

MODERN STUDIES IN PHILOSOPHY is a series of anthologies presenting contemporary interpretations and evaluations of the works of major philosophers. The editors have selected articles designed to show the systematic structure of the thought of these philosophers, and to reveal the relevance of their views to the problems of current interest. These volumes are intended to be contributions to contemporary debates as well as to the history of philosophy; they not only trace the origins of many problems important to modern philosophy, but also introduce major philosophers as interlocutors in current discussions.

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THE PRE-SOCRATICS



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Edited by

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MYSTICISM AND SCIENCE IN THE PYTHAGOREAN TRADITION

F. M. Cornford

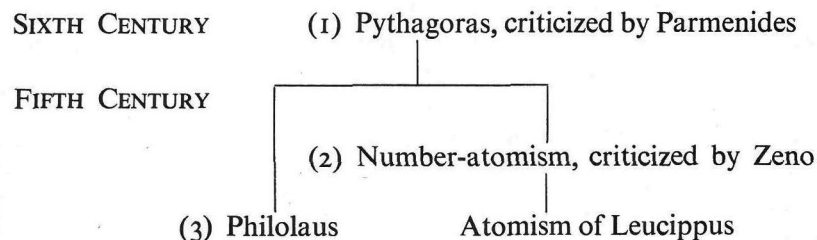
The object of this paper is to show that, in the sixth and fifth centuries B.C., two different and radically opposed systems of thought were elaborated within the Pythagorean school. They may be called respectively the mystical system and the scientific. All current accounts of Pythagoreanism known to me attempt to combine the traits of both systems in one composite picture, which naturally fails to hold together. The confusion goes back to Aristotle, who usually speaks indiscriminately of "the Pythagoreans," though now and then the phrase "some Pythagoreans" indicates that he was aware of different currents within the school.

I shall try to show that the criterion enabling us to distinguish the two systems is furnished by the Eleatic criticism of Pythagoreanism, which can be used as one might use a mirror to see what was happening on the other side of a screen. The history of Pre-Socratic philosophy is divided, circa 500-490 B.C., into two chapters by Parmenides' polemic against any system which derives a manifold world from an original unity. The first chapter contains the two great sixth-century systems of the Milesians and of Pythagoras, both of which fall under Parmenides' condemnation. Parmenides, bred in the Pythagorean tradition, was primarily a critic of the school from which he was seceding. Thus we have a clue to what sixth-century Pythagoreanism must have been, if we ask what is the radical fault found by Parmenides in the system he is criticizing. It will appear that this fault is the attempt to combine a monistic inspiration with a dualistic system of Nature. Parmenides declared for uncompromising monism, and in consequence denied plurality and becoming, including change and motion. The second chapter contains

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the fifth-century pluralist systems of Empedocles, Anaxagoras, and the Atomists, who sought in various ways to restore plurality, change, and motion without infringing the canons Parmenides was believed to have established. It is antecedently probable that some section of the Pythagorean school would attempt a similar answer. Now, in the generation after Parmenides, we find his pupil Zeno attacking a system which appears to be that answer. It is an inchoate form of Atomism—a doctrine that the real consists of an indefinite plurality of units or monads (indivisible points having position and magnitude), which can move in space, and of which bodies can be built up. Of this doctrine there is no trace in Parmenides; it belongs to the early fifth century. Zeno's criticisms, on the other hand, point to this doctrine, and to nothing else. It is not the later developed Atomism of Leucippus and Democritus, from which it differs in various respects. The monads, for instance, do not differ, like the atoms, in shape, but are all alike. I infer that the system in question is another pluralist system, the immediate ancestor of Atomism proper, constructed by the scientific wing of the Pythagorean school as a reply to Parmenides' critique.

Aristotle, when he speaks of "the Pythagoreans," refers sometimes to the original sixth-century system, sometimes to this later doctrine, and probably in his own mind did not clearly distinguish the two. Hence his testimonies, if taken all together, are inconsistent. Here we are told that sensible things "represent" or "embody" (*mimeisthai*) numbers; there, that sensible things or bodies actually *are* numbers, built up of indivisible monads. And so on. But, with the guidance of the Eleatic criticism and our knowledge of the religious antecedents of Pythagoras, we can sort out the testimonies and refer them to the two systems I have mentioned. We can, in a word, distinguish between (1) the original sixth-century system of Pythagoras, criticized by Parmenides—the mystical system—and (2) the fifth-century pluralism constructed to meet Parmenides' objections, and criticized in turn by Zeno—the scientific system, which may be called "Number-atomism." There is also (3) the



system of Philolaus, which belongs to the mystical side of the tradition, and seeks to accommodate the Empedoclean theory of elements. This may, for our present purpose, be neglected. The preceding diagram illustrates the development.

I. THE MYSTICAL SYSTEM OF PYTHAGORAS

We may start from a consideration of the type of society founded by Pythagoras. The beliefs of a religious community in its earliest stages are externalized in its rule of life, and of the Pythagorean fraternity we know enough to guide us. It was modeled on the mystical cult-society, to which admission was gained by initiation—that is, by purification followed by the revelation of truth. To the Pythagorean, "purification" partly consisted in the observance of ascetic rules of abstinence from certain kinds of food and dress, and partly was reinterpreted intellectually to mean the purification of the soul by *theōria*, the contemplation of the divine order of the world. "Revelation" consisted in certain truths delivered by the prophet-founder (*αὐτὸς ἔφα*), and progressively elaborated by his followers under his inspiration.¹

The rise of mystical cult-societies or non-social religious groups seems to coincide with the breaking up, in the sixth century, of the old social units based on the theory or fact of blood-kinship. It had also psychological causes: there was a deepening and quickening of religious experience—the revival associated with the name of Orpheus. These two sets of phenomena lead to certain axioms in any philosophy that arises out of them. There is, moreover, among these axioms a latent contradiction: there is a tendency toward monism and a tendency toward dualism.

Take first the monistic tendency. In the old blood group the social bond, the sense of solidarity (*philia*), had formerly extended to the limits of the blood-kin (*philoï*); beyond were "strangers," if not enemies.

¹ The pious attribution of all discoveries to the Founder may be illustrated by a penetrating observation made in another connection by Auguste Bouché-Leclercq (*L' Astrologie grecque* [Paris, 1899], p. 51 n. 1). He speaks of "a psychological fact, amply demonstrated by the history of apocryphal literature: viz. that every doctrine that appeals to *faith* has an interest in making itself appear of great antiquity, and that those who develop such a doctrine are very careful not to offer their respective inventions as the products of their own genius. They escape from discussion by cloaking themselves with as enormous as possible a mass of unverifiable experiences and revelations."

There had also been a coextensive religious bond in the common worship of some peculiar set of divinities, heroes, or ancestors. The system is naturally polytheistic. The appearance of new religious groups, transcending the limits and ignoring the ties of kinship, is attended by consequences of great importance. On the social side, at least the seed is sown of the doctrine that all men are brothers; the sense of solidarity, set free from the old limits, can spread to include all mankind, and even beyond that to embrace all living things. *Philia* ceases to mean kinship in the ordinary sense, and begins to mean love. At the same time the social basis of polytheism is undermined. Either monotheism, in some form, must take its place, or at least the belief (essentially true) that the mystery gods worshipped by different groups, whether called Dionysus or Adonis or Attis, are really the same god—one form with many names. There emerges the axiom of monism: *All life is one and God is one.*²

On the other hand, there is a no less significant change in the psychology of the individual.³ The old solidarity of the blood group had entailed that diffusion of responsibility for the actions of any one member among all the other members which still survives in the vendetta. When collective responsibility goes, individual responsibility is left. The guilt of any action must now attach personally to its author. It cannot be expiated by another, or by the blood group as a whole. The punishment must fall upon the individual, if not in this life, then in the next, or perhaps in a series of lives in this world. When the Pythagoreans reduced justice to the *lex talionis*, the effect was that it applied to the guilty person only, not to his family. The doctrine of transmigration completes the scheme of justice for the individual soul. The mere idea of reincarnation was no novelty. What is new in transmigration is the moral view that reincarnation expiates some original sin and that the individual soul persists, bearing its load of inalienable responsibility, through a round of lives, till, purified by suffering, it escapes for ever.

