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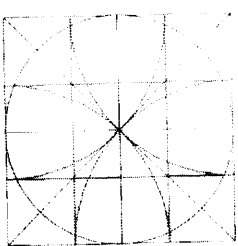
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Ancient Geometry

- AND ITS USE



VOLUME II

RHODOS

INTERNATIONAL SCIENCE PUBLISHERS, COPENHAGEN

ing. The same division was applied to the half circle that surrounds the altar.

But the main aim of our search has (I trust, convincingly) been fulfilled: to illustrate that the design of Cologne Cathedral was built up according to geometric principles that were already hoary with age and use hundreds of years before the cathedral was built.

The Pantheon

WALK THROUGH the older quarter of Rome, capital of the ancient world, and you are bound sooner or later to come face to face with one of the most impressive buildings from antiquity: the Pantheon.

The building rests like a giant animal amid a conglomeration of tightly packed houses and apartments, which seem loath to yield room for the great structure. From floor to its beautiful, domed ceiling the Pantheon measures almost 45 m. (approx. 140 ft), and from side to side almost 56 m.

The main entrance to the building faces a small *plaza*, and from this opening in the dense housing mass one can stand back and see the grand approach and entrance to the Pantheon in its entirety.

The complete frontage, as it stands today, is quite obviously inspired by early Greek temple structures, having eight frontal pillars and richly decorated capitals.

Above the columns the sloping roof rises at a somewhat steeper angle than one is accustomed to seeing in a Greek temple, but the style is the same.

But comparison with a Greek temple goes no further than the door of the Pan-

theon. The analysis has picked out in the main the dimensions and proportions that give the cathedral its distinctive appearance and character. It pointed out the framework round which the details were built, and it brought clearly to notice the definite link between the design of the facade and that of the ground-plan, familiar from previous analyses.

theon. The frontage appears almost to be stuck on as an afterthought to a rounded temple that has an all-enclosing wall instead of a series of columns. We would look in vain therefore for the typical cloister effect found in Greek buildings.

As in the case of most monumental structures from early Christian times, the Pantheon has had a dappled history of reconstruction and no longer bears its original appearance.

In *Fig. 237* we have a picture of the Pantheon, showing its present face.

The name Pantheon denotes that it was a temple dedicated to all gods in the community, and history says it was built originally by Agrippa in 27 B.C. as a ten-column temple, but nothing remains now of the original structure apart from one or two pieces of foundation. These show that the original building was 3.7 m. lower at its base than the present version.

The old, original temple burned to the ground about 81 A.D., and Domitian built a completely new style of temple in the ruins of the old.

The apparent reason for the new building being placed so much higher than the older seems to be that the burned-out shell



Fig. 237.

of the former temple was simply flattened, rolled and used as the foundation.

The new building, too, has since disappeared almost without trace. Historical records indicate that the building had a large circular courtyard at its centre. It would appear to have covered a larger area than the present Pantheon.

Its life was short. In 110 A.D. the building, like its predecessor, was razed by fire. The job of planning and rebuilding a new temple was given to Hadrian.

Hadrian was not content to produce a carbon copy of the previous temple, but planned a new structure from foundation to roof, taking as his starting point the dimensions of the old inner courtyard. This he made into a circular temple which he topped with a fine domed roof, the ceiling of which he decorated with a peculiar stucco design.

The ornamental ceiling does not how-

ever bear the same design as the temple capitals or later decorative panels. Its lines and circle look more like a geometric diagram than an architectural decoration.

A large hole was left in the centre of the domed ceiling, and through this the light of day streams into the church producing a most unusual shadow effect on the plaster ceiling. We see this in *Fig. 238*.

Hadrian's reason for erecting a temple which in height exceeded its predecessors but which occupied less area of ground was perhaps that Rome even then was densely built up, and crowded so close to the site of the Pantheon that he was forced to restrict the ground area in order to create a sense of space.

There is little doubt that the new structure is smaller than the old in ground area. In the previous building the round courtyard was a part of the building, but only part; the same area is not a part of

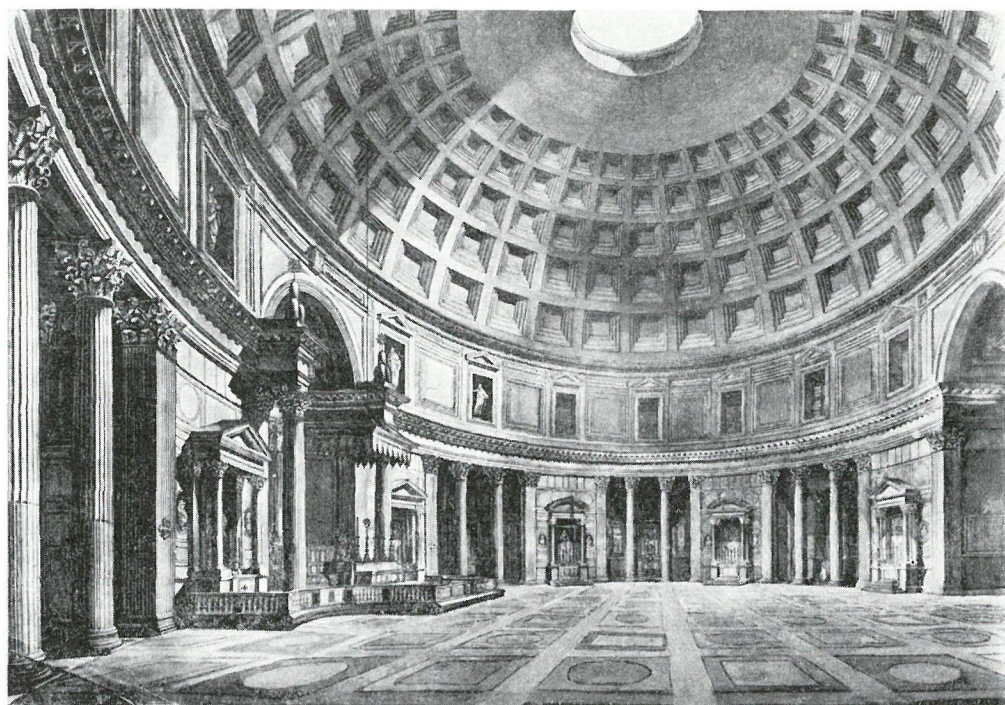


Fig. 238.

the new building—it is the new building.

Historians are not too sure who built the present entrance. It may have been Hadrian, but may possibly have been a later builder, Antonius Pius. Or even a third party.

But the builder of the entrance is not of great interest to a geometric study such as ours, since the original facade probably exists no longer. As late as 1606, for example, Pope Barberini had the Pantheon's bronze porch beams removed and melted down to cannon. The building suffered another "face-lift" in 1747 when, among other things, the windows were altered in size.

In spite of these attacks on its originality, the church has retained a composed beauty. It strikes one immediately and inevitably. It is an impressive experience passing from the darkened hall through a narrow opening in the beaten-

copper door into the dome-shaped church hall. Light pours down from the gaping roof, reflecting on the stucco ceiling, and providing considerably more natural lighting than usual for a Catholic church.

The floor is laid in variously shaded marble, shown at its best in the strong daylight. The design resembles a giant chess-board, the squares in turn being laid in mosaic form.

There are two motifs: a square with a circle inscribed, and a square with a smaller square inside. The patterns alternate as do the black and white squares on the chess-board (see Fig. 238). Already one senses the presence of ancient symbolism and geometry.

