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THE HERITAGE OF WESTERN CIVILIZATION

Series Editors

Heather L. Reid, *Morningside College*
Davide Iamasi, *University of South Florida*

The cultural and intellectual heritage of Western Greece—the coastal areas of Southern Italy and Sicily settled by Hellenes in the 8th and 7th centuries BCE—is sometimes overlooked in academic studies. Yet evidence suggests that poets, playwrights, philosophers, and other maverick intellectuals found fertile ground here for the growth of their ideas and the harvesting of their work. The goal of this series is to explore the distinctive heritage of Western Greece from a variety of disciplinary perspectives including art history, archaeology, classical literature, drama, epigraphy, history, philosophy, and religion.

Dr. Grant, if
the tradition of monumental
learning - through Greece
will

Philosopher Kings and Tragic Heroes

Essays on Images and Ideas from Western Greece

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Parnassus Press
2016

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several qualified scholars. For a survey of this cultural complex see my contribution "ΕΙ ΚΩΜΩΙΔΕΪΝ de Epicarmo", cit.

- ⁴⁰ Zeno 29B2DK. English translation based on that of Lee (op. cit. p. 19) with important modifications mainly in the understanding of the genitival clause *μεγέθους γὰρ μηδενὸς ὄντος*, which I take as a second hypothetical clause ("for if a magnitude were null"). Only in that way does the argument makes any sense at all, as Gomperz (*Hellenika*, Leipzig, 1912, pp. 297 f.), Fränkel (cit.) and Colli observed (*Zenone di Elea. Lezioni* 1964-1965, Milano, 1998), even though I do not agree with their emendation proposals to the text, which I keep as transmitted.
- ⁴¹ Even Melissus in the far-away Samos apparently also found that Epicharmus's conception of material increase and decrease demanded a detailed refutation. Circumstantiated proof of the logical impossibility of growth and decay, envisaged as a particular case of physical alteration and rearrangement of parts, can be seen in 30B7DK and 30B8DK by Melissus. The last fragment contains a profuse argumentation leading to the conclusion that the transformations we perceive in natural things with our body senses, although seeming to confirm the reality of change, should be excluded by reason on the grounds that they would eventually bring the total ruin of material objects. This train of thought admits of no other explanation than as a reaction against a conception according to which change was an unceasing, blind, fully mechanical process, exactly as we know Epicharmus maintained. On this, see O. Álvarez, *Epicarmo ed i Presocratici. Interazione e polemica* (forthcoming).
- ⁴² Lee, op. cit. pp. 100 ff.
- ⁴³ For the relevant testimonies and a commentary, see Lee, op. cit. pp. 52-55 and 78-83. For a critical examination of this argument, see G. Vlastos, "A Note on Zeno's Arrow", *Phronesis* 11 (1966), pp. 3-18.
- ⁴⁴ Arist. *Phys.* Z 9. 239b 14: "The second [argument of Zeno] is the so-called Achilles. This is that the slower runner will never be overtaken by the swiftest, since the pursuer must first reach the point from which the pursued started, and so the slower must always be ahead. [...] it proceeds on the same lines as the dichotomy argument [...] only in the Achilles a *dramatic effect* is produced by saying that not even the swiftest will be successful in its pursuit of the slowest (English translation by Lee, op. cit., p. 50; italics are mine).

The Tradition of Mathematical Learning in Magna Graecia

From the distance of millennia, mathematical thought in Magna Graecia of the 5th-3rd centuries BCE looks genuinely grand. Its history has suffered, however, from a paucity of source texts. We have a few fragments of Philolaus and Archytas, somewhat more in the way of reports and testimonies about these two, and a wealth of doxographical material about the Pythagorean connection with mathematics. The doxography for early mathematicians has of late received even more criticism than usual for its myth-making and misleading character.²

In this context, a *clue* to the nature of the mathematics of this period is that many of its innovations were designed for mathematical natural science, in particular mechanics and astronomy. This means its mathematical achievements were geared to an account of *movement*. In the 5th-early 4th centuries BCE, Plato had not yet asserted the importance of separating mathematics as a discipline from movement.³ *Μαθήματα* was a plural noun for different inquiries pursued with a similar method, learning *μαθηματικῶς*. Arithmetic and geometry seem not to have been considered as prior or superior to *μαθήματα* like astronomy and mechanics, both of which involved movement.⁴

A coherent episode of Greek mathematical thought—stemming from the treatment of motion—can be discerned in the historical course extending from the work of the mathematicians, Archytas of Taras (b. late 5th century BCE) in southern Italy and Eudoxus of Cnidos, his student, (b. circa 400 BCE) through to the treatise *Mechanics*, also called *Mechanical Problems*, whose venue is unknown,⁵ and then on to Archimedes of Syracuse (b. 287 BCE) with his mechanical method of theorems. What unites these mathematicians in a progression of thought are two things. The first and most important is that all pursued solutions to questions about movement by seeking a sameness of ratio—what is called proportion (*ἀναλογία*). In addition, it was important that the proportion was itself *invariant* through a range of related cases. The related cases

were places along a movement or different parts of an on-going movement.

