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Cubes of Sound, Tonal Space: A Geometric Representation of Consonances and other Tone Intervals

Halvard Hårklau 2010 (revised 2012)

Abstract

A while ago, I needed to compare an intriguing natural phenomenon with musical harmonies, and different systems of tuning were tested. As it turned out, both the traditional Pythagorean tuning and the just tuning of the Renaissance seemed to be relevant. Hence, there was a need to construct a system that compiced both these “musical paradigms”. For the sake of a comprehensive overview, a geometric representation was worked out, and surpicing and beautiful features developed as the system crystallized, litterally into a square lattice.

While it is customary to represent tone ratios as direct functions of frequencies, with higher number representing higher pitch, in this essay the ratios relate to string lenghts, as in a monocord (which are resiprocally proportional to the frequencies); hence, unless otherwise stated, lower numbers represent higher pitches; i.e. 1:2 is one octave higher than the fundamental, while 2:1 is one octave lower.

Introduction

[yet to be written]

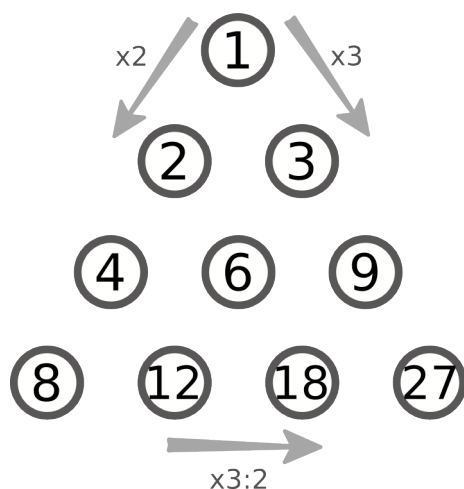


Figure 1: "The Platonic Lambda", or an interpretation of the Tetractys using the series of tone intervals from Plato's *Timaios* (odd series 1-3-9-27 and even series 1-2-4-8) and filling in the gaps. Each step downwards towards the left can be conceived as performing an operator of doubling (x2, musical octaves), while each step downwards towards the right performs the operator of tripling (x3, musical duodecims or 12ths). A horizontal step towards the right performs the operator $x3:2$, musical fifths.

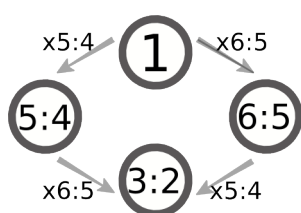


Figure 2: A musical fifth (3:2) can be defined as a harmonious combination of thirds: Either a major (5:4) or minor (6:5) third in sequence or vice versa. If we let the numbers correspond to relative string lengths, larger numbers correspond to lower pitch; hence, this example illustrates a "downward tuning" from the fundamental, as was the custom in ancient Greece

