



PROJECT MUSE®

Pythagorean Pipe Dreams? Vincenzo Galilei, Marin Mersenne,
and the Pneumatic Mysteries of the Pipe Organ

Brandon Konoval

Perspectives on Science, Volume 26, Number 1, January-February 2018, pp.
1-51 (Article)

Published by The MIT Press



➔ For additional information about this article

<https://muse.jhu.edu/article/686155>

Pythagorean Pipe Dreams? Vincenzo Galilei, Marin Mersenne, and the Pneumatic Mysteries of the Pipe Organ

Brandon Konoval

The University of British Columbia

The pipe organ presented early modern science with a pneumatic black box of suggestive dimensions: while producing musical pitches and intervals that corresponded with those of an acoustic device like the monochord, pipe dimensions approached, but yet confounded clear association with the behavior of strings. Nevertheless, investigators like Vincenzo Galilei (c.1520–1591) and Marin Mersenne (1588–1648) continued to rely conceptually upon the monochord and the traditional ratios associated with it in their attempts to discipline the complex variables attending the acoustic properties of pipes. Thus, while certain conventions of historiography associate Vincenzo and Mersenne with a “disenchantment” of Pythagorean traditions that ostensibly retarded the development of an early modern physico-mathematics, their ratios of pipe scaling reveal instead a robust and evolving contribution of Pythagoreanism to mathematical reading of the Book of Nature.

1. Introduction

Sometime between 1589 and 1591, a momentous discovery was announced in Florence; or, at least, a discovery thought to be momentous by its promoter: “The *true* form of the octave is the octuple [ratio 8:1] and not the

My sincere thanks go to the dedicated reviewers and editors of *Perspectives on Science* for their thoughtful attention to an extended manuscript. I would also like to thank Ian Hacking and Carlos Urreiztieta for stimulating and helpful discussion of these matters in Halifax and Oxford. Finally, I owe a particular debt of gratitude to Stephen Straker (1942–2004), Ernie Hamm, and Arcadi Konoval for their longstanding support of this research.

duple [2:1]" (Palisca 1989: p. 189; emphasis added).¹ Thus taken out of context, we might be forgiven if we failed either to share the author's enthusiasm or to recognize the importance of his finding. But the fact remains that, sadly, nobody else did either: not only did his contemporaries decline to adopt this dramatic solution to the problem of the musical consonances,² but even those who may take an almost perverse interest at times in the forgotten or the misbegotten—among others, historians of science—have allowed this singular achievement to fall deep within the shadows of scholarly neglect. And yet, if left undisturbed in quiet obscurity, we would miss out on one of the more curious and informative early modern attempts to read the Book of Nature in the language of mathematics, an attempt that was undertaken by a figure prominently associated with the origins of opera and the birth of the Baroque: namely, Vincenzo Galilei (c.1520–1591)—conveniently, the father of that other Galilei, whose *Il Saggiatore* (*The Assayer* [Rome: Mascardi, 1623]) was to provide a particularly memorable mathematical characterization of the 'great book of the universe.'

Vincenzo has enjoyed some measure of recognition amongst historians of science in recent decades, particularly with his promotion through the ranks of notable if under-appreciated figures of early modern science, above all by the eminent musicologist, Claude V. Palisca, and the influential Galileo scholar, Stillman Drake (Palisca 1961).³ On their account, as a champion

1. Vincenzo Galilei, "Discourse Concerning the Diapason." "Octave" derives from the Latin *octavus* or "eighth"—that is, forming the interval that encompasses a sequence of eight diatonic pitches, by which one returns to the same pitch-class (e.g., "C"); "diapason" comes from the Greek, referring to "through all tones" in the same sense. It should be noted that terms like octave and diapason are categorically distinct from ratios, as they do not specify any precise tuning (unlike a frequency ratio of 1:2). Thus, the conventional string length ratio, 2:1, is not in fact "the octave" or "the perfect octave," but only "the perfect octave in just intonation" (the form of tuning which preserves the whole number ratios for consonant intervals, as well as the dissonant intervals derived from them): this is what Mersenne refers to when using a term like "*diapason tres-juste*" (cf. n. 50, v.i.).