I contend that the purpose and meaning of Eudoxus's ratio and proportion appear in a new light when considered in relation to motion. Both Eudoxus in his astronomy and the author of *Mechanics* in his tour de force of applications of the lever principle were dealing with paths or loci of movements that could be rectilinear, circular, or simply curved. Furthermore, Archytas pursued sameness of ratio as a strategy for mathematical explanation of movement. Sameness of ratio as ascribed to Eudoxus is called ἀναλογία by Euclid, who reports and systematizes the theory in *Elements*, Book 5. Archytas's account of growth is given in a testimonium using this term.⁶ In *Mechanics*, a proportion that does not vary over time during a movement yields either rectilinear or circular movement. Finally, both Archytas and the author of *Mechanics* understand the invariant ratio as accounting for the shape of movement. Let us consider each of these points in turn.

Archytas and the shape of growth

Eudemus, the historian of mathematics in Aristotle's school, recounts Archytas's criticism of definitions of motion that made it the unequal, uneven (ἀνῶμαλον), or indefinite.⁷ Archytas said that these do not characterize what movement itself is but are rather causes or origins of movement. For him, movement in nature when left to itself tends toward a self-regulating orderliness. Movement is in some way a reconciling of disparities.

In a puzzling but well-accepted testimonium, Archytas is reported as having an answer to the question of why unspecialized parts of living things grow in a circular shape:

Why is it that the parts of plants and of animals that are not organs are all round—of plants the stalk and shoots, of animals the calves, thighs, arms, and chest? Neither a whole nor a part is triangular or a polygon. Is it, just as Archytas said, because there is in natural movement the proportion of equality (for he said all things move in proportion), but this [movement] is the only one that returns to itself, and so makes circles and curves, when it comes to be? (915a28–33)

The last sentence reads in Greek:

... Πότερον, ὥσπερ Αρχύτας ἔλεγεν, διὰ τὸ ἐν τῇ κινήσει τῇ φυσικῇ ἐνεῖναι τὴν τοῦ ἴσου ἀναλογίαν (κινεῖσθαι γὰρ ἀνάλογον πάντα), ταύτην δὲ μόνην εἰς αὐτὴν ἀνακάμπτειν ὥστε κύκλους ποιεῖν καὶ στρογγύλα, ὅταν ἐγγένηται;

Archytas says that, without over-riding specialization of functions, parts of living things grow into circular shapes. A ready illustration of his meaning is found in the growth-rings of trees (Fig. 1).

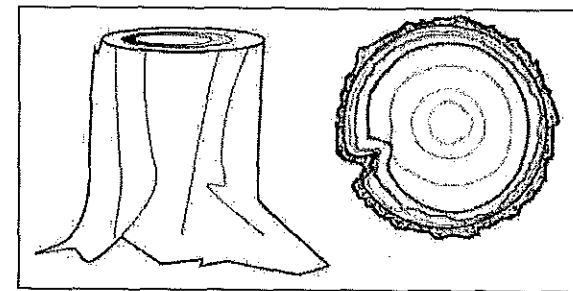


Fig. 1: The growth-rings of trees

In the first part of the passage, Archytas says that growth is circular because there is a proportion of equality (τὴν τοῦ ἴσου ἀναλογίαν) in circular motion. The διὰ and γὰρ clauses suggest that the proportion of equality is a term for any constant proportion in movement and that all natural motion has *some* sort of invariance. Of course, to have a proportion, there must be things entering into ratio. What would these be for circular growth? Developments both before and after Archytas should shed some light on the question. Efforts to square the circle (i.e., produce a rectilinear figure with the same area as a curved figure) took chords or diameters of the circle as starting points. The quadrature of lunes of a circle proven by Hippocrates of Chios (5th century BCE) drew upon rectilinear elements constructed within and around a segment of a circumference. The demonstration of the principle of the lever in *Mechanics* 1 takes perpendiculars (semi-chords) dropped to a particular point on a radius. Elsewhere, I have argued at length that it is reasonable to think the proportion in the Archytas testimonium means a sameness of ratio of rectilinear elements belonging to the circle (Fig. 2).

The ratio $\frac{G}{P}$ remains the same at different points along a surface of growth. The lines in the ratio $\frac{G}{P}$ signify increase of living matter in two directions. One is a straight line extended from an origin, the

center of the circle. The other is a straight line normal to the line from the center. If the ratio of these two lines, the radius and a normal to the radius, is invariant all around the surface of growth from one growth season to the next as represented in Figure 2, then the shape of growth is circular. Circular growth just is characterized by invariance in the ratio of these elements of growth. This is a mathematical answer to the question of why growth of unspecialized parts is circular.

