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A modern approximation to Pythagoreanism: Boscovich's "point atomism"

Las especulaciones de los grandes sabios tienen la virtud de arañar, por debajo de la contingencia del tiempo en el que viven, la dimensión universal y perenne de los verdaderos problemas humanos. (Bernardino Orio de Miguel)

Introduction

In a paper published in a scientific journal on physics, the Austrian physicist Karl Svozil asserted that "one of the most radical metaphysical speculations concerning the interrelation between mathematics and physics is that they are the same, that they are equivalent. In other words: the only 'reasonable' mathematical universe is the physical universe we are living in! As a consequence, every mathematical statement would translate into physics and *vice versa*" (Svozil 1995, pp. 1556–57). After quoting from Aristotle's *Metaphysics* (Book I, 5¹; Book XIII, 6²), Svozil states that "the Pythagoreans must have been the first to believe in this equivalence" between the physical universe and its mathematical translation. In the same line of thought, John Losee, in his book *A Historical Introduction to the Philosophy of Science*, defines the "Pythagorean orientation", "a way of viewing nature which has been very influential in the history of science", as a belief that "the 'real' is the mathematical harmony that is present in nature", next declaring that "the committed Pythagorean" is someone "convinced that knowledge of this mathematical harmony is insight into the fundamental

1 985b33–986a3 "(...) since, then, all other things seemed in their whole nature to be modeled on numbers, and numbers seemed to be the first things in the whole of nature, they [the Pythagoreans] supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number" (Aristotle 1953, *Metaphysics*, 2 vols., tr. W D Ross, revised text, Clarendon Press, Oxford).

2 1080b16–21: "And the Pythagoreans, also, believe in one kind of number – the mathematical; only they say it is not separate but sensible substances are formed out of it. For they construct the whole universe out of numbers – only not numbers consisting of abstract units; they suppose the units to have spatial magnitude. But how the first 1 was constructed so as to have magnitude, they seem unable to say" (tr. W D Ross).

structure of the universe” (Losee 2001, pp. 14–15).³ Losee also speaks of a “Pythagorean commitment” by Copernicus (1473–1543) and Kepler (1571–1630), since the first “sought mathematical harmonies in phenomena because he believed they were ‘really there’” (p. 40).⁴ while Kepler “devoted his life to the discovery of the mathematical harmony according to which God must have created the universe” (p. 39).⁵

Roger Penrose began his massive exposition of the state of modern physics, *The Road to Reality: a complete guide to the laws of the universe* (Penrose 2005), with a defense of Pythagoreanism as a way of looking for “a deeper universal order in the way that things behave” (p. 5), and moreover of Pythagoras as an originator of the notion of mathematical proof⁶ (p. 10),⁷ helping to establish the foundation of mathematical understanding and therefore of science. In the view of Arran Gare, from Swinburne University (Australia), “Pythagoreanism underpins the quest by physicists for an ultimate ‘theory of everything’, that is [according to John Barrow in his book *Theories of Everything*] ‘a single all-embrac-

3 Losee pointed to Harré, R 1965, *The Anticipation of Nature: a study of apriorism as a philosophy of science* (Hutchinson & Co., London), Chapter 4, “The Pythagorean Principles”, as an analysis of the Pythagorean orientation.

4 Copernicus and his disciples referred to the Pythagoreans to show that the notion of a moving Earth wasn’t a new or revolutionary proposition. See Casini, P 1994, “Copernicus, Philolaus and the Pythagoreans”, *Memorie della Società Astronomia Italiana*, vol. 65, pp. 497–507, and the references therein.

5 See also Harburger, W (ed. & trad.) 1925, *Johannes Keplers kosmische Harmonie*, Insel, Leipzig; Werner, E 1966, “The Last Pythagorean Musician: Johannes Kepler”, in J LaRue (ed.), *Aspects of Medieval and Renaissance Music: a birthday offering to Gustave Reese*, W W Norton & Co., New York (reprint 1978, Pendragon Press, Hillsdale, New York), pp. 867–82; Walker, D P 1967, “Kepler’s Celestial Music”, *Journal of the Warburg and Courtauld Institutes*, vol. 30, pp. 228–50 (reissue 1978 in *Studies in Musical Science in the Late Renaissance*, E J Brill, Leiden, pp. 34–62); Field, J V 1988, *Kepler’s Geometrical Cosmology*, University of Chicago Press, Chicago; Stephenson, B 1994, *The Music of the Heavens: Kepler’s Harmonic Astronomy*, Princeton University Press, Princeton. For some remarks on certain anti-Pythagorean positions of Kepler’s musical theory see Pesic, P 2005, “Earthly Music and Cosmic Harmony: Johannes Kepler’s Interest in Practical Music, Especially Orlando di Lasso”, *Journal of Seventeenth-Century Music*, vol. 11, no. 1, URL = <<http://sscm-jscm.press.illinois.edu/v11/no1/pesic.html>>.

6 See Proclus, *In primum Euclidis elementorum librum commentarii* [= *In Eucl.*], ed. G Friedlein, B G Teubner, Leipzig, 1873, p. 15 ff.

7 A view that was in disagreement with Walter Burkert’s *Weisheit und Wissenschaft: Studien zu Pythagoras, Philolaus und Platon* (1962, revised version translated into English: Burkert 1972), so much so that Reviel Netz proclaimed: “Pythagoras the mathematician perished finally A.D. 1962” (Netz, R 1999, *The Shaping of Deduction in Greek Mathematics: a study in cognitive history*, Cambridge University Press, Cambridge/New York/Melbourne/Madrid/Cape Town, p. 272).

ing picture of all the laws of nature from which the inevitability of all things seen must follow with unimpeachable logic"⁸) (Gare 2006, p. 3).

We have to keep in mind the aforementioned remarks about what it could mean to be a modern or contemporary Pythagorean in order to evaluate a probable approximation to Pythagoreanism by the Dalmatian polymath Roger (Ruggerius) Joseph Boscovich (Ruder Josip Bošković, 1711–1787). This approximation has been discretely suggested by Niccolò Tommaseo (1840, pp. 122–3)⁹ and by Ernest Regnault (1883, p. 354; see below), and was embraced with a greater enthusiasm by Lancelot Law Whyte (1961a-c; see below). This paper aims to offer an appreciation of the putative approximation of Roger Boscovich to Pythagoreanism regarding the form and content of his reasoning about the first principles of physical reality. The word "Pythagoreanism" is used here in a broad sense that includes the late heritage¹⁰ of "the so-called Pythagoreans" of Aristotle (*Metaph.*, 985b23) and of the Platonic-Pythagorean tradition, specially in relation to what Luigi Borzacchini (2005, pp. 148–9) named "the 'Pythagorean program'", a "semiotic triangle" connecting reality, geometry¹¹ ("in the place that from Parme-

8 Barrow, J D 1992, *Theories of Everything: The Quest for Ultimate Explanation*, Vintage, London, p. 1.

9 "Il Boscovich ha dimostrata (sic) rara potenza d'ingegno, sostituendo alla materia le forze che la governano. La solidità de' corpi è resistenza all'attività nostra (...). E la solidità ha inseparabile l'idea dell'unità. Onde i latini dicevan solido per intero. Quest'idea rischiarata e (sic) la fisica e la metafisica e la morale. Sarebbe da fecondare l'idea de' pitagorici: ogni cosa è numero. Estensione riducesi a numero. L'idea de' corpi è idea de molteplicità".

10 In 360 b.c. Aristoxenus (fr. 14 Wehrli, *On Pythagoras and his pupils*) declared that he met (in Phlius) the last Pythagoreans [Wehrli, F 1967, *Die Schule des Aristoteles*, vol. 2: *Aristoxenos* (1945), 2nd ed., Benno Schwabe & Co., Basel/Stuttgart], and with the existing evidence it is impossible to be sure about the precise contents of Pythagoras' original teachings. According to Richard Crocker, in some ways Archytas was the last Pythagorean: up to his time, Greek mathematics was somehow synonymous with Pythagorean arithmetic; after him, the new geometry made possible generalities that reduced arithmetic to a branch of mathematics, and incidentally the Pythagoreans paled into relative insignificance (Crocker, R L 1964, "Pythagorean Mathematics and Music", part II, *Journal of Aesthetics and Art Criticism*, vol. 22, no. 3, 325–333, reprinted in Crocker, R L 1997, *Studies in Medieval Music Theory and the Early Sequence*, Variorum, Brookfield, Vermont). About "Pythagoreanism" in post-medieval times the Israeli scholar Joseph Agassi wrote: "Who was the last Pythagorean? Perhaps it was Newton; perhaps the twentieth-century [Luitzen] Brouwer and [Niels] Bohr shared it. Yet whatever it meant for Galileo, he opened his first great book with an admission of guilt and the promise to clear the Pythagorean house of all mumbo-jumbo" (Agassi, J 2003, *Science and Culture*, Kluwer Academic Publishers, Dordrecht, p. 148).

11 The branch of mathematics concerned with questions of shape, size and relative position of figures, and with the properties of space. For a rationale of different theses about the contribution of geometry to the learners' epistemological development and its relation to their

nides onwards will be occupied by the ‘ideas’”), and arithmetic (as a form of language).

Boscovich’s understanding of physical reality

The year 2011 marked 300 hundred years from the birth of Boscovich, a mathematician, physicist, astronomer, philosopher, poet, diplomat, and theologian, the author of at least 149 published titles.¹² Boscovich’s mature ideas on ultimate microcosmic and macrocosmic physical reality appear in his *Philosophiae naturalis theoria reducta ad unicam legem virium in natura existentium* (Vienna, 1758; 2nd ed. 1759, 3rd enlarged edition *Theoria philosophiae naturalis*, Venice, 1763).¹³ According to Lancelot Law Whyte (1961b, p. 4), “Boscovich’s ‘Theory’ was the formulation of a *programme for atomic physics* which is still being carried out, though some are unaware of this”. The American physicist Leon Lederman, with scientific writer Dick Teresi, stated that

Boscovich had this idea, one that was real crazy for the eighteenth century (or perhaps any century). Matter is composed of invisible, indivisible a-toms (...). Here’s the good part: Boscovich said these particles had no size, that is, they were geometrical points. Clearly, as with so many ideas in science, there were precursors to this – probably in ancient Greece, not to mention hints in Galileo’s works. As you may recall (...), a point is just a place; it has no dimensions. And here’s Boscovich putting forth the proposition that matter is composed of particles that have no dimensions! We found a particle just a couple of decades ago that fits such a description. It’s called a quark. (...) Boscovich would have been pleased; the Manchester experiments¹⁴ backed up his vision (Lederman and Teresi 1993, pp. 103 and 156).

Boscovich suggested that Democritus might have been wrong in believing that his “atoms” of infinite kinds which differ in shape and size are “uncuttable”, proposing that “atoms” contain smaller parts, which in turn contain still smaller parts, and so forth down to “fields of force” of identical point-like particles with-

intuitive or ordinary experience of the world see De Beaugrande, R 1991, “Knowledge and discourse in geometry: Intuition, experience, logic”, *Zeitschrift für Phonetik, Sprachwissenschaft und Kommunikationsforschung*, vol. 6, pp. 771–827 (reissue 1992, *Journal of the International Institute for Terminology Research*, vol. 3, no. 2, pp. 29–125).

¹² See Proverbio, E (ed.) 2007, *Catalogo delle Opere a Stampa di Ruggiero Giuseppe Boscovich (1711–1787)*, Accademia Nazionale delle Scienze detta dei XL, Roma, pp. 24–25 ff.

¹³ Boscovich added an appendix, *De anima et deo*, relating his theory to a metaphysics of God and the soul.

¹⁴ In the Cavendish Laboratory; see Lederman & Teresi 2006, p. 152ff.

out extension. Today, most atomic physicists accept a modern form of this idea, with a limited number of unextended particles in the background of their models of physical reality. The Nobel physicists Murray Gell-Mann (1969) and Leon Lederman (1988) considered Boscovich's theory as an anticipation of the modern theory of quarks. More recently, the Italian physicist Gianpietro Malescio wrote:

The breakthrough in our modern understanding of forces between atoms can be traced back to the introduction of the interparticle force law, first proposed around 250 years ago by Roger Joseph Boscovich to explain the physical property of materials. (...) In 1758 Boscovich published *Philosophiae naturalis theoria*, which can be considered the cornerstone of modern theories of atomic forces. (...) His law on interaction can be considered as the first interatomic model (Malescio 2003, pp. 501–2)

For Boscovich, the primary elements (*prima elementa*) of matter consist of infinite and permanent identical points of matter (*puncta; punctorum materiae*) that are perfectly simple, indivisible, unextended and separated from one another, interacting in pairs under an oscillatory law.¹⁵ The arrangements of *puncta* [through attractive or repulsive actions or forces (*vires*), achieving stable or unstable equilibrium] account for all physical properties. Boscovich's "point atomism"¹⁶ proposed that the interaction between two *puncta* of action at very small

15 There is a lengthy presentation of Boscovich's atomic theory in Brewster, D (ed.) 1830, *The Edinburgh Encyclopaedia*, 18 vols, vol. III, William Blackwood & John Waugh, Edinburgh, pp. 749–768, preceded by a biographical section (pp. 744–749). For other valuable discussions of Boscovich's ideas on point-atoms see Thompson, W 1889, "On Boscovich's Theory", *Nature*, vol. 40, pp. 545–547 (another issue: *Annual Report of the Board of Regents of the Smithsonian Institution, Volume 1889*, Government Printing Office, Washington, pp. 435–439), and Marcović, Z 1961, "Boscovich's Theoria", in: Whyte 1961a, pp. 127–152. For an evaluation of Boscovich's position in relation to atomist and anti-atomist strains see Casado Vásquez, J M 2000, "Ruggero Giuseppe Boscovich y el Atomismo", *Llull*, vol. 23, pp. 551–575. For a contemporary criticism of Boscovich's ideas on the structure of matter see Abramovic', V 2004, *The Problem of Continuity in the Natural Philosophy of Leibniz and Boscovich*, tr. M Čirić, Klub NT, Belgrade (orig. 1985, *Lajbnicovo i Bošković'evo Shvatanje Kontinuiteta*, Doctoral thesis, Skopje Philosophical Faculty), Holden 2004, pp. 236–272, Abramovic', V 2009, "Geometry, Time and the Law of Continuity (*Theoria Philosophiae Naturalis*... and Boscovich's synthesis of the continuous and the discontinuous; criticisms of Boscovich's concepts of motion space and structure of matter)", URL = <www.chronos.msu.ru/EREPORTS/abramovic_geometry/abramovic_geometry.htm>, and Kragh, H 2011, *Higher Speculations: Grand Theories and Failed Revolutions in Physics and Cosmology*, Oxford University Press, New York, pp. 19–26.

16 For this concept see Whyte 1961a, pp. 106–107. Whyte recalls that "Boscovich expressed a doubt whether his theory should be regarded as 'atomic' in the sense then prevalent. For his *puncta* had lost two attributes of matter: spatial extension, which the Greeks, Descartes, and Newton had taken for granted, and mass, in the Newtonian sense of a continuous [dimensional]

distances is repulsive, and tends to become infinite as the distance approaches zero, while an attractive interaction appears as the distance between two *puncta* increases. As the distance increases, the action oscillates between attraction and repulsion, until at a large distance only attraction operates.¹⁷ The situation of stable equilibrium (between repulsion and attraction) at several interpunctual distances accounts for the finite extension of gross material bodies and for different physical states, while the conception of unrigid units of matter allows for the capacity of material change through thermoelastic modifications and through decomposition. Boscovich developed the idea that all phenomena arise from changes of spatial patterns of identical *puncta* interacting in pairs according to an oscillation that determines their relative motion. Space, on the other hand, is only the relation between *puncta* that, though unextended, are centers of exertion of forces that have extension.¹⁸ Space and time are parallel, infinite and continuous, consisting of reciprocally corresponding *puncta*. The complexity of the world arises from the varied arrangement of different numbers of *puncta*, and from the parameters determining the law of oscillation (Boscovich 1966, *passim*).

According to the physicist and scientific historian Lancelot Law Whyte,

One half of Kepler's mind was Pythagorean; the whole of Boscovich's was, if we may credit to that school the great principle of blending number and nature. For Boscovich discarded 'massy matter', and developed a vision of the entire cosmos as a changing tapestry of points, an open network of foci, each active everywhere in the universe except where it was itself. This dream of a universe of fundamental structure preserves the spirit of Pythagoras, but extends it to cover all motions under a generalized super-Newtonian law.

The aspect of Boscovich's theory which is philosophically most important is its rejection of the ancient dualistic view – assumed by Newton – of the existence of two kinds of space: space occupied by matter, and empty space. For this is substituted [by] a monistic conception of a single realm constituted by the spatial relations of the discrete *puncta*. This is Boscovich's great transcending of appearances (...). A monism of relations has replaced the old dualism of matter and void. Even inertia is now relational, for motion is determined relatively to observed objects (...) (Whyte 1961a, p. 107).

quantity determined only by measurement" (Whyte 1961a, p. 108). For Boscovich, "space" is the frame of spatial relations between *puncta*, "time" the succession of their changing spatial patterns, and "mass" the number of *puncta* in a body (Whyte 1961a, pp. 106–107).

¹⁷ See Heilbron, J L 1982, *Elements of Early Modern Physics*, University of California Press, Berkeley/Los Angeles/London, pp. 58–9 with n. 29.

¹⁸ A *punctum* exerts its action as an extended shell of force (Holden 2004, p. 238).

