

The triangle that changed Apollo into a cube

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The 3:4:5-proportion triangle has been the subject of interest for millennia. The interest goes right back to ancient Egypt. At least eight pyramids have the proportions in their design, including the second largest structure at Giza, the pyramid of Khephren, which is adjacent to the Great Pyramid (Petrie, p. 202).¹



Figure 1: Khephren's pyramid

Plutarch (ca. 45–120 AD) is noted for his biographic works and essays on philosophy and ethics. He was a priest at Delphi, a place sacred to the god Apollo. He discusses the divine status of the triangle in *On Isis and Osiris* in *Moralia* V. He writes:

One might conjecture that the Egyptians hold in high honour the most beautiful of the triangles, since they liken the nature of the Universe most closely to it, as Plato in the *Republic* seems to have made use of it in formulating his figure of marriage. This triangle has its upright of three units, its base of four, and its hypotenuse of five, whose power is equal to that of the other two sides. The upright, therefore, may be likened to the male, the base to the female, and the hypotenuse to the child of both, and so Osiris may be regarded as the origin, Isis as the recipient, and Horus as perfected result. (Babbitt, p. 135)

For Plato's discussion of the 3:4:5 proportions in the *Republic* see Lee, p. 299 (or *Stephanus* 546, a standard reference indicator for all translations of the *Republic*).

¹ Reference sources are detailed at the end of this essay.

Memorable examples of the triangle's properties are given in this essay. The 3:4:5 triangle is illustrated below.

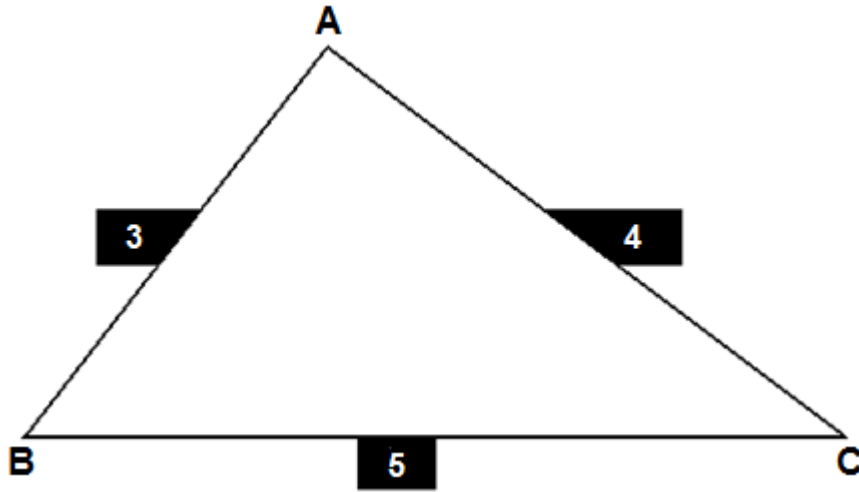


Figure 2

- Location A is the right angle. AB measures 3 units.
AC is 4 units.
BC is the hypotenuse of 5 units.
The sum of all the sides is **12** units.
- AC, as seen, measures 4 units; 4 is a square number: 2×2 .
The sum of AB 3 and BC 5 is 8 units; 8 is a cubic number: $2 \times 2 \times 2$.
The sum of AC 4 and BC 5 is 9 units; 9 is a square number: 3×3 .

Two lines are added to the 3:4:5 triangle. They could be readily created by folding a paper or cloth triangle.

The first addition

D is the middle of BC, the hypotenuse. From D a line is drawn to A.