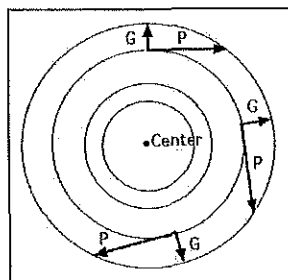


Fig. 2: Circular growth ratio

Eudoxan proportion in mechanics and astronomy

Archytas was the teacher of Eudoxus for some period of time—we do not know how long. Eudoxus' definition of ratio, which plays such a large role in the mathematics of Archimedes, appears in *Elements* 5, Def. 4. It is both elegant and a little obscure. It reads:

Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, whenever the multiples of the first and third alike exceed or alike fall short of multiples of the second and fourth respectively.

In interpreting this definition, we should bear in mind that defining ratio (λόγος) is a foundational move in thought, more fundamental even than ἀναλογία, proportion, which is composed of cohort ratios. Euclid states that magnitudes in ratio have to be capable (δύναται) of the condition specified above, but the statement is also something like an assertion that the condition is fulfilled. The definition says, if things are of the same kind, there are *no* quantities or parts of that kind of thing that *cannot* be brought into relation, whatever the mode of relation. To this extent, it creates the condition that the magnitudes are comparable and thus alike in some way. In understanding the

definition, our attention should be on the multiples, applied respectively to the two candidates for ratio (Fig. 3). The multiples connote that, for any quantities related by greater and lesser, *there will never fail to be a multiple of the lesser quantity that makes it exceed the greater*. Alternatively, there will never fail to be *divisions* of each magnitude that can make the greater to be less than the smaller.

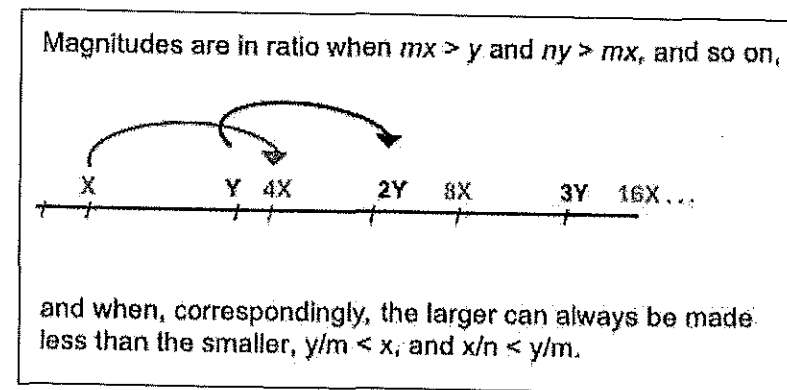


Fig. 3: Magnitudes and multiples

The definition is certainly responding in some way to the issue of incommensurability, what we call the irrationality of numbers or magnitudes. At the same time, the definition ensures that relation will stay within the realm of the finite, however large or small the finite quantities become. Thinking in terms of the jumping arrows in Figure 3, no matter where a multiple or division falls, there is a definite quantity there to correspond to its original. There are no gaps in the continuity of magnitudes being related.

I was forced in explaining the definition to give a negative formulation of the ratio's properties: "there will never fail to be [some quantity] relatable to the other." This is a characteristic of a classic *poria*—a way through a problem. A *poria* is a rationale that sanctions use of mathematical operations we would like to employ. Even though the definition cannot make apparently incommensurable quantities commensurable—that is, make one magnitude divisible without remainder by the other, the definition ensures that we shall not err in proceeding with calculation.

This aspect of the Eudoxan ratio I shall call a *finessing strategy*. The strategy is completed in the method of comparing ratios. The

definition of proportion (ἀναλογία) follows in Definition 5 of *Elements* 5:

Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, whenever the first and third alike exceed or alike fall short of multiples of the second and the fourth respectively.

That is,

$\frac{a}{b} \sim \frac{c}{d}$ when the patterns of exceeding and falling short characteristic of each ratio are just the same when carried on indefinitely.

We need proportion that meets this standard for relations of the very great to the very small—as in astronomy—and for attaining a high level of precision when complete exactitude eludes us—again for astronomy and in mechanics, as well as for the solution of mathematical problems like squaring the circle. The treatise *Mechanics* depends on Eudoxan proportion, but its author seems to pursue an additional finessing strategy to express invariance of the ratio characteristic of circular motion. In other words, there is in *Mechanics* a mathematical trope additional to Archytas' proportion of equality portrayed in Figure 2.