We may summarize the main innovations of Boscovich's indivisibilist theory in three ideas:¹⁹

- 1) Material permanence without spatial extension – rigid finite units of matter of earlier indivisibilists are replaced by quasi-material *puncta* (point-centers) of action without intrinsic size, mathematical points within spheres of force (Mellor 1922, p. 112);
- 2) Spatial relations without absolute space – internal spatial coordinates (the distances between the two members of pairs of *puncta*) are used instead of external coordinates. The idea of continuous extension, impossible to be generated from non-extended point-particles, is abandoned in order to escape the trap of Zeno's paradoxes;
- 3) Kinematic action without Newtonian forces – in modern dimensional terms, Boscovich's theory is kinematic rather than dynamic; it uses only two-dimensional quantities (length and time) instead of the three (mass, length, and time) employed by Newton. Since all *puncta* are identical, the number of point-particles in a system (an integral number) replaces Newtonian mass. Boscovich's reason for rejecting Newtonian mass rests on "Maclaurin's [or Boscovich's] paradox" (Agassi 1996, pp. 225–6).²⁰

Pythagorean "atomism" or "dynamism"?

Borzacchini believes there was a diffuse tendency among ancient Pythagoreans (exemplified by Ecphantus and Eurytus) towards forms of "number atomism",

¹⁹ These "three original features" were proposed by Roger Anderton in an internet Natural Philosophy Alliance (NPA) chat on "Boscovich's atomic theory" on June 17, 2010, URL = <http://worldnpa.org/pipermail/memberschat_worldnpa.org/2010-June/008725.html>.

²⁰ "Boscovich noticed that the grand Newtonian theory was internally incoherent, indeed, self-contradictory. The concept of action, 'force $\times$ time', which was essential to setting up Newton's third law, that in action by contact action and reaction are equal and opposite, required all such action to take place in a finite time. But the Newtonian ontology required the ultimate material particles to be truly hard, that is incompressible. It follows that all action by contact must be instantaneous, since the ultimate contacting surfaces cannot deform. Forces in instantaneous Newtonian impact would, according to the mechanical definition of action, be infinite. But there is no place for infinite forces in the Newtonian scheme. A variety of stratagems were developed to try to resolve the difficulty. In general physicists in France tended to favour theories without forces [D'Alembert (, J-B le R), 1796, *Traité de Dynamique*, Chez Fuchs, Paris (1st ed. 1743; 2nd ed. 1758)], whereas the English and some of their continental allies tended to favour a mechanics without matter [Heimann, P M & McGuire, J E 1971, "Newtonian Forces and Lockean Powers", *Historical Studies in the Physical Sciences*, vol. 3, pp. 233–306], the so-called dynamical interpretation".

that is, “towards the possibility of representing real or ideal objects through little points [dots] interpretable as whole numbers geometrically organized, and towards the fact that the same numbers were ‘figured’; they had an *eidos*, a ‘form’. (...) Even the Pythagorean idea of a ‘point’ as a ‘monad having a position’²¹ reflects the same figured perception of numbers” (Borzacchini 2005, pp. 149 and 150). Are we authorized, in agreement with Whyte (1961a, pp. 106–107), to consider Boscovich’s atomic theory as a descendent of Pythagorean “number atomism”? Edward McKinnon wrote that “the eventual [contemporary] success of atomism provides an abiding temptation to overemphasize the philosophical worth of the original doctrine” (McKinnon 1992, p. 14), but it must be remembered that many sorts of atomist theories have concurred in the prevailing contemporary views on the subject, and that the discussion about the original doctrines of atomism are far from settled. On the other hand, Robert Purington (1997, p. 114) observed that although “it is easy to scorn the idea that modern atomism owes anything to the Greeks”, some ideas from our Greek past may have been “absorbed and accommodated” with the passing of time, even without empirical evidence, by continuing discussion, and that such a process of “preparing the ground for an idea, of making it plausible (...), is neither easily quantifiable nor attractive to methodologists. Nonetheless, it has played an important role in the reception of ideas”, and this is certain in the case of atomism.

The question of a “Pythagorean atomism” is an especially problematic one, and has been the subject of much speculation and polemic. The problem starts by the discussion of the validity of speaking about a “Pythagorean atomism”. David Furley, for example, said: “I do not believe that the early Pythagoreans were atomists, in any but a trivial sense. There were certain important differences between their ideas and those of Leucippus and Democritus, which make it thoroughly misleading to apply the name to both” (Furley 1967, pp. 44). Ernest Regnault (1883) considered Pythagorean “dynamism” as a complete contrary to atomism (“*tout l’opposé de l’atomisme*”; p. 354), with atomism taken in its form of “pure or mechanical atomism” (“*atavisme pur ou mécanique*”; p. 353), that is, considering extension as the essence of bodies and their component elements. For Regnault, in “dynamism” “all bodies resolve themselves into simple or unextended elements, essentially active forces that are called monads. This

21 μονὰς προσλαβοῦσα θέσιν, “monad with a position” or “with position added” [Aristotle, *De Anima*, 409a6, and *Metaph.*, 1016b24 (an equivalent sentence, cf. also 1084b25); *An. Post.*, 87a31, 88a33; Proclus, *In Eucl.*, 95; 21–2 Friedlein].

22 Seen by Regnault (1883, p. 353) as represented mainly by Anaxagoras, Democritus and Epicurus in old times, and by Descartes, Gassendi and Newton in modern times.

system has Pythagoras as a representative in antiquity; in modern times, it was formulated by Leibniz, and later reprised and developed in a far more logical form by Boscovich²³ (Regnault 1883, p. 353). The strict identification of the concept of indivisibles with "extended elements" (and the associated perception of an incompatibility between the "corpuscular theories" of atomism and the "force theories" of dynamism),²⁴ though accepted by some authors other than Regnault, has not received universal acceptance.²⁵ Avoiding the strict taggings of "atomism" and "dynamism", James Ward (1903, p. 124 ff.), Nicolai Velimirović (2008 [orig. 1916], p. 31) and Frederick Copleston (1960, p. 54) preferred to qualify Boscovich's physical theory as "dynamic atomism".²⁶

An identification between indivisibles and a lack of extension has been proposed since Aristotle,²⁷ at least at the mathematical level – and Theodor Gomperz (1912, p. 121) thought the Stagirite may have been "contending for the existence (...) of [indivisible] spatial units having the nature of points, such entities as the 'philosophical' atoms devised by Boscovich".²⁸ Karl Stiegler (1980–81), in turn, sees the birth of the idea of unextended atoms in space as presupposed by the paradoxes of Zeno, which was criticized by Aristotle along with discontinuity.

23 "Tous les corps ... se résolvent en éléments simples ou inétendus, forces essentiellement actives, qu'on a appelées monades. Ce système a pour représentant dans l'antiquité Pythagore; dans les temps modernes, il a été formulé par Leibnitz, puis repris et développé sous une forme beaucoup plus logique par Boscovich".

24 See Meyerson, E 1908, *Identité et Réalité*, Félix Alcan, Paris, p. 60 ff.; Boas, G 1930, *A Critical Analysis of the Philosophy of Emile Meyerson*, The John Hopkins Press/H. Milford & Oxford University Press, Baltimore/London (reprint 1968, Greenwood Press, New York), pp. 14–15, 19–23, and *passim*.

25 Isaac Newton, in his in his *Principia* of 1687, proposed a dynamic atomism replacing pure mechanical interaction (through entanglement and collision) of atoms (whose only fundamental properties are size, shape, and motion) with short-range interparticle forces of attraction and repulsion (see Thackray, A 1970, *Atoms and Powers: an essay on Newtonian matter-theory and the development of chemistry*, Harvard University Press, Cambridge, Massachusetts), while Friedrich Wilhelm Joseph von Schelling [2004, *First Outline of a System of the Philosophy of Nature* (1799), tr. K R Peterson, State University of New York Press, Albany, New York, pp. 20–22 with notes] proposed an "atomic dynamism" that conciliates dynamism and atomism.

26 Others prefer the label "atomic dynamism".

27 *Metaph.*, 1083b11–16: "that bodies should be composed of numbers, and that these numbers should be mathematical, is impossible. For (a) it is not true to speak of indivisible magnitudes (οὔτε γὰρ ἄτομα μεγέθη λέγειν ἀληθές); (b) assuming that this view is perfectly true, still units at any rate have no magnitude; and how can a magnitude be composed of indivisible parts? (εἴ θ' ὅτι μάλιστα τοῦτον ἔχει τὸν τρόπον, οὐχ αἷ γε μονάδες μέγεθος ἔχουσιν: μέγεθος δὲ ἐξ ἀδιαίρετων συγκεῖσθαι πῶς δυνατόν)" (tr. W D Ross).

28 Gomperz's reasoning followed a line previously tackled by the Jesuit Francisco Suárez (1548–1617; see Rossi 1999, pp. 73–6) and the "Zenonists" (see below).

“Zeno’s fundamental paradox”, the logical reasoning that an extended line consists of unextended points, underlies his four paradoxes on motion (Ferber 2000, p. 295). Before Aristotle, Leucippus had suggested that in physical reality the atoms, infinite bodies that compose the plenum,²⁹ are indivisible on account of the smallness of their mass.³⁰ After Aristotle, Robert Grosseteste (1168–1253) defended the existence of physical indivisibles without size,³¹ and Gottfried Leibniz (1646–1716) didn’t hesitate in talking in his *Theoria motus abstracti* (1671) about *indivisibilia seu inextensa* (Leibniz 1880b, p. 228), later arguing in the *Monadology* (1714), 3, that “where there are no parts at all, no extension or figure or divisibility is possible”³² (Leibniz 1991, p. 51).

Furley (p. 47) believed Paul Tannery was “the first to attribute a kind of atomism to the Pythagoreans”³³ but this opinion neglects many early suggestions of the existence of a Pythagorean atomism, as those of Giordano Bruno (1548–1600),³⁴ Henry More (1614–1687), Francesco D’Andrea (1625–1698), and Isaac Newton (1643–1727),³⁵ thinkers who belonged to an ancient tradition (represented by Aristotle, for example) that recognized the existence of two opinions about the nature of the ultimate elements of which bodies are constituted: one that proposed that matter is a composition of units incapable of further division (the atoms, hence “atomism”), and another that thought there is no limit to matter’s divisibility (as in Aristotle’s own view).

There have been different forms of atomism since antiquity, and after Aristotle the attachment of Pythagoras’ name to some sort of understanding of

29 παμπλήρες ὄν or πλήθος.

30 Fr. 67 A7 Diels-Kranz (Aristot. *De gen. et corr.* A 8. 325 a23): (...) φησιν εἶναι· τὸ γὰρ κυρίως ὄν παμπλήρες ὄν. ἀλλ’ εἶναι τὸ τοιοῦτον οὐχ ἔν, ἀλλ’ ἅπειρα τὸ πλήθος καὶ ἀόρατα διὰ σμικρότητα τῶν ὀγκῶν.

31 For Grosseteste, “extension itself depends on the infinite multiplication (or replication) of a single dimensionless point of light” [Molland, G 2001, “Roger Bacon’s corpuscular tendencies (& some of Grosseteste’s too)”, in C Lüthy, J E Murdoch & W R Newmann (eds.), *Late Medieval and Early Modern Corpuscular Matter Theories*, Koninklijke Brill NV, Leiden/Boston/Köln, pp. 57–74 (see p. 59)].

32 “Or là où il n’y a point de parties, il n’y a ni étendue, ni figure, ni divisibilité possible. Et ces Monades sont les véritables Atomes de la nature, et en un mot, les Elemens (sic) des choses”.

33 Tannery, P 1887. *Pour l’histoire de la science hellène. De Thalès à Empédocle*, Félix Alcan, Paris, pp. 250–251.

34 See Gemelli, B 1996, *Aspetti dell’Atomismo Classico nella filosofia di Francis Bacon e nel Seicento*, Leo S Olschki, Florence, pp. 146–7; Gatti, H 2001, “Giordano Bruno’s Soul-Powered Atoms: From Ancient Sources towards Modern Science”, in C Lüthy, J E Murdoch & W R Newmann (eds.), *Late Medieval and Early Modern Corpuscular Matter Theorie (op. cit.)*, pp. 163–180 (see pp. 172–173).

35 See ahead about More, D’Andrea, and Newton.

non-infinite divisibility of matter would surprise no one. Thomas Bradwardine (c. 1290 – 1349), in his *Tractatus de continuo* (between 1328 and 1335), mentioned five opinions concerning the composition of *continua*, and their supporters³⁶ (Murdoch 1974b, p. 314):

- 1) a *continuum* is not composed of indivisibles (atoms) [Aristotle, Averroes, and most of the moderns];
- 2) a *continuum* is composed of indivisibles:
 - 2a) a *continuum* is composed of indivisible bodies [Democritus];
 - 2b) a *continuum* is composed of indivisible points:
 - 2b1) a *continuum* is composed of finite indivisible points [Pythagoras, Plato, Walter Chatton];
 - 2b2) a *continuum* is composed of infinite indivisible points:
 - 2b2a) a *continuum* is composed of infinite indivisible points immediately joined to one another [Henry of Harclay];
 - 2b2b) a *continuum* is composed of infinite indivisible points which are mediate to one another (mutually separated) [Robert Grosseteste].

Apart from these opinions, atoms can also be conceived as points without size (opinion ascribed to Pythagoras) or as particles with very small but finite sizes (as in Democritus) (Rosenfeld 1988, pp. 191–2).

Contemporary scientific historiography proposes “there have been only three basically distinct and widely successful conceptions of atomic particles”: hard

36 *Tractatus de continuo* (Ms. Toruń R. 4^o, 2, p. 165; Erfurt, Ms. Amploniana 4^o, 385, ff. 25v-26r): “Pro intellectu huius conclusionis est sciendum, quod circa compositionem continui sunt 5 opinioniones famose inter veteres philosophos et modernos. Ponunt enim quidam, ut Aristoteles et Averroes et plurimi modernorum, continuum non componi ex atomis, sed ex partibus divisibilibus sine fine. Alii autem dicunt ipsum componi ex indivisibilibus dupliciter variantes, quoniam Democritus ponit continuum componi ex corporibus indivisibilibus. Alii autem ex punctis, et hii dupliciter, quia Pythagoras, pater huius secte, et Plato ac Waltherus modernus, ponunt ipsum componi ex finitis indivisibilibus. Alii autem ex infinitis, et sunt bipartiti, quia quidem eorum, ut Henricus modernus, dicit ipsum componi ex infinitis indivisibilibus immediate coniunctis; alii autem, ut Lyncul[niensis], ex infinitis ad invicem mediate. Et ideo dicit conclusionem: ‘Si unum continuum componatur ex indivisibilibus secundum aliquem modum’, intendendo per ‘modum’ aliquem predictorum modorum; tunc sequitur: ‘quodlibet continuum sic componi ex indivisibilibus secundum similem modum componendi’” [apud Murdoch, J E 2002, “Beyond Aristotle: Indivisibles and Infinite Divisibility in the Later Middle Ages”, in C Grellard & A Robert (eds.), *Atomism in Late Medieval Philosophy and Theology*, Koninklijke Brill NV, Leiden, pp. 15–38 (see pp. 26–27 n. 37)].

atoms [Democrit; Newton], point-centers [Boscovich], and wave-particles [De Broglie; Schrödinger] (Whyte 1961c, pp. 22–3).³⁷ We can agree, anyway, with Bernard Pullman saying that

with a small leap of imagination one can detect in (...) Pythagorean physics a premonition of an atomism of sorts. Such is the opinion of Pichot,³⁸ who writes: “(...) While it does not qualify as genuine atomism, the notion that things are made of particles could be construed as presaging it. (...) The only concrete aspect of Pythagorean physics would be the outline of a physical of particles, in which the units (which constitute numbers and things) have a thickness and a consistency, and are indivisible; hence they are atoms” (Pullman 1998, p. 26).

Pullman (pp. 28–9) believes “a little bit of imagination” reveals in Pythagoras’ doctrine “a glimpse of what amounts to a corpuscular physics, a sort of arithmetic atomism”.

Understanding physics through mathematics: background

For the purpose of our investigation about a possible Boscovichean approximation to “Pythagorean atomism”, it will suffice to remember from ancient philosophy a few passages that deal with the relation between numbers and reality: “(...) the so-called Pythagoreans applied themselves to mathematics, and were the first to develop this science; and through studying it they came to believe that its principles (*archês*) are the principles of everything. (...) They [the Pythagoreans] assumed the elements (*stoicheîa*) of numbers to be the elements of everything, and the whole universe to be a proportion (*harmonía*) or number”³⁹ (Ar-

37 “To these three primary ideas must be added various special conceptions of less value: vortex rings and other rotating units, twists in a rotationally elastic jelly, dislocations or holes in a close packing of spheres, negative atoms of various kinds, and so on. There is also Eddington’s ghost-particle, from which he excluded all vestiges of materiality, so that it became merely a ‘carrier of variants’ or ‘a conceptional unity whose probability function’ is specified by certain wave vectors” (Whyte 1961c, p. 24).

38 Pichot, A 1991, *La Naissance de la Science*, 2 vols., vol. 2, *Grèce Presocratique*, Gallimard, Paris, page not mentioned.