Thus, while God becomes one in the inclusive sense of monotheism—in religious terms, the Father,⁴ not of this household, clan, or city, but of all mankind and of all living things—and his children become, on

² Cf. Sext. Emp. *M.* IX.127; Iamb. *VP* 108.

³ Cf. Gustave Glotz, *La Solidarité de la famille dans le droit criminel en Grèce* (Paris, 1904), p. 587.

⁴ The Pythagorean term was rather *δεσπότης*—the father and master of the household, or *κύριος* (Euxitheos *Athen.* IV.157c).

their side, one all-inclusive group, conversely the soul acquires a unity in the exclusive sense. The individual becomes a unit, an isolated atom, with a personal sense of sin and a need of personal salvation, compensated, however, by a new consciousness of the soul's dignity and value, expressed in the doctrine that by origin and nature it is divine. From God it came, and to God it will return.

But only on condition of becoming pure. So long as it is imprisoned in the bodily tomb, it is impure, tainted by the evil substance of the body. Psychologically—in terms of actual experience—this means that the soul is profoundly conscious of an internal conflict of good and evil, the war in the members. This conflict dominates religious experience. In philosophical expression, it gives rise to the axiom of dualism: *In the world, as in the soul, there is a real conflict of two opposite powers—good and evil, light and darkness.*

Both the axiom of monism and the axiom of dualism are implicit in the doctrine of transmigration, which was certainly taught by Pythagoras. All souls come from one divine source and circulate in a continuous series of all the forms of life. Each soul, involved in the conflict of good and evil, seeks escape from the purgatorial round of lives and deaths into a better world of unity and rest. Any philosophy that arises from a religion of this type is threatened with internal inconsistency. On the one hand, it will set the highest value on the idea of unity, and, at this stage and long afterwards, the notions of value and of reality coincide. Unity is good; reality must be one. On the other hand, Nature will be construed in terms of the inward conflict of good and evil, appearing in the external world as light and darkness. Light is the medium of truth and knowledge; it reveals the knowable aspect of Nature—the forms, surfaces, limits of objects that are confounded in the unlimited darkness of night. But it is hard to deny reality to the antagonistic power of darkness and evil. Hence the tendency to dualism—to recognize, not the One only, but two opposite principles.

Now, if we bring this preliminary inference to the test of the Eleatic criticism, it seems to be confirmed. The gist of Parmenides' doctrine is that we must choose between monism and dualism. If we assert that the real is one, we cannot logically maintain a dualistic system of Nature. And the particular form of dualism he attacks is the doctrine that in Nature there are two opposite "forms"—light and darkness—equally real. So far, then, it appears (1) that the religious experience which underlies the doctrine of transmigration would naturally give rise to a philosophy combining a monistic tendency with a dualistic; and (2)

that the latent conflict of these two tendencies is the radical fault found by Parmenides in the Pythagoreanism of his time.

The reconstruction of Pythagoras' system may be approached through the analysis of certain pivotal conceptions which all admit to be characteristic of the Italian tradition. These are: the ideal of "becoming like God" and the notion of *mimēsis*; the correspondence of macrocosm and microcosm; the conception of harmony; the doctrine of numbers; the symbol known as the *tetractys*.

Aristoxenus⁵ says of Pythagoras and his followers: "Every distinction they lay down as to what should be done or not done aims at communion (or converse, *homilia*) with the divine. This is their starting-point; their whole life is ordered with a view to following God, and it is the governing principle of their philosophy." This "following" or imitation of God was to end in a purification of the soul from the taint of its bodily prison-house so complete that there should be no need of any further reincarnation. Pythagoras was believed to have reached this threshold of divinity;⁶ Empedocles later made the same claim for himself: "And I, among you an immortal god, no longer mortal,"⁷ echoed in the Orphic grave tablets, where the dead soul is addressed: "From a man thou hast become a god."

The means of rising to this condition was "philosophy," the contemplation of the cosmos in which God was contained or embodied. It was assumed, moreover, in sharp contradiction to orthodox Olympian religion, that there was no insuperable gulf between God and the soul, but a fundamental community of nature. The same order (*kosmos*) or structural principle is found on a large scale in the universe and on a small scale in individuals, i.e. those parts of the universe which are themselves wholes, namely living things. The living creature (soul and body) is the individual unit or microcosm; the world, or macrocosm, is likewise a living creature with a body and soul.⁸ Individuals reproduce

⁵ Iamb. *VP* 137 (= DK 58D2). Arius Didymus (?) ap. Stob. *Eth.* VI.3 Σωκράτης Πλάτων ταῦτα τῷ Πυθαγόρᾳ, τέλος ὁμοίωσιν θεοῦ.

⁶ Aristotle *Fr.* 192 (Rose) τοῦ λογικοῦ ζῶον τὸ μὲν ἐστὶ θεός, τὸ δὲ ἄνθρωπος, τὸ δὲ οἶον Πυθαγόρας.

⁷ *BI* 12.4. Cf. *BI* 46, 147. "At the last they appear among mortal men as seers, singers, physicians, and leaders of men" (Empedocles was all these), "and then they spring up as gods highest in honor, sharing the hearth of the other immortals, free from human sorrows, from destiny, and from all harm."

⁸ The expression "microcosm" first occurs in Democritus, B34 ἐν τῷ ἀν-

the whole in miniature; they are not mere fractions, but *analogous parts* of the whole which includes them.

This relation of the many analogous parts to the including whole is very important. It is implied in the term *mimēsis*, by which, as Aristotle remarks, the Pythagoreans meant the same relation that Plato called "participation" (*methexis*).⁹ In Plato it is the relation of a number of similar individual things to the supersensible Idea whose nature is communicated to them. The things "participate" in that Idea, but in such a way that the whole Idea is represented in each, and yet not used up by any one. This meaning of *mimēsis* goes back to the original sense of the word. *Mimos* means an actor. A whole succession of actors may embody or reproduce a character, say Hamlet; but none of them is identical with Hamlet. Each represents the character, which yet is not used up by any one impersonator. The actor was, in the earliest times, the occasional vehicle of a divine or legendary spirit. In Dionysiac religion this relation subsists between the *thiasos* or group of worshippers and the god who takes possession of them (*katechein*). "Blessed," say the chorus in the *Bacchae*,¹⁰ "is he whose soul *thiasseutai*"—is merged in his group, when the whole group is possessed by one spirit, which, not being a fully developed, atomic personality, can alike penetrate the whole group and dwell in each of its members. At that stage "likeness to God" amounts to temporary identification. Induced by orgiastic means, by Bacchic ecstasy or Orphic sacramental feast, it is a foretaste of the final reunion. In Pythagoreanism the conception is toned down, Apollinized. The means is no longer ecstasy

θρόνῳ μικρῷ κόσμῳ ὄντι, but the conception is much older and akin to the astrological premise of a "sympathy" between the heavenly bodies and earthly life.