In our study of the Pantheon through the eyes of the ancient geometer, we must ignore the two previous buildings that stood on this site. Of them there is virtually no trace. We may consider the pre-

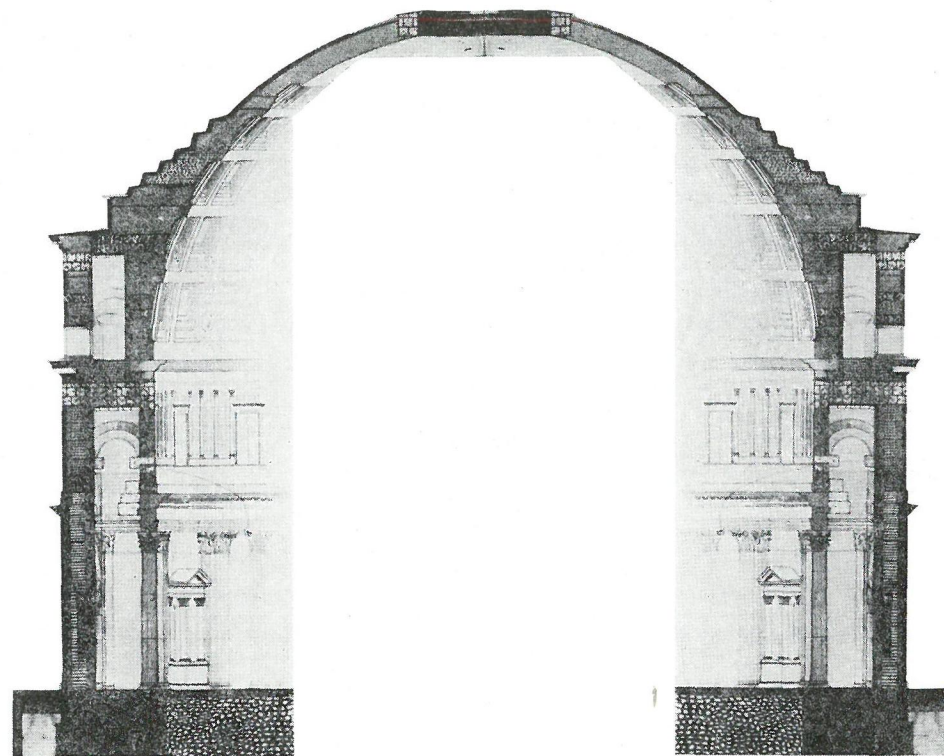


Fig. 239.

sent building to be a brand-new erection by Hadrian around 117—138 A.D.

Hadrian, whatever else he may have been, was an ingenious planner and designer. In addition to a wonderful gift of unfettered imagination, he had at his fingertips "the royal art" of geometry. We ought thus to expect to find this monument to his expertise laid out in accordance with ancient geometric principles.

The rather severe alterations to which the building was subjected in 1606 and later in 1747 leave some doubt about the original design and dimensions of the entrance and windows. These will therefore be omitted from the survey, and we shall stick as close as possible to the building thought attributable to Hadrian alone.

It was a rather difficult task tracing suitable material for an analysis, since most available drawings and plans are in

freehand with no indication of source or authority, and cannot therefore be used in a critical geometric study.

However, I came across the best material on site, i.e. in the front hall of the Pantheon in a booklet describing the building.

Called quite simply *The Pantheon*, the booklet is the work of Roberto Vichi. Its illustrations are apparently based on accurate measurement of the building, and the sources are quoted of both drawings and photographs.

One sectional drawing shows the interior of the building and the structure of the outer walls. It also shows the passages that lie between the outer wall and the temple hall.

The drawing gives only one (the left) side of the building, as far as the vertical axis. But as we require the whole cross-

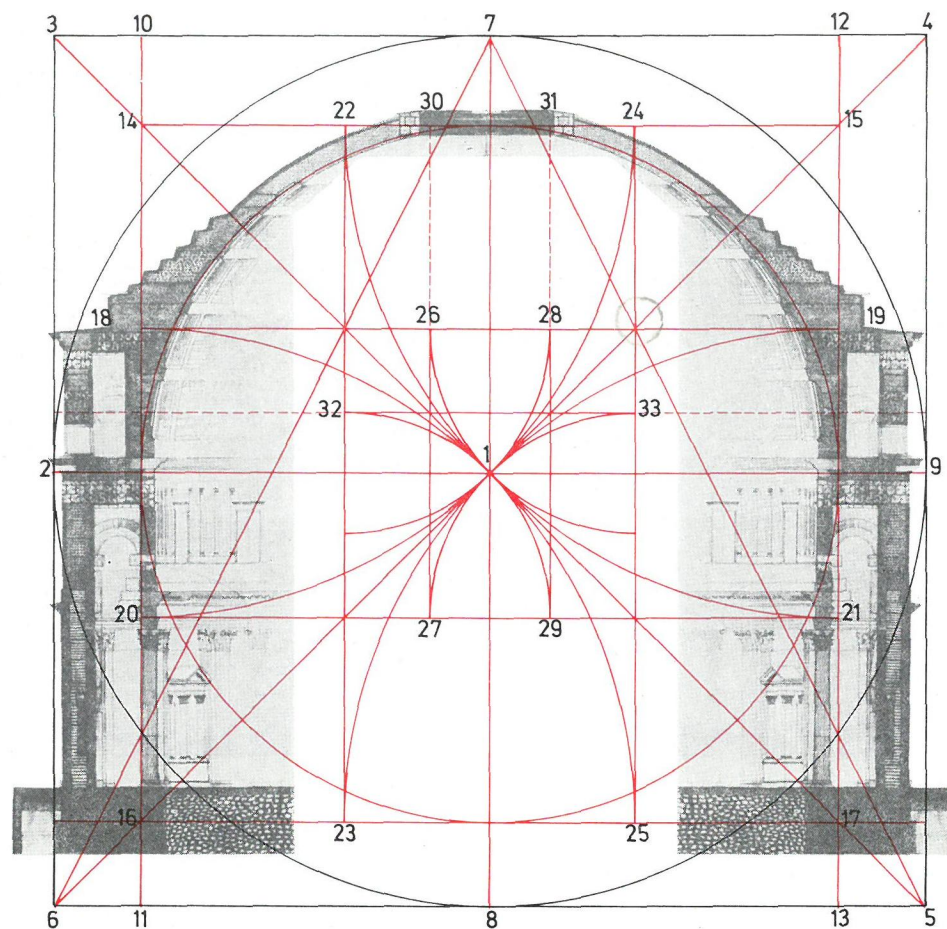


Fig. 240.

section, the illustration was photographed and repeated on the right of the axis in order to produce a complete picture of a slice of the Pantheon. Apart from the double width, the illustration in the present book, Fig. 239 is almost identical in size to the original.

As usual, the first step is to find the constructive basis of the building's plan. As with most of the other buildings we have examined, there are a number of possibilities in the Pantheon.

Its width exceeds its height, and the natural (and correct) assumption is that the basic square's vertical sides run flush

with the building's outer wall. We saw this applied to samples of Grecian temples.

In Fig. 240 the two verticals were entered flush with the walls, and the problem then remained to find the Pantheon's horizontal axis. I found this to be at line 2-9, where it is marked both outside and inside the building by a frieze. The inside frieze marks the level at which the inner hall wall changes from the vertical to the curved dome.

Having fixed the horizontal axis, we can go ahead and construct our basic square: 3-4-5-6.