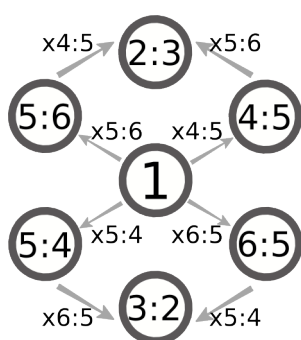
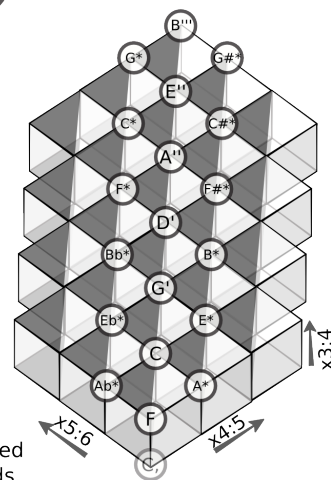
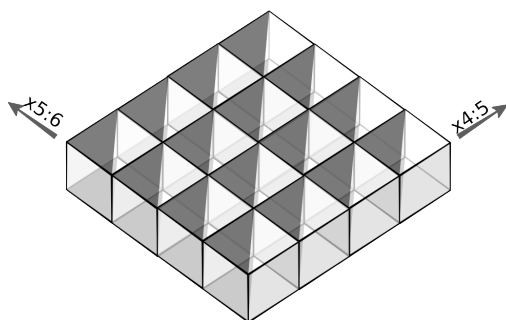
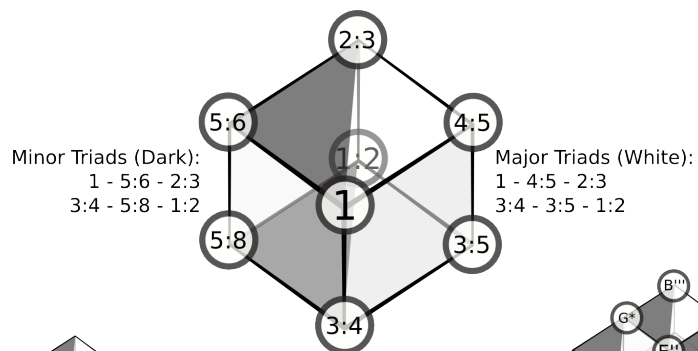
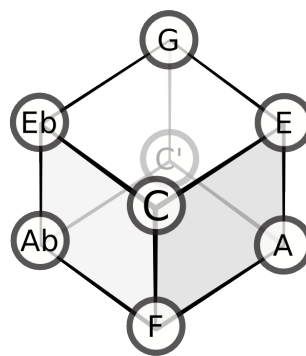
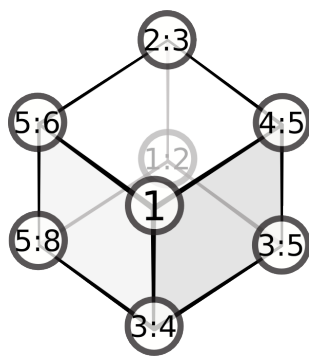
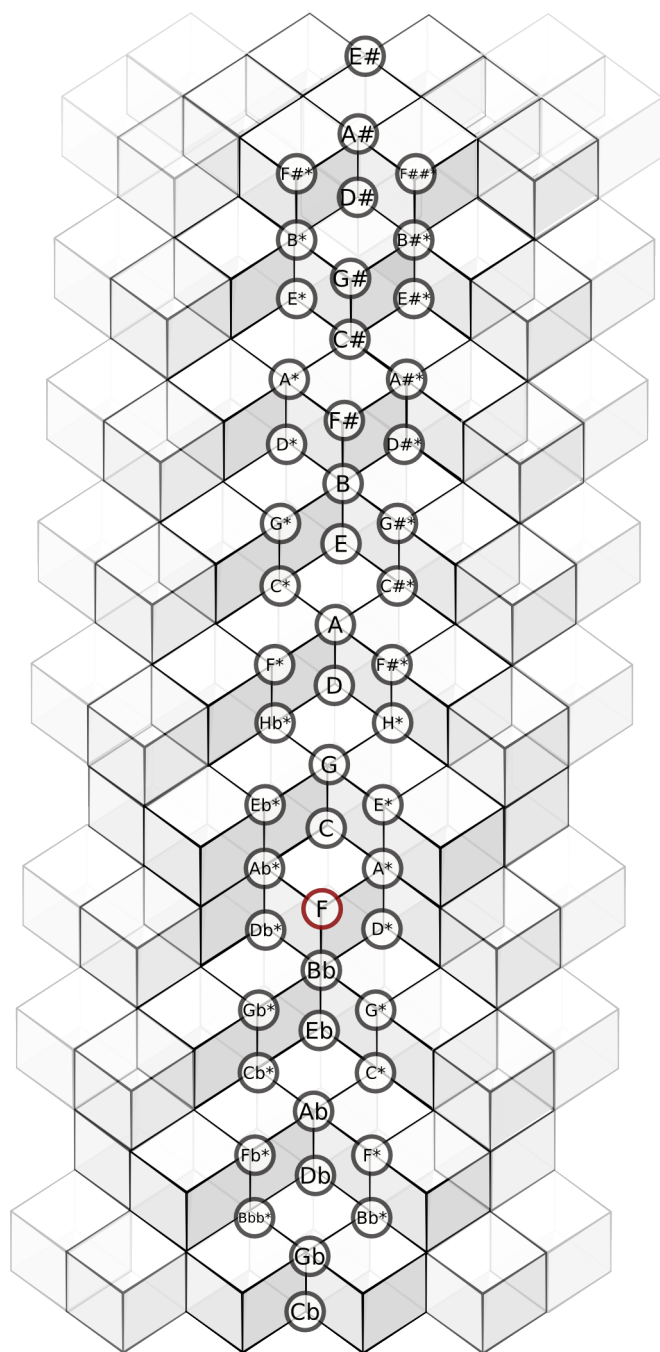