⁹ *Metaph.* I.6.987b10. Otto Gilbert ("Aristoteles' Urteile über die pythagoreische Lehre," *AGP*, 22 [1909], 40 ff.) rightly urges that *μίμησις*, *ὁμοίωμα*, *ὁμοιοῦν*, etc., imply a relation between two *different* things, and holds that Pythagoras and his school (I should say, the Pythagoreans other than the "mathematicians" or number-atomists) did not identify numbers with things in *ihrer stofflichen Grundlage*. At *Metaph.* V.14.1020b4 Aristotle uses *μίμημα* for the plane or solid figure, which is the graphic "representation" of a number. In this case any number of similar figures can "represent" the same number.

¹⁰ Eurip. *Ba.* 72. Cf. A. W. Verrall, *The Bacchantes of Euripides and Other Essays* (Cambridge, 1910), p. 30, who translates "whose soul is congregationalized."

or sacrament, but *theōria*, intellectual contemplation of the universal order, whereby the microcosm comes to reproduce (*mimēisthai*) that order more perfectly and becomes *kosmios*, attuned to the celestial harmony.

From the analogy of macrocosm and microcosm certain cosmological premises follow. The One or All must be perfect (*teleion*) and limited. The Unlimited, the *apeiron* which Anaximander had called divine, cannot be reproduced in a miniature whole. To the Pythagorean it is an evil principle of disorder, the opposite of the good principle of Limit. Again, the derivation of the many from the One cannot be merely a splitting of the One into fragmentary parts. It must be such that the nature of the whole can be reproduced in each subordinate whole or analogous part.

The formula of that identical structure which is repeated in the universe and in its analogous parts is "harmony." This word meant, first, the "fitting together" or "adjustment" of parts in a complex whole; then, specially, the "tuning" of an instrument; and hence the "musical scale" which results therefrom. There was, from the first, the implication of a "right," or tuneful adjustment.

This conception adds to the notion of the mixture of opposites as it occurs in Ionian science, the notions of order, proportion, measure. Each mixture, resulting in a thing, conforms (or ought to conform) to a definite law which characterizes it and distinguishes it from another mixture of the same ingredients. The proportion comes to be regarded as the determining essence of the compound, in which its goodness and reality lie.

This doctrine that the goodness of a compound depends on exact numerical proportions is thus referred to by Aristotle: "One might raise the question what the virtue (*to eu*) is that things get from numbers because their composition can be expressed by a number, either by a simple proportion (*eulogistōi*) or by an odd number. For in fact honey-water is no more wholesome if it is mixed in the proportion of three times three, but it would do more good if it were in no particular proportion but well diluted than if it were numerically expressible but strong."¹¹ Though Aristotle's common sense rejects this theory in the

¹¹ *Metaph.* XIV.6 init. Alexander (ad loc.) explains: "Some Pythagoreans did not hesitate to say that the virtue or good (*τὸ εὖ καὶ τὸ ἀγαθόν*) in everything in this world results from numbers, when the mixture of ingredients is expressed by an even or an odd number, and most of all when the result is

case of the wholesomeness of honey-water, he suggests elsewhere a similar view in the case of colors. The multiplicity of colors, over and above black and white, may, he says, be due to differences in the proportion of their composition. The combination of black and white in the intermediate colors may be in the proportion 3:2 or 3:4 or according to other ratios. Other colors may be compounded in no commensurate proportion, and "colors may be analogous to concords (*symphōniais*). Thus, the colors compounded according to the simplest proportions (*τὰ ἐν ἀριθμοῖς εὐλογίστοις χρώματα*), exactly as in the case of concords, will appear to be the most pleasant colors, e.g. purple, crimson, and a few similar species. It is an exactly parallel reason that causes concords to be few in number" (i.e. the "simplest proportions" are few).¹²

Aristotle here applies to the explanation of the agreeableness of certain colors the Pythagorean doctrine that the virtue of a compound lies in the exact and simple numerical proportion of the ingredients. The analogy with concords points clearly to the original source of the theory, Pythagoras' discovery that the concordant intervals of the musical scale or harmony could be expressed exactly in the terms of the "simple" ratios, 1:2 (octave), 3:2 (fifth), and 4:3 (fourth), and that, if the smallest whole numbers having these ratios to one another (viz. 6:8:9:12) are taken, the internal terms are the means (arithmetic and harmonic) between the extremes. Thus was the principle of harmony revealed as an unseen and unheard principle of order and concord, identical with a system of numbers bound together by interlocking ratios. The system, moreover, is limited, both externally by the octave (for the scale ends, as we say, "on the same note" and begins again in endless recurrence), and internally by the means. The introduction of this system marks out the whole unlimited field of sound, which ranges indefinitely in opposite directions (high and low). The infinite variety

perfect and similar (*τέλειον καὶ ὁμοιον*) and obtained by multiplication, e.g. 3×3 , not $3 + 3$." Alexander wrongly interprets *εὐλογίστω* to mean *ἀπρίω*. The true meaning is clear from Arist. *De Sensu* 439b34: *εὐλόγιστοι ἀριθμοί* = "proportions where the division of one term by the other takes very little trouble" (G. R. T. Ross [Cambridge, 1906] ad loc.), i.e. the simplest proportions.

¹² *De Sensu* 439b21. Cf. Arist. *Pr.* XIX.35.920a27 ff.): The octave is the most beautiful concord because the terms of the ratio (1:2) are whole numbers, and the division leaves no remainder.

of quality in sound is reduced to order by the exact and simple law of ratio in quantity. The system so defined still contains the unlimited element in the blank intervals between the notes; but the unlimited is no longer an orderless continuum; it is confined within an order, a *kosmos*, by the imposition of Limit or Measure.

The mathematical genius of Pythagoras was capable of abstracting this complex of conceptions from the particular case of sound. It must have been by a flash of inspired insight that he saw in it a formula of universal application—the union of the two opposite principles of Limit and the Unlimited to form the Limited. To the microcosm it was immediately applied in the doctrine that the good state of the body (health, strength, beauty) is the proportioned mixture or temperament of the physical opposites, hot and cold, wet and dry, etc. This conception, stated by Alcmaeon,¹³ a junior fellow-citizen of Pythagoras at Croton, persists throughout ancient medical theory.

There is no reason to doubt that the application to virtue, the health or good condition of the soul, is equally old. The distinction between soul and body was not so sharply drawn as to prevent the Pythagoreans from practicing psychotherapy. As they used charms for physical ailments, so they cured the sick soul by music and the recitation of poetry.¹⁴ Protagoras in Plato's dialogue treats as a commonplace of educational theory the effect of music in producing *euharmonia*, "good temper," and *eurythmia*, "good rhythmical order," in the soul, and its result, virtuous conduct.¹⁵ This was not invented by Plato. The doctrine, indeed, only gave an exact and abstract expression to the popular notion that self-control (*sōphrosynē*, *kosmiotēs*) is moderation, the imposition of Limit or Measure upon turbulent passion that runs to excess—a notion that lay at the center of Greek morality. To say that virtue is *euharmonia* is only to restate this in terms suggested by the musical discovery of Pythagoras.

Does the doctrine that the soul itself is a harmony go back to Pythagoras himself? This is commonly denied on the ground that, if the soul is a harmony or *krasis*, "blending," of the bodily opposites, it cannot survive the dissolution of the body: the doctrine is inconsistent with transmigration or any form of survival.¹⁶ This is, of course, the argu-

¹³ Alcmaeon B4. Cf. Plato *Symp.* 186d.

¹⁴ Iamb. *VP* 164.

¹⁵ *Prt.* 326a. Cf. Eryximachus in *Symp.* 187d.