The first case of invariant proportion that the author of *Mechanics* puts forward is familiar to us as the parallelogram of forces, though for the author, it is a parallelogram of movements (Fig. 4).⁸

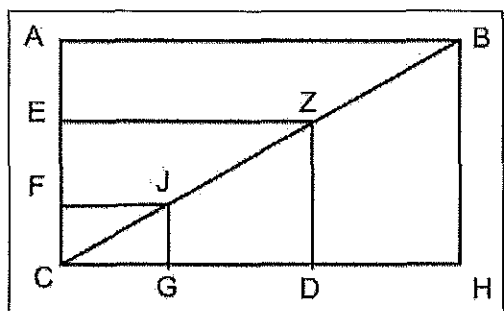


Fig. 4: Parallelogram of movements

The author of *Mechanics* says that any movement, in this case CB, can be represented as a diagonal of a rectangle (τετραπλευρον). So represented, if an object at C travels in its movement as far as J, it has also gone as far as the point F on CA and as far as point G on CH. We can form a ratio $\frac{FJ}{JG}$ and, for a rectilinear movement, this ratio will be

the same as the ratio of sides of rectangles formed in a similar way at any further point along the trajectory, CB. (848b14–24) The author stresses that it is impossible (848b26) that a line is straight without this sameness of ratio being there when complementary sides are constructed around it. The straight in movement is defined, we might say, by this invariance in the sameness of ratio.

The sameness of the ratio along the path of a rectilinear movement constitutes a sort of base camp for the author of *Mechanics*, because in contrast a movement tracing a circular path does not have a single ratio for complements constructed around an arc (Fig. 5). If we take complement sides around each progressively longer arc of Figure 5, AB, AC, AD, these rectilinear elements will not have the same ratio. Furthermore, there is no same ratio for arc AB compared to BC or CD nor for parts of the curve defined by $\frac{AE}{EB}$ or any of the other similar ratios for the other arcs. Indeed, the very movement of a point along an arc ensures that the ratio of rectilinear elements is continually changing.

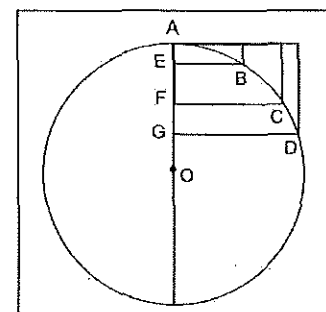


Fig. 5: Circular path.

The search for an invariance—and hence the fruitfulness of sameness of ratio—seems to run aground on the case of circular movement. Bear in mind, though, that the context for movement of points along arcs in *Mechanics* is the lever. Because the account is of mechanical experience, we are interested in *comparing* two arc-movements, and this is where the Eudoxan proportion comes into its own for circular movement. In a physical situation of this sort, says the author of *Mechanics*, the movement of the balance beam can be explained by taking back the system to a special property of the circle. In Plato, Aristotle, and the treatise *Mechanics*, the property is

described either in terms of a moving radius or the revolution of concentric circles bound to a single center:

Moving radius (MR): Points on a moving radius all move at different speeds proportional to their distances from the center of the circle. Points further away from the center move faster. (Fig. 6a)

Concentric circles (CC): Of concentric circles, a point on the circumference of the smaller circle moves in the same time a shorter distance than a point on the circumference of the larger circle. (Fig. 6b)

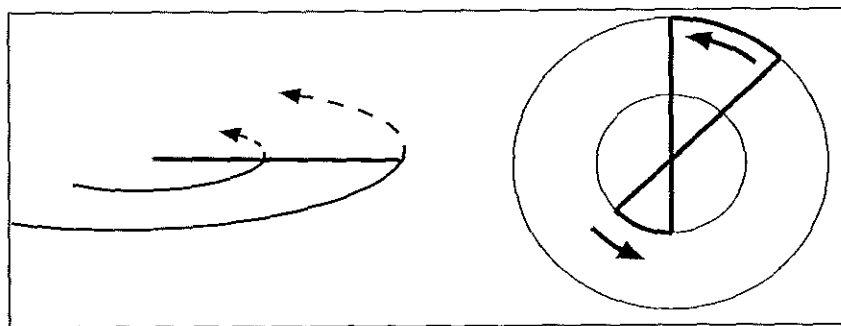


Fig. 6a: Moving radius.

Fig. 6b: Concentric circles.

CC, considered in terms of opposite arcs, accounts for the ability of a lever to move a weight. A weight at the shorter end of the lever is moved by a force that would not be enough to move the weight without the beam and a fulcrum point. In this formulation, arcs must simply be bound to the same center as if on a disc or in a circulating fluid. The concentric circles formulation as a phenomenon would have been observable in the whirlpools readily formed on the coast of the Mediterranean and in the rotation of stars on their apparently fixed paths around the north celestial pole.

By means of the moving radius principle, the author of *Mechanics* is able to show that a sameness of ratio remains in effect at any corresponding points along particular arcs of *different* circles. These are arcs covered in the same time. The proportion holds even though the ratios that constitute the proportion on each arc are continually changing. The ratios change in concert. The physicist thus bypasses having to say what the value of the ratio is at any point

along a single circular path. This is because the undeterminable ratio anywhere along the circular line is the same for corresponding arcs. This is a finessing strategy in the spirit of Eudoxus, because it applies to any slice of an arc, however small. The author conceives of coordinate movements that from moment to moment do not have one single ratio but which assuredly always have the same ratio.

