

Let us now calculate the same succession of intervals beginning from the dominant G, in such a way as to realize a scale of G major similar to the previous one. If we bring these new notes within the range C-2C we observe that, compared to the scale of C major, only the notes F and A have changed:

C	D	E	α F	G	β A	B	2C
1	9/8	5/4	45/32	3/2	27/16	15/8	2
	9/8	10/9	9/8	16/15	9/8	10/9	16/15

where: $\alpha = 135/128$ (chromatic semitone), $\beta = 81/80$ (syntonic comma).

If we carry out the same operation, though this time beginning from the subdominant F (to give the scale of F major), we will note that, again compared to C major, the only two notes changed are D and B (same symbols):

C	D/ β	E	F	G	A	B/ α	2C
1	10/9	5/4	4/3	3/2	5/3	16/9	2
	10/9	9/8	16/15	9/8	10/9	16/15	9/8

If we extend these operations to the 15 major keys contained in the chain of 5ths C $\flat$ -C $\sharp$ we obtain the 15 scales of Fig. B.4.7c. This figure also provides us with the structure of Brown's "Natural Finger-board", given that the rectangles represent the individual keys (those with four letters are white, those with only three are black). The little circles at the top left represent further small cylindrical button keys (coloured in red, visible also in Figs. B.4.7a-b) that serve to provide the relative minor keys for every major scale with a leading tone (e.g. G $\sharp$ for A minor). Between these leading tones and the respective tonic the interval is a diatonic semitone (e.g. G $\sharp$ -A = 16:15). It goes without saying that with this symmetrical arrangement the fingering was the same in all keys.

This keyboard also had didactic aims, for on it, apart from the interval of the syntonic comma, one can also find the following two microintervals:

- the schisma, i.e. the amount by which the Pythagorean comma is greater than the syntonic comma. It is found, for example, between keys such as G $\sharp$ (= α G) and A $\flat$ (= A/ α): if from the minor tone (10/9) we subtract two chromatic semitones (135/128) we obtain: $10/9 \times (128/135)^2 = 2^{-15} \times 3^8 \times 5 = 1.9537... \text{ cents}$.
- the Pythagorean comma, i.e. the amount by which six major tones are greater than an octave. Given that intervals of the type D/ $\alpha\beta$ -2 $\alpha\beta$ C are precisely equal to six major tones, it is immediately evident that this comma is found between D/ $\alpha\beta$ and $\alpha\beta$ C.

With Brown the discoveries of the English-speaking theorists in the field of just intonation seem to come to an end. They continued, however, though for a short time only, in Germany. Joachim Steiner's harmonium, described in 1888, was based

on Liston's system and equipped with an ordinary keyboard. Its harmonic range – again of the syntonic type – could be mechanically shifted through twelve pure 5ths in such a way as to adapt to different keys. Following in his tracks was Shohè Tanaka, who had a more ambitious "Enharmonium" built in Berlin by Johannes Kewitsch in 1890, again provided with an ordinary keyboard, but with 'split' chromatic keys.¹¹⁹

B.5. The Pythagorean and syntonic systems reconciled, 1780-1875

B.5.1. *Roussier*. Well-known is the fundamentalist stand taken by the Abbé Pierre-Joseph Roussier (1716/7-1792), who upheld the Pythagorean tuning 'of the ancient Greeks', in opposition to just intonation (Section C.6.1). In 1782 Georges Cousineau and son were completing their new harp with 7 $\times$ 2 pedals, by means of which each of the seven notes of the octave could be turned into flat, natural or sharp; this was done by modifying the vibrating length of the corresponding string: the range thus increased to (7 $\times$ 3 =) 21 notes per octave (F $\flat$ -B $\sharp$). For the temperament to be used, the Cousineau family turned to Roussier, who obviously instructed them to make the twenty 5ths of the chain F $\flat$ -B $\sharp$ perfectly pure.¹²⁰

In imitation of this harp – and again following Roussier's instructions – in 1782 Jacques Görmans (better known as "Germain") built for Jean-Benjamin de Laborde (1734-1794) a similarly tuned harpsichord (F $\flat$ ⁰-B $\sharp$ ⁰). In this case each of the seven notes of the octave was provided with split keys for the sharps and flats. Roussier justified this arrangement also as a means of ensuring that "all the modes must be equal to one another", which was something that did not occur in the irregular temperaments then used in France.¹²¹ Baron de Prony and Pierre Erard, who had occasion to hear the instrument, agreed that the effect was good when played melodically (thanks also to the raised leading notes, they point out), but instead when played harmonically "the harshness of the thirds wounded the ear" ("la dureté des tierces blessait l'oreille"). De Prony – who, like Erard, wrote in 1815 – also mentions that he had heard the instrument "less than thirty years earlier". It was to be found in a "room full

¹¹⁹ On these instruments see: Tanaka 1890: 18-27 (a work in which the exponential notation applied to notes arranged hexahedrally, also adopted in the tables of the present article, is used for the first time); Barbour 1951: 113; Vogel 1975: 317-23. Three usual keyboards had on the contrary the just intonation harmonium, with 48 notes per octave, that the firm Appunn (Hanau) built in 1887 for Pietro Blaserna, an Italian physicist who had studied in Austria and France; this large instrument is still preserved in Rome, University "La Sapienza", Physics Department, Museum (Conti 2008: 51, photo).