39 (...) οἱ καλούμενοι Πυθαγόρειοι τῶν μαθημάτων ἀψάμενοι πρῶτοι ταῦτά τε προήγαγον, καὶ ἐντραφέντες ἐν αὐτοῖς τὰς τούτων ἀρχὰς τῶν ὄντων ἀρχὰς ᾗθησαν εἶναι πάντων. (...) τὰ τῶν ἀριθμῶν στοιχεῖα τῶν ὄντων στοιχεῖα πάντων ὑπέλαβον εἶναι, καὶ τὸν ὅλον οὐρανὸν ἁρμονίαν εἶναι καὶ ἀριθμόν. Quotations are from Aristotle 1989 (orig. 1933–1935), *Metaphysics* (Loeb

istotle⁴⁰, *Metaph.* 985b23–26; 986a1–3); “(...) they construct the whole universe of numbers, but not of numbers consisting of abstract units; they suppose the units to be extended (*monádas hypolambánousin échein mégethos*)”⁴¹ (1080b18–20)⁴²; “(...) they construct natural bodies, which have lightness (*kou-phótêta*) and weight (*báros*), out of numbers which have no weight or lightness” (1090a32–34)⁴³; “(...) from numbers [spring] points; from points, lines; from lines, plane figures; from plane figures, solid figures; from solid figures, sensible bodies”⁴⁴ (Alexander Polyhistor, *FrGrHist* 273 F 93).⁴⁵ The link between all these

Classical Library), Books I–IX, in *Aristotle in 23 Volumes, Aristotle XVII*, and Books X–XIV, in *Aristotle XVIII*, tr. H Tredennick, William Heinemann/Harvard University Press, London/Cambridge, Massachusetts, 1989.

40 Aristotle's criticism of “Pythagorean atomism” results from his distinction between mathematical unit (*monás*), geometric point (*stigmê*) and a body with magnitude (*Anal. Post.* 87a36, 88a33; *Physica*, 227b27–35; *De Anima*, 409a; *Metaph.*, 1016b24–26, 1080b, 1083b, 1069a12; 1090b16–21). The same applies to his remarks about necessary properties of a spatial continuum (*Physica*, 213b, 231a–b, 237a).

41 τὸν γὰρ ὅλον οὐρανὸν κατασκευάζουσιν ἐξ ἀριθμῶν, πλὴν οὐ μοναδικῶν, ἀλλὰ τὰς μονάδας ὑπολαμβάνουσιν ἔχειν μέγεθος.

42 See also 1080b32–33: “But all who hold that Unity is an element (*stoicheion*) and principle (*archên*) of existing things regard numbers as consisting of abstract units, except the Pythagoreans; and they regard number as having [spatial] magnitude (*mégethos*), as has been previously stated” (μοναδικούς δὲ τοὺς ἀριθμοὺς εἶναι πάντες τιθέασι, πλὴν τῶν Πυθαγορείων, ὅσοι τὸ ἐν στοιχεῖον καὶ ἀρχὴν φασιν εἶναι τῶν ὄντων· ἐκεῖνοι δ' ἔχοντας μέγεθος, καθάπερ εἴρηται πρότερον).

43 (...) κατὰ μέντοι τὸ ποιεῖν ἐξ ἀριθμῶν τὰ φυσικὰ σώματα, ἐκ μὴ ἔχόντων βάρος μηδὲ κουφότητα ἔχοντα κουφότητα καὶ βάρος.

44 (...) ἐκ δὲ τῶν ἀριθμῶν τὰ σημεῖα ἐκ δὲ τούτων τὰς γραμμάς, ἐξ ὧν τὰ ἐπίπεδα σχήματα ἐκ δὲ τῶν ἐπιπέδων τὰ στερεὰ σχήματα ἐκ δὲ τούτων τὰ αἰσθητὰ σώματα (...). The translated quotation comes from Diogenes Laertius 1925, *Lives of Eminent Philosophers*, 2 vols. (Loeb Classical Library), VIII, 25, tr. R. D. Hicks, William Heinemann/Harvard University Press, London/Cambridge, Massachusetts.

45 In Alexander's full quotation, Pythagorean and Platonic ideas (from the indirect tradition) are not distinguished; see Riedweg, C 2008, *Pythagoras: His Life, Teaching, and Influence* (2007), tr. S. Rendall, Cornell University Press, Ithaca, New York, p. 23. Similar ideas appear already in Speusippus (c. 408–339/8 b.C.) and Xenocrates (c. 396/5–314/3 b.C.). In a fragment of the treatise “*On the Pythagorean numbers*”, preserved by Nichomachus *apud* Iamblichus, Speusippus matches one, two, three and four respectively with the point, line, plane, and solid (fr. 44 A13 Diels-Kranz = fr. 28 Tarán; see also Aristotle, *Metaph.*, 1085a31–b4 = fr. 51 Tarán; *Topics*, 108a–b). Xenocrates thought that numbers and forms (ideas) have the same nature, and “it appears that Xenocrates pictured the universe as unfolding in the sequence: (1) forms = numbers; (2) lines; (3) planes [surfaces]; (4) solids; (5) solids in motion, i. e. astronomical bodies; ...; (n) ordinary perceptible things. Solid shapes aren't mentioned in this sentence [*Metaph.*, 1028b24–27; Asclepius' commentary on this passage tells us that it deals with Xenocrates], but they were earlier,

passages and Boscovich's thought occurs through an identification between elements (units) of matter and "space". According to Giovanni Casertano, "Pythagoreans had a 'spatial idea of number', so the number turns into a concept with dimensions; and the word *mégethos* is just the right one to describe this ambiguous nature, including both *mathêmatiká* and *aisthêtà sômata*".⁴⁶ If the property of a number to be endowed with magnitude (*mégethos*) is associated with materiality (corporeality),⁴⁷ a number (or a monad) can be taken as a primary indivisible "body",⁴⁸ and an interpretation more consonant with the Aristotelian pas-

in 1028b17–18, and they are a standard stage in this sequence" [Dancy, R 2009, "Xenocrates", in E N Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, Fall 2009 Edition, URL = <<http://plato.stanford.edu/archives/fall2009/entries/xenocrates/>>]. Xenocrates and some associated thinkers "construct spatial magnitudes out of matter and a number – two in the case of lines, three, presumably, in that of planes, and four in that of solids" (*Metaph.*, 1090b21–23, tr. W D Ross). On differences (already voiced in *Metaph.*, 1076a20–21) between Speusippus and Xenocrates on these topics see Cherniss, H F 1959, "Review of H. D. Saffrey, 'Le Περὶ φιλοσοφίας d'Aristote et la théorie platonicienne des idées et des nombres'", *Gnomon*, vol. 31, pp. 36–51 (see p. 41 n. 2); reprinted in Cherniss, H F 1977, *Selected Papers* (ed. L Tarán), E J Brill, Leiden, pp. 423–430 (see p. 428 n. 2). About approximations between Speusippus and Xenocrates on these topics see Dillon, J 2002, "Theophrastus' Critique of the Old Academy in the *Metaphysics*", in W W Fortenbaugh & G Wöhrle (eds.), *On the Opuscula of Theophrastus – Akten der 3. Tagung der Karl- und Gertrud-Abel-Stiftung vom 19.–23. Juli 1999 in Trier*, Franz Steiner, Stuttgart, pp. 175–187 (see p. 178).

46 In the article published in the present volume.

47 Aristotle 1953. *Metaphysics*, 2 vols., tr. W D Ross (*op. cit.*), vol. 2, p. 429. See also Raven, J E 1954, "The Basis of Anaxagoras' Cosmology", *Classical Quarterly*, new series, vol. 4, pp. 123–137 [p. 133: "(...) whereas, with the exceptions of the Milesians at one end of the story and the Atomists at the other, every single one of the pre-Socratics was striving after an incorporeal principle, their minds were yet so firmly possessed by the preconception that the only criterion of reality was extension in space that one and all they ended in failure"]; Guthrie, W K C 1962, *A History of Greek Philosophy*, vol. 1: *The Earlier Presocratics and the Pythagoreans*, Cambridge University Press, Cambridge [pp. 234 & 280: "(...) the notion of incorporeal reality was not yet grasped by the Pythagoreans or any of their contemporaries... the only form of existence so far conceivable is bodily substance; hence it {in Pythagoreanism, the void or air} is thought of as a particularly tenuous form of matter"]; Pitagorici 1958–1962–1964, *Testimonianze e Frammenti*, ed. M Timpanaro Cardini, 3 vols., La Nuova Italia, Florence, vol. 3 [p. 92: "(...) per i Pitagorici la realtà corporea era tale, che la sua esistenza era condizionata dalla presenza del numero; e questo a sua volta trovava la sua espressione nella realtà corporea" ("to the Pythagoreans, corporeal reality was such that its existence was conditioned by the presence of number, and this, on its turn, found its expression on corporeal reality")].

48 See Aristotle, *De Anima*, 409a §19 [Aristotle 1902, *Aristotle's Psychology: A Treatise on the Principle of Life (De Anima and Parva Naturalia)*, tr. W A Hammond, S Sonnenschein & Co., London (reissue 2009, Cornell University Press, Ithaca, New York), p. 30]: "(...) there's no difference in speaking of monads and of small bodies" (οὐθὲν διαφέρειν μονάδας λέγειν ἢ σωματία μικρά)].

sages just quoted can be given to a famous sentence of Ecphantus (a contemporary of Archytas?): "(...) the first bodies are indivisible and there are three differences between them: magnitude (*mégethos*), shape (*schêma*) and power (*dýnamis*). And the number of them is limited (*plêthos hôrisménon*) and [the space?] infinite (...)"⁴⁹

According to Aetius, Ecphantus was the first Pythagorean to believe that the monads are corporeal.⁵⁰ It is believed today that in early Pythagorean philosophy the concept of space was confounded with that of matter; only later did Archytas⁵¹ establish a distinction between place (*topos*)⁵² and matter (Jammer 1993, pp. 9 and 10). As for the idea of unextended indivisibles as the minimal elements of reality, it may have been proposed before Aristotle without originally having any direct link with the paradoxes of Zeno: According to George McLean and Patrick Aspell, after the discovery of irrational numbers,

Eventually the mathematicians incorporated the irrationals into their general number theory, but to minimize this "scandalous discovery" it was accepted as a lesser evil to produce rational solutions that could approximate to any desired degree the exact irrational one. This meant shrinking the original monadic point beyond any assignable limit, until it became an actual infinitesimal (a unit-point-atom) (McLean and Aspell, 1971, p. 43).

In Boscovich's thought, as we have seen, matter and space are different aspects of a single reality, and because he rejected the absolute identification between matter and extension (accepted by Descartes), the primary units of physical (corporeal) existence are seen as unextended, thus providing an intermediary reality between spirit and matter, and presenting the *puncta* as associated with materiality through their grouping in increasing orders of magnitude (starting from a pair of *puncta*, as in the atomist theory of the *mutakallimûn*).⁵³ The atomic theory

⁴⁹ Fr. 51 A1 Diels-Kranz: (Hippol. *Ref.* I 15 p. 18 [*Dox.* 566, W. 18]) (...) τὰ μὲν πρῶτα ἀδιαίρετα εἶναι σώματα καὶ [I 442. 10 App.] παραλλαγὰς αὐτῶν τρεῖς ὑπάρχειν, μέγεθος σχῆμα δύναμις, ἐξ ὧν τὰ αἰσθητὰ γίνεσθαι. εἶναι δὲ τὸ πλῆθος αὐτῶν ὀρισμένον καὶ τοῦτο [?] ἄπειρον (...).

⁵⁰ Fr. 51 A2 Diels-Kranz (Aët. I 3, 19 [*Dox.* 286]): (...) τὰς γὰρ Πυθαγορικὰς μονάδας οὗτος πρῶτος ἀπεφρήνατο σωματικὰς.

⁵¹ See Simplicius, *Physics*, 108a. Aristotle praises Archytas for having offered definitions which took account of both form and matter [fr. 47 A22 Diels-Kranz (*Metaph.*, 1043a14–26); the words for "form" (εἶδος; μορφή) and "matter" (ὕλη) are Aristotelian].

⁵² The Greeks had no word for the notion of space in the modern cosmological sense; see Santillana, G 1961, *The Origins of Scientific Thought: from Anaximander to Proclus, 600 B.C. to 300 A.D.* (The History of Scientific Thought, vol. 1), University of Chicago Press/Mentor Books/Weidenfeld & Nicolson, Chicago/New York/London, p. 65.

⁵³ According to the *Mutakallim* Abû al-Hasan 'Alî ibn Ismâ'il Al-Ash'arî (873/4–935/6), the juxtaposition of two atoms devoid of dimensions produces a mono-dimensional structure, the

of the *mutakallimûn*⁵⁴ exerted an influence on Leibniz,⁵⁵ who was an important influence on Boscovich (see ahead). Furthermore, Boscovich may have had some knowledge of the atomic theory of *Kalâm* through the work of Moses Maimonides (1135–1204). Many ideas from the *mutakallimûn* reached the Latin West through translations of Maimonides' *Dalâlât al-Hâ'irîn* ("Guide of the Perplexed"), starting with a first complete translation printed in Paris in 1520⁵⁶ by Augustinus Justinianus (Agostino Giustiniani) and ascribed to Jacob Mantino. The Greek background of the "atomism" of the *mutakallimûn*, though generally accepted, is far from elucidated. Neopythagorean influences cannot be discarded; logical parallels with Zeno's ideas have been pointed out by Andrey Vadimovich Smirnov (2000).

Boscovich's "point atomism" is a kind of middle term between physical atomism (disavowed by Aristotle) and mathematical indivisibilism (prevalent in the earlier fourteenth century) – "mathematical in the philosophical sense of the atoms (...) being points or extensionless indivisibles" (Murdoch 1974b, p. 312)⁵⁷; an indivisibilism or atomism proposed as "an anti-Aristotelian answer

juxtaposition of two mono-dimensional produces a duo-dimensional structure, and the juxtaposition of two duo-dimensional produces a three-dimensional structure [Al-Ash'arî 1929–1933, *Kitâb Maqâlât al-Islâmiyyîn wa Ikhtilâf al-Muṣallîn* ("Book of the Sayings of the People of Islam and Controversies Among Those Who Pray"), ed. H Ritter as *Die dogmatischen Lehren der Anhänger des Islam*, 2 vols. and index, Leipzig/Istanbul, 1929–1933; reprint 1963, 1980, Franz Steiner, Wiesbaden, pp. 316–318]. In Friedländer's translation of Moses Maimonides' *Dalâlât al-Hâ'irîn*, chapter 73, first proposition, we read: "The universe, that is, everything contained in it, is composed of very small parts [atoms] which are indivisible on account of their smallness; such an atom has no magnitude; but when several atoms combine, the sum has a magnitude, and thus forms a body'. If, therefore, two atoms were joined together, each atom would become a body, and they would thus form two bodies, a theory which in fact has been proposed by some Mutakallimim. All these atoms are perfectly alike; they do not differ from each other in any point. The Mutakallimim further assert, that it is impossible to find a body that is not composed of such equal atoms which are placed side by side" [Maimonides, M 1904, *Guide for the Perplexed*, tr. M Friedländer, revised 2nd edition (1st ed. 1881), G Routledge & Sons/E P Dutton, London/New York (reprint 2008, Forgotten Books, Charleston, South Carolina), p. 261].

54 See Dhanani, A 1997. "Atomism is Islamic Thought", in H Selin (ed.), *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*, Kluwer Academic Publishers, Dordrecht, pp. 139–142; Rosenfeld, B 1997, "Geometry in the Islamic World", in H Selin (*op. cit.*), pp. 375–377, and Rosenfeld 1988, pp. 193–5.

55 See Jammer 1993, pp. 62–64.

56 Maimonides, M 1520, *Rabbi Mossei Aegyptii Dux seu Director dubitantum aut perplexorum, in treis libros diuisus & summa accuratione*, Ab Iodoco Badio Ascensio, Paris (reprint 1964, Minerva, Frankfurt am Main).

57 A theory "in which atoms were held to be indivisible because they were nothing more than geometrical points, without any extension. But such non-extended atoms could not easily be

to the question of the composition of continua" and "designed simply to explain the structure of magnitudes, and specifically of space, time, and motion as magnitudes" (Murdoch 1974b, p. 312).⁵⁸ Intellectual circumstances marking the transition from medieval times to modernity prepared the way for Boscovich's "point atomism":

Natural philosophy asserted that quantities – including indivisibles such as points, lines, and surfaces – really exist in substances as their accidental forms, although they are considered mathematically in abstraction from such substances. But when geometric indivisibles such as points and lines, or purported physical indivisibles such as instants or exact degrees of quality, were supposed to exist in reality (*in esse*), paradoxes ensued. (...) The previously standard Aristotelian understanding of the relation between mathematics and physics broke down: geometry could no longer be understood to deal with quantities existing in, but considered in abstraction from, physical bodies (Sylla 1997, pp. 148 and 149).

A new mathematization of nature occurred in the Renaissance,⁵⁹ and it may be worth mentioning that an influential historian, the Franciscan scholar André Thevet (1502–1590), in his *Vrais Pourtraits et Vies des Hommes Illustres*⁶⁰

conceived as taking part in physical explanations of extended entities" (Henry, J 2008, *The Scientific Revolution and the Origins of Modern Science*, 3rd ed., Palgrave Macmillan, New York, p. 71).