It might seem that the argument of *Mechanics* is just a downstream application of ideas originally part of a Eudoxan theoretical geometry. This cannot be the case, since Eudoxus makes the moving radius the starting principle of his own astronomy. Aristotle testifies to this in his *De caelo* Book 2, and Simplicius provides portions of Eudoxus' *On Speeds* that show the centrality of the principle.⁹

Archimedes and the method of mechanical reasoning

What do these related developments—Archytas to Eudoxus to *Mechanics*—have to do with Archimedes? Our first stop in answering that question must be Archimedes' letter to Eratosthenes *Concerning the Mechanical Method of Theorems*. In the letter, Archimedes says that some things were first clear to him through reasoning by what he calls "mechanics" (διὰ μηχανικῶν). Demonstration by geometrical methods could be accomplished *after* being first made clear by this method (τρόπος).¹⁰ Archimedes explicitly mentions Eudoxus and borrows from his own *Equilibrium of Planes* a version of the Eudoxan ratio now called the Axiom of Archimedes.¹¹

As historians of mathematics speak of it, mechanical method in geometry involves most basically lines that move, usually in rotation, so as to find something like the single point equidistant from given points on two lines. This could mean moving a ruler on a diagram. There are also reports of using a movable material construction to draw a figure.¹² In Archimedes' *Equilibrium of Planes*, however, the mechanical method utilizes the conceptual feint of weights positioned along a line in order to show how differences in the ratios of weights and distances are responsible for movement of a beam. Archimedes assumes that equal weights at equal distances from a fulcrum balance, and on this assumption deduces by *reductio* that equal weights at unequal distances do not balance. He then proceeds to the famous Law of the Lever, which states that unequal weights

will balance if the ratio of their weights is in inverse proportion to the ratio of their distances from the fulcrum (Fig. 7). It is reasonable, however, to consider these initial propositions of *Equilibrium of Planes* as being aimed not at a theory of equilibrium (ισορροπία) but at establishing centers of gravity for figures, a project that Archimedes discloses in Proposition 8.

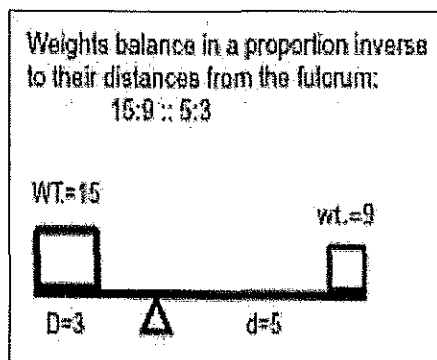


Fig. 7: The "Law of the Lever".

The center of gravity of a weight on a line is a point. Once such points are located along lines or linked to points within a plane figure, one can proceed to a variety of geometrical results by means of proportions that began in ratios of weights and distances. In Fig. 8, if AB is a magnitude whose center of gravity is C, and AD a part of it whose center of gravity is F, then the center of gravity of the remaining part will be G, produced on FC such that line GC is to CF as magnitude AD is to magnitude DE.

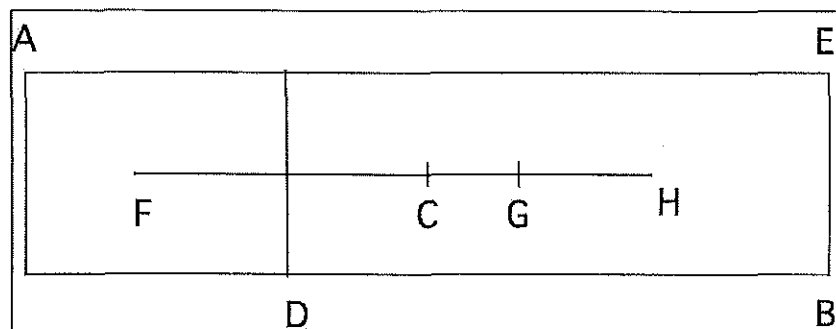


Fig. 8: Center of gravity.

The method can seem odd to modern mathematicians who nevertheless regard Archimedes as the first real practitioner of mathematics as we now know it. Archimedes, however, blends mechanical and geometrical reasoning in a way fully consistent with the practice of his predecessors. He wrote a treatise, now lost, called *Περί ζυγῶν*, *On Balances*, which included the moving radius principle.¹³ His uses of the lemma, or axiom, bearing his name involve weight and suppose the opposition of balance and movement.

As a last example, I will note a few aspects of the propositions Archimedes establishes on the way to proving that the area of any segment of a parabola is $\frac{4}{3}$ of ($\frac{1}{3}$ more than) the triangle which has the same base as the segment and is equal in height.¹⁴ This is one of Archimedes' noted achievements, the squaring of the parabola. In propositions 14–16, he establishes that the area of a segment of a parabola inscribed in a triangle is $\frac{1}{3}$ of the triangle one of whose sides is the base of the segment. The penetration of mechanical thinking is readily seen in the diagrams he describes leading up to and including propositions 14 and 16.