¹²⁰ Roussier 1782a: 7, 29-30; *Nouvelle harpe* 1782. The size of the chromatic-enharmonic semitones was determined by the position of the mobile bridges moved by the pedals; this position, however, could be regulated beforehand and hence adapted to any type of temperament, even equal temperament.

¹²¹ Roussier 1782b: 4 ("le rapport de cette tierce sera de 64 à 81, comme l'ont défini les Grecs, et non de 64 à 80, comme l'évaluent faussement les Modernes") and in the preface by Laborde: "tous les modes [...] doivent être égaux entre eux"; *Clavecin* 1782: "Ce nouveau clavecin est accordé comme la harpe de M. Cousineau, par une suite de quintes justes".

of curious objects collected by the late Monsieur de la Blancherie":¹²² most likely it had been deposited there at the outbreak of the Revolution, when Laborde had to abandon Paris to take refuge in Rouen (a measure that did not save his life, however, for in 1794 he had the misfortune to be one of the last people to be guillotined, only five days before the similar fate of Robespierre).¹²³ Of the harpsichord we have no further news.

B.5.2. Sanchez and Hosseschrueders. In 1823 a certain Joaquin Sanchez de Madrid proposed a similar instrument, though without showing signs of knowing the one made by Roussier (Fig. B.5.1).¹²⁴ On a manuscript sheet he adds that, following his instructions, a square fortepiano of this type had just been made in Spain.¹²⁵ The builder was Juan Hosseschrueders, who was of Dutch origin and had been in Madrid, at the address indicated in the manuscript, since 1807.¹²⁶ Sanchez includes a drawing showing that every diatonic key is followed by a chromatic one similar to that of an ordinary keyboard, but divided lengthwise into two parts, each of different length and colour. The overall arrangement of the keys, which strangely does not adopt a strict order of increasing frequency, is the following: C^0 - $C^\sharp$ ⁰- D^0 - $D^\sharp$ ⁰- E^0 - $E^\sharp$ ⁰- F^0 - $F^\sharp$ ⁰- F^0 -...¹²⁷ Sanchez points out that this arrangement is totally symmetrical, and he also shows an awareness that intervals such as C^0 - F^0 ($\cong C^0$ - E^{-1}) are equal to almost pure major 3rds. The practical application of this observation, however, is the preserve of the authors that we shall now analyze.

B.5.3. Von Wiese. Like Abbé Roussier, Baron Christian Ludwig von Wiese (1732-1800) was also a convinced 'new Greek' of the classical period, even though (as we shall see) he showed greater pragmatism and acumen than his French predecessor. His adherence to the Pythagorean system led him to consider the just major 3rd (5:4) as nothing but a diminished 4th, adding: "I believe that posterity will laugh loud at our idolatry for this major mediant consonant with a diminished fourth", which he

¹²² *Rapport* 1834: 17 (in this *Rapport*, drawn up 1815, the harpsichord was in a "cabinet d'objets curieux rassemblés par feu M. de la Blancherie"); Prony 1834: 26-7. Énard, however, says that each "touche noire" was divided "dans sa longueur", whereas for De Prony it was divided "transversalement".

¹²³ Fend 2001.

¹²⁴ Sanchez 1823, with a plate showing the arrangement of the keys.

¹²⁵ In a copy of the same work preserved in Madrid, Biblioteca Nacional (shelf-mark M.849), an autograph dedication of about ten pages is inserted after the title-page, addressed to the "Serenísimo S.^{or} D.ⁿ Francisco de Paula Infante de España". Here, among other things, we read: "Ultimamente en esta sorte ha construido el S.^{or} Hoseschruders constructor de pianos en la Calle de la Luna Casa q.^a fue Banco, un fortepiano perfecto baso mi sistema y direccion en la clase de los llamados quadrilongos".

¹²⁶ Information on Juan Hosseschrueders is to be found in Clinkscale 1995: 153 (though no mention is made of this particular instrument).