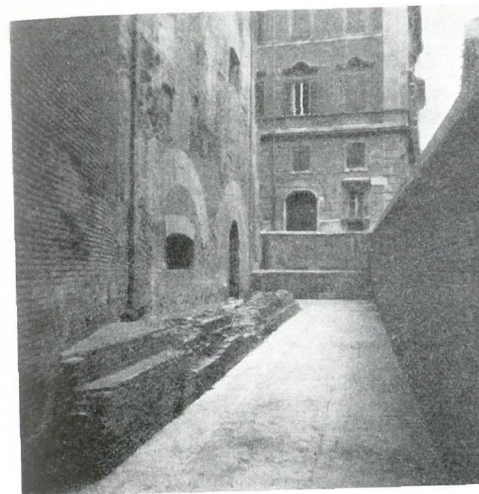


Fig. 241.



Fig. 242.

We notice immediately that neither the upper side nor the base of this square has any contact with the building in our diagram. As regards the upper horizontal (3-4), we are accustomed to seeing it placed well above the roof of the building or temple under discussion. But the square's base-line usually has its place at the foot or base of the building.

This is in fact also true of the Pantheon. But whereas we normally analyse a *facade* drawing or photograph of a particular building, with the Pantheon we have a *sectional* drawing, a slice down through the middle. And therein lies the explanation.

Facade plans normally show the outside steps leading up to the actual temple entrance, but these steps do not show in a sectional drawing.

We may therefore assume that base-line 5-6 marks the bottom of these steps that lead up to the temple.

But there are no steps leading up to the front of the Pantheon! We saw this clearly in the photograph in Fig. 237 taken from the square in front of the building.

This apparent absence of approach steps is caused by the fact that the square has

been raised and surfaced so often in the past that it is now level with the floor of the Pantheon's porch. A further inspection reveals however that there were steps round the building at one time.

The Pantheon is built in a fairly undulating area of the city, there are steeply sloped streets all around. The street, for example, running behind the Pantheon is about 10 m. higher than the church's base.

To protect the building against encroachment from surrounding houses, the authorities built a wall. Outside it lies bustling Rome.

In Fig. 241 we see a view of the right-hand side of the Pantheon towards the rear of the building, illustrating clearly the "moat" between Pantheon and the adjacent neighbourhood.

Fig. 242 is a photograph of the building's left side, showing a similar variation in ground level.

In Fig. 243 the camera has caught a view along the right side of the building towards the front, and we see here that recent authorities have placed steps from ground level up to the temple's porch—proving that the base of the building lies

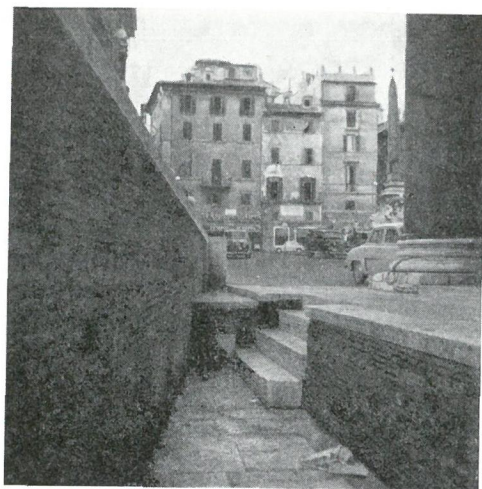


Fig. 243.

in effect much lower than the level of the front plaza.

This is evidence that our base-line 5-6 in Fig. 240 is properly positioned, and proof, too, that a geometric analysis can uncover or rediscover factors that only an intimate search on site can establish. Factors which may be totally absent from the drawing we analyse.

We now add to Fig. 240 the acute-angled triangle 6-7-5 in order to mark off the circle's rectangle. The latter is seen as 10-11-12-13.

The basic square's proportions were selected so as to run flush with the extreme outside of the building, i.e. the projecting sills that encircle the Pantheon in the form of a frieze.

Now that we have entered the circle's rectangle we can establish another part of the plan: the rectangle's vertical sides form the basis of the positioning of the ring of pillars that run around the outside edge of the main hall.

If concrete indication were required that the Pantheon was planned according to the principles of ancient geometric diagrams, the positioning of these pillars certainly whets the appetite. Hadrian was

obviously a planner of the ancient geometric school.

Our survey continues with the entry of the square on the circle's rectangle, which we place centrally in the diagram. Even more lucidly we see the building's plan open before our eyes.

The square is marked by lines 14-15 and 16-17. The former, we see, indicates the internal height of the main hall, passing along the uppermost curve of the dome.

When we inscribe a circle in square 14-15-16-17 we see that it follows exactly the sweep of the dome and takes in part of a colonnade halfway up the inside of the hall.

Other drawings of the Pantheon exist, showing a circle drawn in a similar position, but that circle lies entirely within the hall and does not touch the surrounding columns. But if the arc of a circle is to follow precisely the curve of the dome, it must be drawn as indicated here. A circle with either a shorter radius or a different centre will not match the curve of the ceiling exactly.

The other drawings are not in detail, and give the distinct impression that the hall has been sketched round a circle rather than a circle placed within the building.

Later analysis will prove that the circle I have arrived at is more likely than any other to be correct, since it marks a number of factors both in the sectional view of the building and in its ground-plan.

We enter the sacred cut in the square on the circle's rectangle. The respective lines are 18-19, 20-21, 22-23 and 24-25.

Line 18-19 plays an important part in the building's structure, representing the height of the outside vertical walling. This is the level from which the domed roof starts.

The lower horizontal sacred cut, 20-21, is also marked both inside and outside the

building. Inside, we see it as the height of the niches placed between the columns bordering the main hall. The actual marking is represented by a frieze and can perhaps best be seen in Fig. 238.

The external marking is the edge of the sill that encircles the building.

Fig. 238 also illustrates clearly the stucco ceiling in the main hall. The design is composed of five concentric rings of geometric figures, the rings reducing in size towards the centre. The smallest of the five stops short some distance from the hole in the centre of the roof.

As near as I can ascertain, the point at which the fifth and inner ring is met by a smooth area of plaster is marked by the vertical sacred cut, lines 22-23 and 24-25. The junction is indicated at the intersection of the curved dome and the two sacred cut lines.

The combination of the sacred cut in square 14-15-16-17 creates another smaller square. If we again enter the sacred cut in this latter square, we find that the two vertical lines, produced upwards to the ceiling, mark the diameter of the large "sky-light" in the centre of the roof.

Line 27-26 is produced to point 30, and 29-28 is produced to point 31 showing the diameter.

The upper sacred cut (horizontally) is 32-33 and indicates the top of the upper row of windows in the building.

The analysis of Fig. 240 has provided us with details of a number of important features in the Pantheon. Most of the diagram's lines were used by the architect to place some factor or other in the building. We may thus assume that our choice of basic square was correct.

Our next diagram, Fig. 244, is constructed in the same manner as Fig. 240. The basic square is 3-4-5-6, in which we inscribe the circle and acute-angled triangle 6-7-5, following this with the circle's rectangle 10-11-12-13. The square on the

latter is again 14-15-16-17. Thus far the diagram is identical with Fig. 240.

The next step is to construct the basic square's half-size version in the centre of the diagram. This is done by joining the intersections of the basic square's inscribed circle and diagonals. The required square is 34-35-36-37.

We execute the sacred cut in this square and observe first the placing of the upper horizontal cut. It is produced across the building as line 38-39.