58 The medievals perceived quite well that "normal [mathematical] definitions of points, lines, surfaces, and instants in no way meant this had any effect upon points and the like in geometry and even upon arguments against indivisibilism or atomism of any sort" [Murdoch, J E 2002, "Beyond Aristotle: Indivisibles and Infinite Divisibility in the Later Middle Ages" (*op. cit.*), p. 25]. Cristophe Grellard thinks the cases of Gerard of Odo, William Crathorn, Nicholas of Autrecourt, John Wyclif, and even Walter Burley, "seem to suggest that we should be more cautious" about seeing the mathematical dimension of medieval atomism as a purely intellectual reaction to Aristotle's conception of the continuum ["Nicholas of Autrecourt's atomistic physics", in C Grellard & A Robert (eds.), *Atomism in Late Medieval Philosophy and Theology*, 2002 (*op. cit.*), pp. 107–126 (see p. 107 n. 3)]. See also Rosenfeld 1988, p. 190 ff.

59 See Roux, S 2010, "Forms of Mathematization (14th–17th Centuries)", *Early Science and Medicine*, vol. 15, no. 4–5, pp. 319–337; Goulding, R 2010, *Defending Hypatia: Ramus, Savile, and the Renaissance rediscovery of mathematical history*, Springer, Dordrecht/Heidelberg/London/New York, p. 68 ff.

60 Thevet, A 1584, *Les Vrais Pourtraits et Vies des Hommes Illustres Grecz, Latins et Payens recueilliz de leurs tableaux, livres, médailles antiques et modernes*, 2 vols, La Veuve I Keruert (J Kervert) et Guillaume Chaudière, Paris (reprint 1973 with an introduction by R C Cholakian, Scholars' Facsimiles & Reprints, Delmar, New York); 2nd ed. 1671 as *Histoire des Plus Illustres et Sçavans Hommes de leurs Siècles, Tant de l'Europe, que de l'Asie, Afrique & Amerique*, 8 vols., Chez François Mavger (Mauger), Paris; see vol. 1, pp. 199–206. Thevet's work was written in imitation of Plutarch's Βίοι Παράλληλοι. A partial English translation of the *Vrais Pourtraits et Vies* appeared in 1657 with the title *Prosopographia: Or, Some select pourtraitures and lives of*

(1584), treated Pythagoras as the “inventor of mathematics”.⁶¹ Inappropriate as this opinion may be to readers of Walter Burkert, it has enjoyed a broad acceptance: recently a respected scholar, doctor in applied mathematics by the California Institute of Technology, considered Pythagoras as the “father of logical proof,”⁶² because of his proof of the well-known Pythagorean theorem”, and “father of mathematics” as well, “because logical proof is the heart of mathematics” (Ellison 1999, p. 429), while in a publication sponsored by the “Real Sociedad Matemática Española” to mark the “International Year of Mathematics” the mathematician and prolific writer on mathematical history Pedro Miguel González Urbaneja (2000, p. 30) called Pythagoras “the true creator of pure mathematics, transforming it into a liberal art”.⁶³

Links connecting Boscovich to the Pythagoreans: language

Can we speak of an uninterrupted discussion linking Pythagorean number atomism to Boscovich’s atomic theory? To answer this question, we may first separate the idea of a discussion in its two components, form and content. One of the undoubted contributions of Pythagoreanism to science has been what Ladislav Kvasz called the move “from symbolic language of arithmetic to the iconic language of geometry” (Kvasz 2008, p. 24), resulting in an increase “in logical as well as in expressive power”. As stated by Borzacchini (2005, p. 150), Pythagorean thought conceives “on one side a relation between *arithmo-geometry* and *being*, and on another side an almost ‘modelistic’ idea, in Archytas, of the possibility of representing *being* in geometric form in that which will become the mental world: not by chance, terms of visual origin provide the initial vocabulary both of the ‘verbs of knowledge’ and of ‘theoretical geometry’”.

Although the new language of geometry was originally developed in close connection with arithmetic, “the discovery of incommensurability led the Greeks

ancient and modern illustrious personages, tr. G Gerbier, alias D’Ovilly, *et al.*, Abraham Miller for William Lee, London (other ed. 1676, John Hayes for George Sawbridge, Cambridge).

61 “(...) *il a esté inventeur, à tout le moins principal illustre de toute la Philosophie, spécialement de celle que nous apellons Mathématique*” (1671, vol. 1, p. 199).

62 I have already pointed earlier to Proclus, *In Eucl.*, as influential in the presentation of Pythagoras as originator of the notion of mathematical proof. Robert Goulding has shown how influential Proclus was on Renaissance humanists [Goulding, R 2010, *Defending Hypatia* (*op. cit.*), p. XVff., 6ff.].

63 González Urbaneja’s opinion also stems from Proclus’.

to abandon the Pythagorean arithmetic basis of their new geometrical language⁶⁴ and to separate geometrical forms from the arithmetic content" (Kvasz 2008, p. 24). But before that separation took place, the Pythagorean theory of figurate numbers (or polygonal numbers) was able to convey an arithmetic content in a geometric form. Kvasz explains thus that theory and its implications:

Using small dots in sand or pebbles (*psêphos*) it represents numbers geometrically – as square numbers (i.e., numbers the *psêphoi* of which can be arranged into a square, like 4, 9, 16, ...), triangular numbers (like 3, 6, 10, ...) and so on.⁶⁵ With the help of this geometrical form, arithmetical predicates can be visualized.⁶⁶ (...) This very fact, that arithmetic properties become expressible in the language, makes it possible to prove universal theorems (and not only particular statements, as was the case until then). (...) The language of geometry is able to do this, thanks to an expression of a new kind – a *segment of indefinite length* (Kvasz 2008, pp. 24–5).

Another advantage of geometrical language is that "geometry allows, in effect, the generalization of arithmetic calculations and the inclusion of irrational quantities in those generalized calculations" (Michel 1950, p. 646).⁶⁷

64 "The language of geometry is more general than that of arithmetic. In arithmetic the side and diagonal of a square cannot be included in one calculation. We can either choose a unit commensurable with the side, but then it will be impossible to express the length of the diagonal by a number, or we can choose a unit commensurable with the diagonal, but then we will be unable to express the length of the side. So the incommensurability of the side and diagonal of the square reveals the boundaries of the expressive power of the language of elementary arithmetic" (Kvasz 2008, pp. 22–3. The original quotation appeared in Kvasz, L 2000, "Changes of Language in the Development of Mathematics", *Philosophia Mathematica*, vol. 3, no. 8, pp. 47–83).

65 Pentagonal numbers are those which can be arranged into a pentagon (5, 12, 22 ...). Oblong numbers are those that can be arranged in a rectangle one unit wider than it is high; each is twice a triangular number. Oblong numbers have sides in the ratios 1:2, 2:3, 3:4, 4:5, 5:6, and so forth. The difference between two positive triangular numbers is a trapezoidal number.

66 "For instance, an even number is a number the *psêphoi* of which can be ordered in a double row. (...) The theorem that the sum of two even numbers is even can be easily proved using this Pythagorean language. It follows from the fact that if we connect two double rows, one to the end of the other, we will again get a double row. Therefore the sum of *any two* even numbers must be even. (...), because the double row which represented an even number could be of any length [– a *segment of indefinite length* (...); this was the essence of the Pythagorean innovation]. The geometrical form is independent of the particular arithmetical value to which it is applied" (Kvasz 2008, pp. 24–5).

67 See Kvasz 2008, pp. 25–6.

While developing a qualitatively new kind of formal language,⁶⁸ the Pythagoreans connected this “geometrical language” with an interesting kind of “arithmetical atomism”: “The Pythagoreans supposed every quantity, among others also the side and the diagonal of a square, comprise a finite number of units. So the proportion of the lengths of the side and the diagonal of the square equals the proportion between the numbers of units, from which they are composed” (Kvasz 2008, p. 22). In Pythagorean geometric representations, the basic finite units of every quantity were points (dots), primitively represented by pebbles (*psêphoi*, latin *calculi*)⁶⁹; the early doctrine of odd and even numbers also developed from arrangements of *psêphoi*, as can be seen in Epicharmos’ fragment B2.⁷⁰ It is believed by some authors that the mental association between the minimal units of reality as points (dots) and the transposition of the geometrical reality of lines, planes and solids to empirical existence led to Pythagorean “arithmetical atomism”, in which the minimal units of physical reality are numbers, “and since a physical existent is necessarily extensive, number is extensive, which is to say that it is body or bodily (...). Numbers have the essential features of body, namely, extensiveness, boundedness or limitedness, and fullness” (Leclerc 1972, p. 46 with n. 14).

In Walter Burkert’s words,

The “number atomism” interpretation goes back to Cornford.⁷¹ In his account of Pythagorean doctrine, Aristotle speaks of a plurality of extended monads,⁷² and he often alludes to the definition of the point as a “monad having position”.⁷³ If we interpret this as a comprehensive key idea, to be taken along with the pebble figures, the “star pictures” (constellations), and the procedure of Eurytus, who would determine the “number” of a man or horse

68 In this developmental process, “*mathêmata* changed its meaning from ‘doctrines’, i. e., matters being learned, to ‘mathematics’, no later than the fifth century b.C.” (Høyrup, J 1994, *In Measure, Number, and Weight: studies in mathematics and culture*, State University of New York Press, Albany, New York, p. 10).

69 Corresponding with the early Pythagorean view of number as a pattern of pebbles, prime numbers were called rectilinear because they can only be represented as a line of pebbles, compared to the composite numbers, which can also be arranged into equal size groups of pebbles. Composite numbers were further distinguished as plane numbers (those containing two dimensions, length and breadth) or solid numbers (those containing three dimensions, length, breadth and depth).

70 Fr. 23B2 Diels-Kranz (Diogenes Laertios, *Vitae philosophorum* III, 10 – 13).

71 Cornford, F M 1922–1923, “Mysticism and Science in the Pythagorean Tradition”, *The Classical Quarterly*, vol. 16, no. 3–4, pp 137–150; vol. 17, no. 1, pp. 1–12; Cornford, F M 1939, *Plato and Parmenides*, Kegan Paul, London, p. 56ff.

72 *Met.* 1080b19, 1083b15.

73 See above and Burkert 1972, p. 67 nn. 86 & 87.

by making an outline picture with pebbles⁷⁴ – the result is the thesis that the Pythagoreans understood the materialized point as a kind of atom.⁷⁵ They thought of all bodies as consisting of such point-atoms, and therefore things “are” numbers in the most literal sense; that is, they are the number of atom-point-units which they at any given moment contain. Does not Aristotle himself say that the Atomists “in a way” claim that things “are numbers or composed of numbers” (*Cael.* 303a8)? Still, though Aristotle’s refutation presupposes an atomistic view [at *Met.* 1083b8 ff., Aristotle asserts, as refutation of Pythagorean views, that there are no “indivisible magnitudes”], Cornford’s theory cannot claim to give the final answer. (...) If Ephantus was indeed “The first” to attempt an atomistic interpretation of the number theory,⁷⁶ this is an attempt to modernize the theory, rather than a revelation of its original significance (Burkert 1972, pp. 41 [with n. 70] and 42).

Let me go back to my proposal of separating the idea of a discussion linking Pythagorean number atomism to Boscovich’s atomic theory in its two components, form and content – an intention not easily affordable due to the intimate relation between these two parts of speech, a relation that must not be neglected since it is known that in the mathematical domain the arbitrary separation between form and content leads to errors and distortions of the “true character of ancient mathematics” (Unguru 2004, p. 383).⁷⁷ We saw that Kvasz mentioned that there was an evolution, promoted by the Pythagoreans, “from symbolic language of arithmetic to the iconic language of geometry”. According to Sabetai Unguru,

There is (broadly speaking) in the historical development of mathematics an *arithmetical* stage in which the reasoning is largely that of elementary arithmetic (...), a *geometrical* stage, exemplified by and culminating in classical Greek mathematics (...), and an *algebraic* stage, the first traces of which could be found in Diophantus’ *Arithmetic* [3rd century], (...) but which did not reach the beginning of its full potentiality of development before the sixteenth century in Western Europe (Unguru 2004, p. 396).

74 *Met.* 1092b10 ff.; Theophr. *Met.* 6a19 ff., after Archytas. According to Frans de Haas, for the Pythagoreans and some Platonists the notion of *limit* is representative of the general notion of *determination*, and we often find together references to the *limitation* as well as the *determination* of physical entities, and sometimes *determination* and *limitation* are completely confused (De Haas, F A J 1997, *John Philoponus’ New Definition of Prime Matter. Aspects of its background in Neoplatonism and the ancient commentary tradition*, E J Brill, Leiden, p. 49).

75 This may be exemplified by a connection between the “Pythagorean” definition of the point as a “monad having position” (according to Aristotle and Proclus) and Euclid’s definition of a point as “that which has no part (*méros*)” (I, Definition 1).

76 Fr. 51 A2; 51 A4 Diels-Kranz.

77 “Unguru’s article entailed a whole recalibration of the historiographical attitude towards mathematics as done in the past” (Acerbi, F 2006, “Classics in the History of Greek Mathematics, edited by Jean Christianidis” [review], *Aestimatio*, vol. 3, pp. 108–113; p. 110).

Unguru (2004, *passim*) warns that, by “reading ancient texts through modern glasses”, we may illegitimately trace modern views back to antiquity. It is not legitimate to visualize anything such as an “algebraic geometry” or “geometric algebra” (algebra in a geometric disguise) in ancient Pythagoreanism; therefore, atomism had to wait until the sixteenth century in order to be translatable into symbolic algebra,⁷⁸ in due time giving rise to theoretical quantitative atomism, initiated by Isaac Newton (1643–1727), Johann Bernoulli (1667–1748) and his son Daniel (1700–1782). Unguru nevertheless acknowledges that

The “figurative” numerical approach of the Pythagoreans contained somehow in germ another possibility of generalization (and, potentially, of removal of contradictions) than that actually taken by classical Greek mathematics (i.e., the purely geometrical approach), and this is the possibility of distinguishing visually relations between numbers of the same kind, by means of the gnomonic differences in their punctiform representation (...). For the Greek mathematician living before the discovery of the irrational [numbers] and working within the tradition of arithmetical geometry, the very way of representing numbers geometrically by points and punctiform figures contained intrinsic possibilities of grasping visually numerical relations; in other words, the Pythagorean way of representing numbers gave the Pythagorean mathematician an intuitive, visual means of generalization which, undoubtedly, contributed to the progress of mathematics (Unguru 2004, p. 411).

If in the above sentence we substitute the “visual means of generalization” by mental means of generalization (therefore shifting its intuitive content to a more abstract one),⁷⁹ we move from mathematics to speculative philosophy, and it is in the form of philosophical works that we may look for evidences of a continued discussion connecting Pythagorean number atomism to Boscovich’s atomic theory.

78 See Mahoney, M S 1971, “Die Anfänge der algebraischen Denkweise im 17. Jahrhundert”, *RETE: Strukturgeschichte der Naturwissenschaften*, vol. 1, pp. 15–31. According to Mahoney, the “algebraic mode of thought” has three main characteristics: 1) it is characterized by the use of an operative symbolism that represents the workings of the combinatory operations; 2) it deals with mathematical relations rather than with objects (even when certain relations become themselves objects), resting more on a logic of relations (because of the central role of combinatory operations) than on a logic of predicates; 3) it is free of ontological commitment; concepts like “space”, “dimension”, and even “number” are understood in a purely mathematical sense, without reference to their physical interpretation.

79 According to Mahoney, the algebraic mode of thought can be characterized as an abstract mode of thought (where existence depends on consistent definition within a given axiom system), in contrast to an intuitive one.

Links connecting Boscovich to the Pythagoreans: history of ideas

It is generally accepted that modern discussions of atomism resulted mainly from the rediscovery in 1417 of Lucretius' *De Rerum Natura*,⁸⁰ but Lucretius' poem was uninterruptedly discussed in the Middle Ages from the time of the Church Fathers until the 12th century,⁸¹ However, "despite the fact that many indirect sources were present during the Middle Ages, there were no new atomist theories of matter, nor detailed exegesis of ancient ideas, until the 12th century (...) in the works of William of Conches"⁸² (Grellard and Robert 2002, p. 4). According to Murdoch (1974a, p. 313; 1974b, p. 27), later medieval atomism was mainly an intellectual reaction to Aristotle's analysis of continuous quantity in the 6th book of *Physics*, and not a development from ancient atomism or the result of physical observations and experiments. Following Aristotle's track, late medieval and early modern speculations on the fundamental principles of the natural world tended to conflate Pythagorean and atomist ideas in relation to arguments about non-divisibility and infinite divisibility first developed in detail by Zeno of Elea (born c. 490 B.C.) in the form of his famous paradoxes.⁸³ Some interpreters⁸⁴ think that at least in the last of his four arguments about motion – the one about the bulks or masses (*onkoi*) in the stadium⁸⁵ – Zeno seems to have been arguing against the Pythagorean opinion about the existence of indivisible corpuscles,⁸⁶ and Henry D. P. Lee (1936, p. 34), believed Zeno attacked "a

80 See Serres, M 1977, *La Naissance de la Physique dans le Texte de Lucrèce: Fleuves et turbulences*, Les Editions de Minuit, Paris; Brown, A 2010, *The Return of Lucretius to Renaissance Florence*, Harvard University Press, Cambridge, Massachusetts/London.

81 Phillipe, J 1896, *Lucrèce dans la Théologie Chétienne du IIIe au XIIIe Siècle, et Spécialement dans las Écoles Carolingiennes*, Ernest Leroux/Félix Alcan, Paris.

82 As "a result of different traditions, Platonist and medical", and not as "a strict reading of Ancient atomism" (Grellard & Aurélien 2002, p. 5).