¹²⁷ It is worth stressing that these chromatic keys appear as divided lengthwise, like those of Roussier's harpsichord (at least judging from Énard's evidence, see fn. 122).

also called "sweetened major third" ("adoucie").¹²⁸ In fact we have just seen, for example, that C^0 - $F^0 = C^0$ - $E^{-12/11} \cong C^0$ - E^{-1} (approximating the Pythagorean comma to 11/12 of a syntonic comma). Von Wiese is therefore the first to call explicit attention to what less than a century later would be known as "Helmholtz's Theorem": in a succession of pure 5ths, every two notes separated by eight 5ths give rise to an almost just major 3rd (i.e. narrowed by only 1/11 of comma, a quantity equal to 2 cents).¹²⁹

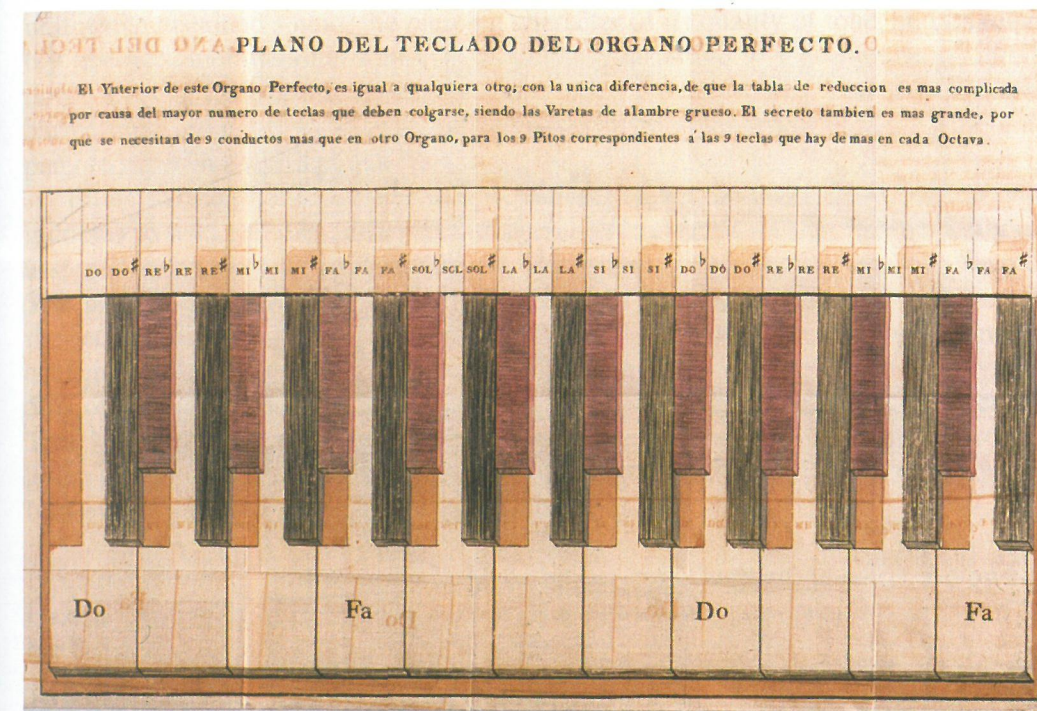


FIGURE B.5.1. New keyboard proposed by Joaquin Sanchez for a "Perfect organ", with Pythagorean tuning (Sanchez 1823: plate). In the wording above, he states that the instrument's structure is standard, except that – owing to the nine keys per octave more than the ordinary keyboard – "the roller-board is more complicated than the others" and "the pallet-box is also bigger". With the same key arrangement, Sanchez had already had a fortepiano constructed at Madrid.

On these foundations von Wiese had a harpsichord with 17 keys per octave built by Carl Gottlob Sauer, who was also from Dresden, in around 1791.¹³⁰ Though tuned with strictly pure 5ths, its harmonic range (A^0 - B^0) and a special arrangement of split

¹²⁸ Wiese 1795: 18 ("je crois que la posterité rira bien de nôtre idolâtrie harmonique pour cette médiate maieure en assonance d'une quarte diminuee"), 14.

¹²⁹ Bosanquet 1876: 10 ("Helmholtz's Theorem").

¹³⁰ Wiese [1791]: 27-31. In the works cited this writer also proposes certain new temperaments (Barbieri 1987a: 245-8).

keys pragmatically also made it possible to put at the player's disposal a syntonic version of the main triads always neglecting the schisma (Table B.5.1 and Fig. B.5.2). Although no document points this out, it is curious to note that this could have easily been obtained on the harpsichord designed by Roussier by simply exchanging the sharps and flats. But unlike Roussier's instrument, Sauer's cannot have been widely known, given that von Wiese is the only person to mention it.¹³¹