In the previous analytical diagram we found that line 18-19 marked the upper edge of the frieze or sill that tops the outside walling. Here we find that 38-39 indicates the underside of the same sill. The thickness of this projection is thus determined by the distance between the two upper horizontal sacred cuts in squares 14-17 and 34-36 respectively.

The lower horizontal sacred cut is also produced across the building as 40-41. It indicates the top of a lower projecting frieze. The underside of the same frieze was marked in the previous analytical diagram by line 20-21, i.e. the sacred cut in square 14-17. Thus the thickness of this sill, too, is determined by the distance between the two (lower) horizontal sacred cuts in squares 14-17 and 34-36.

The base of the latter square, line 36-37, forms the floor of the main hall.

The vertical sides of the half-size square, lines 34-37 and 35-36, appear to have been the determining factors in fixing the net width of the main hall just as the sides of the basic square marked the gross width of the whole building. We notice that the first-mentioned set of lines touch the projecting sills of the inside columns in the same way as lines 3-6 and 4-5 run flush with the friezes of the outer columns.

In the horizontal lines of the sacred cut in the inner square, lines 38-39 and 40-41, we can find the relationship with the ba-

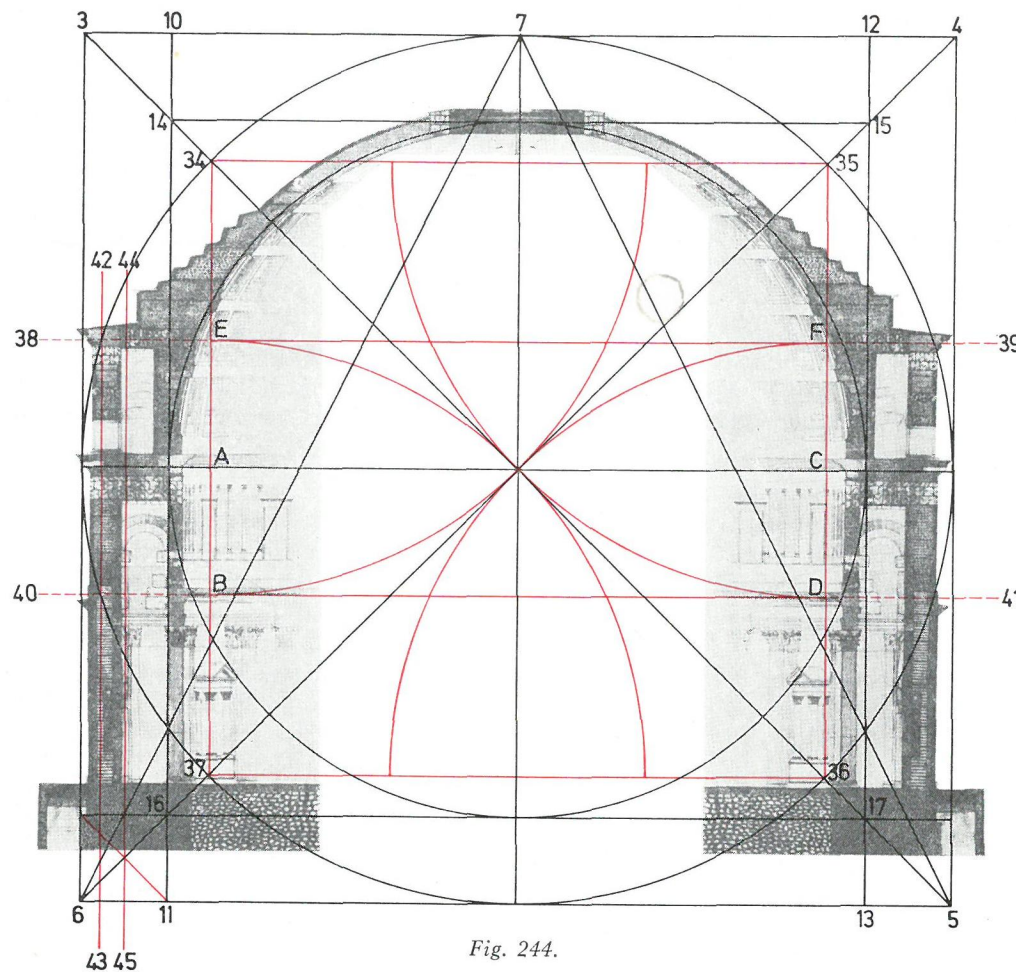


Fig. 244.

sic square's inside circle that decided the dimensions of the temple's outer walls. And the factor that determined the width of the upper "shelf" or sill from which the domed roof rises.

The sides of the basic square coincide with the extreme tip of this ledge and, as we have seen, indicate the temple's total width.

To decide on a thickness for the outer wall Hadrian then drew a vertical line through the intersection of sacred cut 38-39 in the half-size square and the basic square's inside circle. The vertical is

seen as line 42-43 and marks the outside of the main wall.

That determines the outside of the wall, but how did he fix the inside? He took the area between the basic square and the circle's rectangle (area 3-10-11-6) and split it in two pieces down the centre. The dividing line is seen as 44-45. The actual process of division was carried out in square 16-11-6, by means of the simple diagonal cross.

Thus we have the five outer vertical lines of the Pantheon: line 3-6 is the temple's total width; line 42-43 the outside

of the wall; 44-45 is the inside of the wall; 14-16 the outside dimensions of the colonnade in the main hall; and 34-37 marks the effective width of the main hall.

This is a typical example of geometric diagrams being applied as a form of static curve, in the same way as a present-day engineer uses graphs to make rapid calculations of height/weight/strength ratios, etc.

The ratio within the diagram remains constant, of course, irrespective of the size of the finished building. The deciding factor is the length of the basic square's baseline. The smaller the basic square, the slimmer the wall; the larger the square, the thicker the wall will be.

The ability to appreciate these ratios depended to a great extent on the experience and wisdom of the master builder, and it was also a matter of routine in applying these experiences. But I have no doubt at all that builders in ancient times followed certain rules and ratios laid down by ancient geometry when they had to determine bearing thickness, strength, proportions, etc., in a building. It was a matter of knowing the properties of the basic square.

It was naturally a stage that he and his predecessors had approached very slowly over the centuries.

Whereas in the dawn of geometric thinking and draughtsmanship the actual lines and diagrams were a sacred subject, revealing wisdom of occult geometry and numbers, it gradually over thousands of years became a subject applied to more practical spheres—such as the building site.

For thousands of years Temple brethren had gone about the business of fixing the dimensions of a building by the principles of ancient geometric symbols. But in the beginning they knew nothing of (and gave little regard to) the most economic form of structure, wall thickness, arch curve,

etc. They went ahead with geometric symbols, diagrams, plans and finished structures that could support ten, twenty times the required weight—and more.

The Great Pyramid of Egypt, for example, is more a mammoth memorial to ancient geometry than a building planned for the sake of economics. For its effective interior is nothing compared with the tremendous mass of material used in the building's construction.

As the religious builders gained more experience, studied their finished efforts, returned for a second look at their geometric plans, and speculated on whether such massive walls, roofs, etc., were strictly necessary, so the style of building altered. More space was allowed within the structure, dimensions were slimmed down, columns became tapered, new materials were made available—and geometric diagram became even more than before a tool of the builder. Once he had experimented with a particular height/thickness ratio and had found a suitable set of lines in his favourite geometric symbol, the builder stuck to it. He had found a successful shape from which to advance. He could afford to be bold. But always he came back for a second look at the geometric diagram.