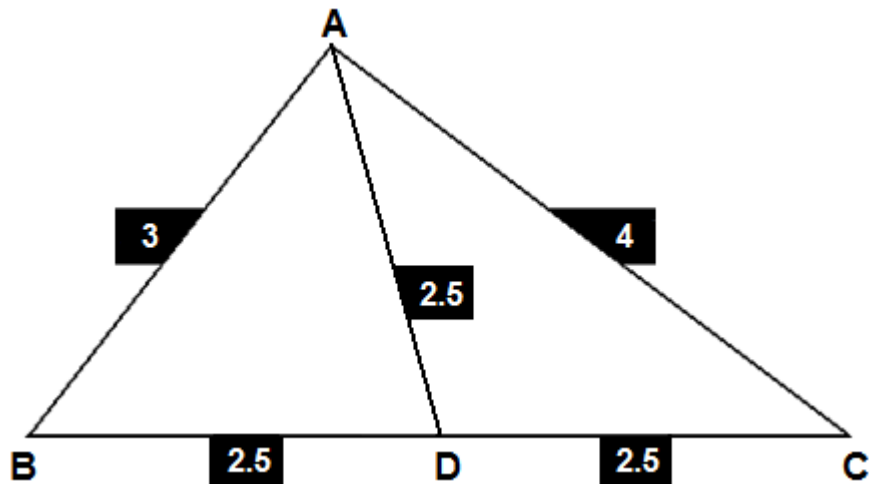


Figure 3

AD measures 2.5 units. DC measures 2.5 units. BD also measures 2.5 units.

The second addition

A line perpendicular to **BC** is drawn from **A** to **E**. The new triangle **AED** emerges.

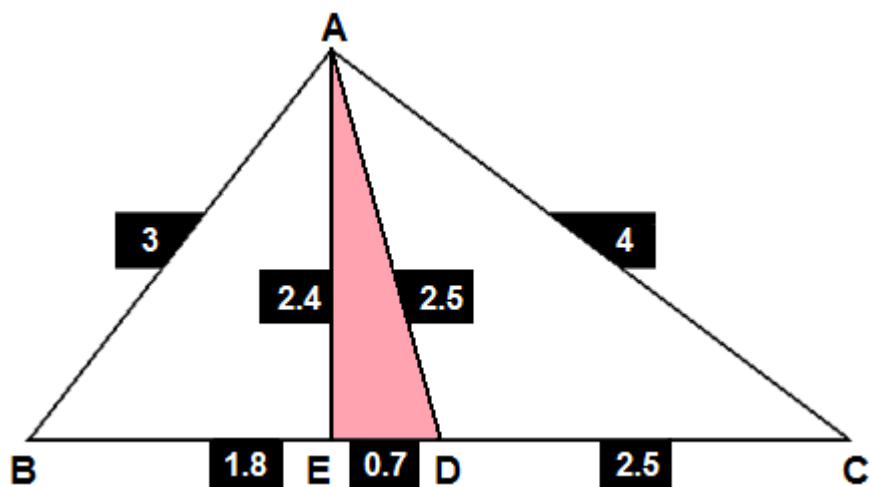


Figure 4

Three measures are of interest in figure 4. They are easily established:

- **AB** 3 units is the hypotenuse of the new 3:4:5 proportion triangle **AEB**. The angle at **B** has not changed.

Angle **AEB** is a right angle, like angle **BAC**. Consequently angle **BAE** must be the same as the angle at **C**.

So, the three measures of interest are:

- (1) **BE** is three-fifths of **AB** 3 units and measures **1.8** units.
- (2) **AE** is four-fifths of **AB** 3 units and measures **2.4** units.
- (3) Since **BD** measures 2.5 units and **BE** is 1.8 units, then **ED** must measure **0.7** of a unit: $2.5 \text{ minus } 1.8 = 0.7$.

ED 0.7 is one-tenth the measure of the sum of **AB** 3 and **AC** 4 ($3 + 4 = 7$).

The highlighted triangle **AED** is a new Pythagorean triple, a special class of right angle triangles like **ABC**. Pythagorean triples are right angle triangles with whole number sides and interesting characteristics.

Triangle **AED** has the proportions **7:24:25**.

The measures in **AED** reflect its proportions:

- **ED** is **0.7** of a unit
- **AE** is **2.4** units
- **AD** is **2.5** units.

In a triangle with **7:24:25** proportions (illustrated in figure 5 below), the sum of the two sides connected to the “7” side add to **49**: **AE 24 + AD 25 = 49** units.