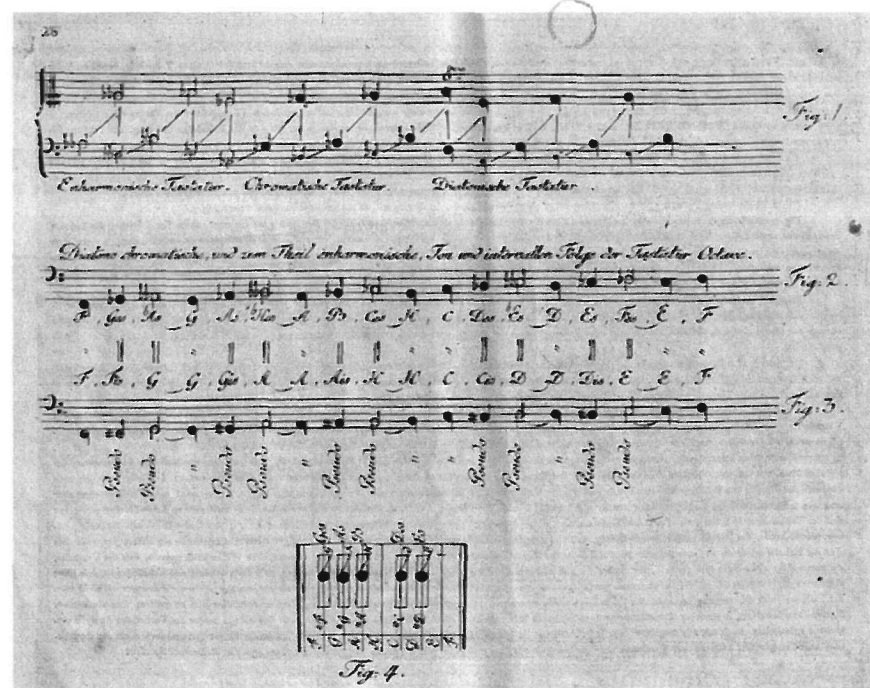


FIGURE B.5.2. Enharmonic harpsichord made in Dresden by Gottlob Sauer; designed by Baron von Wiese, it was tuned in perfect 5ths (Wiese [1791]: 26). The Pythagorean-syntonic equivalences are: $A\flat^0 = G^{-12/11}$, $E\flat^0 = D^{-12/11}$, $B\flat^0 = A^{-12/11}$, $F\flat^0 = E^{-12/11}$, and $C\flat^0 = B^{-12/11}$. See Table B.5.1.

$G^{-12/11}$	$D^{-12/11}$	$A^{-12/11}$	$E^{-12/11}$	$B^{-12/11}$	$F\sharp^{-12/11}$	$C\sharp^{-12/11}$	$G\sharp^{-12/11}$	$D\sharp^{-12/11} = E^0$
$[E^0]$	B^0	F^0	C^0	G^0	D^0	A^0	E^0	B^0

TABLE B.5.1. Baron von Wiese's harpsichord (1791) was tuned with a chain of sixteen pure 5ths, from $A\flat^0 (= G^{-12/11})$ to B^0 . The table also shows the enharmonic equivalences that allowed the player to avail himself of a series of almost pure triads, i.e. with major 3rds nar-

¹³¹ It was still in his possession in 1795: "l'instrument à touches enharmoniques que j'ai fait construire et que je possède" (Wiese 1795: 23). At present Carl Gottlob Sauer is known only for a "two manual harpsichord and tangentflügel" of 1786 (Boalch 1995: 166).

rowed only by only 2 cents ($1/11$ of syntonic comma) and the minor 3rds widened by the same amount.

B.5.4. *Helmholtz*. In his celebrated *Tonempfindungen* moves further along the path indicated by Baron von Wiese, an author whom he shows no sign of knowing, however. His experiments were carried out on an instrument then of recent invention, the harmonium, since "among musical instruments, the harmonium, on account of its uniformly sustained sound, the piercing character of its quality of tone, and its tolerably distinct combinational tones, is particularly sensitive to inaccuracies of intonation". Among the various models on the market he resorted to a large two-manual harmonium produced by J. & P. Schiedmayer of Stuttgart and retuned by the company to his own specifications.¹³²

From Tables B.5.2-3-4 we easily observe that his tuning follows von Wiese's guidelines, with the only difference that von Wiese tuned all the 5ths perfectly and assigned the schismatic deviation to the 3rds, whereas Helmholtz prefers to keep the 3rds perfect and narrow by a schisma just the two 5ths that connect the first line with the second and the second with the third. At a later stage he advises distributing this schisma uniformly among the eight 5ths of each line; in this way all the twenty-three 5ths of the chain would be narrowed by only $1/8$ of schisma, i.e. about 0.25 cents.¹³³ This last operation, of course, is purely nominal and extremely difficult to realize, given that in the central area of the keyboard each of them should generate about one beat every nine seconds!

B.5.5. *Guérout*. A similar path was taken by Georges Guérout, to whom we owe the French translation of Helmholtz's work. His harmonium presented the tuning of Table B.5.5.¹³⁴

Guérout made use of this instrument to disprove the claim made by the two French physicists Cornu and Mercadier in 1869. Their findings led to the conclusion that musicians preferred Pythagorean to syntonic tuning melodically, whereas the second of the two was preferred harmonically.¹³⁵ His harmonium was perfectly suited to making such a test, given that the same melody could be compared on it in the two types of tuning, under identical conditions. From the tests carried out in Paris around 1870 it turned out that the just scale was preferred melodically as well. The judges consisted of various well-known musicians, such as the baritone Enrico Delle Sedie, then a singing teacher at the Paris Conservatoire (1867-71), the singer "Madame Delle Sedie" and M.E. Souzay, professor of violin at the Conservatoire.¹³⁶

¹³² Helmholtz 1885: 316.

¹³³ Helmholtz 1885: 317.

¹³⁴ Guérout 1874; [Id.] *Instruction*.