Hadrian was certainly a brilliant and experienced builder. A less experienced, more timorous planner would have been tempted to select line 3-6 as the outside of the main wall and not, as we have seen in the analysis, line 42-43. But the difference between the two lines meant, for example, a considerable saving in building materials. And cost was presumably also a factor worth remembering in those days, too.

In Fig. 245 the basic square is subdivided 10×10 in the same manner as we saw in Chapter Ten. Plato, we remember, split the square on the circle's rectangle 8×8 , and by producing his

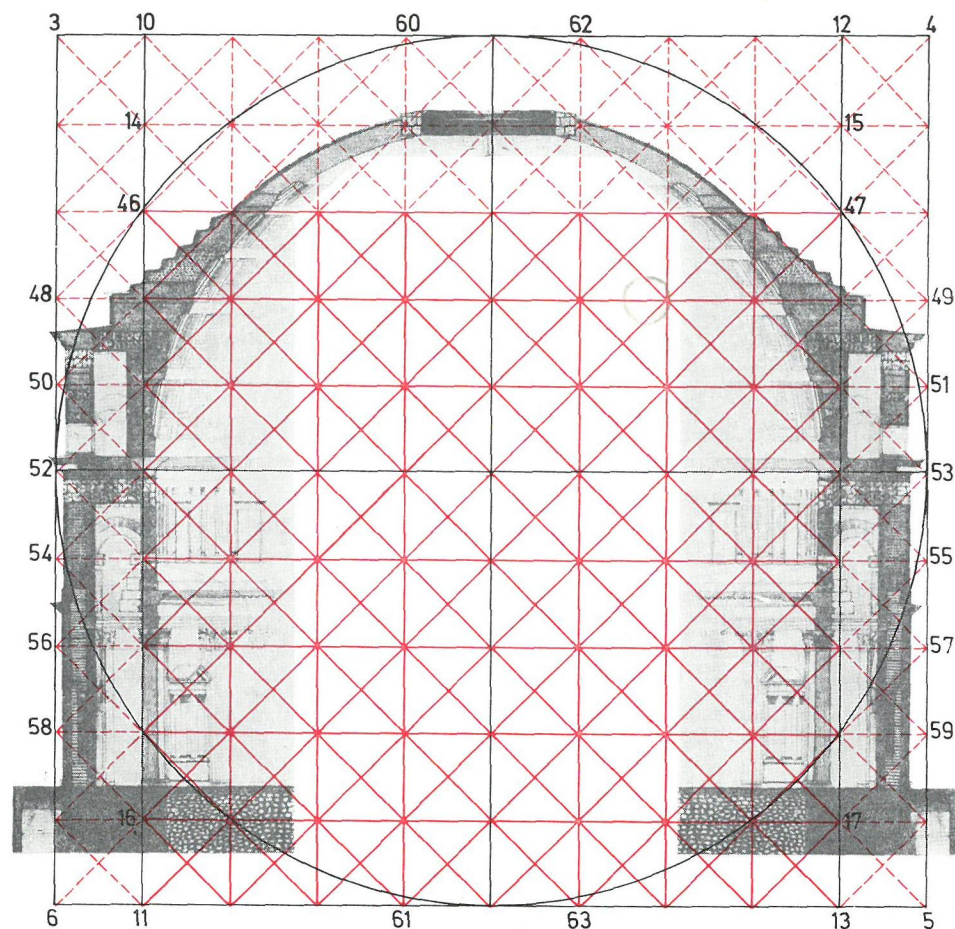


Fig. 245.

lines of division to the basic square, divided the latter 10×10 .

In the present diagram the circle's rectangle is 10-11-12-13, and its square lies at the bottom of the figure: 46-47-13-11.

We divide the latter square with verticals, horizontals and diagonals as shown previously, and produce these to meet the sides of the basic square. The extensions are shown as broken lines.

We shall now examine the horizontal lines of 10-part division—and the result is surprising. Almost all of them are responsible for marking some dimension or other in the building.

The top line, 3-4, is the constructive starting point. It lies outside the building and therefore has no place in the actual structure.

The next line, 14-15, runs flush with the inside ceiling of the dome, and is in fact the same line (in Figs. 240 and 244) as the top of the centrally placed square on the circle's rectangle.

We examine next line 46-47, which runs through the upper half of the dome. From this level down to the next line, 48-49, we note that the outside of the dome is broken in a series of seven steps.

Line 50-51 has apparently no special

place or value in the diagram, running merely through a frieze on the outside.

We are already familiar with line 52-53 as the horizontal axis of the basic square. In Fig. 240 it was numbered 2-9 and its value was illustrated in that analysis.

Line 54-55 indicates the lower sill of the windows and niches backing into the gallery above the main hall. These are more clearly seen in Fig. 238. As far as can be ascertained, the distance between points 52 and 54 is equal to the height of the windows.

Line 56-57 marks, as we can see, the lower edge of the corinthian-style capital in the inner colonnade.

The representative marking of line 58-59 is most interesting. The line lies so low in the building's plan that it plays no constructive part in the structure, but

nevertheless, to point out the existence of the line, a mark has been made in the waist of the pillars that surround the main church hall.

Line 16-17 is the same as seen in Figs. 240 and 244. It is the base of the square on the circle's rectangle when this is placed centrally in the diagram.

Finally we come to line 6-5, the basic square's base-line, which is part of the original construction. Since it lies somewhat lower than the building itself, it has no real marking there, but perhaps acted as the level of the foundation or the steps which originally led up to the temple.

These three surveys of the building's sectional view ought to be sufficient to demonstrate the guidance given by ancient geometry in planning the structure of the Pantheon.

Pantheon's ground-plan

THE CHOICE of starting point for an analysis of the Pantheon's ground-plan must, as we know from past experience, be geometrically linked with the building's facade (or in this case its cross-section).

Since the Pantheon is a circular building and since the building's complete elevation was contained by the basic square in the earlier analysis, it would be natural to transfer the dimensions of this square to the ground-plan.

In Fig. 246 we see this taken as the basic square: 1-2-3-4. It embraces the entire circular part of the building. Additional structures at the front and rear of the main building are excluded from the basic square. In order to include these in a geometric study, we construct the basic square's double-size version: 5-6-7-8.

We notice first that lines 1-2 and 3-4 were used as the determining factor in fixing the depth of the triangular rooms at the front and rear of the building on either side of the vertical axis.

We see, too, that line 5-6 completely encloses the ground-plan at the rear of the building, while the lower line of the outside square, 7-8, appears to run aimlessly through the porch. Later analysis, however, will prove that the depth of the porch was determined by other factors than the basic square's double-size version.

The sacred cut is executed in the large outer square as 9-10 and 11-12 (vertically) and 13-14 and 15-16 (horizontally).