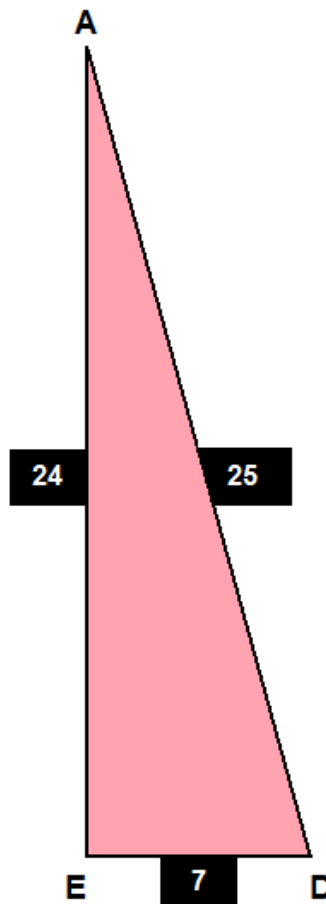


Figure 5

Forty-nine is a square number: **7** times **7**. Seven is the square root of 49. **ED** measures 7 units.

Interesting matters regarding Roman and Greek measures worth contemplating arise:

- a) The Roman foot was divided into 12 Roman inches (*uncia*) or 16 Roman digits (Rowland and Howe, pp. 189 – 192). The Roman foot measured about 296 mm or approximately $11 \frac{2}{3}$ British imperial/US inches.

A Roman inch was $\frac{4}{3}$ times the length of a Roman digit. A 4:3 ratio is found in a 3:4:5 triangle.

- b) A Roman stade distance measure of **625** Roman feet was equal to a Greek stade² of **600** Greek feet (Rowland and Howe, p. 167, footnote 90). The relevant Greek foot measure, found in the Parthenon in Athens, was, accordingly, $\frac{25}{24}$ times the length of a Roman foot (Wilson Jones, p. 74).

The “Parthenon” foot measured about 308 mm or about 12.15 British imperial/US inches. Information given by Herodotus in his book *The Histories* provides supporting evidence for the existence of the 308 mm Greek foot.

The Roman writer Pliny the Elder, who died in the dramatic eruption of Mt. Vesuvius in 79 AD, is one of the sources for information about the 25:24 proportional relationship between the Greek foot and the Roman foot.

Put simply, the “Parthenon” Greek foot was equal to 12 $\frac{1}{2}$ Roman inches. The Roman foot, as noted in point (a) above, contained 12 Roman inches.

The **25:24** ratio as it manifests in a 3:4:5 triangle can be seen in figure 4.

The number **625** mentioned earlier can be expressed as 5 x 5 x 5 x 5, or 25 x 25.

Note that a square drawn on hypotenuse AD 25 in the Pythagorean triple illustrated in figure 5 would contain **625** square units. The Roman stade contained **625** Roman feet.

Furthermore, in the same triple, the product of **25** (AD) multiplied by **24** (AE) is **600**. The Greek stade contained **600** Greek feet.

The data in points (a) and (b) above is explored further from radically different perspectives in forthcoming essays by the present writer.

The 3:4:5 triangle and the Greek god Apollo

The god Apollo is mentioned numerous times in the renowned treatise *The Ten Books on Architecture* by the Roman architect Vitruvius, notably in relation to the replacement of a cracked pedestal for a statue of the god. Vitruvius lived in the first century BC. As can be seen in his book, he was an admirer of Plato and Pythagoras.

² The English word “stadium” derives from the ancient Greek word for “stade”.



Figure 6: statue of Apollo
(image courtesy Wikipedia)

The description of Apollo's pedestal is given in the last book of the *The Ten Books on Architecture*, that is, in Book 10. Vitruvius writes that the pedestal is **twelve** feet long, **eight** feet wide and **six** feet high (Morgan, p. 289/Book 10.2.13).