¹³⁵ Cornu & Mercadier 1869.

¹³⁶ Guérout 1874: 622-3. However, on the problem of intonation in these years, see Section C.6.3.

$G\sharp^{-2}$	$D\sharp^{-2}$	$A\sharp^{-2}$	$E\sharp^{-2}$	$B\sharp^{-2}$	$F\sharp\sharp^{-2}$	$C\sharp\sharp^{-2}$	$G\sharp\sharp^{-2}=A^{-10/11}$
E^{-1}	B^{-1}	$F\sharp^{-1}$	$C\sharp^{-1}$	$G\sharp^{-1}$	$D\sharp^{-1}$	$A\sharp^{-1}$	$E\sharp^{-1}=F^{+1/11}$
C^0	G^0	D^0	A^0	E^0	B^0	F^0	C^0

TABLE B.5.2. Tuning of Helmholtz's experimental harmonium, equipped with two ordinary keyboards of 12 notes per octave. Of the resulting 24 notes per octave, those printed in roman type in the table are found on the lower manual, those in *italics* on the upper. The twenty-three 5ths from $G\sharp^{-2}$ to C^0 are all pure, except for $A^{-10/11}$ - E^{-1} and $F^{+1/11}$ - C^0 , which are narrowed by a schisma, i.e. like those of ordinary equal temperament. For a different 'reading' of the same system see the two following tables.

$A\flat^{-10/11}$	$E\flat^{-10/11}$	$B\flat^{-10/11}$	$F^{-10/11}$	$C^{-10/11}$	$G^{-10/11}$	$D^{-10/11}$	$A^{-10/11}=B\flat^{+2/11}$
$F\flat^{+1/11}$	$C\flat^{+1/11}$	$G\flat^{+1/11}$	$D\flat^{+1/11}$	$A\flat^{+1/11}$	$E\flat^{+1/11}$	$B\flat^{+1/11}$	$F^{+1/11}=G\flat^{+13/11}$
$D\flat^{+12/11}$	$A\flat^{+12/11}$	$E\flat^{+12/11}$	$B\flat^{+12/11}$	$F\flat^{+12/11}$	$C\flat^{+12/11}$	$G\flat^{+12/11}$	$D\flat^{+12/11}$

TABLE B.5.3. Same tuning as Table B.5.2, rewritten in such a way as to demonstrate the practicability of the keys with flats. See also the following Table B.5.4.

$F\flat^0$	$A\flat^{-1}$	$E\flat^{-1}$	$B\flat^{-1}$	F^{-1}	C^{-1}	G^{-1}	D^{-1}	$A^{-1}=B\flat^{+1/11}$
$D\flat^{+1}$	$A\flat^{+1}$	$E\flat^{+1}$	$B\flat^{+1}$	$F\flat^{+1}$	$C\flat^{+1}$	$G\flat^{+1}$	$D\flat^{+1}$	$F^0=G\flat^{+12/11}$

TABLE B.5.4. Same as Table B.5.3, in which all the notes have been lowered by a schisma (1/11 of syntonic comma), in a way that simplifies reading.

C^{-1}	G^{-1}	D^{-1}	A^{-1}	E^{-1}	B^{-1}	$F\sharp^{-1}$	$C\sharp^{-1}$	$G\sharp^{-1}=A\flat^{+1/11}$
$E\flat^0$	$B\flat^0$	F^0	C^0	G^0	D^0	A^0	E^0	B^0

TABLE B.5.5. Tuning of Guérault's harmonium (Paris, c1862-70). Like Helmholtz's it had two ordinary manuals; the 24 notes per octave were tuned with a chain of 5ths ($F\sharp^{-2}$ - B^0), all pure except for $F^{-10/11}$ - C^{-1} and $A\flat^{+1/11}$ - $E\flat^0$, which were narrowed by a schisma.

B.5.6. *Bosanquet*. At a meeting held by the London Musical Association on 1 May 1875 an enharmonic organ was presented by Bosanquet (1841-1912), professor of acoustics at Oxford University. It had an innovative keyboard and only two stops, each consisting of a metal "stopped diapason" with caps that could be adjusted by screws for fine-tuning.¹³⁷ In this way two different temperaments could be selected:

1. "Positive stop" (by this term Bosanquet designates the regular temperaments with 5ths wider than those of equal temperament, i.e. ETS 12). He claims to

¹³⁷ Bosanquet 1876: 38-9, 56-9.

have adopted the "Helmholtz system" – with 5ths narrowed by 1/8 of schisma – and to have doubled the harmonic range, taking it to 48 notes per octave. However, he observes that, if anything, the effort of tempering the 5ths in this way would be justified on the harmonium, with its timbre rich in harmonics, but not on the organ – especially (we might add) if stopped pipes are used, given the lack of beats from the 5ths and 3rds, seeing that in this case the emission is limited to just the odd-numbered harmonics! As a result of this experiment on this instrument he concludes that in future he would always make use of the version with pure 5ths (Table B.5.6).