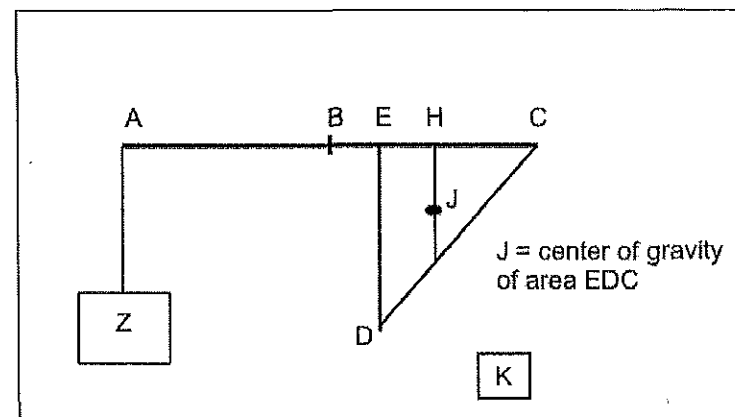


Fig. 9: Proposition 8

He begins by supposing the horizontal line ABC to be a balance (Prop. 8) with both a weight Z suspended on the beam AB and a triangle EDC suspended from BC (Fig. 9). The center of gravity J of the triangle balances weight Z and so has the same ratio to Z that the length AB has to BH (not to BC).¹⁵ If the line to D were suspended from B, the triangle would outweigh the weight Z by an amount K

smaller than Z but of the same kind. In proposition 14 (not pictured), a triangle BDC has a vertex at B along BC and is divided into vertical segments. Archimedes adds straight lines radiating from C to BD making trapezoidal segments. These trapezoids of the triangle BDC correspond to a series of magnitudes substituted for Z and suspended at A, but the sum of the trapezoids is still taken in relation to the whole of what is added at A.¹⁶ The segment of a parabola is inscribed with the base of the segment along BC. In proposition 16 (Fig. 10), triangle BDC is displaced below the line AC. E and other equal line segments remain along BC with the trapezoidal segments of the triangle they define. In proof of Proposition 16, he utilizes the progressively smaller trapezoids converging on C to argue that the inscribed segment of the parabola with base BC cannot be less than 1/3 of the triangle BDC nor can it be greater than 1/3 of the triangle BDC. (This numerical claim relies on Proposition 7). He has reached this point by way balancing segmented areas to weight-magnitudes ($\chi\omega\rho\acute{\iota}\alpha$), a comparison based on the conceptual feint of movement of weights on a balance.

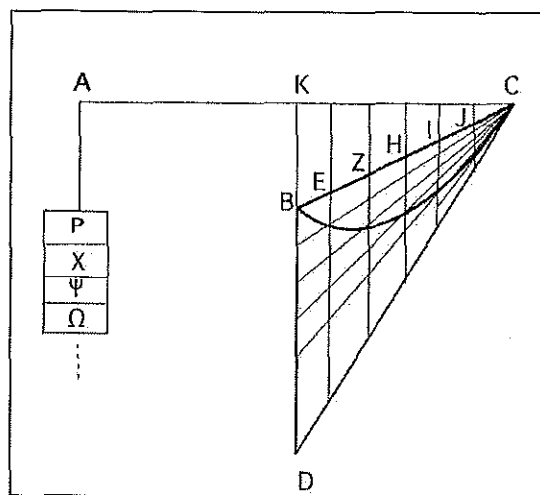


Fig. 10: Proposition 16.

In Archimedes's long and complex argument squaring the parabola, the weights that balance areas and centers of gravity serve as a sort of control or constraint on the geometrical constructions introduced from B to C. Archimedes's geometrical ratios that balance ($\iota\sigma\omicron\rho\eta\sigma\pi\epsilon\iota$) weight-magnitudes are ratios that forestall or prevent

motion being initiated along ABC. His procedure amounts to a penetrating distillation of the method of proportion, transforming ratios involving motion into mathematical constraints progressively limiting (boxing in) curvilinear segments whose areas are to be determined.

Connections can be established between Archimedes and the earlier mathematicians of Magna Graecia in a number of ways. First, like them, Archimedes did not regard the mathematics of mechanics as an application of geometry but as prior to it in some ways. Mechanics is a legitimate starting point for mathematical thinking in general. Secondly, he had little need for theoretical concepts when using mechanics. Ordinary notions like weight, length, and inclining in one direction or another sufficed to support the mathematical realities he pursued. There was no tangle of ontology to work through concerning weight, different kinds of matter, or the reality of relations. Although the efflorescence of Platonic mathematics was underway in Ptolemaic Alexandria and Archimedes knew of it, a great part of his originality still lay in his inheritance from the past. He used proportions in the context of a mathematics of motion, which had come to him from his predecessors in Southern Italy. It seems clear that these methods were handed down within Magna Graecia itself and without the mediation of Athenian natural philosophy.

