken as being at 20π modules (116.55").

⁴ *Op. cit.*, II, p. 192: "Albert von Sachsen . . . und mit ihm das ganze Mittelalter hielten $\pi = 3\frac{1}{7}$ nicht etwa für einen Näherungswert, sondern für genau richtig. Von dieser Meinung zurückzukommen war schon ein Fortschritt, und Cusanus machte denselben. Erleichtert war er ihm allerdings durch den Umstand, dass, . . . gerade damals eine Übersetzung des Archimedes in Lateinischer Sprache verfasst und Cusanus in die Hände gegeben worden war. So musste er die beiden Grenzen $3\frac{1}{7}$ und $3\frac{10}{71}$ kennen lernen, zwischen denen π sich befindet, so musste er zugleich die genaue Bestimmung von π als eine noch nicht gelöste Aufgabe erkennen."

any solution reached by it, e.g. in calculating a circumference is not “pontalmente la verita: ma e molto presso.”¹

But more relevant here are the researches of Nicholas of Cusa who dedicated the first of his treatises (1450) on the quadrature of the circle to Paolo Toscanelli. Although in one treatise he came nearer by 0.00052 than $3\frac{1}{7}$ to the true value of π ,² subsequently, by using a different method, he reached a solution of π as $3\frac{8}{25}$ approximately.³ Piero may well have been acquainted with these new studies⁴ through Toscanelli, since it was partly through his teachings that he learnt mathematics. Or he may even have known Nicholas of Cusa personally. In any case Pacioli in his dedicatory epistle to *De Divina Proportione* mentions having met, amongst other famous scholars at the court of Milan, the “molto in tutte premesse admirato e venerato Nicolo cusano.”

From the conclusions reached in the foregoing analysis, although perhaps incomplete, and in one important instance based upon an unverifiable assumption, it appears that Piero used perspective in this picture for portraying his three-dimensional design with mathematical accuracy, and furthermore, that this design is infused with mathematical symbolism.

It may well be that Piero revealed the exact nature of these *arcana* to certain of his friends. For the uninitiated, however, the clear spatial order of the ‘Flagellation’ must always have held an inexplicable element of strangeness.⁵

¹ *Tractatus Geometrie*, fol. 30^r-32^r.

² On this Cantor remarks, “Die Mangelhaftigkeit der Schlüsse ist so augenscheinlich, dass es verwundern muss, wie wenig mangelhaft das Ergebniss ausfällt.” *Ibid.*, p. 194.

³ *Ibid.*, p. 197.

⁴ Montucla (*Histoire des recherches sur la quadrature du cercle*, 1754) refers very slightly to Cusanus’ attempts to square the circle. He writes: “Regiomontanus . . . mérite il est vrai des éloges pour le soin qu’il prit de combattre les prétendues quadratures du cercle du Cardinal Cusa, homme célèbre de son

temps et qui en aurait imposé, si l’on peut en imposer, aux géomètres” (ed. 1831, p. 57); and again (p. 202): “. . . le fameux cardinal de Cusa . . . prétendait avoir réussi à quarrer le cercle par deux voies différentes.”

⁵ I am indebted to Mr. W. T. Monnington who made the first steps of this analysis possible by putting at my disposal a full-scale photograph of the ‘Flagellation,’ which I was later able to check against the original, and also to Mr. R. Nuttall-Smith for the many clarifying discussions on some of the abstruse perspective problems encountered.