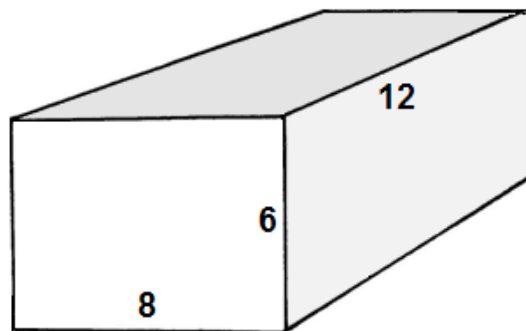


Figure 7: Representation of Apollo's pedestal
(measures in feet)

The volume is easily established as being **576** cubic feet: $12 \times 8 \times 6 = 576$. The number 576 is a square number: **24 x 24**.

Especially note that the ratio **8:6** on the front face of the illustrated pedestal is the same as **4:3**. Consequently, on the faces of the pedestal that have these dimensions (shown below), the diagonal must measure 10 feet because it is the hypotenuse, the “five” side, of a 3:4:5-proportion triangle. The 3:4:5 triangle has often been linked to Pythagoras and the Pythagoreans—for example, see Morgan, pp. 252–3/Introduction to Book 9.6–7.

The sum of $6 + 8 + 10$ is **24**.

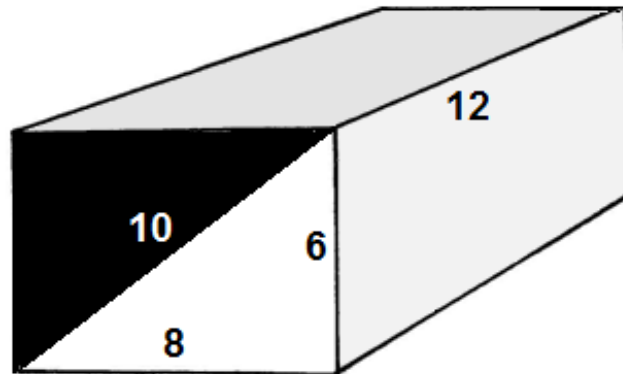


Figure 8: Representation of the pedestal that includes the ten-foot diagonal measure

It is possible to discern now why the pedestal was made to be twelve feet long: it ensures the volume becomes **576** (24×24) cubic feet. This is not the only 24×24 formulation in Vitruvius's book. Another is detailed in other work by the present writer.

On manifestations of 24

This discussion begins with a return to the 3:4:5 triangle illustrated in figure 4, which is repeated below.

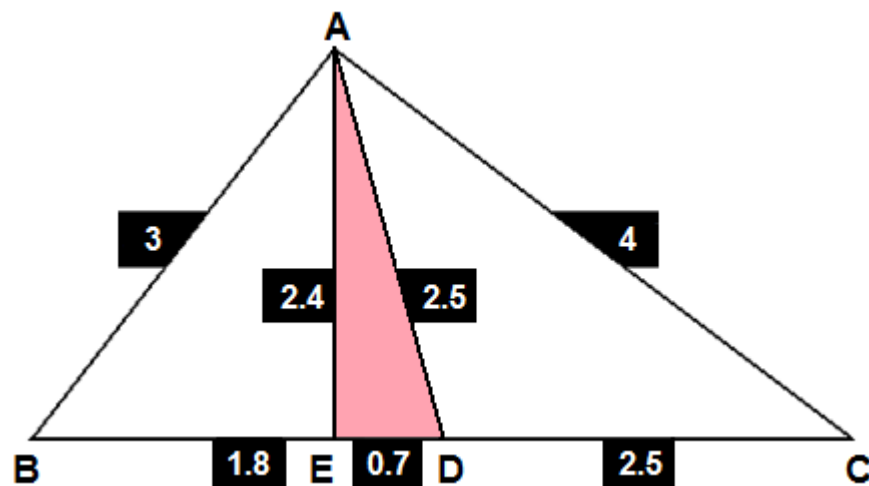


Figure 9 (formerly figure 4)

One additional line is added to the geometry. **EF** is drawn perpendicular to **AB**. The new 3:4:5-proportion triangle **AFE** emerges.

