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ON THEORY AND PRACTICE: PYTHAGORAS, EUCLID AND ARCHIMEDES AND THEIR INFLUENCE ON NAVIGATION

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This paper provides an overview of some significant theoretical and practical contributions of three historical figures to modern-day navigation. As we trace our move from navigating using stars in the sky to artificial satellites deployed by man thousands of kilometres above the Earth's surface (i.e. Global Navigation Satellite Systems, or GNSS), it is evident that the main principles of navigation remain unchanged. Thus, it is relevant to study the key contributions of historical figures—the three fathers of geometry: Pythagoras, Euclid and Archimedes—who influenced the navigational principles that are embedded in science, mathematics and thought. These principles form the backbone of even the most sophisticated satellite-based navigation systems.

1. Introduction

In the year of the 50th anniversary of the successful launch of the first artificial satellite (*Sputnik*), one can take a step back and reflect on the circumstances that led to today's modern satellite-based technologies. Although modern society is somewhat shielded from its beginnings, satellite positioning and navigation play a major role in the advancement of discovering and mapping our planet. Several of the greatest discoveries in a wide variety of disciplines, from astronomy to philosophy, were made during ancient times or have their roots in antiquity.

Many achievements of ancient intellectuals significantly affected the lives of people in the following centuries, forming a framework for various sectors in academia, research and industry.

Our account touches on some key points in history, to demonstrate the role of the so-called "three fathers of geometry" from antiquity—Pythagoras, Euclid and Archimedes—with regard to determining three-dimensional positions and current navigation systems. The fundamental beginnings of terrestrial survey instruments and satellite geodesy are also investigated relative to the contributions of the three mathematicians.

The relationship of man with the ocean is a relationship with life. It has been documented since many centuries BC in an attempt to discover new places, explore and conquer new lands, and pursue means of survival. From the relationship between man and the oceans came the art of marine navigation, based on empirical observations and measurements. The act of navigating/piloting by the first explorers of the closed seas and open oceans was the root of all empirical experience and primary data source; the knowledge gained became the basis of academic disciplines such as geodesy, hydrography and cartography. At first, man tried and mastered the oceans to provide food, through fishing. Later, through navigation, it became a way to systematically expand our knowledge and map the Earth. In this pursuit, man encountered various problems whose solutions can be attributed to the intellectual ideas of Pythagoras, Euclid and Archimedes. Their work laid the foundation for amazing discoveries and the scientific pursuits of modern man.

2. The Fathers of Geometry

2.1 Pythagoras (580-500 BC)

Pythagoras was born on the island of Samos. Although his work has not been recovered, his contributions in mathematics and geometry, physics, astronomy, engineering, music—and, in particular, philosophy—are considered significant. He was the first to link mathematics with physics and is considered the father of mathematical proofs. He championed the beginning of "Everything could be predicted and measured in rhythmic patterns and cycles." This thought was later taken up by scientists like Newton and Einstein. He is considered the father of numbers and coined the phrase "All things are numbers and are expressed as numbers."

The well-known Pythagorean theorem uses fundamental definitions and meanings of topology, such as 'metrics' or a 'distance measurement' between points. If an orthogonal system is applied in practice through Euclidean geometry, we have a 'metric' that arises from Pythagorean theorem. The fundamental application of Pythagorean thought can be found in the application of satellite location positioning, while the Pythagorean beginnings of stereometry are manifested in positioning and navigating in three-dimensional space.

Pythagoras was the first intellectual to give a mathematical and harmonious dimension to the universe

and to establish a connection between astronomy, mathematics and music. He discovered arithmetic relationships between the planets, the Moon and the Sun, and brought forth the idea that the heavenly spheres of the planet produce a harmony, namely "the music of the spheres". To illustrate, he brought forth the concept that the Moon, the celestial body nearest to the Earth, represented the smallest period (corresponding to the shortest string of a lyre); while the furthest planet, Saturn, represented the largest period (corresponding to the longest string of a lyre); and so forth. He believed that the Earth rotates and generated the idea of perfect solids, claiming that the Earth was in the shape of a sphere.

He supported the idea that the Moon's orbit around the Earth was inclined towards the Earth's equator, and realized that the morning star, Venus, and the evening star were the same planet, bringing forth the fourth dimension in navigation—time. He empirically derived the relationship between harmonious musical spaces with integer numbers and derived relative distances between satellite planets and the Earth. In modern navigation, the basis of this aspect of Pythagorean thought can be found in mathematical derivations of the Doppler effect. His fundamental observations of the harmonic properties in nature can also be linked to the cyclical explanations for the 11-year solar cycle which affects the propagation of the GPS/GNSS satellite measurements. Overall, Pythagoras' efforts brought us closer to realizing satellite-based positioning systems.