2. "Negative stop" (i.e. with 5ths narrower than those of equal temperament). For his second stop Bosanquet instead opted for an "extended meantone", i.e. for the 'quarter comma' of Table B.5.7, extended to 36 notes per octave. The use of this temperament is not as surprising as it might seem; for in Great Britain up to the 1860s it was still used, for example, on the "concertina", a type of accordion equipped with the enharmonic doublings $E\flat/D\sharp$ and $G\sharp/A\flat$.¹³⁸

$D\sharp^{-3}$	$A\sharp^{-3}$	$E\sharp^{-3}$	$B\sharp^{-3}$	$F\sharp\sharp^{-3}$	$C\sharp\sharp^{-3}$	$G\sharp\sharp^{-3}$	$D\sharp\sharp^{-3}=E^{-21/11}$
B^{-2}	$F\sharp^{-2}$	$C\sharp^{-2}$	$G\sharp^{-2}$	$D\sharp^{-2}$	$A\sharp^{-2}$	$E\sharp^{-2}$	$B\sharp^{-2}=C^{-10/11}$
G^{-1}	D^{-1}	A^{-1}	E^{-1}	B^{-1}	$F\sharp^{-1}$	$C\sharp^{-1}$	$G\sharp^{-1}=A\flat^{+1/11}$
$E\flat^0$	$B\flat^0$	F^0	C^0	G^0	D^0	A^0	$E^0=F\flat^{+12/11}$
$C\flat^{+1}$	$G\flat^{+1}$	$D\flat^{+1}$	$A\flat^{+1}$	$E\flat^{+1}$	$B\flat^{+1}$	$F\flat^{+1}$	$C\flat^{+1}=D\flat^{+23/11}$
$A\flat^{+2}$	$E\flat^{+2}$	$B\flat^{+2}$	$F\flat^{+2}$	$C\flat^{+2}$	$G\flat^{+2}$	$D\flat^{+2}$	$A\flat^{+2}$

TABLE B.5.6. Tuning of the "Positive stop" of Bosanquet's experimental organ (1875). It extended the tuning already used by Helmholtz on his harmonium (see Table B.5.2) to 48 notes per octave. Again in this case, all the forty-seven 5ths (from $D\sharp^{-3}$ to $A\flat^{+2}$) are pure, except for those of the passage from one line to another (e.g. $D\sharp\sharp^{-3}$ - B^{-2} = $E^{-21/11}$ - B^{-2}), which were narrowed by a schisma. Initially, following Helmholtz, Bosanquet had narrowed all the forty-seven 5ths by 1/8 of schisma, but he later abandoned this solution after finding the effort unrewarding.

¹³⁸ Ellis also made use of it for his experiments (Ellis 1885: 470). The "English concertina" had been invented around 1820-30 by the physicist Charles Wheatstone (1802-75) and had become very popular in a short time (Atlas 2001). Free-reed instruments emit a high number of harmonics, as a result of which in Great Britain equal temperament was avoided because it produces an effect that was disturbing to many: in any case, by the early 1860s, Wheatstone & Co. were obliged to convert it to equal temperament, a conversion that was also happening for flue-pipe organs (Berlioz 2003: 466, fn. 2). On the contrary, the "German concertina" never presented the above enharmonic doublings, being tuned to equal temperament from its very start (Berlioz 2003: 471). The conservative spirit of the English is also confirmed by the fact that William Wheatstone, the brother of Charles and a builder of musical instruments, at the Great Exhibition held in London in 1851 displayed "an enharmonic tonometer, which produces any sound in the enharmonic scale" (*Great Exhibition* 1851: 469, no. 526).