83 See Furley 1967, pp. 63–78; Kenyon Jr, R E 1994, *Atomism and Infinite Divisibility*, Doctoral dissertation in Philosophy, University of Massachusetts Amherst, URL (in html format) = <<http://www.xenodochy.org/rekphd/contents.html>>.

84 See Tannery, P 1885, "Le concept scientifique du continu: Zenon d'Elea et Georg Cantor", *Revue Philosophique de la France et de l'Étranger*, vol. 20, no. 2, pp. 385–410; Tannery, P 1887, *Pour L'Histoire de La Science Hellène (op.cit.)*, p. 250 ff.; Matson, W I 2001, "Zeno Moves!", in A Preus (ed.), *Before Plato* (Essays in Ancient Greek Philosophy VI), SUNY Press, Albany, New York, pp. 87–108.

85 Fr. 29 A28 Diels-Kranz (Arist. *Physics*, 239b33).

86 For a broad view of the related polemics see Booth, N B 1957, "Zeno's Paradoxes", *Journal of Hellenic Studies*, vol. 77, part II, pp. 187–201 (see pp. 193–194); Booth, N B 1957, "Were Zeno's

system which made the fundamental error of identifying or at any rate confusing the characteristics of point, unit and atom”, accepting former identifications⁸⁷ of the Pythagorean thesis of “a monad having a position” as Zeno’s target.

There’s also a possibility that late medieval and early modern appreciations of continuous quantity owe something to mereological speculations⁸⁸ about the eight hypothesis of Plato’s *Parmenides*: Jean Wahl (1969, p. 553) thinks that those speculations put us into contact with a sort of Pythagorean atomism by which the others (*tà alla*) are so different from the One that constitute blocks or masses, of which each set is an unlimited plurality. Another approach to understanding the connections between ancient atomism and modern atomic theories considers the impact on atomism of the invention and use of optical microscopes in late sixteenth and early seventeenth centuries in crystallographic studies, revealing crystal structures that would have been associated with the Platonic solids⁸⁹

Arguments Directed Against The Pythagoreans?”, *Phronesis*, vol. 2, pp. 95–103; Guthrie, W K C 1965, *A History of Greek Philosophy*, vol. 2: *The Presocratic Tradition from Parmenides to Democritus*, Cambridge University Press, Cambridge, pp. 94–96; Furley 1967, pp. 72–75; Vlastos, G 1967, “Zeno of Elea”, in P Edwards (ed.), *The Encyclopedia of Philosophy*, vol. 8, Macmillan, New York/London, pp. 369–79, especially p. 375 ff. (reprod. in Vlastos, G 1993, *Studies in Greek Philosophy*, vol. 1: *The Presocratics*, ed. D W Graham, Princeton University Press, Princeton, pp. 241–63); Faris, J A 1996, *The Paradoxes of Zeno*, Avebury, Aldershot, pp. 114 ff.; Matson, W I 2001, “Zeno Moves!” (*op. cit.*), especially p. 96 ff.

87 Milhaud, G 1900, *Les Philosophes-Géomètres de la Grèce. Platon et ses prédécesseurs*, Félix Alcan, Paris, Tannery 1887 (*op. cit.*), Cornford 1922–1923 (*op. cit.*).

88 “Mereology” is the theory of the relations of part (Greek μέρος) to whole and of part to part within a whole. Franz von Kutschera has a strong point in proposing that the eighth hypothesis can be understood in the light of a logical theory regarding parts and wholes (Von Kutschera, F 2002, *Platons Philosophie II: Die mittleren Dialoge*, Mentis, Paderborn, pp. 185–198).

89 See Burke 1966, pp. 14, 15, 18, 29, 57. In the footsteps of Anton van Leeuwenhoek (1685, “Concerning the Various Figures of the Salts Contained in the Several Substances”, *Philosophical Transactions of the Royal Society of London*, vol. 15, p. 1073), who reported on the geometrically regular structure of particles of many salts, a detailed geometrical theory of crystal structure was proposed by the Italian physician and mathematician Domenico Guglielmini (1655–1710), who asserted in his *Riflessioni Filosofiche Dedotte dalle Figure dei Sali* (1688, Eredi d’Antonio Pisarri, Bologna), and specially in his *De Salibus Dissertatio Epistolaris Physico-medico-mechanica* (letter to Cristino Martinelli, August 4, 1704 [publ. Venice, 1707]; an extension and complementation of the former work. Reissued 1719 in *Opera Omnia*, Sumptibus Cramer, Perachon & socii, Geneva, vol. 2, pp. 73–200) that there are four basic forms for the particles of salts (the cube for the common salt [“natrium muriaticum”], the hexagonal prism for potassium nitrate [“nitrum”], the rhombohedron for copper sulfate [“blue vitriol”], and the octahedron for aluminium nitrate [“alum”]), which combine to form other salts [Senechal 1990, p 44; Guareschi, I 1914, “Domenico Guglielmini e la sua opera scientifica”, *Supplemento annuale all’Enciclopedia di Chimica* (Turin), vol. 30, pp. 7–33, appended with Guareschi’s transcription of the *Riflessioni Filosofiche Dedotte dalle Figure dei Sali* at pp. 35–52, and with Mario Zucchi’s translation of the introduction and

and giving support to the idea that the minimal units of crystalline matter were geometrically shaped structures (Burke 1966, p. 43).

Some portion of the "Pythagorean" content of Boscovich's "atomic" theory can possibly be traced to a less hypothetical ancient source. In 1744, the year in which he completed his theological studies, was ordained priest and became a full member of the *Society of Jesus*, Boscovich was accepted to the *Accademia degli Arcadi*⁹⁰ as Numenius Anigraeus, after the second-century Pythagorean and Platonist Numenius of Apamea [Apama] (Hill 1961, p. 38). No written account of why this particular name was chosen is known, though Boscovich's biographer Elizabeth Hill says without explanation that "the choice of such a name for Boscovich is revealing". Numenius' ideas were available to Boscovich through testimonies and fragments of his works mainly preserved by Origen Adamantius, Proclus, Theodoret of Cyrrhus, and especially by Eusebius of Caesarea in his *Praeparatio Evangelica*.

Numenius is one of the persons thought to have introduced Plotinus to Neopythagoreanism,⁹¹ and since it is known that by 1744 Boscovich was already working toward the development of a middle way between Isaac Newton's physical theory based on "hard atoms" and Leibniz's metaphysical theory of monad-points (theme of his *De Viribus Vivis*, of 1745), it is interesting to observe that Numenius first god⁹² (the monad⁹³; fr. 26 Guthrie⁹⁴/11 Des Places⁹⁵), which offers a

sections I-CXXXI of the *Riflessioni Filosofiche De Salibus Dissertatio Epistolaris Physico-medico-mechanica* at pp. 52–66].

90 In 1656 a society of men of letters, poets and scientists gathered around Queen Christina of Sweden (who had abdicated the Swedish crown in 1654, and converted to Catholicism in the same year), in her palace in Rome (the *Palazzo Farnese*), and a cultural society, the *Arcadia* (inaugurated on January 24), flourished informally under her auspices. After her death (1689), that cultural society was named *Pontificia Accademia degli Arcadi* (in 1690), and officially founded to combat the corruption of public taste and to revive Italian poetry from baroque barbarisms.

91 Guthrie, K S 1917, *Numenius, the father of Neo-Platonism: works, biography, message, sources, and influence*, George Bell and Sons, London; Dodds E R 1957, "Numenius and Ammonius", *Les Sources de Plotin* (Entretiens sur l'Antiquité classique, No. 5), Fondation Hardt, Vandoeuvres-Geneva, pp. 1–32; Schroeder, F M 1987, "Ammonius Saccas", *Aufstieg und Niedergang der römischen Welt II*, vol. 36, no.1, pp. 493–526; Narbonne, J-M 1994, "Plotinus and the Secrets of Ammonius", *Hermathema*, vol. 157, pp. 117–153.

92 A development of Xenocrates first god/monadic male principle (Heinze, R 1892, *Xenocrates*, Teubner, Stuttgart, Fr. 15; Senocrate & Ermodoro 1982, *Frammenti*, ed. M Isnardi Parente, Bibliopolis, Naples, Fr. 213).

93 The monad/first god "exists in himself", "is simple", "absolutely deals with none but himself", "is in no way divisible" (Fr. 26 Guthrie/11 Des Places). For Numenius, the dyad is

possible conceptual link between the Platonic idea of creation as a composition of universal order (*toû kósmou sýstasis*)⁹⁶ and Plotinus' view of the universe as separation from the One (*apóstasis toû henòs*),⁹⁷ is both cause (*aítion*) of, and connatural (*sýmphyton*) with *ousía* (fr. 25 Guthrie/16 Des Places), having the paradoxical characteristic of *stasis*, ontological stability (as being),⁹⁸ and *kinêsis* *sýmphytos*, innate motion (as principle of change)⁹⁹ (Slaveva-Griffin 2009, pp. 13 and 24).

Boscovich's point-centers (*puncta*) of action harmonize the ideas of *stasis* and *kinesis* by proposing these two conditions as different possibilities or moments of the same minimal unit of reality. As intermediaries between spirit and matter, Boscovich's *puncta* also help overcome an objection of Numenius against the existence of material principles of matter, since matter, which is changeable, unstable and disordered, cannot be explained by matter, but only by something with permanence, stability and order (fr. 11–12 Guthrie/3–4a Des Places). Boscovich's presentation of matter as formed by the grouping of *puncta* in increasing orders of magnitude, starting from a pair, may have been influenced by Numenius, who (in Calcidius' report) calls God *singularitas*, and matter *duitas* (fr. 14, 3–6 Guthrie/52, 2–6 Des Places), though moving away from Numenius radical dualism.¹⁰⁰

identified with matter in its disorganized state; the demiurge (second god) organizes matter by looking to the first god (Fr. 32 Guthrie/18 Des Places).

94 Numenius of Apamea, "The extant works", in Guthrie, K S 1917, *Numenius, the father of Neo-Platonism: works, biography, message, sources, and influence* (op. cit.), pp. 2–93.

95 Numénus 1973, *Fragments*, ed. É des Places, Société d'Édition Les Belles Lettres, Paris.

96 *Timaeus*, 32c5–6.

97 *Enneads*, VI.6.1.1 ff.

98 The life of the first god is firm (ἔστως; fr. 30 Guthrie/15 Des Places).

99 It precedes the moving (κινούμενος) life of the second god (fr. 30 Guthrie/15 Des Places). "Although Numenius does not further describe the innate motion of the First God, there can be little doubt that this 'motion' is the activity of thought. That is required by his identification of the First God as both intellect and idea, and it also explains how the inherent motion of the First God can be the source of cosmic order and stability" (Bradshaw, D 2004, *Aristotle East and West: metaphysics and the division of Christendom*, Cambridge University Press, Cambridge, p. 66).

100 The dualist Numenius presents his own doctrine of matter (clearly developed out of Plato's *Timaeus*) as the work of Pythagoras. He considered (fr. 14–16 Guthrie/52 Des Places) *duitas* (doubleness) as a sort of matter that, while indeterminate (*duitatem indeterminata*), is characterized as unborn and ungenerated (*sine ortu et generatione*), and having the same age as the god by which it is adorned/ordinated (*aequavam deo a quo est ordinatum*). While adorned/ordinated or illuminated (*illustratam*) by the adjusting god (*digestore deo*), alternatively, the *duitas* is determinate (*limitata*; limited) and generated (*generata*), therefore occupying a time that is *posterior*. Sensible matter is bad due to the existence of a bad providence, a precedently

What sort of development was, then, Boscovich's "atomism" in relation to his precursors? If we acknowledge the fact that Roger Boscovich's mother was a member of an Italian merchant family, that in his native city (Ragusa/Dubrovnik) Italian was the "language of culture", and that he received all his formal education from Italian Jesuits,¹⁰¹ it's wise to look for antecedents of Boscovich's ideas on point-atoms not only in antiquity, but also in Church (especially late Scholastic) sources and their Italian contributions – but not exclusively in them, since one of the characteristics of modernity was the wide circulation of ideas (favored by the use of Latin as the cultured language of philosophy and science).

To a certain extent, Boscovich's thinking was "an attempt (...) to go beyond various kinds of *Zenonism*" (Pearson 2000, p. 7). The Jesuits Rodrigo de Arriaga (1592–1667), Francisco de Oviedo (1602–1651), Juan de Ulloa (1639–c. 1725) and Luis de Losada (1681–1748) were the main "Zenonists" in late scholasticism.¹⁰²

existing evil nature (*providentia mala... de existente olim natura maligna*) according to "Pythagoras" or an evil world-soul (*maligna anima*) according to "Plato" (fr. 15–16 Guthrie/52 Des Places).

101 First in Ragusa (in St. Nicholas' Church and later in the *Collegium Ragusinum*) and then, from October 31, 1765 (at the age of 15), in Rome (in the *Collegium Romanum*, predecessor of the present *Pontificia Università Gregoriana*, and in other establishments of the *Societas Iesu*). On the role of the Jesuits in mathematical teaching and studies see MacDonnell, J F 1989, *Jesuit Geometers. A study of fifty-six prominent Jesuit geometers during the first two centuries of Jesuit history*, The Institute of Jesuit Sources/The Vatican Observatory, Vatican; Gorman, M J 1998, *The Scientific Counter-Revolution. Mathematics, natural philosophy and experimentalism in Jesuit culture 1580–c. 1670*, EUI PhD theses, Florence; Romano, A 1999, *La Contre-Réforme Mathématique. Constitution et Diffusion d'une Culture Mathématique Jésuite à la Renaissance (1540–1640)*, École Française de Rome, Rome [reviewed by Schubring, G 2003, "'Reformation' and 'Counter-Reformation' in Mathematics – The Role of the Jesuits", *Llul*, vol. 26, no. 57, pp. 1069–1076], and many articles of the same author; Díaz, E A 2009, *Jesuit Education and Mathematics: review of literature on the history of Jesuit education and mathematics*, VDM (Verlag Dr. Müller), Saarbrücken.

102 "*Sin embargo, entre los escolásticos, se llamaban 'zenonistas' a aquellos que defendían la tesis de que el mundo se compone de indivisibles*: [Rodrigo or Roderigo de] Arriaga, [Francisco de] Oviedo, [Juan de] Ulloa, [Luis de] Losada" (Gustavo Bueno, G 1974, *La Metafísica Presocrática*, Pentalfa, Oviedo, p. 264. Bueno does not mention the Valencian "Zenonist" Benedictus Pererius/Benito Pereyra/Benedetto Pereira; traces of "Zenonism" have also being pointed in Francisco Suárez (Adams H P 1970, *The Life and Writings of Giambattista Vico*, Russell & Russell, New York, p. 22; Rossi 1999, pp. 73–76). See also Beeley, P 1995, *Kontinuität und Mechanismus: zur Philosophie des jungen Leibniz in ihrem ideengeschichtlichen Context*, Franz Steiner, Stuttgart, pp. 298–300 with nn. 65–70; Rossi P 1999, pp. 67–8, 76–65 & 89 ff.; Solère, J-L 2006, "The question of intensive magnitudes according to some Jesuits in the sixteenth and seventeenth centuries", *The Monist*, vol. 84, no. 4 ("Physics Before Newton"), pp. 582–616.

According to Massimo Lollini (2002, p. 60 n. 20), “the Zenonists of the sixteenth century held that matter is composed of mathematical points to avoid the difficulty implicit in the notions of atoms conceived simultaneously as particles and as unextended points”. In his *Theoria philosophiae naturalis* Boscovich (§139; “Synopsis of the whole work”, 131) explained his own position in these terms: “From the idea of non-extension of any sort, and of contiguity, it is proved by an argument instituted against the Zenonists many centuries ago that there is bound to be compenetration; and this argument has never been satisfactorily answered”. “By rejecting the idea of continuous extension, I remove the whole of the difficulty, which was raised against the disciples of Zeno in years gone by, and has never been answered satisfactorily; namely, the difficulty arising from the fact that by no possible means can continuous extension be made up of things with no extent” (Boscovich 1966, pp. 59 and 13).

The Pisan Galileo Galilei¹⁰³ (1564–1642) argued that the continuum is composed of indivisibles, and that they are physical realities¹⁰⁴ whose properties can be studied mathematically,¹⁰⁵ while his Milanese disciple Bonaventura Cavalieri (Cavalerio)¹⁰⁶ (1598?–1647), a Jesuate,¹⁰⁷ proposed a theory of indivisibles restrict-

103 See Galileo 1843, “Postile di Galileo alle Esercitazioni [filosofiche] di Antonio Rocco [contro il *Dialogo dei Massimi Sistemi*]” (1633). *Le Opere di Galileo Galilei, prima edizione completa condotta sugli autentici manoscritti palatini*, 15 vols. plus a supplement (1842–1856), ed. E Albèri, Società Editrice Fiorentina, Florence, vol. 2, pp. 290–335 (see p. 330); Galileo 1638, *Discorsi e Dimostrazioni Matematiche, intorno à due nuove scienze attenenti alla meccanica & i movimenti locali*, Elsevier, Leiden (reprint 1966, Culture et Civilisation, Brussels), *passim* (in Salviati’s words); Predari, F 1842, “Rassegna Critica Italiana. I. Nuova enciclopedia popolari”, *Rivista Europea. Giornale de scienze, lettere, arti e varietà*, ano V, parte I, Vedova di A F Stella e Giacomo Figlio, Milan, p. 329.

104 See Cirino, R 2006, *Dal Movimento alla Forza: Leibniz, l’infinitesimo tra logica e metafisica*, Rubbettino, Soveria Mannelli, pp. 203–04.