2.2 Euclid (320–275 BC)

Euclid was born in Alexandria and remains something of a mystery, with few details uncovered about his life. He is believed to have been educated in Plato's Academy in Athens and he is the most famous mathematician of ancient times. He invented the first spatial science—geometry—and is known as the Father of Geometry, who coined the

well-known phrase "There is no royal road to geometry." He connected planar geometry in 3D space. All problems for position determination and navigation are directly linked with Euclidean geometry (e.g., see Figure 1, hydrographic vessel position determination).

Euclid's most famous book is *The Elements*. It comprises a synopsis of geometrical thought of the time, as well as an anthology of his original work in geometry. This book was the focus of mathematical teachings for more than 2000 years. Fragments of the Middle Ages edition and a Greek manuscript of the first edition can be found at the Bodleian Library of the University of Oxford. His other works include *Data*, which deals with the properties of planar shapes. *On Divisions of Figures* deals with the division of geometrical figures into two or more equal parts or into parts in given ratios. *Optics*, which is the earliest surviving Greek treatise on perspective, contains information on apparent sizes and shapes of objects viewed from different distances and angles; and *Phenomena* applies spherical

geometry to Astronomy (star shots, etc.). The well-known Euclidian axioms for a line, circle, sphere and angle, are the basic geometric figures and measures used to define the position of a point in three dimensions, the position of a ship on the ocean, the position of an aircraft in the sky, and the position of a satellite in space.

2.3. Archimedes (287–212 BC)

Archimedes was born in Syracuse, Magna Graecia, and is considered to be one of the greatest mathematicians and engineers of all time. He contributed to differential (gear) calculus and derived the areas and volumes of conoids (cones). He constructed a machine for lifting ships with levers and pulleys, and coined the famous phrase in engineering "Give me a place to stand on, and I will move the Earth." He perfected methods for finding areas and volumes

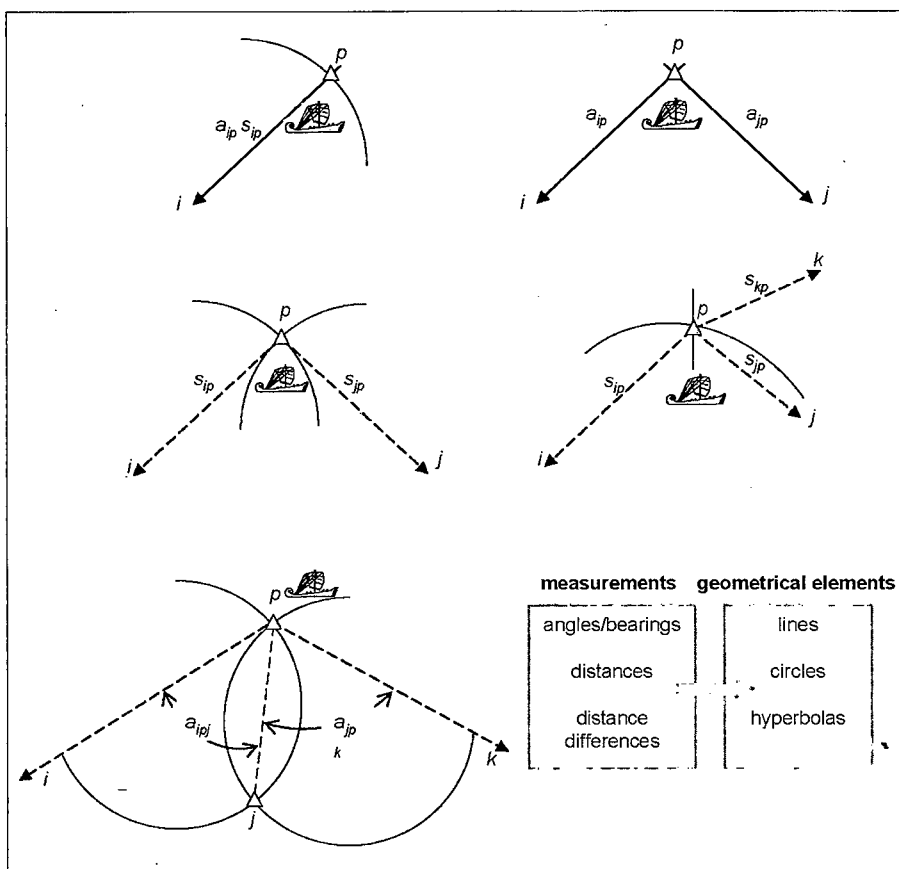


Figure 1: Euclid's axioms and fundamental measurements for ship positioning.

of different shapes, and formulated fundamental theorems for the determination of the centre(s) of gravity of various geometrical figures, laws of equilibrium of fluids and buoyancy.