Note that location **F** stems from the creation of a continuous line, bent like a lightning flash, drawn from location **D**: **D** to **A** to **E** to **F**.

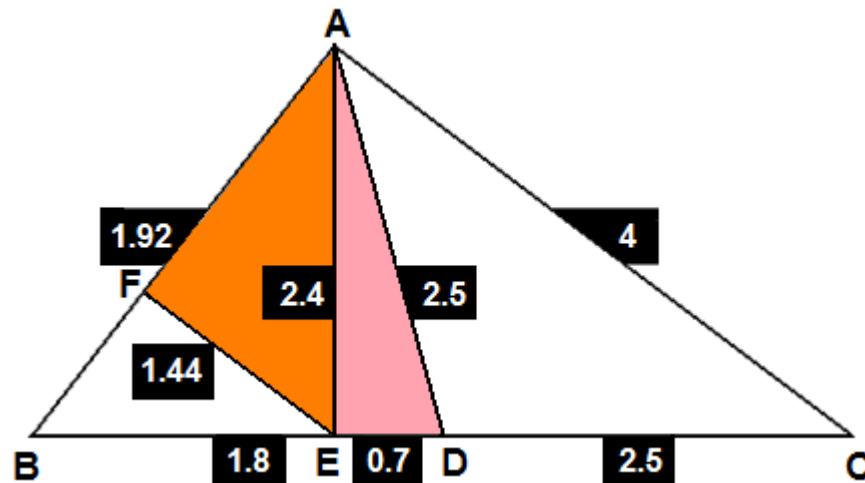


Figure 10: the “lightning flash” DAEF

Features of interest

(a) **EF** measures 1.44 units: 1.44 is 1.2 squared.

(b) The perimeter of triangle **AFE** measures **5.76** units:
(**EF** 1.44 + **AF** 1.92 + **AE** 2.4 = 5.76 units).

The number 5.76 is 2.4 times 2.4, that is, 2.4 squared. Compare this with the material on the volume of Apollo’s pedestal (**576** cubic Roman feet) in the preceding section *The 3:4:5 triangle and the Greek god Apollo*.

(c) Imaginatively, in triangle **AFE**, the hypotenuse **AE** 2.4 units can be said to be the square root of the triangle’s perimeter of 5.76 units.

(d) The sum of **EF** 1.44 units and **BE** 1.8 units is 3.24 units: 3.24 is a square number, 1.8 x 1.8.

The cube in the triangle

A cube created from characteristics of the 3:4:5 triangle provides a notable conclusion to this essay.

Below is the triangle illustrated in figure 9 and figure 4. It is now labelled figure 11.

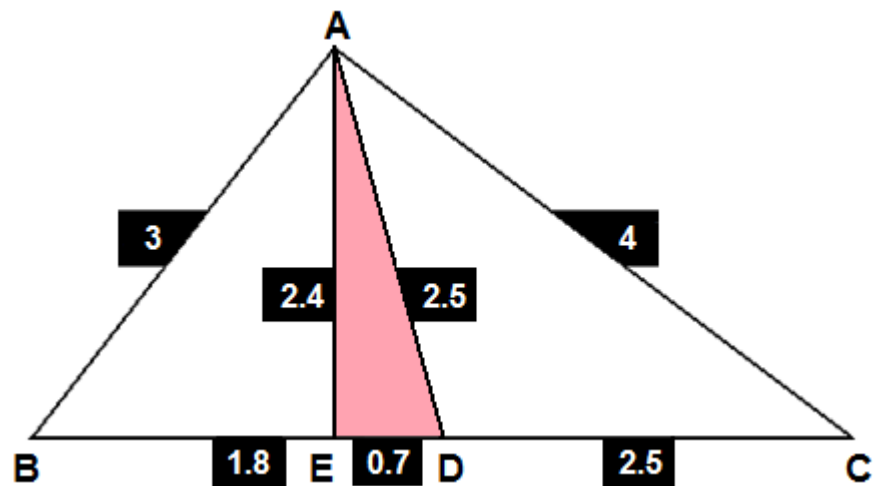


Figure 11

Line AD is deleted from the triangle and this leads to figure 12.