105 Edmund Husserl claimed that Galileo was the first to mathematize nature, substituting concrete things of the intuitively given surrounding world by mathematical idealities [Husserl, E 1970, *The Crisis of European Sciences and Transcendental Phenomenology. An introduction to phenomenological philosophy* (1954), tr. D Carr, Northwestern University Press, Evanston, Illinois, section 9, pp. 23–59], and according to Raffaele Cirino (*op. cit.*, p. 204), “Galileo was the first to consider the infinitesimal entities as simple ‘artifices’ of calculation”.

106 Cavalerio, B 1635, *Geometria Indivisibilibus Continuatorum Nova Quadam Ratione Promota*, Clemente Ferroni, Bologna (revised edition 1653, Ex Typographia De Duciis, Bologna); Cavalieri, B 1666, *Geometria degli Indivisibili* (1635), ed. & trad. L L Radice. UTET, Turin; Cavalerio B 1647, *Exercitationes Geometricae Sex. I. De priori methodo indivisibilium. II. De posteriori methodo indivisibilium. III. In Paulum Guldinum e’ Societate Iesu dicta indivisibilia oppugnantem. IV. De usu eorumdem ind. in potestatibus cossicis. V. De usu dictorum ind. in unif. diffor. gravibus. VI. De quibusdam propositionibus miscellaneis, quarum synopsis versa pagina ostendit*. Typis Iacobi

ed to geometrical realities.¹⁰⁸ The Neapolitan natural philosopher and jurist Francesco D'Andrea (1625–1698),¹⁰⁹ author of an *Apologia in Difesa degli Atomisti* (1685),¹¹⁰ in his letters¹¹¹ in favor of the atomist ideas embraced by Leonardo di Capua (1617–1795) in his *Parere* (1681),¹¹² defended Pythagoras' "atomism" against the Jesuit scholar Giovanni Battista Benedetti (Giovanni Battista de Benedictis; "Benedetto Aletino"), author of the (five) *Lettere Apologetiche in Difesa della Teologia Scolastica e della Filosofia Peripatetica* (1694).¹¹³ By that time, there existed some attempts to conciliate atomism and scholasticism, while an ecclesiastic reaction ensued: in August 5, 1693, the Inquisition proscribed Di Capua's work and condemned the atomism of Democritus and Epicurus as contrary to faith, but in the same occasion assured that this would not mean a damage to the "doctrine of the Zenonists", which postulates that bodies are constituted of infinite indivisible parts (Beretta 2007, pp. 59–60). According to Paolo Rossi

Montii, Bologna. See also Andersen, K 1985, "Cavalieri's Method of Indivisibles", *Archive for History of Exact Sciences*, vol. 31, no. 4, pp. 291–367.

107 Member of a religious order founded in 1360 by Giovanni Colombini of Siena.

108 Cavalieri modified Galileo's ideas on indivisibles according to the classical "method of exhaustion" (from Eudoxus, Archimedes, and others) and to the principle of indivisibles crudely used by Kepler in 1604 (*Astronomiae Pars Optica*), 1609 (*Astronomia Nova*) and 1615 (*Nova Stereometria Doliorum Vinariorum*) while considering geometric figures in terms of the infinitesimal.

109 See Mastellone, S 1962, "Note sulla cultura napoletana al tempo di Francesco d'Andrea e Giuseppe Valletta", *Critica Storica*, vol. I, pp. 369–398; Borrelli, A 1995, *D'Andrea Atomista: L'“Apologia” e altri inediti nella polemica filosofica della Napoli di fine Seicento*, Liguori, Naples, and the bibliography mentioned in Stone, H S 1997, *Vico's Cultural History: The production and transmission of ideas in Naples, 1685–1750*, E J Brill, Leiden/New York/Koln, p. 55 n. 11.

110 Naples, Bibl. Oratoriana dei Gerolamini, ms. XXVIII.4.1; Bibl. Nazionale di Napoli, ms. I D 4, f. 286–317. The *Apologia in difesa degli atomisti* was published in Borrelli A 1995, *D'Andrea Atomista* (op. cit.), pp. 59–109. Borrelli also published, in the same book, other atomist texts from D'Andrea: *Dubii de' quali si desiderarebbe maggior esplicazione nella scrittura formata contra gl'atomi e gl'atomisti* (pp. 111–130), *Riflessione sopra la seconda scrittura circa la materia degl'atomi* (pp. 131–140), and *Lezioni* (pp. 141–160).

111 *Risposta a favore del sig. Lionardo di Capoa contro le lettere apologetiche del p. De Benedictis gesuita*, 1695–7 (Bibl. Nazionale di Napoli, ms. I D 4; Bibl. Angelica di Roma, ms. 1340); *Risposta del signor Francesco d'Andrea a favore del signor Lionardo di Capoa contro le lettere apologetiche*, 1697–8 (Bibl. Nazionale di Napoli, ms. IX A 66; and ms. Brancacc. I C 8).

112 Di Capua, L 1681, *Parere del Signor Lionardo di Capoa, divisato in otto ragionamenti, ne' quali partitamente narrandosi l'origine, e'l progresso della medicina, chiaramente l'incertezza della medesima si fa manifesta*, Antonio Bulifon, Naples.

113 Aletino, B 1694, *Lettere Apologetiche in Difesa della Teologia Scolastica e della Filosofia Peripatetica*, Giacomo Raillard, Naples. Cf. a.inda Aletino, B 1703, *Difesa della Scolastica Teologia* [part I *Lettera di Benedetto Aletino in difesa della teologia scolastica* (reissue of the *Lettere* from 1694, with some modifications); part II *Difesa della lettera precedente*], Antonio de' Rossi, Roma.

(2001, p. 470 [16]), “‘Zenonist’ (like ‘Scotist’) is not a term with univocal meaning. But everybody knows that the Zenonists,¹¹⁴ beyond their differences, opposed the Aristotelian theory of the continuum, made varied uses of the concept of point, [and] believed in indivisible entities that are real and not constructed by thought”.¹¹⁵

Giuseppe Valetta (1636–1715), another Neapolitan, also defended atomism; in his *Istoria Filosofica* (1697–1704)¹¹⁶ he suggested that this was a distinctive characteristic of Magna Graecia, and presented Pythagoras as an atomist. This was not an uncommon view at that time, and also not an original one:¹¹⁷ in England, Henry More (1614–1687)¹¹⁸ considered the Pythagoreans to be the founders of Greek atomism (Hall 1990, p. 111), and (accompanied by his close friend Ralph Cudworth¹¹⁹) that atomism was initially an immaterialist tradition (Hall 1990,

114 “Una setta di filosofi gesuiti spagnoli e di Lovanio che vissero tra la fine del sedicesimo e l’inizio del diciassettesimo secolo” (“a sect of Jesuit philosophers of Spain and Louvain that lived between the end of the sixteenth and the beginning of the seventeenth century”) (Arthur 2003, p. 335).

115 “(...) zenonista (come scotista) non è un termine dal significato univoco. Tutti però sanno che gli zenonisti, al di là delle differenze, si oppongono alla teoria aristotelica del continuo, fanno variamente uso del concetto di punto, credono a entità indivisibili reali e non costruite dal pensiero”.

116 Included in Valetta, F 1975, *Opere Filosofiche* (ed. M Rak), Leo Olschski, Florence. See also Piaia, G 2010, “The General Histories of Philosophy in Italy in the Late Seventeenth and Early Eighteenth Century”, in G Piaia & G Santinello (eds.), *Models of the History of Philosophy, Volume II: From the Cartesian Age to Brucker*, Springer, Dordrecht/Heidelberg/London/New York, pp. 213–297 (see pp. 252–8).

117 See Diogenes Laertius, *Vitae*, VIII, 25 (Alexander Polyhistor, *FrGrHist* 273 F 93); Stobaeus, *Eclog.*, I.16 (= Diels-Kranz 51 A2; 51 A4 on Ecphantus)

118 More, H 1653, “Appendix to the Defence of the Philosophick Cabbala”, in *Conjectura Cabbalistica, or a Conjectural Essay of interpreting the Mind of Moses in the first three chapters of Genesis, according to a threefold Cabbala, viz. Literal, Philosophical, Mystical* [dedicated to Ralph Cudworth], William Morden, London (2nd edition 1662).

119 Cudworth, R 1678, *The True Intellectual System of the Universe*, Printed for R Royston, London. See Sailor, D B 1964, “Moses and Atomism”, *Journal of the History of Ideas*, vol. 25, pp. 3–16; Rodney, J M 1970, “A Godly Atomist in Seventeenth Century England: Ralph Cudworth”, *The Historian*, vol. 32, no. 2, pp. 243–249. More and Cudworth have taken some of their ideas on the early history of atomism from Robert Boyle (Boyle, R 1661, *The Sceptical Chymist: or Chymico-physical doubts & paradoxes, touching the spagyrist’s principles commonly call’d hyposstatical, as they are wont to be propos’d and defended by the generality of alchymists*, J Cadwell for J Croke, London, p. 120).

p. 112).¹²⁰ Newton, on his turn, wrote in a scholium intended for a new edition of the *Philosophiae Naturalis Principia Mathematica*.¹²¹

That all matter consists of atoms was a very ancient opinion. This was the teaching of the multitude of philosophers who preceded Aristotle, namely Epicurus, Democritus, Ecphantos [sic], Empedocles, Zenocrates [sic], Heraclides, Asclepiades, Diodorus, Metrodorus of Chios, Pythagoras, and previous to these Moschus the Phoenecian [sic] whom Strabo declares older than the Trojan War. For I think that same opinion obtained in that mystic philosophy which flowed down to the Greeks from Egypt and Phoenecia, since atoms are sometimes to be found to be designated by the mystics as monads (*apud* Guicciardini 1999, p. 101).

Thomas Holden (2004, pp. 239 and 245) believes in a "strong likelihood of an *indirect* influence" of Henry More on Roger Boscovich, through Samuel Clarke and other Newtonians that accepted the existence of extended and metaphysically indivisible ("indiscernible"), though formally-divisible, spiritual substances¹²² (instead of material atoms).

Gianbattista Vico (1668–1744), a Neapolitan, embraced in his *De Antiquissima Italorum Sapientia* ("*Liber Metaphysicus*", 1710)¹²³ the suggestion that atomism was a peculiar trait of Magna Graecia, but instead of material atoms proposed, as Henry More (1614–1687) in England had done before him, spiritual ones (like Leibniz' monads),¹²⁴ calling to his support Zeno (Vico 1944, pp. 127 and 319)¹²⁵ – who had been misinterpreted by Aristotle, according to Vico (1944, pp. 151–2)¹²⁶ – and Pythagoras. In a trend not uncommon in his time, Vico attributed to the Stoic Zeno ideas more appropriate to the Eleatic one,¹²⁷

120 See More, H 1646, *Democritus Platonissans or, An essay upon the infinity of worlds out of Platonick principles*. Roger Daniel, Cambridge, and see also relevant sections of *Conjectura Cabbalistica*.

121 University Library, Cambridge, Ad. Ms. 3965.6, folio 270r.

122 On the idea of substance in Boscovich's "atomic" theory see Holden, 2004, pp. 250–252.

123 Vico, G 1979, *Liber Metaphysicus/De antiquissima Italorum sapientia liber primus* (ed. S Otto & H Viechtbauer), Wilhelm Fink, Munich; see also Otto S & Viechtbauer H (eds.) 1985, *Sachkommentar zu Giambattista Vicos Liber Metaphysicus*, Wilhelm Fink, Munich.

124 See Grimaldi, A A 1958, *The Universal Humanity of Giambattista Vico*, S F Vanni, New York, p. 106; Santillana, G 1968, *Reflections on Men and Ideas*, MIT Press, Cambridge, Massachusetts, p. 208.

125 See also Caparelli 1944, pp. 145–6 & 584; Stone, H S 1997, *Vico's Cultural History* (*op. cit.*), p. 185.

126 See Grimaldi, A A 1958, *The Universal Humanity of Giambattista Vico* (*op. cit.*), pp. 104–6.

127 See Rossi, P 1998, "I punti di Zenone: una preistoria vichiana", *Nuncius*, vol. 13, no. 2, pp. 377–426 (reissue in Rossi 1999, pp. 55–107); Rossi, P 2000, "Ritratto di uno zenonista da giovane" (1998), in F Ratto (ed.), *Il Mondo di Vico/Vico nel mondo, in ricordo di Giorgio Ta-*

but this was probably an intentional act. Aristotle's objections to Zeno enforced in Vico an interpretive approximation between "Zeno" and Pythagoras:

Zeno, a supreme metaphysician, accepts the hypotheses of the geometers and, as Pythagoras did through numbers, interpreted the principles of things through points. (...). Aristotle employs geometric demonstrations to deduce that any particle, no matter its minimal extension, is divisible to the infinite.¹²⁸ But Zeno remains undisturbed, and from those same demonstrations confirms his metaphysical points (Vico 2005, p. 66).

Vico's reasoning was that the geometric point resembles the metaphysical point in that it is indivisible¹²⁹ (Vico 1971, p. 157); physical extension is an attribute, and therefore divisible, as Aristotle said, while Pythagorean/Zenonian points pertain to the essence, that is indivisible (Vico 1971, 159).¹³⁰ According to Massimo Lollini (2002, pp. 59–60), "in the *De antiquissima* Vico intends to restore to Zeno of Elea and to the Italic Pythagoras the theory of metaphysical points, as a support to an animistic philosophy of nature in polemic either with the Aristotelian tradition or with the modern corpuscular theory of Descartes and Gassendi".¹³¹

Leibniz had also proposed (after 1695)¹³² "metaphysical points", "points of substance", "formal atoms", or simply "force"¹³³ (the *vis insita rebus*),¹³⁴ as the

gliacozzo (Roma, April 15–16, 1999), Edizioni Guerra, Perugia, pp. 181–191 (another issue Rossi 1999, pp. 109–154); Rossi, P 1999, "Dimenticare Zenone? Conati e punti nella Scienza nuova", in F Ratto (ed.), *Alfombra di Vico, Testimonianze e saggi vichiani in ricordo di Giorgio Tagliacozzo*. Edizioni Sestante, Ripatransone, pp. 327–334 (another issue Rossi 1999, pp. 155–164); Mazzola, R 2000, "Vico e Zenone", in M Sanna & A Stile (eds.), *Vico tra l'Italia e la Francia*, Alfredo Guida, Naples, pp. 311–341. Vico had access to the 1st edition of Bayle's *Dictionnaire Historique et Critique*.

128 See Aristotle, *Phys.* VI, 2, 233a21; VI, 9; VIII, 8, 263.

129 "(...) che il punto geometrico sia una simiglianza del metafisico, cioè della sostanza; e che ella sia cosa che veramente è, ed è indivisibile (...)" [Seconda Risposta: Risposta di Giambattista Vico all'articolo X del tomo VIII del *Giornale de' letterati d'Italia* (1712)].

130 "Aristotile sconvien da Zenone in cose diverse, convien nel medesimo: egli divide in infinito l'estensione, l'attributo; Zenone dice indivisibile la sostanza, l'essenza" [Seconda Risposta (1712)]. See also Otto S & Viechtbauer H 1985, *Sachkommentar zu Giambattista Vicos Liber Metaphysicus* (*op. cit.*), p. 65.

131 "Nel *De antiquissima* Vico intende restituire a Zenone di Elea e all'italico Pitagora la teoria dei punti metafisice, come supporto ad una filosofia animistica della natura in polemica sia con la tradizione aristotelica che con la teoria corpuscolare moderna di Descartes e Gassendi".

132 See Anapolitanos, D 1999, *Leibniz: representation, continuity, and the spatiotemporal*, Kluwer Academic Publishers, Dordrecht, pp. 78–93, especially p. 88 ff.; Garber, D 2009, *Leibniz: body, substance, monad*. Oxford University Press, New York, pp. 303–349 and ff.

basic unit of reality, but Vico's atoms, whose metaphysical status lay between Leibniz's *monads* and Boscovich's *puncta* (Whyte 1961a, p. 118), "are more physical than Leibniz's monads, for they possess location, give rise to extended forms, and display tendencies to movement" (Whyte 1961c, p. 53). Outside Italy, in 1734 Emanuel Swedenborg (1688–1772), also following Leibniz and Vico, published in his *Principia*¹³⁵ and in his *The Infinite and Final Cause of Creation*¹³⁶ a doctrine of dimensionless material points, with a tendency to motion, as the source of all physical phenomena. Apart from Leibniz and Swedenborg, Lancelot Law Whyte (1961a, pp. 118–9; 1961c, pp. 55–6) saw other non-Italian thinkers, John Michell¹³⁷ (1724–1793) and Immanuel Kant¹³⁸ (1724–1804), moving in parallel directions with respect to their theories of the constitution of matter.

In Boscovich's time, two kinds of indivisibilism were recognized: metaphysical and physical. Metaphysical atomism dealt with monads, physical atomism

133 On his turn, Leibniz, in a letter of May 29, 1716, to Bartholomew des Bosses, refers explicitly to "Zenonian *puncta*" ("*de punctis Zenoniis*") [1875–1890. *Die Philosophischen Schriften von Leibniz*, 7 vols. (ed. C I Gerhardt), Weidmann, Berlin (reprint 1971, George Olms, Hildesheim), vol. 2, p. 520; cf. Corsano, A 1956, *Giambattista Vico*, Bari, Laterza, p. 126 n. 21].