Immediately we notice that the verti-

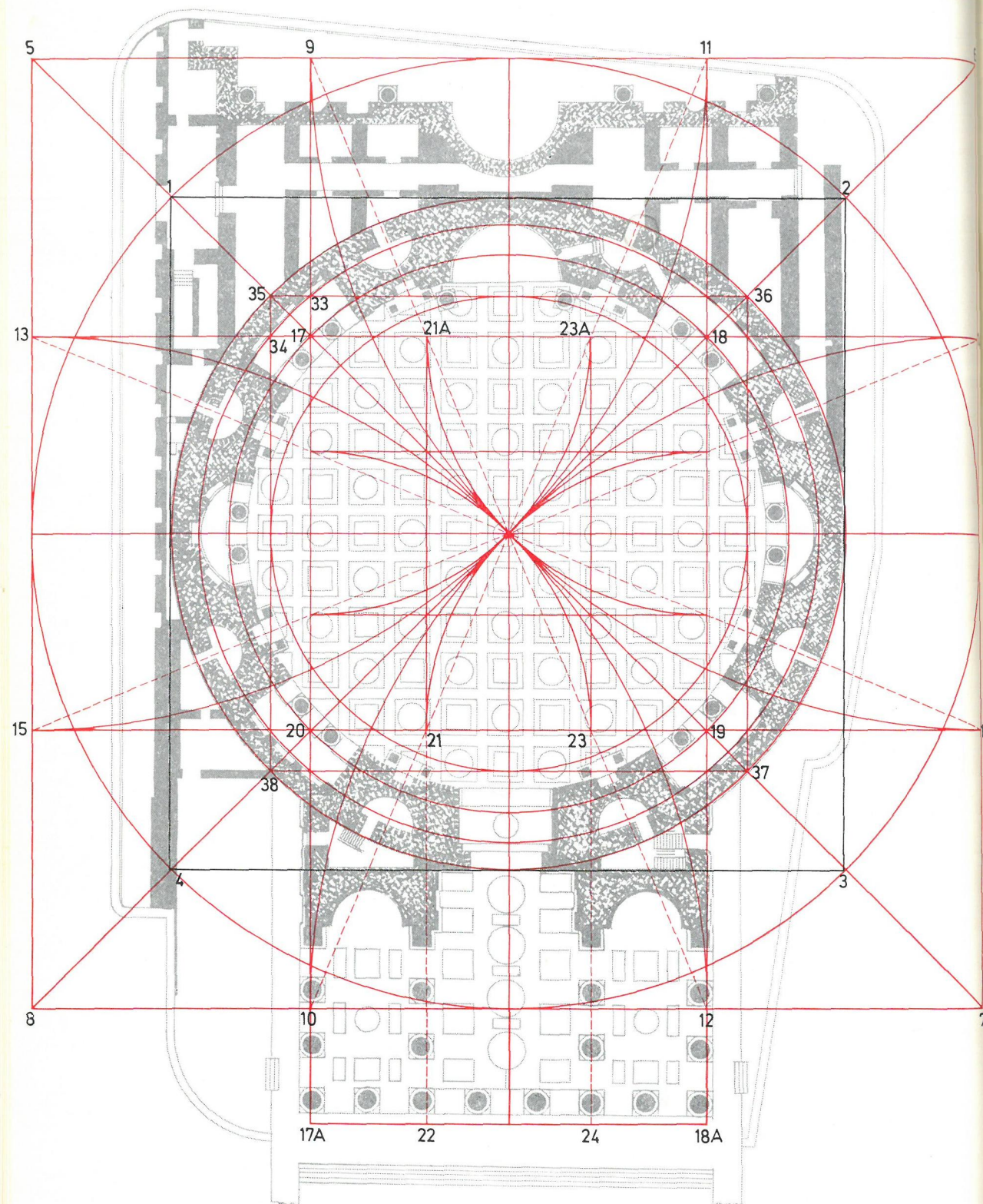


Fig. 246.

cals indicate the width of the porch. They run precisely through the centre of the two outer rows of columns.

The combination of the four sacred cuts creates as usual a smaller square in the centre of the diagram: 17-18-19-20.

If we can imagine this square flipped over in the direction of the Pantheon's entrance, with line 19-20 as the "hinge" or axis, we achieve a new square: 19-20-17A-18A. And we see that the new square takes in exactly the complete entrance arrangement, including the part that lay outside square 5-6-7-8.

Thus we have covered the area of the whole ground-plan. Line 5-6 contains the rear, the vertical sides of the basic square (1-4 and 2-3) contain the long sides of the building, and line 17A-18A contains the entrance.

In square 17-18-19-20 we execute the sacred cut. We are particularly interested in the two vertical lines. These are 21A-21 and 23A-23. Produced to the entrance of the building, they mark the total width of the approach—passing through the middle of the two rows of pillars that line the entrance and through the square butt-end pillars on either side of the doorway.

The extensions are to points 22 and 24 respectively.

We have concentrated in the Pantheon's ground-plan so far on the horizontal and vertical lines of the diagram and the interplay of the various squares.

But a circular building of this type is naturally influenced considerably in dimensions by the circles within and out-with the respective squares. A square, we recall, has two constructive circles: one described around the outside, the other inscribed within the square.

Before examining the applications of the circle, however, we require to add one more square to our diagram. It is the basic square's half-size version (seen as

35-36-37-38), and its associate circles have an important role to play in the building's lay-out.

We now have four squares placed concentrically within each other, and we begin our study of their circles with that of the outside square.

Square 5-6-7-8's inner circle was a deciding factor in planning the rear of the ground-plan, being the constructive link between the basic square and its double-size version.

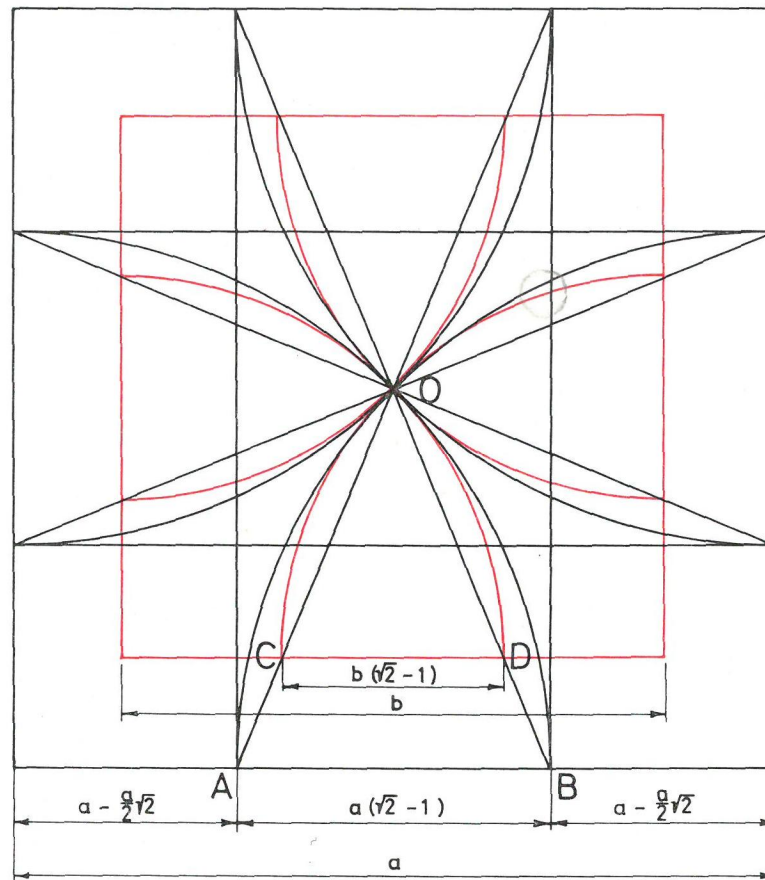
Being the inside circle of the large, outside square, it is the outside circle of the next (basic) square: 1-2-3-4. This same square's *inside* circle, we see, follows the curve of the outer wall of the Pantheon and was obviously the factor that determined the outer dimensions of that wall.

This circle is simultaneously the outside circle of the basic square's half-size version: 35-36-37-38. This square's *inner* circle provides the absolute net floor area in the large main hall. The circle touches the outside of the two large columns on either side of the altar (on the left of Fig. 238). The actual perimeter of the inner hall is broken by a series of recesses and projecting walls.