Among Archimedes' top inventions is the astronomical collimator and the ship's log (also known in Ancient Egypt and by the Phoenicians) for measuring distances in the ocean. In his time, and primarily due to his discoveries, the interest in determining the size of the earthly sphere intensified and gave rise to an estimate of the circumference equal to 300,000 stadia (a value later refined by Eratosthenes that resulted in an estimate within 10% of today's accepted value).

Recovered works of Archimedes with direct application to position determination and navigation are: *On the Equilibrium of Planes*, that deals with the law of the lever and uses it to calculate areas and centres of gravity of geometrical figures; *On Spirals*; *On the Sphere and the Cylinder*; *On Conoids and Spheroids*; *On Circle*, which calculated the value of π with great precision; and *The Sand Reckoner*, a number system that is capable of expressing numbers up to 8×10^{63} (the number of grains of sand required to fill the universe).

A particular example of Archimedes' contribution to modern-day navigation through his developments in volume computations is depicted in Figure 2. The calculation of dilution of precision values, which is essential for determining position accuracy and ambiguity resolution techniques adopted for carrier phase positioning with GPS, rely on Archimedes' principles.

3. Historical Outline

A chronological survey of navigation is instilled with human vision and driven by dreams of the unknown. The history of navigation is inherently linked with transportation in civilization and, simultaneously, with

economic, cultural, political and military campaigns, and overseas commodity trading. However, the greatest achievements are international trade, the skill of sailing and the birth of scientific knowledge in subjects such as geometry, astronomy, geodesy, geography, cartography, surveying and engineering. Since ancient times, the travelling adventurers and the first sailing trips were connected with the stars and observations towards the fixed star. According to Homer, the goddess Athena told Odysseus to "have the big star to his left" during his journey from Calypso's island to Ithaca. The Chinese, Indians and Egyptians also turned their interest towards the starlit sky, resulting in significant discoveries and inventions at almost the same time as the Greeks. At first, the Greeks distinguished themselves as skilled seafarers by observing the stars and developing systematic techniques for directional positioning, orienteering and mapping of safe routes in the ocean. Later, they extended their interest and skill on land, and the terms *trip* and *discovery* took on a broader meaning.

Our historical outline takes us from some of the key instruments developed and used for positioning, which fundamentally applied Euclidean geometry; Pythagorean metrics and findings on extra-terrestrial spheres; and Archimedes' contributions from pure mathematics to applied engineering. The significance of nautical charting as the first form of navigation taking place on the open seas is

highlighted, with particular emphasis on the mapping and cartographic milestones leading up to the 21st Century.

3.1 Instruments and Navigation

From the 6th Century BC up to the 1st and 2nd centuries AD, from the Iones up to the Alexandrinus, navigation for trade morphed into systematic observation and established an advanced scientific base. This coincided with the invention of the first instruments for observing stars and terrestrial targets. Later, reliable marine charts emerged.

The sky and constellations are not the only means for orienteering, positioning and mapping routes. Flaming fires lit along the coasts functioned as lighthouses for tracing safe passages and pinpointed areas used for dispatching messages, first arrivals/departures and served as the first benchmarks.

In the beginning, the developed instrumentation gave a significant boost to navigation as a skill, which later cultivated a scientific discipline. During this time, a number of instruments appeared for measuring Euclid's axioms, angles and distances, that not only covered the needs for navigation, but also for cartography,