134 Universal internal force of all things, somehow related to alchemical and modern conceptions of *conatus* (or of being as inner *potentia* or "*conatus agendi*") and to Newton's "*vim penetrantem spiritus*". A body and its force are related to the idea of mind (*mens*); in an undated letter to Antoine Arnauld, Leibniz defined a body as "*mens momentanea*", and mind as the central point from where a body occupies space [*Die philosophischen Schriften*, ed. C I Gerhardt (*op. cit.*), vol. 1, p. 73].

135 *Principia rerum naturalium sive novorum tentaminum phaenomena mundi elementaris philosophice explicandi* (1734. Friedrich Hekel, Dresden/Leipzig), the 1st volume of his *Opera Philosophica et Mineralia* (3 vols.) and an improvement upon his *Prodromus principiorum rerum naturalium: sive novorum tentaminum chymiam et physicam experimenta geometrice explicandi* (anonymously published, 1721, John Oosterwyk, Amsterdam) and upon an unpublished ms., *Principia Rerum Naturalium ab experimentis et geometria sive ex posteriori et priori educta* (mentioned in a letter from 1729).

136 1734. *Prodromus Philosophiae Ratiocinantis de Infinito, et Causa Finali Creationis; deque Mechanismo Operationis Animae et Corporis*, Friedrich Hekel, Dresden/Leipzig.

137 "Perhaps independently", in a letter to Joseph Priestley (Whyte 1961a, pp. 118 & 125 n. 12), probably from "around 1760, the year in which he met Boscovich in Cambridge". "Michell's fertile ideas were neglected and forgotten; he was too modest for his colleagues to take seriously" (Whyte 1961c, p. 56).

138 See his *Monadologia Physica*, 1756, and a late reappraisal at *Metaphysische Anfangsgründe der Naturwissenschaft*, 1786. Some authors have suggested that Kant may have been influenced by Boscovich [see Cassirer, E 1981, *Kant's Life and Thought* (1918), tr. J Haden, Yale University Press, New Haven, p. 42; Supek, I 1976, "Boscovich's Philosophy of Nature", *Poznan Studies in the Philosophy of the Sciences and the Humanities*, vol. 2, pp. 112–120 (see p. 114)].

dealt with corpuscles (Kant 1997, p. 230). Kant thought that metaphysical atomism is the same as mathematical (or absolute) atomism; Leibniz, however, had a different view on mathematical atomism. Leibniz believed that

Atoms of matter are contrary to reason, quite apart from the fact that they are still composed of parts (...). Only *atoms of substance*, that is to say real units [monads] absolutely devoid of parts, can be the sources of actions, and the first absolute principles of the composition of things, and, as it were, the ultimate elements in the analysis of substantial things. They might be called *metaphysical points* (...). Mathematical points (...) are nothing but modalities; only metaphysical or substantial points (...) are exact and real, and without them there would be nothing real (...) (Leibniz 1880a, pp. 482–484).

In Richard Arthur's interpretation, this implies that Leibniz's unextended monads are presupposed by the physical continuum, but are not parts of it, and that according to Leibniz physical points or atoms are only apparently indivisible, while mathematical points, strictly indivisible and unextended, cannot be thought as parts of the continuum without falling into contradiction (Arthur, 2003). The Leibnizian continuum, composed of infinite unextended points that, although smaller than any sensible magnitude, yet have parts that lie in a certain situation and order, was criticized by Boscovich (*Theoria philosophiae naturalis*, §138): "Those arguments that some of the Leibnitian circle put forward are of no use for the purpose of connecting the indivisibility and nonextension of the elements with continuous extension of the masses formed from them" (Boscovich 1966, p. 59). Boscovich rejected the idea of continuous extension (absolute space) while maintaining the existence of unextended point-centers of action (real monads absolutely without parts), actively extended (in a bodyless vacuum) to form space (a "kind of" physical continuum). This was an attempt to reconcile metaphysical and physical atomism, and to connect the indivisibility and nonextension of the minimal elements of reality with the continuous extension of the masses they form when grouped. Boscovich's points differ from mathematical points in that they possess the property of inertia, and in that there is a force acting between them.

Boscovich (1966, p. 59) said in his *Theoria philosophiae naturalis* (§138): "I was not the first to introduce the notion of simple non-extended points into physics. The ancients [*veteres*] after the time of Zeno had an idea of them, and the Leibnizians indeed suppose that their monads are simple and non-extended". The Ragusan argued (§§138–139) that Leibniz remained a Zenonist, and admitted ("Synopsis of the whole work", 2) that his *puncta* were somehow similar to "those simple and non-extended elements upon which is founded the theory of Leibniz" (Boscovich 1966, p. 19). He thought, however, that it is necessary to

give up the idea of continuous extension – which cannot be generated from non-extended particles – in order to escape the trap of Zeno's paradoxes.

Since the publication of the *Dictionnaire Historique et Critique* by Pierre Bayle (1647–1706), the "doctrine of the Zenonists" was associated with Pythagoras and Plato:

[Zeno of Elea] is regarded as the founder of the sect that maintains that the continuum is made up of mathematical points. (Arriaga¹³⁹ and a hundred other Spanish Scholastics call 'Zenonists' those who assert that the continuum is composed of indivisible and unextended parts, an opinion that is very different from that of the Atomists.) It would be more reasonable to attribute this view to Pythagoras and Plato as [David] Derodon has done, basing himself on the testimony of Sextus Empiricus for [the attribution to] Pythagoras, and on the testimony of Aristotle for [the attribution to] Plato (Derodon, *Disp. de atomis*, pages 4 and 5. He quotes Sextus Empiricus, *Adv. math.*, book IX, and Aristotle, *De generat.*, book I, text VII) (Bayle 1720, p. 2916, with notes 135 and 136).

Bayle (*loc. cit.*) considered that Zeno of Elea did not maintain that the continuum is made up of indivisible parts,¹⁴⁰ and mentioned an error that was very common in his time, the belief that "the Zeno who denied motion, and whose arguments Aristotle examined, was the leader of the Stoics".¹⁴¹ Bayle cited the "Zenonist" Francisco de Oviedo as someone who believed Zeno of Citium, the founder of the Stoic school, to be the one against whom Aristotle had argued¹⁴² in defending the idea that a continuum is not composed of indivisibles. Bayle believed that behind Aristotle's argument against indivisibility one should see an attack on a Pythagorean and Platonic assertion that "the continuum is composed of indivisible and unextended parts", so what Rodrigo de Arriaga and many other

139 Arriaga, 1632. See also Luna Alcoba, M 1994, "El problema del continuo en la Escolástica Española: Rodrigo de Arriaga", *Fragmentos de Filosofía*, vol. 4, pp. 137–150; Beeley, P 1995 (*op. cit.*); Feingold, F 2003, "Jesuits: Savants", in M Feingold (ed.), *Jesuit Science and the Republic of Letters*, MIT Press, Cambridge, Massachusetts, pp. 1–45 (see p. 28 ff.); Solère, J-L 2001 (*op. cit.*).

140 "(...) *il ne paraît pas que Zénon d'Elée ait enseigné que le continu fût composé de parties indivisibles. Il se contentait de se prévaloir de la doctrine contraire, pour montrer que le mouvement était impossible. Il disait même qu'un corps indivisible ne diffère point du néant (...)*".

141 "(...) *le Zénon qui niait le mouvement, et dont Aristote examine les raisons, fût le chef des stoïciens*".

142 "*Continuum ex partibus indivisibilibus constare contra Aristotelem constanter defendebat Zeno stoicorum princeps (...)*" (*Physic.*, controvers. XVII, section 1, p. 334, col. I [Bayle's reference is to Oviedo, F 1651, *Cursus Philosophicus ad unum corpus redactus* (1st ed. 1640): *Tomus primus: Complectens Summulas, Logicam, Physicam, Libros de Caelo, & de Generatione...*, 2nd ed., Sumpt. Philippi Borde, Laurentii Arnaud & Claudii Rigaud, Lyon]). See also Luna Alcoba M, 1996, "El problema del continuo en la escolástica española: el *Cursus Philosophicus* de Francisco de Oviedo", *Δαίμων. Revista de Filosofía*, vol. 12, pp. 37–47.

Spanish Scholastics (followed by citizens from other countries) treated as “Zenonism” was really, in its background, Pythagoreanism or Platonism. In fact, Arriaga (*disp. phys.* XVI, §2) mentions Zeno, Pythagoras, Leucippus, all the Stoics and many Jesuits as sharing the same opinion that the finite continuum is composed of finite indivisibles¹⁴³ (Arriaga 1632, p. 460). While accomplishing a critical evaluation of “Zenonism”, Boscovich reintroduced the simple, indivisible and unextended points of the Pythagorean/“Zenonist” tradition, but had to abandon the idea of continuous extension because in his model of identical point-particles interacting in pairs, the non-extension of *puncta* “is the direct outcome of the unlimited increase in repulsive force when the distance between bodies or particles becomes very small (...); if such elementary particles were extended, they would have to scatter, for the repulsive force would make it impossible for any extended particle, however small, to persist” (Dadić 1987, p. 76).

Paolo Rossi thinks “a reading of Boscovich’s text through post-Newtonian and post-Leibnizian eyes leads almost fatally to put in the shade the terrain – frequently uncertain and always heavily scholastic – that constitutes the ‘tradition’ from which Boscovich’s synthesis received leavening” (Rossi 1999, p. 90). Though “Zenonism” may have reached Boscovich through overt “Zenonist” texts, he may also have read Bayle. However, his quotations about Zeno suggest Vico as his intermediary and main “Pythagorean” and “Zenonist” source (Caparelli 1944, pp. 146 and 584), with Leibniz, “the last great philosopher and scientist who felt himself to be in the Pythagorean tradition” (Thesleff 1974, p. 581),¹⁴⁴ as another important “Pythagorean” precursor. According to Whyte,

143 “*Celebris est enim sententia Zenonis, Pythagorae, Leucippi, omniumque Stoicorum doctum, continuum finitum componi ex indivisibilibus finite. Eam defendunt recentiores multi et graves e nostra Societate [Gesu]*”.

144 “The last philosopher to consider himself a Pythagorean was Gottfried Leibniz” [Honigsberg, A 1999, “Pythagoras”, in C D von Dehsen (ed.), *Philosophers and Religious Leaders (Lives & Legacies)*, Oryx, Phoenix, p. 157]. Leibniz said in the “Preface to the General Science” of *The Method of Mathematics* (1677): “There is nothing which is not subsumable under number. Number is therefore, so to speak, a fundamental metaphysical form, and arithmetic a sort of statics of the universe, in which the powers of things are revealed” [Leibniz, G W 1951, *Selections* (ed. P P Wiener). Charles Scribner’s Sons, New York, p. 17]. Working out the classical idea of *mathesis*, he thought he had found a universal and exact system of notation, a symbolic language, a “universal characteristic”, “an infallible method of calculation” (an algebra), “the true principle, namely, that we can assign to every object its determined characteristic number” (*Selections*, p. 18). Dietrich Mahnke claimed that Pythagoreanism is a fundamental aspect of Leibniz’s metaphysics [Mahnke, D 1939–1940, “Die Rationalisierung der Mystik bei Leibniz und Kant”, *Blätter für deutsche Philosophie*, vol. 13, pp. 1–73 (see p. 21)].

Boscovich had made a thorough study of the Greek geometers. And the ideas of the Pythagorean school, of Democritus, Descartes, and Locke,¹⁴⁵ had helped to form his mind. But his main conscious sources were Newton and Leibniz; Newton the geometer of forces and Leibniz the philosopher of space, time, and the *monads*. (...) Boscovich may have received some stimulus from Vico, either by personal contact or by reading. But it is equally possible that he did not; that Leibniz's monads did the trick, and by fertilizing the Newtonian matrix in Boscovich's mind produced the most original¹⁴⁶ and influential work on the mathematics of atomism (Whyte 1961a, pp. 117 and 118).

At least part of the "Pythagorean" content of Boscovich's "atomic" theory can, therefore, be traced back to Vico and Leibniz. Vico's ideas on Pythagoreanism were principally on the line of philosophical historiography,¹⁴⁷ while Leibniz was influenced by "Pythagorean" ideas from Johann Heinrich Bisterfeld (1605–1655)¹⁴⁸ and Franciscus Mercurius van Helmont (1614–1699),¹⁴⁹ and, in a less specific way, from the alchemist and Hermetic traditions.¹⁵⁰ Ancient Pytha-

145 Locke minimized the importance of the concept of substance. Furthermore, Boscovich, in the same way as Locke, held truth to be an unattainable goal in natural philosophy, focusing instead on convenience, conviction, elegance, utility, etc. (cf. Nedeljković, D 1922, *La Philosophie Naturelle et Relativiste de R. J. Boscovich*, Editions de la Vie Universitaire, Paris, pp. 13–18 & 189).

146 On the aspects of this originality see Whyte 1961a, p. 118.

147 In the tradition of Johannes Scheffer; see Micheli, G 1993, "Johannes Gerhard Scheffer (1621–1679)", in G Santinello *et al.* (eds.), *Models of the History of Philosophy*, vol. 1: *From its origins in the Renaissance to the "Historia Philosophica"* (1981), English language editors C W T Blackwell & P Weller, Kluwer Academic Publishers, Dordrecht, pp. 150–154.

148 Willy Kabitz discovered, in the Hannover library, a copy, annotated by Leibniz, of Bisterfeld's works. For Leibniz's relationship with Bisterfeld and the presence of Pythagorean ideas in Leibniz's writings see Kabitz, W 1909, *Die Philosophie der jungen Leibniz. Untersuchungen zur Entwicklungsgeschichte seines Systems*, Carl Winter, Heidelberg; Mugnai M 1973, "Der Begriff der Harmonie als metaphysische Grundlage der Logik und Kombinatorik bei Johann Heinrich Bisterfeld und Leibniz", *Studia Leibnitiana*, vol. 5, pp. 43–73.

149 See Orio de Miguel, B 1993, "Leibniz y la tradición teosófico-kabbalista: Francisco Mercurio van Helmont" (Doctoral thesis in philosophy, 1988), 2 vols., Universidad Complutense de Madrid, Madrid; Coudert A P 1995, *Leibniz and the Kabbalah*, Kluwer Academic Publishers, Dordrecht, pp. 35–77 and *passim* (especially pp. 70–71).

150 For the relations between Leibniz and the Hermetic (or Teosophic-Kabbalist) tradition see Orio de Miguel, B 1993, "Leibniz y la tradición teosófico-kabbalista" (*op. cit.*); Orio de Miguel, B 1987, "Leibniz y la *Philosophia Perennis*", *Estudios Filosóficos*, vol. 101, pp. 29–57; Orio de Miguel, B 1992, *Leibniz y el Hermetismo*, 2 vols., Universidad Politécnica de Valencia, Valencia; Orio de Miguel, B 1994, "Leibniz y la Tradición Neoplatónica", *Revista de Filosofía*, 3ª época, vol. 7, no. 12, pp. 493–527; Orio de Miguel, B 2005, "Leibniz. Hermetismo y Ciencia Circular: Una carta a Burcher de Volder (20 de junio de 1703)", *Thémata*, vol. 34, pp. 297–338; Orio de Miguel, B 2009, "Leibniz y la tradición hermética", *Thémata*, vol. 42, pp. 107–122; Orio de Miguel, B

gorean ideas had passed through many channels, esoteric or scientific, pagan¹⁵¹ or Christian,¹⁵² until they reached modern times. Though Boscovich apparently did not see himself as an overt Pythagorean like Leibniz, some aspects of his biography point to a life with traces that could be seen as not dissonant with the traditional view of a modern heir of the Pythagoreans.

It is said that Boscovich discovered by himself the proof of the Pythagorean theorem while still a teenager (Hill 1961, p. 29). He was interested in archaeology, “his archeological involvement having always been related to mathematical and astronomical interests” (Dadić 1987, p. 202), and he believed that a sundial he had excavated at the western slopes of the Alban Hills, above Frascati (ancient Tusculum), was the one mentioned in the writings of Vitruvius.¹⁵³ He was very skilled in mathematics, and published numerous Latin dissertations that show an important concern for geodesical and astronomical problems.¹⁵⁴ His opinions

2008, “Some Hermetic aspects of Leibniz’s mathematical rationalism”, in M Dascal (ed.), *Leibniz: What Kind of Rationalist?* (Logic, Epistemology, and the Unity of Science, vol. 13), Springer Science + Business Media, Heidelberg, pp. 111–124.

151 As in the fusion between revived Pythagorean doctrines with Platonism and Stoicism in Neo-Pythagoreanism.

152 In the 2nd century, Justin Martyr, in his *Discourse (or Exhortation) to the Greeks*, had already moved towards a Christianization of Pythagoras [see Roberts, A & Donaldson, J (eds.) 1867, *Ante-Nicene Christian Library: Translations of the Writings of the Fathers down to A. D. 325*, vol. II – Justin Martyr and Athenagoras, T and T Clark, Edinburgh, p. 305; Heniger Jr, S K 1974, *Touches of Sweet Harmony: Pythagorean Mythology and Renaissance Poetics*, The Huntington Library, San Marino, California, p. 202], broadly accepted in Patristic literature and continued in modern times (see Heniger, pp. 203–3 & 229 n. 9).