Our attention turns now to the inner square created by the combination of sacred cuts in the large outer square. The small square is 17-18-19-20. We observe how its outside circle forms the back of the small niches cut into the projecting wall all the way round the main hall. The arc of the same circle positions the pillars at the mouth of each of the six main recesses in the hall.

As we saw with earlier Greek temples, the intersections of existing lines and squares often provide opportunities for entering new lines.

One such intersection in the Pantheon's ground-plan is the meeting of the square created by the sacred-cut combination (17-18-19-20) and the basic square's half-



$$\triangle AOB \sim \triangle COD \Rightarrow a(\sqrt{2} - 1) \frac{b}{\frac{a}{2}} = b(\sqrt{2} - 1)$$

Fig. 247.

size version (35-36-37-38). Their junction produces four tiny squares, one of which is 33-17-34-35.

Taking the diagonal (33-34) of this square as the basis of a new circle with the same centre as the previous, we see how the new circle follows the inside sweep of the main wall of the circular hall. And the distance between this circle and that drawn within the basic square gives the total thickness of the wall.

The various concentric circles we have

examined are so much a part of the structural plan of the Pantheon that they represent a reality that cannot be ignored. There can be no doubt that they were part of the original ground-plan sketched out by Hadrian and his fellow-builders. Not only was Hadrian well aware of the structural value of geometric symbols, he also made full use of them in this sample of his work.

At the centre of the large square the sacred cut forms a rectangle. Vertically

this is 9-11-12-10, and horizontally 13-14-16-15. When we enter the diagonals in these two rectangles (i.e. 9-12, 11-10, etc.) we observe that they pass exactly through the middle of the eight windows in the outer wall of the church.

This is a construction we have not seen previously. What are its characteristics? Is it not usual for lines to possess a certain geometric property?

A close examination will, however, reveal that these lines do indeed have a purpose, a geometric birthright. Hadrian did not include them in his plan accidentally.

The diagonal lines under discussion have the following property: their intersections coincide with the sacred cut in any square sharing the same centre and axis as the basic square.

Whether this discovery was made by Hadrian or whether by some earlier geometer is difficult to state categorically. Only the existence of another, earlier building showing the same construction would prove the latter.

That the discovery is correct is illustrated in Fig. 247 which shows the geometric construction and corresponding arithmetical calculation.

We move on in the analysis of the ground-plan to a new diagram, having obtained the majority of the principal dimensions in the church from the preceding diagram.

The object of the next two analyses is to demonstrate how a similar analysis of the same squares provides further information on the recesses and column spacing within the main hall.

In Fig. 248 we start with the basic square 1-2-3-4, and construct its half-size version 35-36-37-38. This is a repeat of the previous analytical diagram, but in this case we go down one stage further, constructing yet another half-size square: 39-40-41-42.

This new square is divided 3×3 in the same manner as executed in the Greek temple analyses. The simplest method of doing this is to enter the diagonal cross and the acute-angled triangle. The intersections of these two figures indicate the 3-part dividing lines. Vertically these lines are 47-48 and 49-50. We see immediately that they indicate in the rear wall the width of the semi-circular niche in which the altar is situated. The lines run through the centre of the two columns flanking the entrance.

At the opposite side of the temple the corresponding lines mark, as I believe was intended, the net width of the hall entrance.

The same lines horizontally (43-44 and 45-46) show the same thing: they indicate the width of the two niches at right and left of the main hall.

The 3-part division just executed constructs a new small square at the centre of the diagram, and in this square, too, we enter the 3-part dividing lines. They are (horizontally) 51-52 and 53-54. The vertical lines are not required.

We recall that 3-part division was the most common mode of spacing pillars in earlier Greek temples.

If we produce lines 51-52 and 53-54 across the diagram, we see that they were apparently responsible for positioning the columns in the niches on the right and left of the hall.

Fig. 249 shows in effect the same diagram, the difference being that the inner square has been swung through 45° . In other words, square 39-40-41-42 corresponds in the new diagram to square 55-56-57-58.

We see how the 3-part division of this square was used to mark the maximum width (at the rear) of the four rectangular recesses in the main hall. When this width is linked to the centre of the diagram in the form of radii we see that the

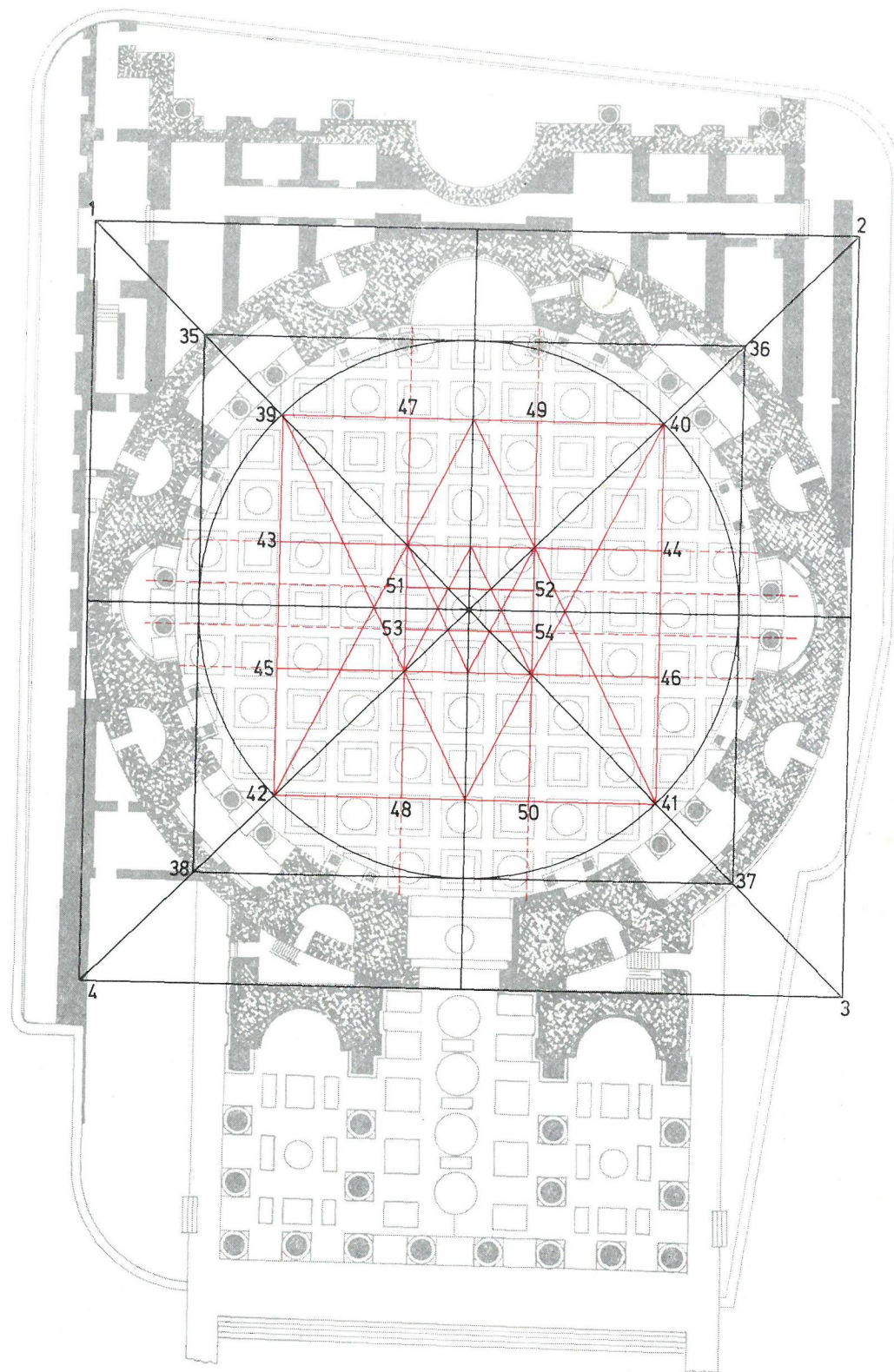


Fig. 248.