A note on usage: I will reserve double quotation marks for quoted material in its original (if translated) wording, and when introducing recognized terms of art (like "octave"). I will use single quotation marks when highlighting a word or phrase to capture the characteristic usage or intentions of a source, but where that precise wording was not used by the original author (for example, Galileo's 'great book of the universe' to represent "this great book [I speak of the universe]"); or, conversely, to refer to a conventional usage that has no single source (such as the legend of the 'harmonious blacksmiths', who were not described as such by Nicomachus). These distinctions aim to bring greater transparency to source use in this discussion.

2. That such a solution was indeed known, if not explicitly associated with Vincenzo, is attested to by Kepler in the *Harmonices mundi*, as we will see.

3. The foundational, and deeply influential account by Palisca is "Scientific Empiricism in Musical Thought" (1961); Palisca returned to these themes in several subsequent articles, to be addressed here when pertinent to the context of discussion. Stillman Drake's enthusiastic embrace of Palisca's vision of the relationship between music and early modern science was inaugurated with "Renaissance Music and Experimental Science" (Drake 1970).

of empirical acoustic investigations, Vincenzo confronted hoary Pythagorean tradition like an early modern Hercules preparing to clean out the Augean stables: “the new acoustics replaced that elaborate conglomeration of myth, scholastic dogma, mysticism and numerology that had been the foundation of the older musical theory,” with that “older musical theory” recognized as Pythagorean (Palisca 1961, p. 137). And yet, despite his own enthusiasm for it, Vincenzo’s discovery of the true ratio of the octave—and the corresponding privilege he placed on the number 8—played no role in the feats recounted by Palisca and Drake.

Although by now several decades old, this account continues to resonate with more recent commentators, sometimes in bold relief, at other times in subtler but still telling detail. In *Absolute Music And the Construction of Meaning*, for example, Daniel K. L. Chua promotes a corresponding perspective that Chua claims as Weberian: namely, that Vincenzo’s acoustic investigations somehow ‘disenchanted’ a Pythagorean universe otherwise metaphysically beholden to simple numerical ratios for consonant intervals that were derived from the divisions of the monochord, an abuse of mathematical representation that bestowed wholly unempirical powers on simple numbers that simply measured things—whether vibrating lengths of string, or the weights hung from them to produce variable tensions, or the dimensions of pitch-producing pipes.⁴ Thus,

to disenchant the world, modernity had to sever the umbilical link of the monochord, disconnecting itself from the celestial realms in order to remove music as an explanation of the world. With its supernatural aura demystified as natural, and its inaudible, invisible essences dismissed as non-existent, modern music [*sic*] became an autonomous object open to the manipulations of instrumental reason. (Chua 1999, p. 18)

Chua’s enlistment of Weber for such an account is problematic in several respects, not least due to the questionable representation of Weber’s music study, *Zur Musiksoziologie*,⁵ as a text concerned with *Entzauberung* or disenchantment, a term which Weber did not introduce to his writings until several years later, and which cannot be treated as equivalent to Weber’s “rationalization”—several varieties of which provide the genealogical framework for Weber’s music study. Furthermore, Pythagorean theory and attunement fit within Weber’s account precisely as a departure from

4. “Numbers [are] significant only when applied to certain material relationships in sounding bodies, but are meaningless as abstractions.”

5. Published posthumously in 1921; translated into English as *The Rational and Social Foundations of Music* (1958).

chant formulas “addressed to magical ends, particularly apotropaic (cult) and exorcistic (medicinal) ends”: thus, “rationalization proper commences with the evolution of music into a professional art [...] reaching beyond the limited use of tone formulae for [such] practical purposes, thus awakening purely aesthetic needs” (Weber 1958, pp. 40–2, 2002, pp. 187–88). If there is a “disenchantment” to be found anywhere in Weber’s music study, therefore, it is courtesy of Pythagorean rationalization.⁶ Moreover, when claiming that Vincenzo “Galilei wanted to ‘demonstrate real things’ [...] in the spirit of Aristotle and not the numerological abstractions of Pythagorean mysticism” (Chua 1999, p. 18), Chua refers to the same manuscript quoted at the start of this essay, citing passages that contain no references to “Pythagorean mysticism” whatsoever, and to “real things” (“*cose reali*”) through which Vincenzo precisely sought to attack the use of number as formal cause in Aristotle’s *Physics*, as it had been deployed by Vincenzo’s polemical target, the music theorist Gioseffo Zarlino (1517–1590).⁷