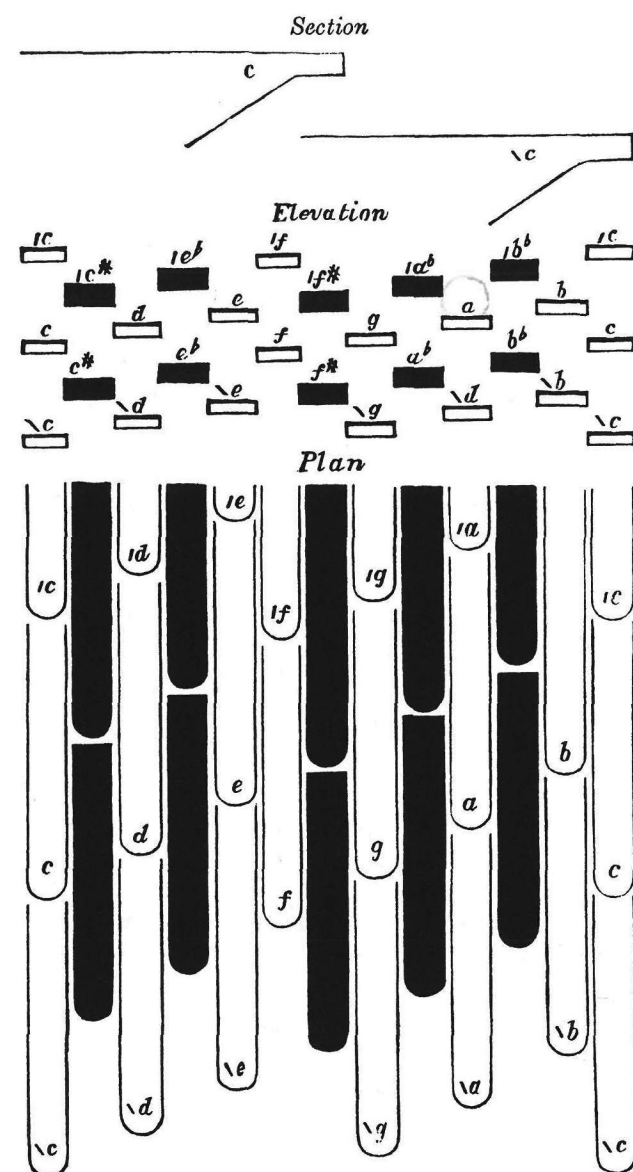


FIGURE B.5.3a. "Generalized keyboard" of Bosanquet, 1872 (Helmholtz 1885: 429 = Bosanquet 1876: 23). As one sees, the keys are arranged symmetrically, while the black and white colouring serves only to make identification easier. This keyboard was common to the instruments that Bosanquet based on just intonation, meantone and – as we shall see in Section B.6 – ETS 53.

The fingering was thus the same for all keys. Following the notation in the figure, for the succession of the major triads we have (both in just intonation and in its approximation with ETS 53): $c-e-g$ ($=C^0-E^{-1}-G^0$), $c^\sharp-f-a^\flat$ ($=C^\sharp^0-E^\sharp^{-1}-G^\sharp^0 \cong D^\flat^{+1}-F^0-A^\flat^{+1}$), $d-f^\sharp-a$ ($=D^0-F^\sharp^{-1}-A^0$), $e^\flat-g-b^\flat$ ($=E^\flat^0-G^{-1}-B^\flat^0 \cong D^\sharp^{-1}-F^\sharp^{-2}-A^\sharp^{-1}$), $e-a-b$ ($=E^0-G^\sharp^{-1}-B^0$), Similarly, for the minor triads we have: $c-e^\flat-g$ ($=C^0-E^\flat^{-1}-G^0$), ...

$D^\flat^{+3}$	$F^\flat^{+2}$	$A^\flat^{+1}$	C^0	E^{-1}	$G^{-1/4}$	$B^{-5/4}$	$D^{-9/4}$	$F^{-6/4}$	$A^{-10/4}$	$E^{-11/4}$	$G^{-15/4}$	$B^{-19/4}$	$D^{-23/4}$	F^{-5}	$C^{-21/4}$	$G^{-22/4}$	$D^{-23/4}$
$A^\flat^{+11/4}$	$C^\flat^{+7/4}$	$E^\flat^{+3/4}$	$G^\flat^{+1/4}$	$B^{+2/4}$	$D^{+5/4}$	$F^{+1/4}$	$A^{-3/4}$	$C^{-7/4}$	$E^{-11/4}$	$G^{-15/4}$	$B^{-19/4}$	$D^{-23/4}$	F^{-5}	$C^{-21/4}$	$G^{-22/4}$	$D^{-23/4}$	

TABLE B.5.7. Tuning of the "Negative stop" of Bosanquet's experimental organ (1875). It extends the temperament of the 'quarter comma' to the thirty-five 5ths $D^\flat^{+3}-D^\sharp^{-23/4}$. It is worth noting that this involves a return to the typical Renaissance scheme of four notes per line (see Table B.1.3), as opposed to the eight notes per line of the "Positive stop" of Table B.5.6.



FIGURE B.5.3b. "Generalized keyboard" of Bosanquet's harmonium (courtesy of Science Museum, London, photo no. 10213657).

As already noted, the organ was also equipped with a symmetrical keyboard of a new type ("generalized keyboard"), by means of which it was possible to play in the

various keys availing oneself of just one type of fingering (Fig. B.5.3a-b).¹³⁹ Given that Poole's has survived only on paper, Bosanquet's was therefore the first enharmonic symmetrical keyboard to be made.¹⁴⁰ To give a demonstration of its practicality, in the course of the meeting Bosanquet played three preludes from the *Wohltemperirte Clavier* (nos. 1 and 2 from Book I, and no. 9 from Book II), not followed, however, by their respective fugues, given that the keyboard was limited to just three octaves.¹⁴¹ In his paper he says that on both stops it was also possible to find good approximations of the harmonic minor 7th (7:4) by choosing either the necessary commatic scaling on the "positive stop", or enharmonic equivalences of the type C-A# (instead of C-Bb) on the "negative stop".¹⁴²