¹⁶ Professor Burnet, who uses this argument, is hardly entitled to do so,

ment urged by Simmias in the *Phaedo* (85e and 92); but the inconsistency does not seem to have been perceived by Philolaus, whom the young Simmias had heard at Thebes. There is no doubt that Philolaus held both that the soul is, in some sense, a harmony and that it is immortal.¹⁷ If Philolaus felt no inconsistency, it is still less likely that the earliest Pythagoreans would have been aware of any. It is probable that the objection was first raised by Plato; and his argument apparently was not accepted as conclusive, for Aristotle speaks of "soul = harmony" as an opinion that has been "handed down" (*paradedotai*) and that "commends itself to many minds as readily as any of those previously mentioned."¹⁸ . . .

That the doctrine of the three parts of the soul goes back to Pythagoras himself we are told on the authority of Posidonius, who, as Professor Burnet says, was "not likely to have been mistaken on such a point."¹⁹ Pythagoras must, indeed, have thought of the soul as in some sense divided into at least two parts. This follows from the central religious experience of the divided self, the internal warfare between good and evil, the Orphic double nature of man, the sense of sin combined with the consciousness of inward good and light taking part against inward evil and darkness. And with this must go also the possibility of internal reconciliation and concord, when the man, as Plato says, becomes φίλος ἑαυτῷ, "a friend to oneself," and εἰς ἐκ πολλῶν, "a one (man) out of many." If virtue is this concord (*symphōnia*) or peace of the soul gained by the mastery of passion and animal desire, what could be more natural to the Pythagorean mathematico-musical mode of thought than to conceive the soul itself as the

since he regards inconsistency between religious and scientific beliefs as normal in the Pre-Socratic philosophers (*EGP*, p. 295). Cf. p. 250: "All through this period there seems to have been a gulf between men's religious beliefs, if they had any, and their cosmological views."

¹⁷ Cf. Erwin Rohde, *Psyche*, 2d ed., Freiburg i.B. (Leipzig and Tübingen, 1898), vol. 2, p. 169.

¹⁸ *De Anima* I.4, init.

¹⁹ John Burnet, *Plato's Phaedo* (Oxford, 1911), note on 68c, where the passages cited by Eduard Zeller (*Die Philosophie der Griechen*, vol. 1, 5th ed. [Leipzig, 1892], p. 447) are quoted. Professor Burnet also points out that the doctrine of the tripartite soul agrees with the Pythagorean apologue of the Three Lives, compared to the three classes of men who go to Olympia (1) *θέας ἐνεκα*, (2) to compete (*δόξα*), (3) to buy and sell (*κέρδος*), Iamb. *VP* 58.

harmonia, and the parts of the soul as the terms to be harmonized and brought into concord? . . .

I conclude, then, that there is good reason to regard "soul = harmony," thus interpreted, as an original Pythagorean doctrine. Indeed, if virtue is a *symphōnia*, I do not see how the soul can be anything but the *harmonia* which contains it. It was, moreover, because the human soul contained both the divine and the irrational parts that it could, if purified, become wholly divine, or, if still impure, sink into the lower forms of life. So far from contradicting transmigration, the doctrine of the tripartite soul, under the image of the charioteer and his horses, is seen in the *Phaedrus* as part of the scheme of transmigration. The sense given to "soul = harmony" by Simmias in the *Phaedo* is quite different. Not that it is necessarily incompatible with the other. It is possible to regard the soul both as the vital principle which, during earthly life, maintains a healthy balance of the opposite elements in the body, and also, in its other aspect, as a harmony of its own three parts, with its own peculiar concord, virtue. When disembodied, it would temporarily lose the former function, but would remain a harmony in the second sense, more or less well tuned according as it departs this life more or less "pure."

The fact is that in dealing with the doctrine of the soul in philosophies of the religious type, we are dealing with a thing that exists, as it were, upon two different planes—the spiritual plane and the natural. On the natural plane the soul acts as a vital principle, distinguishing organic living things from mere casual inorganic masses of matter. In that aspect it is conceived in Pythagorean mathematico-musical terms as a harmony or ratio, expressible in numbers. It is the element of proportion in an ordered compound. But on the spiritual plane, it is itself a compound of good and evil parts—of the element of limit, order, proportion, reason, and the disorderly unlimited element of irrational passion. So considered, it is a permanent immortal thing. The question how exactly this spiritual thing is related to the vital principle which distinguishes a living from a dead body is a question that might be put to any modern believer in immortality without the expectation of any very clear and precise answer.

The other argument urged in the *Phaedo* against "soul = harmony" seems to be fallacious. It is that, if you say that the soul is a harmony, you must not also say that virtue is a harmony, for then, in the virtuous soul, you would have a harmony of a harmony. The answer is simple:

"harmony" is ambiguous.²⁰ It may mean merely an adjustment, or a tuneful adjustment. A lyre that is out of tune (*anharmosteī*)²¹ is still adjusted, though wrongly. Virtue is not so much *harmonia* as *euharmonia*; vice is *anharmostia*. The virtuous soul is the well-tuned adjustment; the vicious, the ill-tuned. No doubt, a strictly logical and literal consideration will discover what seem to us to be obscurities and even contradictions in the conception. But the greater, in that case, the likelihood that it did not date from the end of the fifth century, when men were thinking much more clearly than they had a hundred years earlier.²²

The reason for supposing that the doctrine "soul = harmony" goes back to Pythagoras is that it seems to follow from the correspondence of macrocosm and microcosm and to be required by the fundamental conception of the imitation of God, considered as the tuning of the soul into consonance with the celestial harmony, which alone will manifest *euharmonia* in perfection. "The whole Heaven is harmony and number." The macrocosm is a living creature with a soul, or principle of life, and a body. It is an easy inference that the soul of the world is a harmony or system of numbers (as it is described by the Pythagorean Timaeus in Plato)—that very harmony which is manifest to sense in the order of the heavenly bodies and is to be reproduced in the attunement of the individual soul.

We can now approach the interpretation of the famous symbol called the *tetractys* or "tetrad," which is a compendium of Pythagorean mysticism. The *tetractys* is itself a system of numbers. It symbolizes the "elements of number," which are the elements of all things. It contains the concordant ratios of the musical harmony. It might well be described in the Pythagorean oath as "containing the root and fountain of everflowing Nature." In one of the *akousmata* preserved in Iamblichus

²⁰ Hence the doctrine vacillated: Arist. *Pol.* 1340b17.

²¹ Plato *Grg.* 482b.

²² There is the same confusion and obscurity about the λόγος τῆς μίξεως of Empedocles, which Aristotle suggests that he identified with ἁρμονία καὶ ψυχὴ (*De Anima* I.4.408a13 ff.). I believe that Empedocles' physical doctrine of the nature of soul was consistent (to his mind) with transmigration. See *From Religion to Philosophy* (London, 1912), p. 239. Since the peculiar features of Empedocles' physical system can only be explained by the desire to accommodate his religious doctrines, the common view that the religion and science are incompatible must be rejected.

it is identified with the cosmic harmony.²³ It was also called *kosmos*, *ouranos*, *pan*.²⁴ Theon says it was held in honor because it contained the nature of the universe.²⁵

The *tetractys*, also called the decad, consists of the first four integers ($1 + 2 + 3 + 4 = 10$), represented in the old fashion by pebbles or

dots arranged in an equilateral triangle . It "represents all the

consonances," in the sense that these four numbers are those which occur as terms in the concordant ratios discovered by Pythagoras in the musical scale. It is "perfect," and "embraces the whole nature of number," because all nations count up to ten and then revert to one; all the other numbers are obtained by repetition of the decad.²⁶ Further, the component numbers symbolize the "elements of number."

"It is clear," says Aristotle,²⁷ "that the Pythagoreans regard number both as the matter of things and as their properties and states. The elements of number are the even and the odd, of which the even is unlimited, the odd limited. The One (or Unity) consists of both, for it is both odd and even. Number (proceeds) from the One, and numbers, as has been said, are the whole Heaven."