Setting aside these scholarly concerns, those who would subscribe to the *Entzauberung* envisaged by Chua must understandably downplay that finding of which Vincenzo was so proud, the seeming capstone to a decade of musical and acoustic investigations: namely, his purportedly empirical discovery that the true harmonic number—or, as he repeatedly characterized it, the “mother” of all the ratios of the consonances—was the number 8.⁸ How did Vincenzo come by this novel insight, and on what basis did he

6. *Entzauberung*, which has become popularly if somewhat confusingly translated into English as “disenchantment,” was introduced by Weber in 1917 with his lecture, “Wissenschaft als Beruf” (“science” or “intellectual work as a vocation”), although he had clearly encountered the term with a similar critical orientation in Georg Simmel’s 1906 monograph on religion, which Weber owned and marked up (my thanks to Thomas Kemple for sharing this research with me). For recent scholarly and interpretive approaches to Weber’s music study itself, see Darmon 2015; Steege 2012, pp. 241–51; Fend 2010; Molino 2008; and Turley 2001. For benchmark scholarship, see the classic monograph by Christoph Braun 1992, and the introduction for the *Musiksoziologie* in the *Max Weber Gesamtausgabe* prepared by Braun and Ludwig Finscher (Weber 2004).

7. Cf. Book II, part 3 (194b25), where Aristotle refers to the ratio of the octave, 2:1, as an example of a formal cause.

8. Scholarship that continues to subscribe to the fundamental account of Palisca and Drake tends to cite source materials without acknowledging such idiosyncratic details that do not fit the Vincenzo-as-Empiricist narrative: cf. Moyer 1992, pp. 259–60, 263; Stephenson 1994, pp. 45; Gozza 2000, pp. 34–5, 60; Chua 1999, pp. 12–22, and 2001; and Heilbron 2010, pp. 9–11. D. P. Walker (1978) offered an important early corrective to Palisca’s claims, a balanced critique adopted and extended by Cohen 1984, pp. 82–5. Cohen acknowledges the problematic nature of the pipe ratios on which Vincenzo’s 8:1 ratio was based (Cohen 2010, pp. 145–46), but straightforwardly contrasts Vincenzo’s claims on behalf of the “octuple” ratio with purportedly purely empirical results for tension that were no less problematic, as Mersenne was to demonstrate and as this study will further examine.

believe others ought to recognize its validity? Far from an umbilical link in which the progress of music theory and musical practice had become hopelessly entangled, the monochord would continue to provide Vincenzo with an Ariadne's thread to which he clung through the labyrinthine passages of acoustic investigation, and which would continue to guide the thought of Vincenzo's great inheritor in acoustic endeavors, Marin Mersenne (1588–1648). For both investigators, the monochord would provide such crucial aid most of all when confronting the mysteries of pitch-producing pipes.

At the same time, the provenance of Chua's 'umbilical monochord' merits careful attention: the imagery was inspired by the well-known illustration of a 'cosmic monochord' tuned by the hand of God as Pythagorean/Platonic demiurge (Chua 1999, p. 17), from the *Utriusque cosmi ... historia* (1617–1626) of Robert Fludd (1574–1637)—an image which might be read as emblematic of “the inaudible *musica mundana* emitted through the invisible chord connecting man with his Creator”⁹ (Figure 1). Fludd as would-be Pythagorean *magus* was to become a *bête noire* for both Mersenne and Johannes Kepler (1571–1630), not least for deploying a theory and application of number derived from the neo-Platonism of Marsilio Ficino (1433–1499) that was associated with the practice of natural magic—on which account, Chua's 'disenchantment' thesis appears to find some measure of historical footing.¹⁰ However, as we will see, the very critique leveled at Fludd's number theory by Kepler in the *Harmonices mundi* (1619) attacked the identical principles that had been deployed by Vincenzo to arrive at his “true ratio” for the diapason or octave. For Kepler, therefore, it would have been Vincenzo's claims no less than those of Fludd that stood in need of a thorough *Entzauberung*.