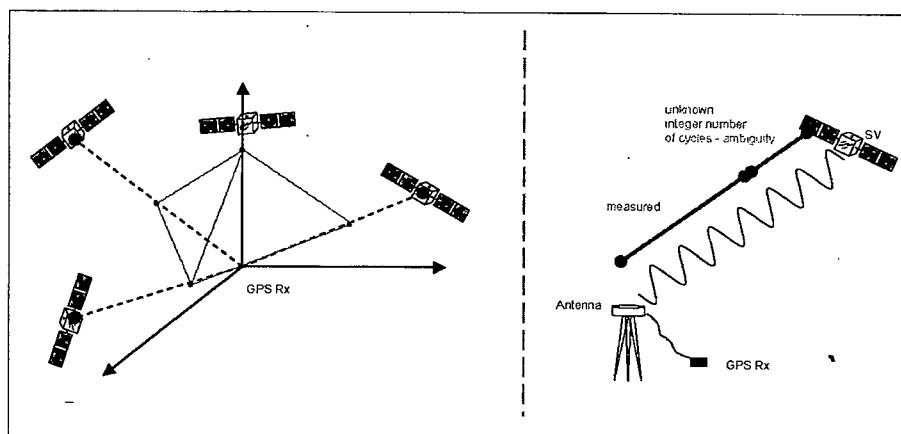


Figure 2: Dilution of Precision (DOP, left) and Ambiguity Resolution (right) implement Archimedes' principles for determining the accuracy of a position and resolving unknown ambiguities.

geodesy and geometry. This included instruments such as Huron's odometer and the corresponding dromometer for the ocean, the cross-staff, the hourglass, the Pole star and the dioptra. In addition, appropriate equations were formulated for measuring angles and bearings and setting out right angles based on the implementation of Pythagorean theory.

As time passed, knowledge replaced trial-and-error and the first basic instruments for surveying appeared: the astrolabe, the cross-staff and the quadrant, which allowed for more systematic exploration, observations and navigation. We still have the surveyor's tape measure, the compass and the triangle. General rules of thumb were adopted for laying out angles, such as using the Sun's rays for determining the line of mid-day and the North-South direction based on the shade cast on the ground. The primary instrument used for navigating between points based on celestial observations was the astrolabe, which was preceded by various prototype-like versions, such as the quadrant or the dioptra, the Pole star (via a circular compass with the needle pointed at Polaris) and others. Advancements in these instruments and their integration would prove useful for generations to come in the invention and construction of complete scientific instruments, such as astronomical theodolites.

Throughout the 1st to 18th century, numerous discoveries were made in parallel that benefitted navigation. In particular, the purely geometrical instruments used for drawing (ruler, bow compass, protractor) and necessary for charting navigational routes, were also instrumental to the evolution of astrogeodetic instruments used for terrestrial and astronomical measurements (i.e. geodetic theodolite). Certainly, the astrogeodetic-goniometer that dominated in the 18th Century is the sextant, which can be categorized as a purely navigational instrument. In its various forms, the sextant is perhaps the only geometrical instrument that can be used to obtain dis-

tances and angle-bearings from one ship towards its target on land, or even astronomical measurements for the determination of the ship's position, or for the geographic coordinates of the ship's destination. However, the exclusive use of the sextant in piloting and navigation began to fade with the appearance of radio-systems (i.e. DECA, LORAN, OMEGA), which were limited by their coverage (DECA and LORAN) and their accuracy (OMEGA).

The Chinese discovery of the compass (11th Century), the application of the magnetic needle (12th Century) on ships for determining and mapping out routes, and orienteering were key discoveries in the history of navigation. The invention of the chronometer in the 18th Century played a significant role in advancing practical navigation, which is vital in determining longitude. The implementation of radio-systems and the birth of the concepts of radio-positioning and radio-navigation ushered in a new age where positioning accuracy, safety and geographic extent significantly improved.

The radio-systems dominated for more than a century in piloting and navigation, until about the mid-20th Century, when the launch of *Sputnik* in 1957 marked the era of artificial satellites, satellite geodesy and the new Space Age. The indirect measurements of distances (through times of signal emission and arrival) to artificial satellites with lasers and the global satellite systems used for positioning (GPS and GLONASS) are fully operational. The planned satellite positioning system, GALILEO, the ongoing deployment of geostationary satellites

of the Chinese regional navigation system, Beidou, and a host of meteorological and other satellite systems have given a different context to navigation with respect to accuracy, efficiency, safety and breadth of applications. Figure 3 provides a brief synopsis of some of these major navigational inventions throughout history.

Navigation is no longer restricted to the oceans, but extends to land, air and space. Modern navigation is used for ships and fleets of vessels, as well as other vehicles on land, aircrafts and satellites. The rapid developments and progress in other scientific areas, such as computer science, automation, spatial data information systems and cartography, also contributed to today's electronic means for navigation, which have led to applications that are dependent on physics and bound only by our imagination.