153 See Boscovich, R 1746, “D’un’antica villa scoperta sul dosso del Tuscolo, d’un antico orologio a Sole, e di alcune altre rarità che si sono tra le rovine della medesima ritrovate. Luogo di Vitruvio illustrato”, *Giornale de’ Letterati* (Roma), April issue, article XIV, pp. 115–135. On Vitruvius’ place within the Pythagorean tradition see McEwen, I K 2003, *Vitruvius: writing the body of architecture*, MIT Press, Cambridge, Massachusetts/London, p. 40 ff.; Joost-Gaugier, C L 2006, *Measuring Heaven: Pythagoras and his influence on art in Antiquity and the Middle Ages*, Cornell University Press, Ithaca, New York, p. 29.

154 Among them can be mentioned: *De Maculis Solaribus* (1736); *De Mercurii novissimo infra Solem Transitu* (1737); *De Aurora Boreali* (1738); *De novo Telescopi usu ad objecta coelestia determinanda* (1739); *De veterum argumentis pro telluris sphaericitate* (1739); *Dissertatio de telluris figura* (1739); *De Circulis oscillatoribus* (1740); *De motu corporum projectorum in spatio non resistente* (1740); *De inaequalitate gravitatis in diversis terrae locis* (1741); *De Annuis Stellarum Fixarum Aberrationibus* (1742); *De observationibus Astronomicis, et quo pertigunt eorundem certitudo* (1742); *De determinanda orbita planetae ope catoptricae ex datis vi celeritate, et directione motus in dato puncto* (1749), *De aberrationibus, quas sibi mutuo videntur inducere Jupiter, et Saturnus* (1752; published as *De inaequalitatis quas Saturnus et Jupiter sibi mutuo videntur inducere praesertim circa tempus conjunctionis*, 1756); *Opera pertinentia ad Opticam et Astronomiam Maxima ex parte nova, et omnia hucusque inedita* (5 vols., 1785), and the poem *De Solis ac*

were essential to the abolishment of papal censorship of the heliocentric model,¹⁵⁵ which lasted until 1757 (Muller 2007, p. 277). Besides his work in astronomy, Boscovich also speculated on the essence of matter and the laws of nature.¹⁵⁶ Furthermore, he was an important writer on geometrical crystallography, and one of the inventors of the prismatic micrometer,¹⁵⁷ and is today considered as the founder of modern refractometry and optical spherometry.

Links connecting Boscovich to the Pythagoreans: essential elements

The language employed by Boscovich in his scientific works is "geometrical", not only in the sense of the geometrical analysis as understood by Pappus of

Lunae defectibus (1760 London, 1761 Venice; final version 1779 Paris, with a French translation by the Abbé de Barruel, *Les Eclipses*). As a homage to Boscovich his name was given to an asteroid (no. 14361) and to a lunar crater.

155 On Boscovich and heliocentrism (accepted by him at least since 1739 with the *Dissertatio de telluris figura*), see Dadić, Ž 1987, "Bosković and the Question of the Earth's Motion", in I Macan & V Pozaić (eds.), *Filozofija Znanosti Rudera Bošković'a/The Philosophy of Science of Ruder Bošković. Proceedings of the symposium of the Institute of Philosophy and Theology, Filozofsko-Teološki Institut Družbe Isusove/Institute of Philosophy and Theology of the Society of Jesus*, Zagreb (distributed by Fordham University Press, New York); Casanovas, J 1988, "Boscovich as an Astronomer", in M Bossi & P Tucci (eds.), *Bicentennial Commemoration of R G Boscovich: Proceedings*, Edizioni Unicopli, Milan, pp. 57–70 (see p. 59).

156 See, for example, *De viribus vivis* (1745); *De materiae divisibilitate et de principiis corporum dissertatio* (1748; publ. 1757 in *Istoria Naturale di diversi Valentuomini*); *Dissertationis de Lumine pars prima* (1748) and *pars secunda* (1748); *Elementorum universae matheseos tomi tres* (1754); *De continuitatis lege et ejus consecrariis pertinentibus ad prima materiae elementa eorumque vires* (1754); *De lege virium in natura existentium* (1755); *Philosophiae naturalis theoria reducta ad unicum legem virium in natura existentium* (1758; 1759; 1763).

157 Boscovich, R 1777, "Account of a New Micrometer and Megameter", *Philosophical Transactions of the Royal Society of London*, vol. 67, pp. 789–798; abridged in C Hutton, G Shaw & R Pearson (eds.) 1809, *The Philosophical Transactions of the Royal Society of London, from their commencement, in 1665, to the year 1800; abridged. Vol. XIV. From 1776 to 1780*, C and R Baldwin, London, pp. 248–250). On Boscovich's place in crystallographic studies see Lord Kelvin 1893, "On the Elasticity of a Crystal according to Boscovich", *Proceedings of the Royal Society of London*, vol. 54, pp. 59–75; McKenzie, A T 1998, "'Nature doth everywhere geometrize': Crystals, Crystallization, and Crystallography in the Long Eighteenth Century", *Studies in Eighteenth Century Culture*, vol. 27, pp. 209–236 (see p. 219); Senechal 1990. On the application of the prismatic micrometer to astronomy see Delambre, J-B 1827, *Historie de l'Astronomie du XVIIIe siècle* [*Historie de l'Astronomie*, v. IV, posthumous], Bachelier, Paris (reprint 2004, Éditions Jacques Gabay, Paris), p. 645.

Alexandria,¹⁵⁸ but also because he was accustomed to employ geometric images to communicate information that in modernity is usually presented in algebraic notation.¹⁵⁹ Boscovich's views on the constituent parts of matter were closely related to hypotheses on the structure of crystals, and according to Marjorie Senechal (1990, p. 43), Boscovich's ideas about point-atoms were compatible, from the geometric point of view, with two previous mathematical representations of crystal structure: theories of crystal structure as space-filling polyhedra of close-packed identical spheres,¹⁶⁰ and theories of crystal structure as space-filling polyhedra composed by particles of various shapes and sizes. All those mathematical representations depended on what Alan L. Mackay called "a Pythagorean strain in our culture", which "continually made congenial the idea that somehow the symmetrical geometrical figures – the Five Platonic Solids in Particular – are at the bottom of things" (Mackay 1986, p. 22).

Boscovich's point-centers share the characteristic aspects of the Pythagorean monad as defined by Clark Butler: "[It] has quantitative aspects: it can be repeated and then counted. Yet it is not a purely quantitative concept. It also has qualitative determinations, namely, variable relations of the limit (finite) and unlimited (infinite)" (Butler 1997, p. 86).¹⁶¹ Boscovich's "point-atomism" is also in agreement with Pythagorean views about the role of the "void" in physical reality, according to Aristotle's testimony: "[For the Pythagoreans] the void distinguishes the nature of things, as if it were like what separates and distinguishes the terms of a series. This holds primarily in numbers, for the void distinguishes

158 See, for example, Robinson, R 1936, "Analysis in Greek Geometry", *Mind*, vol. 45, no. 180, pp. 464–473 (reprinted 1969 in *Essays in Greek Philosophy*, Oxford University Press, Oxford, pp. 1–15); Lafrance, Y 1978, "Aristote et l'analyse géométrique", *Philosophiques*, vol. 5, no. 2, pp. 271–307; Behboud, A 1994, "Greek Geometrical Analysis", *Centaureus*, vol. 37, pp. 52–86.

159 See Iltis, C 1970, "D'Alembert and the *vis viva* controversy", *Studies in History and Philosophy of Science*, vol. I, no. 2, pp. 135–144 (see p. 139); Martinović I 1993, "Boscovich on the problem of *generatio velocitatis*: genesis and methodological implications", in P Bursill-Hall (ed.), *R J Boscovich – Vita e Attività Scientifica/His life and Scientific Work*, Istituto della Enciclopedia Italiana, Roma, pp. 59–79 (see p. 67); Giorgilli, A [2011], "Roger Joseph Boscovich between geometry and astronomy", in P Pareschi (ed.), *Proceedings of the Symposium Ruggiero Boscovich: astronomo, uomo di scienza e di cultura a trecento anni dalla nascita (Memorie della Società Astronomica Italiana)*, Milan, 18 may 2011 [forthcoming].

160 Robert Hooke seems to have been the first person to consider the ultimate spheres to be "atoms" or "atomical particles" (Hooke, R 1665, *Micrographia: or, some physiological descriptions of minute bodies made by magnifying glasses with observations and enquiries thereupon*, J Martyn and J Allestry, London).

161 For more on the equilibrium between quantitative and qualitative aspects of numbers see Butler 1997, p. 114 ff.

their nature"¹⁶² (Aristot. *Phys.*, 213b23–27).¹⁶³ This passage may be compared with a statement from the *Theoria philosophiae naturalis* (§88):

The whole of space is merely a continuous vacuum [with simple, non-extended and non-adjacent points], and, in the continuous motion by a simple point, the passage is made from continuous vacuum to continuous vacuum. The one point of matter occupies but one point of space; and this point of space is the indivisible boundary between the [void] space that precedes and the [void] space that follows. There is nothing to prevent the moving point from being carried through it by a continuous motion, nor from passing to it from any point of space that is in immediate proximity to it (Boscovich, 1966, p. 45).

Lancelot Law Whyte saw Boscovich's "point atomism" as developing from the Pythagorean idea (Pythagorean-Platonic, we may say) that physical reality originates from the dynamics of point-numbers, first to form a line, then a plane, and finally a solid (body),¹⁶⁴ with this thought being changed to include a force emanating from the points, a single general force or "single law of interactions" in a "monism of relations" (Whyte 1961a, 117 and 107) that is the idea behind Boscovich's *magnum opus* – whose title, *Theoria philosophiae naturalis*, can be translated as "Physics reduced to a single law of the forces existing in nature" (*Naturam ex unica simplici lege virium derivandam*, in the words of the typographer of the 1763 edition, p. III). For Whyte, "Boscovich is Pythagoras extended to cover process, Newton generalized, and Mach atomized" (Whyte 1961a, p. 124).¹⁶⁵ It is generally agreed that, improving upon Newtonian and Leibnizian principles, Boscovich provided the first general mathematical theory of atomism, and somehow anticipated the physics of relativity, *quantum* mechanics and modern field theory.¹⁶⁶ He was, nevertheless, continuing a long tradition that, instead

¹⁶² Aristotle 1952, *Physica*, tr. R P Hardie & R K Gaye, Clarendon Press, Oxford, p. 7.

¹⁶³ Fr. 58B30 Diels-Kranz. Cf. also Gigon O (ed.) 1987, *Aristotelis Opera (ex recensione Immanuel Bekkeri, ed. 2), III: Librorum Deperditorum Fragmenta*, Walter de Gruyter, Berlin/New York, pp. 415–6 (fr. 166).

¹⁶⁴ See Alexander Polyhistor, *FrGrHist* 273 F 93, but also Speusippus and Xenocrates.

¹⁶⁵ Boscovich is thought to have anticipated "Mach's principle", the idea that, in a material spatial system, the forces presented by masses (for example, the local motion of a rotating body) are determined by the large scale distribution of matter. All masses and all forces are relative. "As Boscovich suggested in 1758, and Mach in 1872, it may be possible to base theory directly on the changing spatial relations of physical entities, and to dispense with the introduction of co-ordinate systems as a technique for representing observed relations" (Whyte 1961b, p. 13).

¹⁶⁶ Boscovich had the intention of integrating his scientific theories in three groups, a theory of forces existing in nature (*theoria virium in natura existentium*), a theory of transformations of geometric loci (*theoria transformationum locorum geometricorum*), and a theory of infinitesimals (*theoria infinitesimorum; theoria indefinitorum sive indefinite parva sint, sive indefinite magna*).

of clinging to indivisible bodies, as physical atomists did, understood physical reality as primarily composed of indivisible points and units.

The Eleatics abhorred non-being, and due to the metaphysical impossibility of the conversion of being into non-being they confronted the idea of infinite divisibility. They can be thought of as metaphysical atomists. Pythagorean numbers correspond to being in the Eleatic tradition; therefore, the Pythagoreans can somehow be thought of as arithmetical atomists. According to Aristotle, the Pythagoreans supposed numbers to be extended (*Metaph.* 1080b18–20), but without bulk (they have no weight or lightness; 1090a32–34). The transition from arithmetical reality and geometrical extension to physical substance was considered an impossibility by Aristotle; where, then, would the essence of bulk/corporeality find its origin? To the thinkers of the tradition that understood physical reality as primarily not composed of indivisible bodies somewhere between mathematical and physical reality, a sort of pre-corporeal existence [as with the special numbers in the arithmetic atomism of Plotinus (*Enneads*, VI.6 [34])] was proposed for unextended indivisible points (“point-atomism”), in the place that in the geometric atomism of the *Timaeus* was occupied by triangles. Prior to Boscovich, the way in which unextended pre-corporeal units assemble to form extended bodies was speculated by Neopythagoreans in their understanding of the commencement of materiality from a dyad (as in Numenius of Apamea), but also by the *mutakallimûn*, who may have received some Neopythagorean influence in their “atomism”, and by the “Zenonists”. These, according to Bayle, should be understood to be in the same tradition as Pythagoras and Plato – two thinkers who, according to Thomas Bradwardine, believed that a *continuum* is composed of indivisible points.

The theory of forces appeared as his *Philosophiae naturalis theoria/Theoria philosophiae naturalis* (1758/1763), the theory of geometric transformations appeared in the 3rd volume of his *Elementorum universae matheseos* (1754), but Boscovich died without completing his third theory [see Martinović, I 1990, “Theories and inter-theory relations in Bošković”, *International Studies in the Philosophy of Science*, vol. 4, no. 3, pp. 247–262]. Nevertheless, according to Whyte (1961b, p. 15) Boscovich’s ideas may serve as a basis for developments beyond the two great physical theories of the 20th century: the classical foundation on which relativity and quantum theories have been constructed “involves a double parametric redundancy: excessive dimensionality and excessive analysis into separable entities, whereas the observed facts may only compel the assumption of certain changing spatial relations of simpler non-analysable particles, here called primary. These primary particles may be permanent Boscovichian point centres (a class of stable nucleons?), appearing singly in different circumstances as neutrons and protons, and displaying in complex extended systems propagated modes of system deformation identifiable as electrons, photons, and the various other particle-fields. On this view ‘fields’ are modes of deformation of systems of primary particles representable by collective co-ordinates”.

Henry More, the young Immanuel Kant and Boscovich, continuing a tradition widely held to represent Pythagorean-Platonic views, had in common some theories that broke with cospusal metaphysics and introduced a processual (field-theoretic) conception of matter (Holden 2004, pp. 236–272), and therefore the “perfectly solid, sharply defined and – it would seem – metaphysically-divisible ‘atoms’, familiar from the main tradition of Gassendi, Boyle, Locke and Newton, are each replaced with a difused shell of force projected by an unextended central *punctum*” (Holden 2004, p. 238). For Boscovich, the shift from forces to figures, corresponding to a transposition from metaphysical to physical reality, lies behind the origination of a material (corporeal) reality out of point-atoms devoid of corporeal dimensions. In contemporary physics, the conversion from forces (energy) to figures (matter), or vice-versa, has become universally accepted.

Conclusion

Boscovich's approximation to Pythagoreanism is suggested by the form in which he presents his ideas, with statements on physics translating into mathematics and vice versa, and by the content of his ideas about nature; he saw physical reality as originating from the dynamics of point-centers (simple, indivisible and unextended, scattered in a vacuum; Boscovich 1966, pp. 21 and 144) that share the characteristic aspects of the Pythagorean monad, and conceived a “monism of relations” (Whyte) with a single law of forces (Boscovich 1966, p. 8). Boscovich stands in a line of thinkers that connect ancient Pythagoreanism with revolutionary mathematics. The theoretical physicist Frank Tipler, for example sees physical reality as a subset of a much larger mathematical reality: “physical reality is not ‘real’, only number – the integers comprising the true ultimate reality – is actually real” (Tipler 2005, p. 905). Hans Vaihinger (1922, p. 606), believed it is impossible that matter is composed of point-atoms without extension,¹⁶⁷ but that nevertheless it is useful to keep this false hypothesis in order to achieve an easier calculation of the ponderal relations of matter. On

167 This impossibility rests on the opinion that hypothesizing that a continuous body is made of point-sized particles “runs counter to the concept of contact and therefore abolishes precisely what makes up the essence of the continuum” [Brentano, F 1988, *Philosophical Investigations on Space, Time and the Continuum* (posthumous; undated), tr. B Smith, Croom Helm/Routledge, London/New York, p. 147]. However, if Boscovich refuted a coexistent continuum, he proposed a successive one (see D’Ors [d’Ors Rovira], E 2009, *Las Aporías de Zenón de Elea y la Noción Moderna del Espacio-Tiempo*, Ediciones Encuentro, Madrid, pp. 127–128).

the other hand, Dean Zimmerman (1995, p. 98) suggested that “perhaps Boscovich was right and every physical object is ultimately made out of a cloud of disconnected point-sized atoms”. Boscovich’s theory would be consonant with a “fluid dynamic geometry of open space”, in which the central elements “would not be the discontinuous and so lifeless, fixed, dimensionless points of mass or force characteristic of closed space geometry, but *dynamic relational centres of flow*. That is, they would be dimension-full ‘breathing points’ as *dynamic configurations of space*, that is, *point-influences* or *local spheres of non-local influence*, not purely local *point-forces* or *point-masses*” (Rayner 2008, pp. 99 and 100).

Visionary as it may seem, the approximation between Pythagoreanism and contemporary physics isn’t so far-fetched when we have in mind the main innovations of Boscovich’s indivisibilist theory, which incorporates and advances the rich tradition of Pythagorean-Platonic forms of “number atomism” in its “dynamic atomism”: material permanence without spatial extension, spatial relations without absolute space, and kinematic action without Newtonian forces.

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