9. Peter Hauge, “Introduction,” in Fludd [1618] 2011, p. 17. Hauge's description is actually applied to the musical automaton, the design of which Fludd details in the final book of his *Temple of Music*, but the description is possibly even more apt for the cosmic monochord. The span of the monochord is two perfect octaves, 4:1, or the “disdiapason” in the “quadrupla” proportion (Fludd has the names of the proportions marked on the left-hand side of the monochord, and the names of the corresponding intervals on the right.) The sun appears central in this scheme, with a “diapason” or octave on either side, but we are to ‘read’ the monochord from the bottom, where we find the earth at the center of the universe (with its elements suitably stacked above it, according to characteristic Aristotelian properties). Note the representation of a Platonic distinction between Being and Becoming, with the “*Diapason formalis*” corresponding to the former (following the *Timaeus* 37d, where the image of eternity is to be instantiated in the heavens, for the contemplation and guidance of rational minds located in the sublunary realm), and the “*Diapason materialis*” corresponding to the latter.

10. For an engaging introduction to the relationship between music and natural magic in Ficino, see Tomlinson 1993; Tomlinson wrote in part in response to the classic account developed by D. P. Walker cf. Walker [1958] 1985. Recent scholarship may be found in Prins 2015.

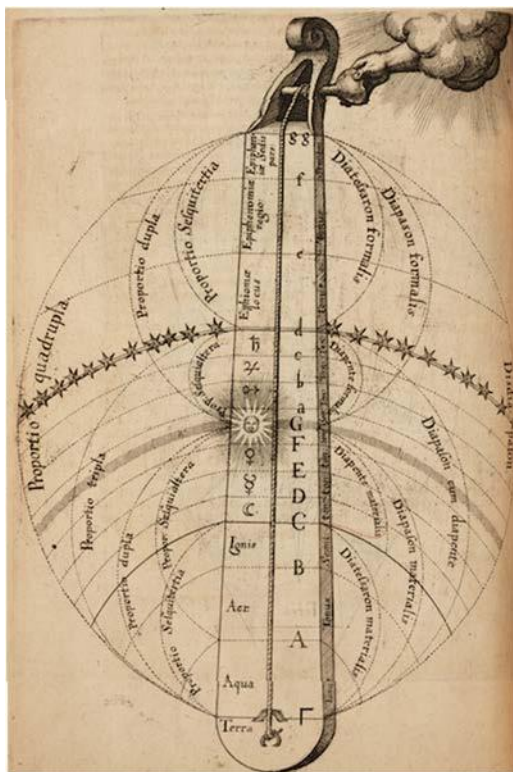


Figure 1. The 'cosmic' monochord. Robert Fludd, *Utriusque cosmi ... historia* (Frankfort, 1617–26). (Linda Hall Library of Science, Engineering & Technology)

2. Pythagorean Lore and Acoustic Interventions

The account of Vincenzo as the outright vanquisher of Pythagoreanism draws in turn upon the popular story of Pythagoras as recounted in Nicomachus' *Enchiridion*, a handbook of harmonics intended as a primer to the field for a non-specialist reader who might benefit from an evocative episode to illustrate a point of interest, without necessarily holding that episode to exacting standards of truth.¹¹ This is the fable of the 'harmonious blacksmiths' (Figure 2, upper left quadrant), in which Pythagoras plays the role of acoustic assayer, weighing the hammers of smiths whose

11. Nicomachus of Gerasa, *Enchiridion*, in Barker 1989, pp. 245–69. On the pedagogical character of the *Enchiridion*, see Creese 2010, p. 89.