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² Walter Burkert, *Lore and Science in Ancient Pythagoreanism*, trans. Edwin L. Minar, Jr. (Cambridge: Harvard University Press), 1972; Giuseppe Cambiano, "Archimede meccanico e la meccanica di Archita," *Elenchos* 19, no. 2 (1998): 291–324; Leonid Zhmud, *The Origin of the History of*

Science in Classical Antiquity, trans. Alexander Chernoglazov (Berlin: Walter de Gruyter, 2006). For Zhmud's most recent reflections on early Pythagoreanism, see *Pythagoras and the Early Pythagoreans*, trans. Kevin Windle and Rosh Ireland (Oxford: Oxford University Press, 2012). In contrast, Huffman argues that the testimony of Aristoxenus of Tarentum, which is likely the source of Iamblichus's account of Pythagoreanism in his *Life of Pythagoras*, is reliable as a source and confirms some of the details about the Pythagoreans offered in Aristotle's *Metaphysics* A (Book review, Bryn Mawr Classical Reviews 2014.08.30). Aristoxenus, who excelled in harmonics, came from Taras in southern Italy to be part of Aristotle's immediate circle in Athens.

³ *Republic* 7, 525a–530c. Note in particular Plato's insistence of the priority of solid geometry to astronomy (528d–e) and his belief that astronomy should be studied by means of problems and not by reference to visible movements of heavenly bodies (530c).

⁴ See Archytas of Tarentum, Fr. 1 and 3 in Hermann Diels and Walther Kranz, eds. *Die Fragmente der Vorsokratiker: griechisch und deutsch*, vol. 1 (Berlin: Weidmann, 1956), 47[35], 431–438, or Carl Huffman, *Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King* (Cambridge: Cambridge University Press, 2005), 103–161, 182–224.

⁵ *Mechanics* is usually ascribed to an Aristotelian writing decades after Aristotle's death. See for example G. E. L. Owen, "Aristotelian Mechanics," in *Logic, Science and Dialectic: Collected Papers in Greek Philosophy*, ed. Martha Nussbaum, 318. The date of the treatise is the subject of renewed scrutiny, however, and some would make it the work of Aristotle himself. For a survey of the evidence, see Peter McLaughlin, "The Question of the Authenticity of the *Mechanical Problems*," www.philosophie.uni-hd.de/md/.../mclaughlin_authenticity_2013_2.pdf (accessed August 16, 2015). Thomas N. Winter ascribes it to Archytas ("The *Mechanical Problems* in the Corpus of Aristotle," iii–ix, *Faculty Publications, Classics and Religious Studies Department* 2007. <http://digitalcommons.unl.edu/classicsfacpub> (accessed August 16, 2015), but Huffman would not endorse this attribution (*Archytas* [note 3], 77). For the later history of the text, see P. L. Rose and S. Drake, "The Pseudo-Aristotelian Questions of Mechanics in Renaissance Culture," *Studies in the Renaissance* 18 (1971), 65–104.

⁶ Euclid, *Elements* 5, in Thomas L. Heath, *The Thirteen Books of Euclid's Elements*, with introduction and commentary, 2nd ed. rev. with additions. 3 vols. (New York: Dover, 1956), vol. 1; Archytas, A23a (Diels-Kranz, *Fragmente*, vol. 1, 430–31, which appears in the

Aristotelian *Problems* 16.9. Huffman treats the passage at length in *Archytas*, 516–540.

⁷ Diels-Kranz, *Fragmente*, 1, 47[35], A23, 430. This view of Eudemus is reported by Simplicius in his commentary on Aristotle's *Physics*. These definitions are ascribed, perhaps inaccurately, to Plato and the Pythagoreans (*Physics* III.2, 201b16–202a2). Simplicius points out that, in the *Timaeus*, Plato says that inequality is the cause of the unevenness (*ἀναμαλότης*) of motion (57e), not that these two are the same. (Simplicius, in *physicorum, Commentaria in Aristotelem Graeca* [CAG], Berlin: G. Reimer, vol. 9, 431.4–16). See Huffman's commentary on this passage (*Archytas*, 508–515). According to Huffman, Archytas believed that motion participates in equality though its causes are inequality, unevenness, and lack of concord (515).