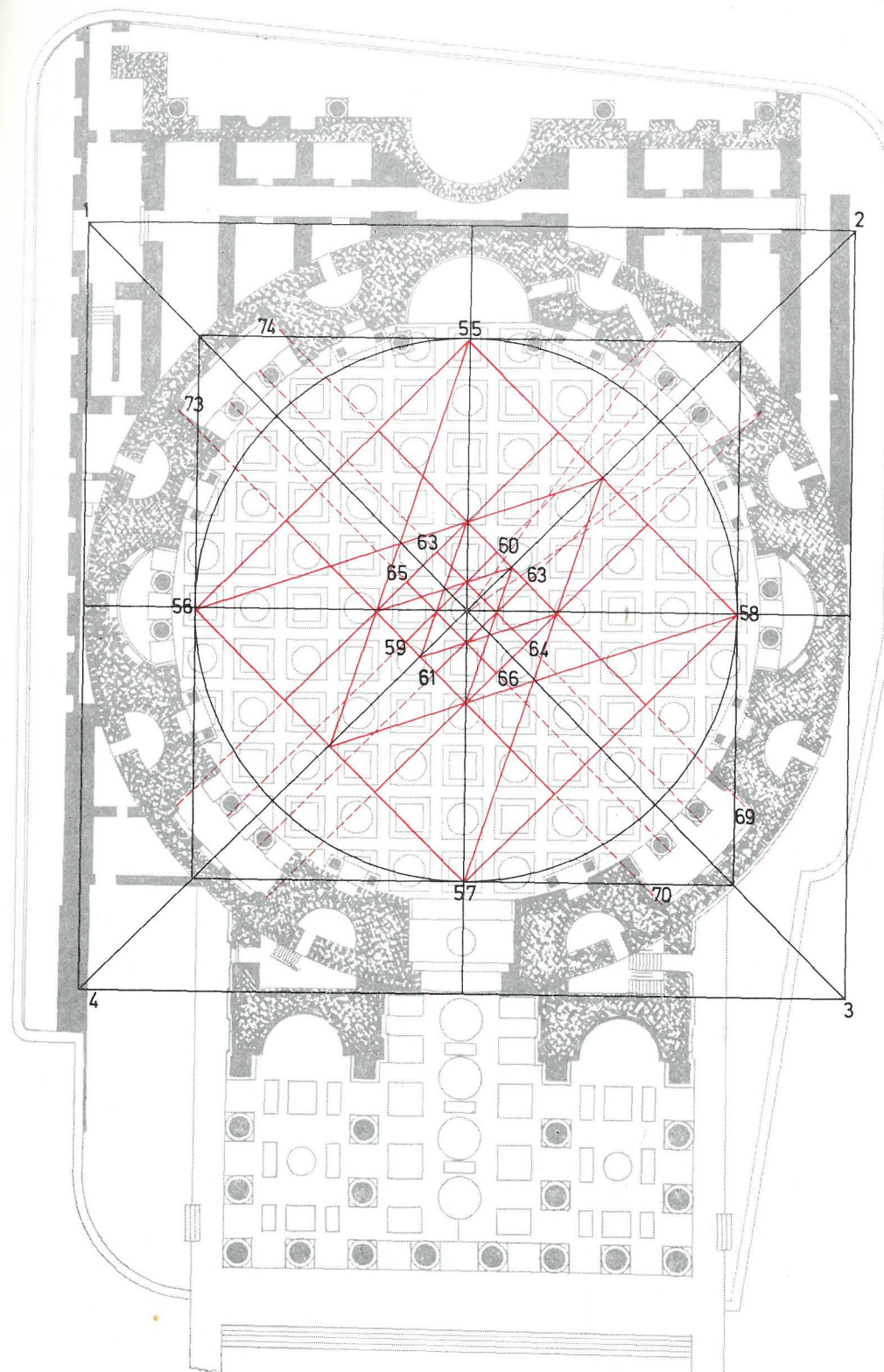


Fig. 249.

angle matches perfectly the angle of the recess wall.

The small inner square created at the centre of the diagram by the lines of 3-part division is also divided 3×3 , and we see how the resultant lines appear to have been the factor that positioned the two columns placed before each of the four rectangular recesses.

We see this carried out, for example at recess 69-70. The dividing lines are extensions of 63-64 and 65-66.

As in previous analysis, a mass of detail is still hidden geometrically in the diagram, but we have nevertheless traced as many of the main lines as permit us—as

long as we remember the order and manner in which Hadrian applied the symbols—to reconstruct the chief structural features of the Pantheon. We require neither measurements nor drawings. Only a full appreciation of the circles and squares of ancient geometry.

A continued analysis would certainly provide us with a wealth of additional information. The diagrams have not been exhausted, possibilities are many. But as emphasised previously, the aim and intention of this book is not to explain every line of any specific building. It is to demonstrate the presence of ancient geometry in planning a particular building.

The Golden Section versus the Sacred Cut

WE HAVE made free and frequent use throughout this book of a completely new term in geometry: the (by now) familiar *Sacred Cut*, a label applied by the author.

This geometric newcomer distinguishes itself from other similar terms in that it is not simply a geometric or mathematical phenomenon; it was a concrete factor in the sphere of building right from the earliest days of religious constructions through time as far as the Middle Ages.

Notwithstanding that the *sacred cut* existed as one of the chief factors in apportioning dimensions to almost all monumental structures of the past and can be traced and revealed in those examples of such structures as (from the point of view of preservation) lend themselves to study, the Temple and later the Church were completely successful until now in keeping secret from uninitiated the mathematical knowledge that the term conceals. Indeed the *sacred cut* and ancient geometry, still hidden from the casual observer, died an unnoticed death the day that building, design and construction work were wrested from the hands of the religious orders and passed instead to professional builders ignorant of the training and tradition of the cloister and temple.

We have studied the practical application of the *sacred cut* in the planning of the Great Pyramid of Egypt, we have picked it out from among the building

instructions for Moses' tabernacle in the Midian desert, and we have seen it employed as undoubtedly one of the principal motives of design in building temples of antiquity and religious structures of the Middle Ages.

In fact the *sacred cut* has proved itself the corner-stone of the entire building industry of ancient times in precisely the same way as it represents the main factor in ancient geometry itself, the subject to which this book is devoted. The ancient system of geometry can almost be regarded as a direct development of meditation on the *sacred cut* and the sacred number seven, and in the same way as these latter it has remained part of the early Church's occult teaching.

The phenomenon arose, as we saw, at an immeasurably early stage in Man's mathematical speculation, and assisted the developing geometer to calculate the circumference of the circle to within an error of less than 1 %. The *sacred cut* has the property of being explicable to any intelligent observer devoid of mathematical knowledge and experience as we understand it today.

The concept of the *sacred cut* is so devastatingly simple, requiring only a primitive system of numbers and a reasonably intelligent operative to apply it successfully, that it is scarcely surprising that it was observed and recorded so early in