Figure 2. Origin and demonstration of the Pythagorean ratios. Franchino Gaffurio, *Theorica musicae* (Milan, 1492).

blows upon the anvil rang out the sounds of the perfect consonances, to discover ratios of weight that could be reproduced in tension ratios of strings.¹² Nicomachus shared his fable in the second century CE: that is, approximately six centuries after the events he describes are conceived

12. Note Gaffurio's apologetic attribution (in the theological sense) of the initial discovery of the ratios of the consonances to Jubal ("the ancestor of all who handle the lyre and pipe," *Genesis* 4.21); Jubal was thus enlisted in music treatises beginning with Isidore of Seville. (Gaffurio cites Josephus, but Walter Kreyszig identifies his actual source as Peter Comestor, *Historia scholastica*, Paleografia Latina 198, col. 1079: see Gaffurio, *Theorica musicae*, I.8, 48.) Gaffurio also includes Philolaus among the *inventores musicae*, a fifth-century B.C. Pythagorean who was the first Western source to have left record of the ratios of the consonances (cf. Huffman 1993).

to have taken place.¹³ Gaffurio's illustration tends to be characterized by modern commentators as representing the fantastical nature and scientific pretensions of pre-modern music theory; and yet, though representing an ancient *scientia*, Gaffurio's illustration was nonetheless remarkably prescient. This image—of the philosopher or mathematician enthusiastically rubbing elbows with the artisan or tradesman—was to become emblematic of early modern science itself, as with Sagredo's nostalgia for his encounters at the Venetian Arsenal in the opening scene of Galileo's *Discorsi* of 1638. If Sagredo's recollection has enjoyed rather more credibility as an emblem of empirical science, Nicomachus' account was nonetheless concerned with a corresponding pursuit: namely, the attempt to find some means by which motion and its products could be empirically registered and mathematically represented.

Nicomachus refers to Pythagoras suspending strings from a ceiling rod with weights attached to their ends, as though the hammers had been transferred from the hands of smiths to the ends of strings. Ptolemy, in the *Harmonics*, imagined more the kind of device portrayed by Gaffurio (Figure 2, lower left quadrant), which in theory makes possible a more precise delimitation of the lengths of string, but which Ptolemy nonetheless rejected as wholly unreliable due to a wide variety of factors that he scrupulously anatomizes in the *Harmonics*, 1.8—sources of imprecision that, as we will see, appear to have been acutely diagnosed. So much for Pythagoras as the *saggiatore* of sound. What Ptolemy promoted instead was the use of the monochord, in view of the variety of ways in which the monochord could be calibrated and subjected to error correction, features he was unable to find in tension experiments.¹⁴

Palisca observed that the monochord—the device on which the ratios of the consonances were typically demonstrated—is not even included in Gaffurio's illustration, which suggests that Gaffurio's intent was to represent the transcendence of the ratios beyond the confines of music theory and its conventions of practice. We can thus read Gaffurio's illustration

13. For close analysis of the earliest extant Pythagorean sources and testimonia, see Huffman 1993, 2005. For overviews of ancient Pythagoreanism, see Burkert 1972, and Zhmud 2012. For a succinct overview of Pythagoreanism that extends into the early modern period, see Kahn 2001; for a scholarly compendium on Pythagoreanism from antiquity to the Renaissance, see Huffman 2014; and for an account of the broader cultural and philosophical impact of the figure of Pythagoras and the doctrines associated with him during the Renaissance, see Joost-Gaugier 2009.

14. The term *monochorda* was already used by Nicomachus, but without unambiguously referring to the device as it came to be recognized: cf. Creese 2010, p. 90, who notes that Nicomachus could be referring to the lute.

as emblematic of Galileo's *grandissimo libro* of mathematics, scored in a language of ratios that was common to each. As Peter Machamer recounts:

Galileo used a comparative, relativized geometry of ratios as the language of proof and mechanics, which was the language in which the book of nature was written. This is very different from what will follow in the eighteenth century and from the way we think of science today. In very few places in his work, and then mostly in talking about astronomical distances, does Galileo attempt to ascertain real values for any physical constant. Nowhere does Galileo attempt to find out, for example, what the real speed or weight of anything is. This proportional geometry is inherently comparative and relational, a matter of ratios. (Machamer 1998, p. 65)