This obscure statement can be interpreted with the help of other authorities.

First, there is the identification of the Even with the Unlimited, the Odd with the Limited, or Limit.²⁸ Euclid's definitions of Even (Book VII, def. 6 "the number divisible in two [parts]") and Odd (def. 7 "the number not divisible in two [parts]") seem to be derived from the Pythagorean definitions given by Aristoxenus:²⁹ "among numbers,

²³ Iamb. *VP* 82 (= DK 58C4): *τί ἐστι τὸ ἐν Δελφοῖς μαντεῖον; τετρακτύς, ὅπερ ἐστὶν ἡ ἁρμονία ἐν ἣ αἱ Σειρήνες.*

²⁴ Plutarch *Isis et Osiris* 75.

²⁵ Theon. Smyrn. π. τετρακτύος 154 (ed. Dupuis).

²⁶ Arist. *Metaph.* I.5.986a8; Aet. I.3.8; Hippol. *Haer.* VI.23.

²⁷ *Metaph.* I.5.986a15.

²⁸ *Metaph.* I.5.990a8 has *πέρας* (not *πεπερασμένον*) and *ἄπειρον* as the equivalents of *περιττόν* and *ἄρτιον*. *Πέρας* (*περαῖνον*, Philolaus) is correct.

²⁹ DK 58B2; Diels compares Arist. *Metaph.* XIII.8.1083b28, q.v. For explanations and other definitions see T. L. Heath, *The Thirteen Books of Euclid's Elements* (Cambridge, 1908), vol. 2, p. 281. The curious and unique use of *ἰσοσκελής* = *ἄρτιος* and *σκαληνός* = *περιττός* in Plato *Euthyphro* 12d may be explained by the diagrams $\cdot\cdot$, $\cdot\cdot\cdot$, $\cdot\cdot\cdot\cdot$, etc., and $\cdot\cdot\cdot$, $\cdot\cdot\cdot\cdot$, $\cdot\cdot\cdot\cdot\cdot$, etc.,

even are those that are divisible in two equal (parts), odd those that are divisible in unequal (parts) and have a middle." Plutarch explains further: "Since even numbers start with 2, odd numbers with 3, and 5 is generated by the combination of these, 5 has rightly received honor as the first product of first principles, and has been named 'Marriage,' because the even is like the female, the odd like the male. For when numbers are divided into equal parts, the even is completely parted asunder, and leaves within itself as it were a receptive principle or space, whereas, when the odd is treated in the same manner, there is always left over a middle (*meson*), which is generative (*gonimon*)."³⁰ And again, "when numbers are equally divided, in the uneven number a unit is left over in the middle, while in the even there is left a masterless and numberless space, showing that it is defective and imperfect."³¹

Thus the Dyad, as the first even number, stands for the female receptive field, the void womb of unordered space, the evil principle of the Unlimited. The Triad is its opposite, the good principle of Limit, the male whose union with the Unlimited produces the Limited. As Aristotle says:³² "The Universe and all things (in it) are limited or determined by three" (the Triad). The numbers 5 ($2 + 3$) and 6 (2×3) are both symbols of the marriage of Even and Odd, Unlimited and Limit.

Such are the two opposite "elements of number" and of all things. In the Monad they are not yet differentiated; it "consists of both," is both odd and even, or, in mythical language, male and female (*arsenothēlys*), like the Orphic Phanes. The Monad, so conceived, is not the first in the series of numbers; indeed, it is not a number at all, but *archē* ("principle" or "origin") of number.³³ It is the original undifferentiated unity, from which emerge the two opposite principles Limit and Unlimited, the elements of number and of all things.

In this interpretation of the Monad in the *tetractys* I have taken the view that the Monad is prior to, and not a resultant or product of, the

which show even numbers when divided as "equal legged," odd numbers as having one leg longer than the other.

³⁰ *De E ap. Delphos*, 388A. On this subject see W. A. Heidel, "Πέρας and Ἀπειρον in the Pythagorean Philosophy," *AGP*, 14 (1901), 384-99.

³¹ Plutarch (Diels, *Dox. Gr.*, 96) ap. Stob. *Ecl. Phys.* I.1.10, p. 22 (ed. C. Wachsmuth).

³² Arist. *De Caelo* I.1.268a10.

³³ Aristoxenus ap. Stob. I.1 pr. 6. DK 58B2.

two opposite principles, Odd or Limit, and Even or Unlimited.³⁴ In favor of this view the position of the Monad at the head of the *tetractys* seems to be decisive. As Theon, discussing the properties of the numbers in the *tetractys*, says: "The Monad is the origin (*archē*) of all and the most dominant of all . . . and that from which all things issue forth (*ἐξ ἧς πάντα*), whereas it does not issue forth out of anything, being indivisible and potentially all things, unchangeable, never transcending (*exhistamenē*) its own nature in the process of multiplication" (i.e. $1^n = 1$).³⁵ This view has also the advantage that it brings the Pythagorean scheme of thought into line with the other early systems, both mythical and scientific. The abstract formula which is common to the early cosmogonies is as follows: (1) There is an undifferentiated unity. (2) From this unity two opposite powers are separated out to form the world order. (3) The two opposites unite again to generate life. This formula is stated clearly by Melanippe the Wise (Eurip. Fr. 484 N²): "The tale is not mine; I had it from my mother: (1) that Heaven and Earth were once one form, and (2) when they had been sundered from one another, (3) they gave birth to all things and brought them up into the light, trees, and winged things, and creatures that the salt sea breeds, and the race of mortal men."³⁶ The same formula, stripped of the mythical imagery of sex, fits the cosmogony of Anaximander. He has (1) the primal undifferentiated *apeiron*, containing in complete fusion the opposites which are to be separated out of it;³⁷ (2) the separating out of these opposites in two pairs—first the Hot (fire) and the Cold (air), and later the Wet (water) and the Dry (earth)—to form the world order; (3) the reunion of the opposites (conceived, not as marriage, but under the alternative symbol of the warfare and aggression of the opposite powers invading one another's provinces unjustly) to form those temporary combinations which are living things. The Pythagorean

³⁴ Hence in the above passage from Aristotle (*Metaph.* I.5.986a19) I translate τὸ δὲ ἐν ἐξ ἀμοτέρων εἶναι τούτων "the One consists of both of these" (odd and even), not (with Ross, e.g.) "the One proceeds from both of these." Cf. Alexander on *Metaph.* 985b26, p. 30, 16 Bz.

³⁵ Ed. Jean Dupuis (Paris, 1892), p. 164.

³⁶ Cf. Appollonius Rhod. I.494. For the separation of Father Heaven and Mother Earth out of a primal unity and their subsequent marriage, see Edward B. Tylor, *Primitive Culture*, 4th ed. (London, 1903), vol. I, p. 325 (parallels from New Zealand, China, etc.), and Arthur Grimble, "Myths from the Gilbert Islands," *Folk-Lore*, 33 (1922), 91-112.

³⁷ So Aristotle, see *Phys.* I.4.187a20.

Monad similarly symbolizes the primal undifferentiated unity, from which the two opposite principles of Limit (physically, light or fire) and the Unlimited (space, air, "void") must, in some unexplained and inexplicable way, be derived. The union of the two opposites, as Plato explains in the *Philebus*, generates *to mikton*, "the mixed," when "the equal and the double and whatsoever puts an end to the mutual disagreement of the opposites, by introducing symmetry and concord, produce number" (25d).