It should be added that, towards the end of the century, Bosanquet's 'positive' tuning was adopted in a harmonium built by Karl Andreas Eitz (1848-1924), a German mathematician and teacher of music. Eitz also adopted Bosanquet's 'generalized keyboard', marking its keys, however, with the syllables of an extremely elaborate solmization system of his invention. Far too complicated, the above Eitz method was formally banned in Prussia between 1914 and 1925. His harmonium, before the Second World War, was in the Hochschule für Musik in Berlin.¹⁴³ Lastly, we should recall that – more than a century after its invention – the principle of Bosanquet's keyboard is still employed and elaborated on, demonstrating its validity.¹⁴⁴

B.6. 'Schismatic' approximations: ETS 53, 1834-80

On some of the instruments described above the divisions of the octave are so densely concentrated that certain notes have only a schisma between them, i.e. just two cents, or even less: see, for example, Section B.4.6. Thompson (1834) and Poole (1867) had already pointed out that their keyboards could be simplified, in such a way that these notes should coincide with those nearest to them in the temperament that divides the octave into 53 acoustically equal parts.¹⁴⁵ This system, already known

¹³⁹ Bosanquet 1876: 23; Helmholtz 1885: 429-30 (which also reproduces the same illustration of the keyboard already published by Bosanquet, "by permission" of the latter); Ellis 1885: 479-81.

¹⁴⁰ As regards ordinary keyboards, on the other hand, the first is that designed by Caramuel around 1650, which prompted many other similar ones, right down to the 20th century (Barbieri 1987d: 153-6; Id. 1990a: 100-03). See also Section F.3.

¹⁴¹ Bosanquet 1876: 39.

¹⁴² Bosanquet 1876: 42.

¹⁴³ Fickenscher 1941: 360; Partch 1974: 438; Rainbow 2001a.

¹⁴⁴ See e.g. (1) the new instrument built by Motorola for George Secor, a composer from Chicago (it used a Bosanquet keyboard design, but based on ETS 31, with keys of five colours), (2) Erv Wilson's elaboration of the 'generalized keyboard', 1973 (Mandelbaum 1974: 225-6), (3) John S. Allen, *The general keyboard in the age of MIDI* (<http://www.bikexpert.com/music/bosanquet.htm>), (4) various Internet sites, e.g. the one of the Ukrainian Mykhaylo Khramov.

¹⁴⁵ [Thompson] 1834: 5; Poole 1867: 19-22. In fn. 32 above we saw that already in 1808 William Hawkes had made use of 'Mercator's comma', corresponding to a 53rd part of an octave.

to the Chinese and Arabs of Antiquity, had been rediscovered in the West by independent routes by various theorists, among whom the German Nicolaus Mercator (c1620-87).¹⁴⁶ Each of the constituent 53 parts was called a 'comma', given that its size lay between the Pythagorean and the syntonic (Table B.6.1). In the technical literature it is therefore often called the 'Arabic comma' or 'Mercator's comma'.¹⁴⁷ Table B.6.1 also shows that this ETS 53 (= Equal Tempered System 53) is capable of approximating with sufficient precision both the Pythagorean and syntonic systems (see also Fig. B.6.1).

Interval	ETS 53		Syntonic (cents)	Pythagorean (cents)
	'commas'	cents		
comma	1	22.64	21.51	23.46
Pyth. min. 3rd	13	294.34	—	294.13
synt. min. 3rd	14	316.98	315.64	—
synt. maj. 3rd	17	384.91	386.31	—
Pyth. maj. 3rd	18	407.55	—	407.82
5th	31	701.89	701.96	701.96

TABLE B.6.1. Size of certain intervals in the Equal Tempered System 53, syntonic system and Pythagorean system. As one sees, the maximum difference between the ETS 53 and the corresponding values in the other two systems is under a schisma (= 23.46 – 21.51 $\cong$ 2 cents).

B.6.1. *Bosanquet*. In 1885 Alexander Ellis reported that this division was called "Bosanquet's cycle", given that Bosanquet was the first to put it into practice, on a harmonium of 1872-73 (Table B.6.2).¹⁴⁸ This instrument was equipped with the above-described "generalized keyboard", extended to four and a half octaves and equipped with as many as 84 keys per octave, distributed on seven rows. In fact, 31 of the 53 notes per octave were replicated, in such a way that – given that the ETS 53 was a circulating system – when the player arrived at the fifth row he could partially continue without having to make an awkward return to the first row.¹⁴⁹

¹⁴⁶ Barbour 1951: 123-5; Barbieri 1987a: 306 The first person to calculate ETS 53 was the Chinese theorist Ching Fang (78-37 BCE), with a deviation of just 0.14 cents. Using a rapid approximation method, Ching Fang continued with his calculations, discovering that a note equisonant with the starting note can be obtained, with still greater approximation, using ETS 306 (McClain 1979: 208-15).

¹⁴⁷ Altwein 1971.

¹⁴⁸ Ellis 1885: 436.

¹⁴⁹ One of the reasons for using a harmonium, as Helmholtz had already observed (Section B.5.4), was its richness of its timbre, which also made it possible to appreciate the "combinational" sounds. Bosanquet, for example, used the beats produced for tuning the major 3rds of the ETS 53, which are slightly narrowed (Bosanquet 1875: 143).