The ratios are implicit in Gaffurio's illustration, encountered through the same series of numbers associated with different resonating bodies, whether strings attached with weights of varying size, pipes of different lengths but apparently uniform diameter, glasses filled with varying volumes or weights of water, bells of different size and weight, and the Pythagorean hammers of legend. The recurrent sequence of six numbers, $\langle 16, 12, 9, 8, 6, 4 \rangle$ ¹⁵ not only contains all of the Pythagorean ratios through twelve combinations of its terms,¹⁶ but are conveniently arrayed within the compass of the double octave ($16:4 = 4:1$), the largest of the traditional ratios, providing a framework within which all of the consonances could be intelligibly mapped, as though onto an actual instrument on which they might be reproduced.¹⁷

In conventional historical narrative, Vincenzo typically rejoins the story at this point as the figure who disrupted the hegemony of the Pythagorean ratios and the mathematical harmony they underwrote, by setting up something like the device portrayed by Gaffurio and disparaged by Ptolemy (to whom Vincenzo frequently referred both in published and unpublished writings, including letters and manuscripts). We are given no actual description of any such device or how it was used, but Vincenzo assured

15. The numerical sequence used throughout the illustration may be understood as an expansion of an important Pythagorean quaternary, the numbers 6, 8, 9 and 12—on one account, the initial tetractys given by Pythagoras to describe the ratios of the consonances (Iamblichus, *In Nicomachi arithmeticae introductionem*).

16. That is, for the consonances, $16:8$ or $12:6$ or $8:4 = 2:1$; $12:8$ or $9:6$ or $6:4 = 3:2$; $16:12$ or $12:9$ or $8:6 = 4:3$; $12:4 = 3:1$; $16:4 = 4:1$ (See Table 1, column 1.); for the dissonant interval of the tone, $9:8$. Another derivation, not found among the traditional Pythagorean ratios, is the controversial $8:3$ from $16:6$, the octave plus fourth or consonant eleventh: cf. Barbera 1984.

17. This double-octave span was likewise the framework for the principal ancient Greek tuning system, the Greater Perfect System, comprised of its four tetrachords, Hypatôn, Mesôn, Diezeugmenôn and Hyperbolaïon.

Table 1. Ratios of the Consonances*

	String Length	Frequency	Tension
<i>Perfect:</i>			
Octave	2:1	1:2	1:4
Fifth	3:2	2:3	4:9
Fourth	4:3	3:4	9:16
<i>Imperfect:</i>			
Major third	5:4	4:5	16:25
Minor third	6:5	5:6	25:36
Major sixth	5:3	3:5	9:25
Minor sixth	8:5	5:8	25:64

*The first term of each ratio refers to the lower pitch.

his readers that he had found the ratios for tension to be in fact the precise inverse squares of those for string length (Table 1, columns 1 and 3). If we have been reassured by some historians that Vincenzo was a pure empiricist—a man who got his numbers only by measuring the phenomena, and not by feigning vain numerological hypotheses—the very neatness of Vincenzo’s result, this perfect (inverse) squaring of Pythagorean ratios, remains curious; and, try though he might, Mersenne was in fact unable to replicate Vincenzo’s results, as he reported in the *Harmonie universelle*: rather than a ratio of 1:4 to produce the perfect octave, Mersenne recorded 1:4¼.¹⁸ But Vincenzo was not content to cease investigation into the cold case of the Pythagorean ratios with the inverse-square ratios: he proposed to deepen his forensic pursuit into features that, within the constraints of his medium, Gaffurio had represented with reasonable accuracy, namely, the dimensions of pipes needed to produce consonant musical intervals (Figure 2, lower right quadrant).