The parallel with Anaximander suggests that, for the interpretation of the fourth number in the *tetractys*, we may use the identification of 4, as the first square number, with Justice.³⁸ In the third stage of the cosmogonical formula above stated, the combination of the sundered opposites to generate life is represented in mythical terms either as a marriage or as a warfare. In the Euripides fragment we have the immemorially ancient symbolism of the marriage of Heaven and Earth, mediated (in the Orphic cosmogony, as in Hesiod) by Eros or Phanes, and, in physical terms, by the rain, the seed of the Sky-father.³⁹ The marriage symbol is appropriate to the elemental forms arranged in concentric regions in the order of space. The two extreme elements, heavenly fire and earth, are united by the intermediate element, water or "air" (mist, etc.) or *to metaxy*, "the in between." The alternative symbol of warfare, on the other hand, fits the same elemental forms (Hot, Cold, etc.), conceived rather as the seasonal powers in the order of time, in which each prevails successively and yields in turn to its antagonist.⁴⁰ The principle of justice is preserved by this balanced alternation of advance and retreat. As Anaximander says, "Things pay to one another the penalty of their injustice according to the order of time" (B1). Now it can hardly be accidental that in the Pythagorean number-symbolism, after the undifferentiated Monad and the numbers 2 and 3 representing the opposites, female and male, the next two numbers, 4 and 5, symbolize Justice and Marriage. Justice, τὸ ἀντιπεπονθὸς ἄλλω, "reciprocal treatment," according to the law of Rhadamanthys,⁴¹

³⁸ Arist. *MM* I.1.1182a11, ἡ δικαιοσύνη ἀριθμὸς ἰσάκεις ἕσος. This interpretation of 4 in the Decad occurs in a Paris MS. published by Armand Delatte, in his *Études sur la littérature pythagoricienne* (Paris, 1915), p. 167: ἡ τετραὰς δικαιοσύνη διὰ τὸ ἰσάκεις ἕσον.

³⁹ Aeschylus *Danaids* 44 N².

⁴⁰ Cf. Empedocles B17.26, speaking of his elements; Alex. Polyhistor ap. Diog. Laert. VII.26 (Pythagorean doctrine).

⁴¹ Arist. *EN* V.5.1132b21.

completes the tetrad, and assures that the opposite tensions of the contraries shall be held together in harmony.⁴² It is easy to see why later authorities also identify the square number with *philia*.⁴³

Such is the meaning of this extraordinary symbol, the *tetractys*, which both contains the elements of number and of all things, and, as "the fountain of everflowing Nature," symbolizes also the evolution of the many out of the One, the cosmogonical process. How was this process conceived?

We have hardly any information about the earliest Pythagorean cosmogony. Pythagoras was the discoverer of the world of mathematics, which was to be conceived later as a supersensible world of concepts related in an infinite system of eternal truths—a timeless world in which no change or process can occur, and which is unaffected by the existence, becoming, or perishing of any sensible thing. But Pythagoras was still far from realizing the nature of this new world of thought. To him numbers and their relations were not only invested with a halo of divine and mysterious properties, but were also implicated in the sensible world, serving as the substructure of reality within that world and occupying space. He could not yet distinguish clearly between a purely logical "process" such as the "generation" of a series, and an actual process in time such as the generation of the visible Heaven, which "is harmony and number." The cosmogonical process was thus confused with the generation of numbers from the One, and will appear to us as a transcription of this (really logical) process into physical terms. The physical system will be determined by the way in which the generation of numbers is conceived. It was at this point, I believe, that the two schools of Pythagoreans—the original sixth-century mystics and the fifth-century mathematicians—parted company. They took very different views of the nature of the Monad, and consequently of the generation of numbers and things.

We have seen how, in the primitive symbolism of the *tetractys*, the Monad was the divine, all-inclusive unity, containing both the opposites, male and female, Limit and Unlimited. According to the old cosmogonical scheme, from the undifferentiated unity emerge the two opposite principles, and these are recombined to generate determinate

⁴² Cf. Plato's description of *δικαιοσύνη* in the *Republic*, 443c–e.

⁴³ Alex. on Arist. *Metaph.* 987a9 (p. 36, 18 Bz). The saying *φιλότης ἰσότης* is attributed to Pythagoras by Iamb. *VP* 162 and Porphyry *VP* 20 (probably following *Timaeus*; Delatte, p. 253).

(limited) things—the series of numbers and the things which represent or embody (*mimēisthai*) numbers. Thus any determinate thing will, like the Orphic soul, contain both principles, good and evil, light and darkness.

How this process was construed in physical terms is obscure. The Unlimited was evidently the unmeasured field of space, which, though called "the void," was filled by "air," the circumambient envelope of the limited Heaven, the breath of the living world. It is the primeval "Night" of the Orphics. The opposite principle of Limit is manifest to sense as light or fire. The product of the two principles is the cosmos or Heaven. As the unlimited range of musical sound is marked off by consonant numbers into the definite intervals of the musical scale, so the blank field of darkness is marked off by those boundary points of heavenly light, sun, moon, and planets, whose orbits (still conceived as material rings) are set at musical intervals to form the celestial harmony or scale, bridging and binding together the visible order from earth at the center to the outermost sphere of the fixed stars. How this majestic order was evolved we cannot say. There is no sign that the earliest Pythagoreanism went further.⁴⁴

The geometrical character of Pythagorean arithmetic must, of course, not be forgotten. Indeed, we are told that Pythagoras identified geometry with science (*historia*) in general.⁴⁵ In the unlimited darkness of night all objects lose to the eye their colors and shapes; in the daily renewed creation of the dawn of light they resume their distinct forms, their surfaces and colors (*chroia* in Pythagorean language means both). Thus in the physical world, light, the vehicle of knowledge, acts as a limiting principle, which informs the blank darkness with bodies bounded by measurable planes and distinguished by all the varieties of color. A body is thus a visible thing in which two opposite principles meet—the Unlimited (darkness, "air," void, space) and Limit, identified with the colored surface (*εἶδος, ἰδέα, μορφή, σχῆμα*). True to its mathematical character, Pythagoreanism tends to conceive a sensible body as essentially a geometrical solid, whose surfaces are ultimately reducible

⁴⁴ Aristotle's obscure remark as to the Pythagorean *κοσμοποιία* (*Metaph.* XIV.3.1091a12) refers, I believe, to the later system of Number-atomism discussed below (see p. 157). At *Metaph.* I.8.990a8 Aristotle remarks that, though the Pythagoreans *γεννώσι τὸν οὐρανόν*, they have no explanation how there is to be motion when only Limit and Unlimited or Odd and Even are posited.

⁴⁵ Iamb. *VP* 89.

to numbers and their relations. It is the mode of conception applied in Plato's *Timaeus* to the atoms of the four elements. In this way things "represent" numbers.

Now in this system of thought the most obscure and inexplicable moment is the evolution, out of the primal unity, of the two opposite principles, the elements of number and of all things.⁴⁶ The lucid and logical mind of Parmenides fastened upon this point. He accepted the premises (ultimately dictated by religious preconceptions) that Limit, Unity, Rest are good, and therefore attributes of the real. But, with a logic that seemed unanswerable, he exposed the latent contradiction in sixth-century Pythagoreanism, which had sought to combine these monistic premises with a dualistic system of Nature.⁴⁷ If the real is indeed one, Nature cannot be a battle-ground of two opposite powers, good and evil, light and darkness, equally real. If the One is at rest—motionless and immutable—it cannot become two, and then many; it must always be One. Plurality, becoming, motion, and change must be in some way unreal. We must choose between monism and dualism.