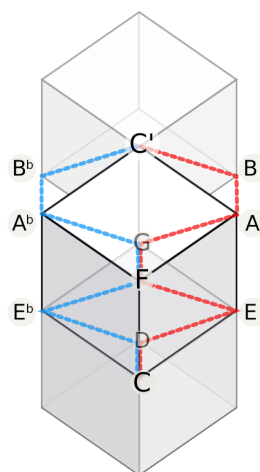
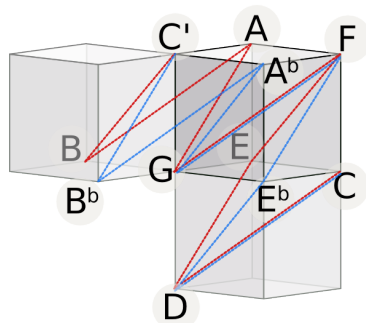
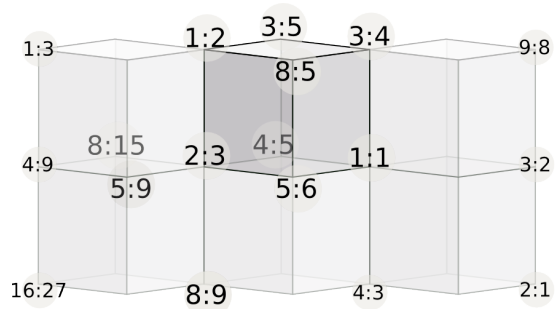
Figure 3: Performing the inverse operations in the opposite directions gives symmetric upper and lower sets of triads, and we obtain a hexagonal figure with point corresponding to six different tone intervals from the fundamental in the centre.



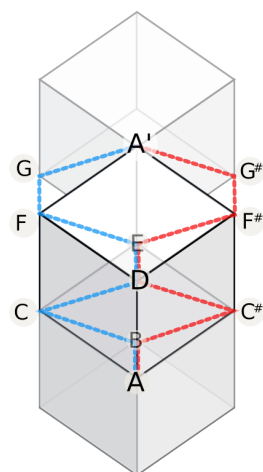
The planes defined by the thirds and the fifths are separated by fourths. Each of these planes can be considered as composed of an infinite system of Major and Minor Triads.



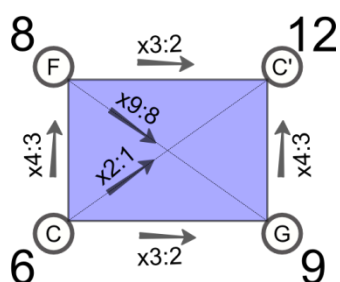
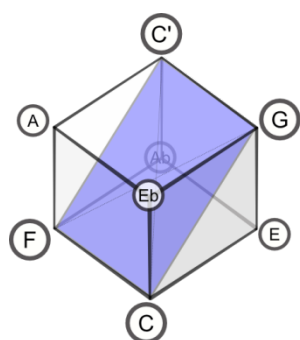
$\swarrow \times 5:6 \searrow \times 4:5$
 $\downarrow \times 3:4$