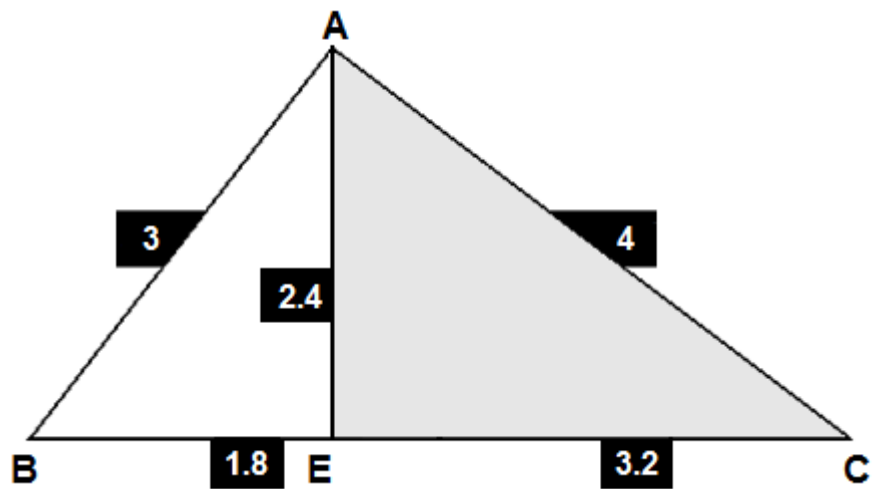


Figure 12

If BE is folded so that it is at right angles to EC , a rectangular solid like that shown in figure 13 below can be created.

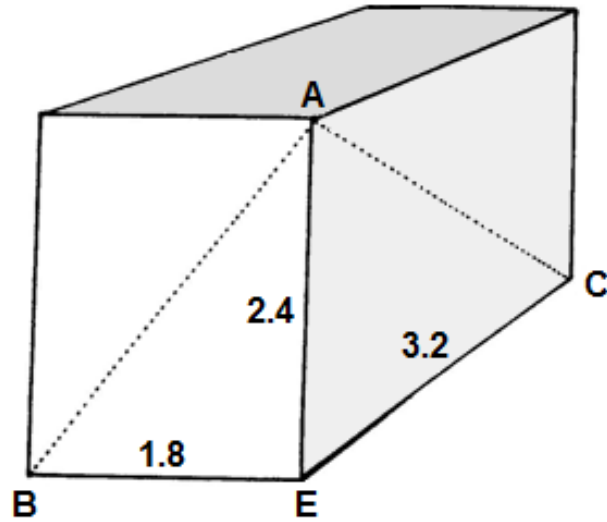


Figure 13: the rectangular solid

The volume of this solid is **13.824** ($1.8 \times 3.2 \times 2.4$) cubic units. The number 13.824 is a cubic number: $2.4 \times 2.4 \times 2.4$. Consequently, the volume of the rectangular solid is equal to the volume of a **cube** that has sides which measure 2.4 units.