Parmenides' own choice is not that of a man of science, prepared to accept and explain the obvious facts presented by the natural world. His preference for unity, rest, limitation (perfection), can be ultimately explained only by the value, and consequent reality, ascribed to these conceptions as divine attributes. Rather than surrender these attributes, he is prepared to set all common sense at defiance. Hence it is in the Eleatic school that the distrust of the senses, so immensely important in later thought, first emerges. This doctrine was indeed latent in the other-worldliness of the Pythagorean type of religion, in the condemnation of the body as a dark prison hiding the light of truth from the soul. Like the appetites, the senses were regarded as bodily and inseparably connected with pleasure, which ascetic religion suspects and denounces. But the philosophic conclusion that the senses are false

⁴⁶ Later mysticism regards the emergence of the Dyad as an act of rebellious audacity: *Theol. Arith.* II.10 πρώτη γὰρ ἡ δυὰς διεχώρισεν αὐτὴν ἐκ τῆς μονάδος, ὅθεν καὶ τόλμα καλεῖται. So Plotinus, who in *Enn.* V.I.1 ascribes the fall of the soul to τόλμα. Proclus on Plato *Alc.* I 104e attributes this use of τόλμα to the Pythagoreans.

⁴⁷ Simplicius *Phys.* 181 (quoting Eudorus): "According to their highest teaching we must say that the Pythagoreans hold the One to be the principle of all things; according to a secondary teaching (δευτερος λόγος) they hold that there are two principles of created things, the One and the nature opposed to it."

witnesses to the external reality they profess to show us was new. It was destined to lead, later on, to the skepticism of the Academy. Thus the first parent of skepticism was not science, but religion.

Here, however, we are not concerned with these developments, but only with the light thrown by Parmenides' criticism upon the character of the original Pythagorean system. The first part of his poem leaves the divine Monad incapable of generating a pair of opposites, and through them the world of appearances. The second part contains a cosmogony on the traditional lines, vitiated by its dualism. In neither part is there any trace of the pluralist system next to be considered. . . .

II. THE SCIENTIFIC SYSTEM: NUMBER-ATOMISM

The existence of such a doctrine in the generation after Parmenides is proved by the critical arguments of Zeno. Zeno did not, like Parmenides, attack the dualistic doctrine of two opposite forms, or the inconsistency of this doctrine with monistic premises. His criticism is directed solely against the pluralist view that a manifold world and motion, denied by Parmenides, can be restored by regarding the real as composed of a plurality of units or monads moving in space. He deduces the absurdity of the hypothesis "if the many are" in that sense. This is not the hypothesis of primitive Pythagoreanism, nor yet the developed atomism of Leucippus. It appears to be an inchoate form of atomism, a reinterpretation of the doctrine of numbers, designed to obviate Parmenides' criticism.

Most of Aristotle's allusions to the doctrine of "the Pythagoreans" refer to this system. At *Metaph.* XIII.6.1080b16, for instance, he attributes to them the following theory. (1) There is only one kind of number—namely, mathematical number. (2) This number does not exist separately, but sensible substances are composed of it; indeed, the Pythagoreans construct the whole Heaven of numbers. (3) These numbers do not consist of abstract units, but the units are conceived as having spatial magnitude. (4) They are described as "indivisible magnitudes" (*atoma megethē*, 1083b13). (5) Things (*ta onta*) or bodies (*sōmata*) are identified with numbers composed of these indivisible magnitudes or monads; "at any rate, they apply their propositions to bodies as if they consisted of those numbers" (1083b12 ff.). (6) The Pythagoreans regarded numbers as generated—the process of generation being, of course, identical with the physical generation of the sensible world (1091a17 ff.).

My contention is that the theory here outlined is not merely not identical with the mystical doctrine reconstructed in the earlier part of this paper, but cannot be reconciled with it. It proceeds from a totally different conception of what is meant by the "monad," and of the way in which numbers are generated from it. In the old mystical system the Monad, standing at the head of the *tetractys*, was the primal all-inclusive unity, both male and female, from which the elements of number, Limit and Unlimited, proceeded before they reunited to generate numbers. Deprived of the mysterious power of generating plurality, this Monad becomes the One Being of Parmenides. It is obviously unique. Numbers cannot consist of a plurality of such units (*plēthos monadōn*) merely added together. Numbers are not obtained in that way, but by the union of the Limit and the Unlimited. But now this whole conception of the generation of numbers has been destroyed by Parmenides' logic. The "mathematicians" with a scientific turn of mind, indifferent to the obscure symbolism of the *tetractys* and to the religious premises of the founder's system, accept Parmenides' criticism of it. The Monad, the "beginning of number," is divested of its mystical properties. Let it be simply an indivisible unit. There is then nothing to prevent our supposing the existence of an indefinite plurality of such units, and saying that any number is simply a *plēthos monadōn*.

On this view, any number is a "finite plurality" or "collection of units." For the process by which numbers are generated we find the expressions "flow of quantity" (*chyma posotētos*) and "progression of multitude from a unit and retrogression of multitude ceasing at a unit." These are objective terms for the subjective processes of adding and subtracting. Thus Theon, after giving the second of the above definitions, proceeds: "A unit is a limiting quantity (*perainousa posotēs*)—a principle or element of numbers—which, when the multitude is diminished by subtraction (*κατὰ τὴν ὑφαίρεσιν*), is deprived of all number and takes an abiding position (*μονήν*) and rest. For the division (*tomē*) cannot proceed further; for even if we divide one sensible thing into parts, that which was one will become again a multitude or many, and, by subtraction of the parts, one by one, will end in unity. So the one, as one, is without parts and indivisible." We should say that any number can be obtained by adding one monad to another as often as is required; but the early Pythagorean mathematicians must have confused the generation of numbers with a real process that occurred in time and space, and was identical with the generation of the cosmos containing sensible bodies, which actually *were* numbers. This seems to follow

clearly from the passage of Aristotle already quoted (1091a13 ff.), where he adds that it is impossible to doubt that the Pythagoreans believed in a generation of numbers, thereby committing the absurdity of holding a becoming of things which are really eternal. "For their language is clear when they say that when the one" (i.e. "the first unit having magnitude," 1080b20) "had been constructed, whether out of planes or of surface or of seed or of some (elements) they cannot describe, immediately the nearest part of the Unlimited began to be constrained and limited by the Limit. Since, however, they are describing the construction of the cosmos and mean what they say in a physical sense," their opinions, Aristotle concludes, need not be further examined here, but belong to physics.

It seems clear from this passage that the Pythagoreans had not yet reached the position of fully developed atomism, which postulates an indefinite plurality of atoms or monads as an ultimate and eternal fact. Such a plurality seems to be required if sensible bodies are to be built of monads or indivisible magnitudes, as they were in both systems. A body is a collection of monads, *systema monadōn*, and so a number. But is there any sense in which one of the monads composing bodies—a "first unit"—can be regarded as prior to, or generating, the collection? For atomism, no; but the Pythagoreans confused the physical process with the so-called "processes" of arithmetical generation and geometrical construction. They had not faced the question which puzzled Socrates: how one and one can "become" two "by addition" (*diaprosthesin*), or how "division" (*schisis*) can be the cause of one becoming two (*Phaedo* 97a). Aristotle's mention of "seed" (*sperma*) suggests that their thought was governed by the analogy of the growth of the living body from its "seed" or "root": both terms are applied elsewhere to the monad as the principle of number.⁴⁸ If the cosmos is a living creature, naturally it also would grow from a seed. This growth is again confused with the generation of the solid by the "flowing" of the point into a line, of the line into a surface, of the surface into a solid. The first or minimum solid is the pyramid,⁴⁹ which is composed of four points having magnitude, and has four equal triangular faces. This could readily be identified with the atom of fire, the sensible manifestation of the principle of the Limit. So we reach Aristotle's alternative suggestion that the original unit was perhaps "constructed of planes or

⁴⁸ Plutarch ap. Stob. *Ecl.* I, pr. 2; Hermes ap. Stob. *Ecl.* I.10.15. Cf. also Arist. *Metaph.* XIV.5.1092a23–32; Theon, ed. Dupuis, p. 158.