C-major red
C-minor blue



A-major red
A-minor blue



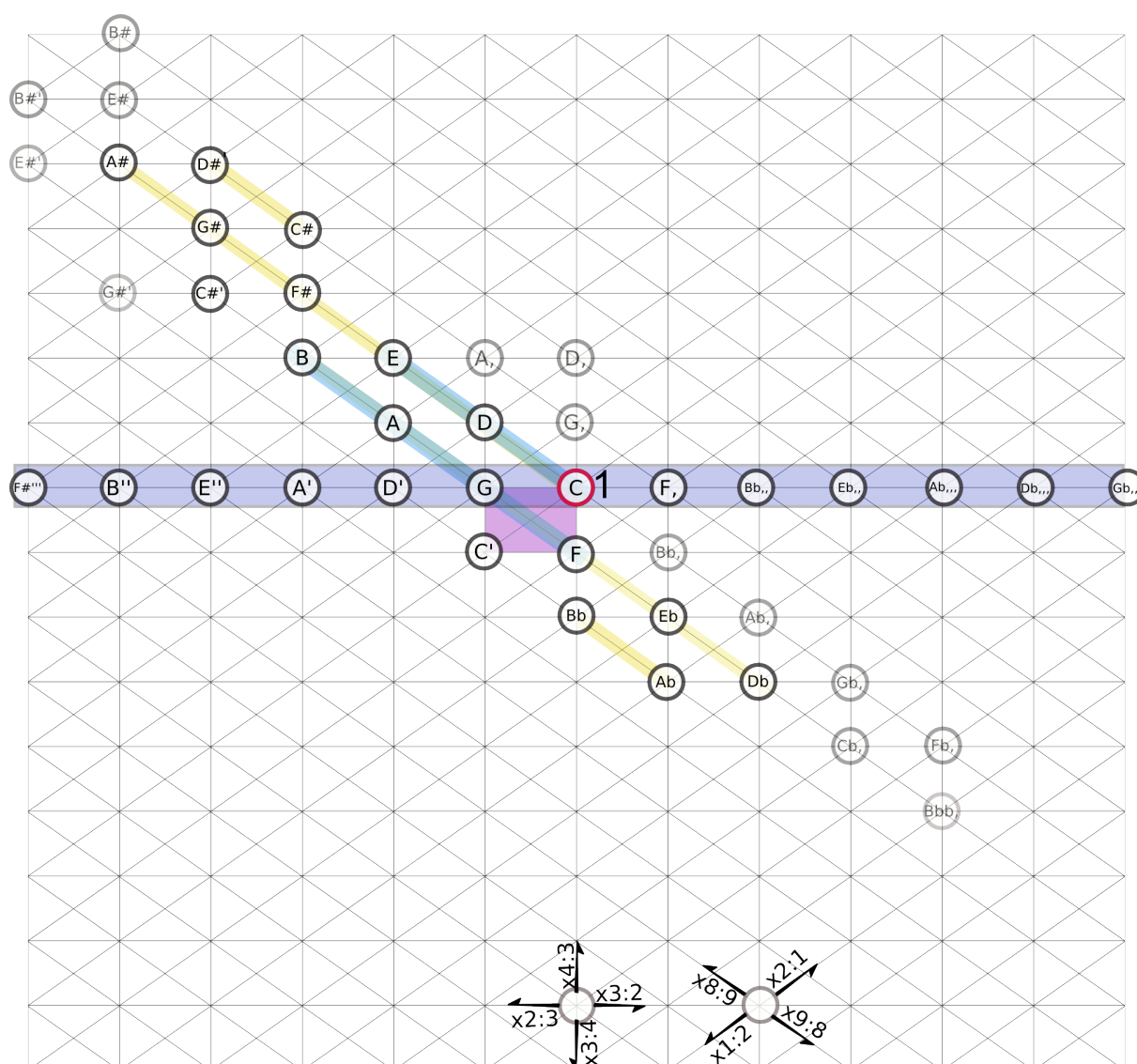


Figure 10: 'The Pythagorean Plane. The horizontal row with bluish background illustrates a series of twelve fifths, which is a way of defining a chromatic Pythagorean scale. The diagonals with greenish background illustrate a diatonic scale (from C to B, corresponding to modern C-major). This together with series marked yellow comprises a complete enharmonic Pythagorean scale of 17 notes. The purple rectangle is the unit, illustrated in the previous figure as a central plane of a unit cube. Note that one diagonal step upwards to the left is a whole tone step upwards, while a diagonal step downwards to the left is an octave upwards. A fourth upwards is one step downwards, while a fifth upwards is one step to the left. We leave it as an exercise to the reader to calculate the exact tone values, with middle C as 1 as reference, and to fill in the remaining tones in the scheme.