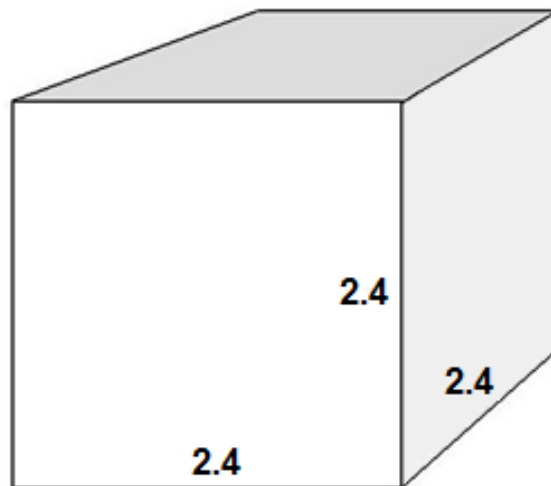
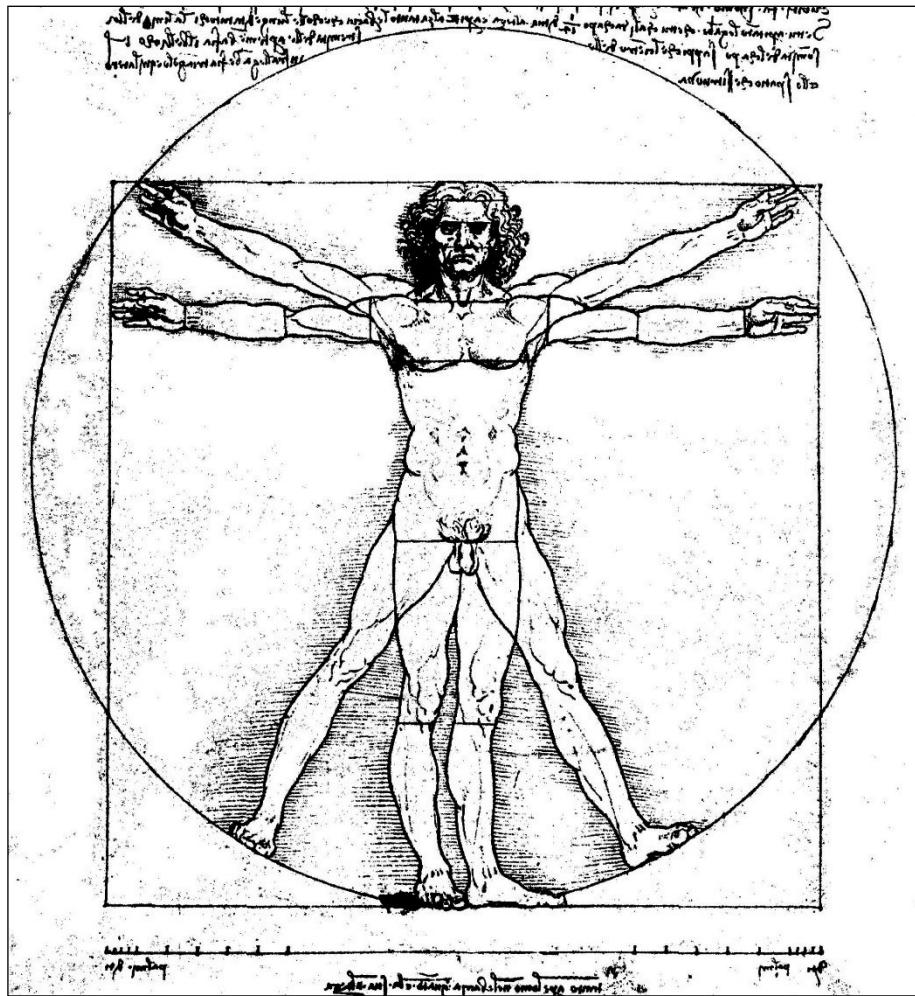


Figure 14: the cube



References

Babbitt, Frank Cole. *Plutarch: Moralia. Volume V*. Harvard University Press, 1999 edition.

Lee, D. (H. D. P) *Plato: The Republic*. Penguin, England, 1987 edition.

Morgan, M. H. *Vitruvius: The Ten Books on Architecture*. Harvard University Press, Cambridge, Mass., 1914. (Republished by Dover Publications)

Petrie, W. M. F. *The Pyramids and Temples of Gizeh*. Field and Tuer (Ye Leadenhalle Presse), London, 1883.

Rowland, I. D. and **Howe**, T. N. *Vitruvius: Ten Books on Architecture*. Cambridge University Press, 1999.

Wilson Jones, M. "Doric measure and architectural design 1: The evidence of the relief from Salamis" *American Journal of Archaeology*. 104 (January 2000) pp. 73–93, Archaeological Institute of America

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