⁴⁹ Speusippus *Theol. Arith.* p. 82 ff. (= DK 44A13).

surface." The doctrine, mentioned by Aristotle (*De Caelo* III.5.304a7), that Fire is the *only* element and has the pyramidal form must be Pythagorean,⁵⁰ though how it is related to Number-atomism we cannot say.

On the whole we are left with the impression of an atomistic type of cosmology struggling to free itself from mythical analogies and elementary confusions of thought. It is obvious that a theory of this kind would be immediately suggested by the practice of representing numbers by pebbles or counters arranged in geometrical patterns. The pebbles may stand for a sort of magnified atoms; the space or "field" (*chōra*) between them is analogous to the void. By adding unit to unit a solid body of any size and shape can be constructed. With this simple materialistic conception of a plurality of monads, the old mystical derivation of the world and its harmony from the divine Monad and the "elements of number" disappears, and with it go all the religious notions of the harmony of warring opposites, good and evil, the correspondence of macrocosm and microcosm, and the ideal of the imitation of God. The real is reduced to discrete quantity with the single purpose of restoring plurality and motion.

Aristotle himself draws attention to the two diverse ways of making numbers "the causes of substances and being," which, in my view, are characteristic of the two different schools of Pythagoreans. At *Metaph.* XV.5.1092b8, he remarks that "it has not been clearly distinguished in which of two ways numbers are the causes of substances and being—whether (1) it is as terms (*horoi*), as points are of spatial magnitudes (as Eurytus used to decide what was the number of what—e.g. of man or of horse—by representing the forms of living things with pebbles, as some people bring numbers into the figures of triangle or square). Or (2) is it that concord (*symphōnia*) is a ratio (*logos*) of numbers, and so is man and everything else?" As an example of the latter view he instances Empedocles' λόγος τῆς μίξεως, "mixture ratio," and objects that, on this view, it is the ratio itself (e.g. 3 parts of fire to 2 parts of earth) that is the essence, whereas the number is "matter." I believe that this second view is the original Pythagorean doctrine, according to which things *embody*

⁵⁰ There is no evidence for attributing more than πύρ ἀρχή to Hippasus, though the story (countenanced by Heath, *A History of Greek Mathematics* [Oxford, 1921], vol. 1, p. 160) connecting his name with the construction of a regular solid may be recalled. Simplicius ad loc. does not know to whom to attribute the doctrine mentioned by Aristotle.

or represent (μυεῖται) numbers, not *are* numbers; and the soul, as the essential reality, is a ratio or harmony, not a mere collection of monads. The other is the crude materialistic view of Number-atomism that things *are* numbers, and numbers consist of monads, which are the terms or boundary-stones (*horoi*) marking out the void "field" (*chōra*) in the geometrical patterns of numbers "figured" by pebbles.⁵¹

The doctrine that the soul is either a harmony or a "mixture ratio" is also foreign to this system. We should expect to find in it a materialistic conception of the soul approximating to the Atomists'. The soul can be nothing but a set of monads, and its chief function would be to cause motion. Now, among the philosophers who say that soul is primarily "a mover" (*to kinoun*), Aristotle mentions first the Atomists, with their soul consisting of spherical atoms or fire, and then remarks that certain Pythagoreans⁵² held a doctrine which appears to mean the same thing—namely, that the soul is "the motes in the air," while others say it is that which moves these motes. It has been observed, he adds, that "the motes are constantly in motion even in a complete calm" (i.e. as if they had the power of self-motion, which he goes on to discuss as an attribute of soul). I suggest that this view is that of the Number-atomists. It is hard to see how it could possibly be combined with any doctrine of the nature of the soul resting on the old conception of the mixture or harmony of opposites. On the other hand, it could easily be connected with the fire-atom whose pyramidal shape, being "sharpest cutting" (*tmētikōtaton*), enables it to penetrate everywhere.⁵³

I need not enter into Zeno's arguments against this view of reality. It is generally admitted that they are directed against "the Pythagoreans";^a and Plato tells us that they were a counter-attack upon the

⁵¹ The Pythagorean Ecphantus of Syracuse is said to have been the first who regarded the Pythagorean monads as bodily (σωματικές) or as ἀδιαίρετα σώματα of which sensible things consist (Aet. I.3.19; Hippol. *Haer.* I.15). His date is unknown; but the testimony supports the view that this Number-atomism was no part of the original doctrine, and that the view that things are related to numbers by μίμησις is older than the identification of bodies with numbers.

⁵² Themistius observes that he does not know which Pythagoreans are meant (π. ψυχῆς I.2, p. 17, Spengel). The doctrine was evidently obsolete.

⁵³ Arist. *De Caelo* III.5.304a7; cf. *De Anima* 404a1 ff.: Democritus and Leucippus made soul consist of spherical atoms "because such shapes are most able to permeate everywhere."

^a [This no longer "generally admitted." See above, pp. 13–15.—Ed.]

hypothesis "if the many are" as held by those who satirized Parmenides' argument and urged that it led to ridiculous contradictions.⁵⁴ This testimony exactly agrees with the view above advocated, that Number-atomism was the form of pluralism put forward by the Pythagorean mathematicians as a reply to Parmenides.

In trying to distinguish the two divergent schools of Pythagoreans I have naturally stressed the fundamental differences. I do not, of course, wish to imply that, for instance, the method of representing numbers by geometrical patterns was not practiced by Pythagoras. But it was the "mathematicians" who, so to say, took this method as giving a literal picture of the structure of reality, and so gave birth to Atomism, which in the series of philosophical systems stands in extreme contrast to the religious tradition continued by Philolaus and Plato.

⁵⁴ Plato *Parm.* 128c. The imaginary date of the dialogue is about 450 B.C. Zeno is "about 40 years old" (127b), and he speaks of his treatise as having been written "when he was young" (128d). This suggests a date about 470, which would be too early for an attack on Atomism proper.

6

PYTHAGOREAN PHILOSOPHY BEFORE PLATO

Charles H. Kahn

The name of Pythagoras is not only the most famous, it is also the most controversial in the history of Greek thought before Socrates and Plato. Since antiquity it has been a name to conjure with: There is such a wealth of conflicting evidence concerning Pythagoras' teaching, but so much of this evidence is unreliable. In 1925 A. N. Whitehead could write, in reference to the function of mathematical ideas in abstract thought: "Pythagoras was the first man who had any grasp of the full sweep of this general principle. . . . He insisted on the importance of the utmost generality in reasoning, and he divined the importance of number as an aid to the construction of any representation of the conditions involved in the order of nature."¹ But just two years earlier Erich Frank had published a book in which he claimed that "all the discoveries attributed to Pythagoras himself or to his disciples by later writers were really the achievement of certain South Italian mathematicians of Plato's time (in the decades before and after 400 B.C.)," that these contemporaries of Plato are the "so-called Pythagoreans" referred to by Aristotle, and that this mathematical school must be sharply distinguished from the "genuine Pythagoreans who are attested in southern Italy since the sixth century as a religious sect similar to the Orphics."² The implication of this view is that, except in connection with religious doctrine concerning the soul, the name of Pythagoras should be struck out of the history of philosophy and replaced by the name of Archytas and other mathematicians of

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¹ *Science and the Modern World*, p. 41.

² E. Frank, *Plato und die sogenannten Pythagoreer* (Halle, 1923), p. vi.