⁸ Editions of *Mechanics* are Maria Elisabetta Bottecchia Dehò, *Mechanica*. Tradizione manoscritta, testo critico, scoli a cura di M. E. Bottecchia. Padova: Editrice Antenore, 1982, and *De Plantis, de Mirabilibus Auscultationibus, Mechanica, de Lineis Insecabilibus, Ventorum Situs et Nomina, de Melisso, Xenophane, Gorgia*, edited by Otto Apelt. Leipzig: B. G. Teubner, 1888. Bottecchia Dehò published the text with translation, and commentary in *Problemi meccanici; Introduzione, testo greco, traduzione italiana, note a cura* (Cantanzaro: Rubbettino, 2000). The Loeb translation by W. S. Hett uses the Teubner edition (*Minor Works*. Loeb Classical Library. Cambridge: Harvard University Press, 1980).

⁹ Simplicius, in *de caelo*, CAG 7, 492.31–497.8.

¹⁰ Archimedes, *Ad Eratosthenem Methodus*, 83.23–29 (ed. C. Mugler, *Archimède*, Paris: Belles Lettres, vol. 3).

¹¹ The basis for the method of exhaustion, one statement of the Axiom is the following: "that the excess by which the greater of two unequal areas exceeds the less can, by being added to itself, be made to exceed an given finite area" (*Quadrature of the Parabola*, ed. J. L. Heiberg [Leipzig: Teubner, 1881]), *Opera Omnia* vol. 2, 296.9–14. Archimedes calls this a lemma. The reader will notice the similarity to the Eudoxan definition of ratio in *Elements* 5.4. Euclid in *Elements* 12.1 gives a version that Archimedes often utilizes without commenting on it: Given two unequal magnitudes, if from the greater [a part] is subtracted greater than the half, if from the remainder [a part] great than the half be subtracted, and so on continually, there will be left some magnitude which will be less than the lesser given magnitude. Archimedes relies on this version in, for example, proposition 16 of *Parabola*, 330.3–6. See also Heath, *Works of Archimedes*, (New York: Dover), xlvii–xlvi.

¹²"... [I]n Apollonius' solution of the problem of the two mean proportional as given by Eutocius a ruler is supposed to be moved about a point until the points at which the ruler crosses two given straight lines at right angles are equidistant from a certain other fixed point." (Heath, *Works*, cv) The invention of a machine for drawing the conchoid is attributed by Pappus and Eutocius to Nicomedes (*Works*, cvii).

¹³ ἀπεδείχθη γὰρ ἐν τῷ περὶ ζυγῶν Ἀρχιμήδους καὶ τοῖς Φίλωνος καὶ Ἡρώωνος μηχανικοῖς, ὅτι οἱ μείζονες κύκλοι κατακρατοῖσιν τῶν ἐλασσόνων κύκλων, ὅταν περὶ τὸ αὐτὸ κέντρον ἢ κύλισις αὐτῶν γίνηται. (Pappus, *Collectio* 8, 1068.20 [F. Hultsch, *Pappi Alexandrini collectionis quae supersunt*, Berlin: Weidmann, 1876, vol. 2).

¹⁴ Archimedes, *Parabola* [note 10], prop. 17, 334. For an explication of the proof, see E. J. Dijksterhuis, *Archimedes*, with a new bibliographic essay by Wilbur R. Knorr (Princeton NJ: Princeton U. P., 1987), 336–345.

¹⁵ Archimedes, *Parabola*, 310.15–22.

¹⁶ Archimedes, *Parabola*, 320.14–23.

Michael Papazian¹

Gods and fossils:

Inference and scientific method in Xenophanes's philosophy

The meager remains and testimony of the pre-Socratic philosophers present a challenge and danger to the scholar of ancient philosophy. There is always the risk of projecting modern philosophical concerns and arguments onto the earliest philosophers of the Greek world. The case of Xenophanes is in this respect typical. While some scholars have depreciated him as a minor poet with no original philosophical ideas, others have celebrated him as an innovative and even revolutionary figure in Western philosophy and science. Indeed two of the major philosophers of science in the 20th century—Karl Popper and Paul Feyerabend—present diametrically opposite assessments of Xenophanes's significance. For Popper, Xenophanes stands at the beginning of Western rationalism and science while for Feyerabend, Xenophanes was a "conceited big mouth" who presented "effective one-liners" rather than any reasoned arguments for his commitments.² The extant fragments, however, support a more sober assessment rather than the extreme views of Feyerabend and Popper. Xenophanes developed important epistemological insights and applied his methods in an empirical and systematic way that anticipates or approximates some of the methods of modern science. In particular his arguments rely on a method of inference to the best explanation, and so Xenophanes stands as an early pioneer in the development of non-deductive logic. This paper attempts to elucidate Xenophanes's epistemology and methodology by examining two theses of his thought and his method of supporting them: his anti-anthropomorphic theology and his theory of cyclic cosmic generation. Casting Xenophanes's arguments in a non-deductive form makes his inferences more plausible and also resolves some of the difficulties and puzzles that commentators have observed in Xenophanes's fragments.

Xenophanes's life and relation to Magna Graecia

According to Diogenes Laertius, Xenophanes was born in the Ionian city of Colophon but after being expelled from his homeland