3.2 Nautical Charts and Navigation

Advancements in the field of navigation are historically allied with the developments in nautical charts and cartographic maps. Initially, nautical data, used for navigation in ancient times, were of a descriptive nature. They marked the hydrographic activities of sea mariners of the time and gave proportionate representations of the relationship of points on land and water.

The field of cartography evolved significantly, along with related disci-

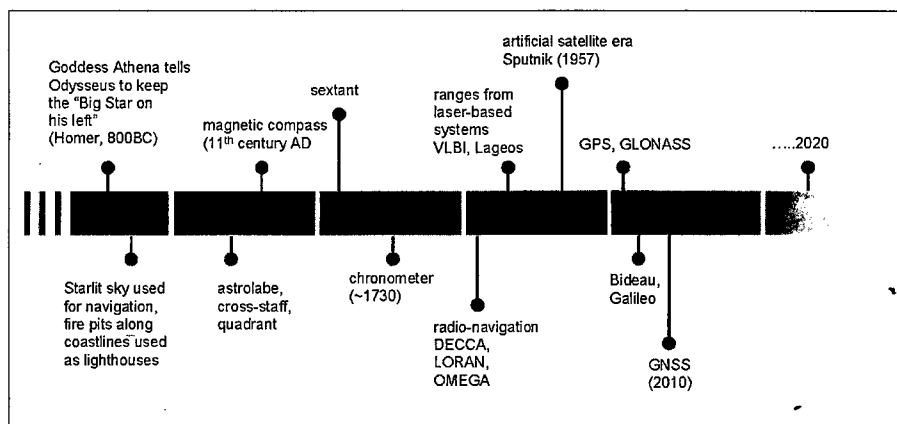


Figure 3: A navigational journey through time from natural stars to artificial satellites.

plines of Earth sciences—geodesy, geography and hydrography—in the first century BC and the first years AD. It was also a time where nautical charts blossomed through increased knowledge and intense scientific investigations. Geographical coordinates, geodetic locations and mid-day appeared as concepts in cartographic studies and relevant cartographic depictions. It marked the beginning of maps that thematically portrayed a greater variety of information than in previous years. For the first time, these charts were considered valuable for safe passage and navigation (also due to the provision of more accurate information through Ptolemy's geography and cartography around the 2nd Century AD).

During the same period as the magnetic compass was invented, nautical charts were equipped with new elements, such as directions and routes for navigating, realistic map scales and other techniques for orientation. The first mass-produced maps were printed in the mid-16th Century on large sheets of paper, replacing hand-made charts/maps. The Atlas was published in Holland and Earth-globes were produced.

The well-known maps of the British navy appeared in the 17th Century. They were drawn in meticulous detail and the routes were drawn with great accuracy. The sources of information used for the construction and drawing of these maps included the observations of mariners, ocean explorers and tradesmen from various expeditions, as well as preceding cartographic editions and relevant texts.

After the 18th Century, the extradition and publication of nautical maps followed specified guidelines that were implemented through organized hydrographic bureaus or similar agencies. Since the beginning of the 20th Century, the International Hydrographic Service and the International Hydrographic Organization have more than 62 member nations that collectively provide meaningful and uniform international descriptions and guidelines for the publication of nautical maps, the registration of hydrographic

data and the printing of publications that concern navigation.

From the mid-20th Century and onwards to the threshold of the 21st, coincident with progress in the fields of satellite geodesy, hydrography and computers, the classical nautical chart gave its spot to the electronic chart; subsequently, classical navigation gave way to electronic navigation. The electronic chart completes any instrument used for navigating a ship within the framework of an integrated navigational system. The electronic map depicts on a computer screen, and accepts as input, geometrical or other forms of information from terrestrial stations and/or satellites, every instant of the day or night and under any conditions. This map is updatable and can be modified in real-time on board the ship.

The modern electronic map comprises the digital version of a typical nautical chart on a computer screen with graphical tools. In addition, through the processing of information from a geometrical foundation and spatial information from positioning systems and ship orienteering (radio-systems, radar, satellite receivers), the line of course (sailing/navigating) is determined, as well as its speed and position relative to other ships in the vicinity.

4. The Geometrical Principle—from Navigation in Antiquity to Navigating with Satellites

Navigation is based on geometrical measurements of size (derived distances and angles), and hydrographic surveying with intersections of geometrical land/spots that are realized via these measurements (see Figure 4).

Over the years, man's occupation with navigation allowed for the basic principles of geometrical range and shapes (defined by Euclid) in order to pinpoint a position at sea, on land and, more generally, in space. This has evolved from empirical measurements (e.g., Pythagoras' notions of celestial spheres and our use of harmonic functions to represent gravitational fields), rudimentary instruments (e.g., Archimedes' collimator) to advanced scientific knowledge and equipment. Angles and directions, distances and relative distances, are the basic geometrical quantities that are defined by

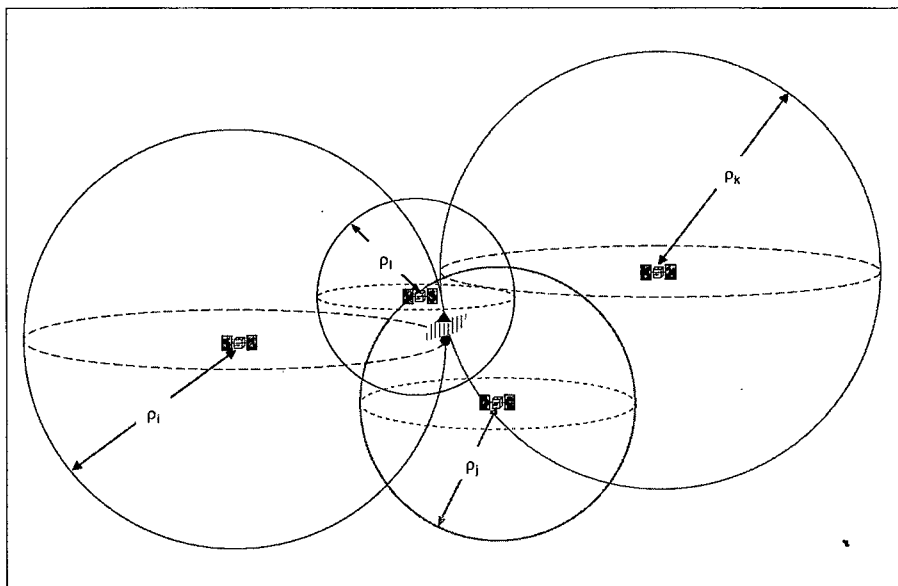


Figure 4: GPS ranging concept—trilateration in space. Each satellite is centred in a hypothetical sphere with a radius equal to the range between the satellite and the receiving antenna. The final 3D position (x, y, z) of the antenna/receiver is indicated by the triangle where all four spheres intersect.

these Fathers of Geometry. These quantities—either measured with compasses, diopters/collimators and sextants; with ropes, chains and odometers; and even with radio-systems, modern geodetic instruments and satellite receivers—have the same practical value in position determination. All are applied to find the intersection where the position of a point is defined on a plane or in space from angles and directions (resulting from straight lines), from the distances of circles and circular arrows in the plane, or spheres in space. Two geometrical points are required to uniquely define a point on a plane, and three for a point in space with respect to a 3D reference system (see Figure 4). Today, that specification is met with respect to a reference system via radio-triangulation or satellite-triangulation, depending on the instruments used for observations and measurements.

Conclusion

We have provided a glimpse of how navigation, from its birth in Ancient times up to today's findings of satellite geodesy, applies the theories of Pythagoras, Euclid and Archimedes. Although skilled practitioners were not familiar with the mathematical works of these ancient mathematicians, it is clear that the fundamental geometrical concepts were learned and adopted for navigating.

The Euclidean works demonstrate the connection of basic concepts of navigation with satellite-determined position through trilateration in 3D-space. The Pythagorean theorem is used in a plethora of applications, from plane geometry to satellite positioning and navigation. Pythagoras demonstrated the strong, yet often overlooked, link between art and science through the theory of music with geometry and the shape of the Earth. The mathematical ideas of Archimedes for the sphere and the ellipsoid of revolution constitute the fundamental mathematical elements of modern-day navigation that are commonly embedded within geoscientific methods.

Thus, these fundamental mathematical beginnings that were first formulated by Pythagoras, Euclid and Archimedes, and developed/exploited/implemented for centuries, were the solution of the geometrical problem of navigation in terrestrial/satellite positioning, in planetary motion, and in the propulsion of revolutionary theories of modern-day science and thought. It seems that as the Earth continues its perpetual rotation, in a mathematical and harmonious environment, the ideas and theories of Pythagoras, Euclid and Archimedes will stand and be continuously applied as well.

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