As we have already seen, in a late, unpublished manuscript (c.1589–1591), Vincenzo claimed that, in terms of pipe volume, the octave would be given by the ratio 8:1—that is, it would be given by the cubes of the terms which otherwise describe the ratio of the octave in string lengths, 2:1, and thus yet again, a strikingly neat exponential extension of the traditional ratios. On this account, however, Gaffurio’s illustration was

18. Marin Mersenne, *Harmonie universelle*, “Des instrumens,” III, Proposition VII, second rule. We will revisit Mersenne’s investigation of this and corresponding matters, likewise in connection with Ptolemy.

in fact more accurate in empirical terms, as Vincenzo himself had attested in the earlier *Dialogue on Ancient and Modern Music* (1581), in which Vincenzo had observed that diameters kept equal will indeed produce a clear enough octave when lengths are in the ratio 2:1.¹⁹ In general terms—with some notable exceptions, to be addressed in due course—pitch produced by forcing air through a pipe is principally determined by the length of the pipe, not its volume, and although width and pipe shape and many other factors can play an appreciable role—in particular, if vibration of the enclosed air column is initiated by a reed, as in a clarinet or oboe, and thus a “lingual” organ pipe²⁰—such influences are not described with any precision by Vincenzo’s cubic ratios. Palisca nonetheless suggested that Vincenzo’s precise cubic ratios corresponded near enough to established conventions of pipe scaling,²¹ and—most importantly—that they were furthermore verified by Mersenne. How well does such an account capture the claims or evidence provided by either Vincenzo or Mersenne?

3. The Music Black Box

The growth of interest in the scaling of organ pipes corresponded with the appreciable growth in range and means of pitch production of the organ itself from approximately the mid-fourteenth century onwards.²² When kept in a more limited range, distinctions in timbre between low and high pipes were sufficiently minimal, such that a simple, relatively uniform scaling could be applied: with pipe lengths approximately adjusted according to conventional string lengths for Pythagorean ratios, pipe width could be kept essentially uniform across ranks and with a straight bore.²³ However, boundary conditions that affect the behavior of air within

19. Vincenzo made many further observations regarding pipe dimensions in the *Dialogo*, to which we will return.

20. A basic distinction is observed between “lingual” and “labial” pipes: the former forces air past a reed (thus, “reed” pipes, for which the reed may be either fixed or free); the latter uses air flowing upwards from a narrow slot or flue in the pipe foot (thus, “flue” pipes) which encounters the upper lip, where a sheer effect creates eddies that initiate vibrations. Feedback stabilizes the initial stochastic behavior, establishing a longitudinal wave that sustains an audible pitch.

21. Cf. Palisca 1989: “Three Scientific Essays,” pp. 158–59, and “Diapason,” p. 187n10 and p. 191n14.

22. For general history of the structural developments of the organ, see Hopkins and Rimbault 1877/1965 (esp. pp. 1–160); Audsley 1905 (following an initial historical survey in vol. I, pp. 1–85, Audsley continually refers to notable English and continental historical organs to highlight different features of structure and design); and Andersen 1969.

23. For a critical edition, and erudite account of the character and development of collected Medieval organ treatises, see Sachs 1970, 1980. Sachs cautions that Medieval *mensura fistularum* treatises cannot be taken at face value as actual organ construction manuals, particularly when dealing with pipe length (where Pythagorean string length ratios tend to be assumed).

the pipe increasingly register as psychoacoustic phenomena when the range of pitch production—and, correspondingly, of pipe dimensions—is extended, phenomena that are typically addressed in modern terms according to the relative presence of partials: in general, a narrower width-to-length ratio tends to emphasize upper partials, producing what is often characterized as a more penetrating, ‘string-like’ sound quality; broader width tends to bring greater presence to the fundamental, registering as a more ‘flutey’ quality in timbre. Likewise, a pipe with one end covered—a “capped” pipe—tends to damp out the nodes that produce even partials, although these can be restored by adjusting wavelength (via a moveable cap or slits in the side of the pipe), or by applying a more conical bore, and so on. This plethora of timbral characteristics thus corresponded with a no less extraordinary variety of physical adjustments, which further include nicks made in the pipe; adjustments to the mouth dimensions of flue pipes and their “beards” (the fins on either side of the flue, geared to separating air currents outside the pipe from those vibrating within); the dimensions, elasticity, and placement of reeds, and so on.

The mysteries of craft that correspondingly accrued to the design and building of organs by the architects, and the art of summoning the “voice” of each of their pipes by their tuners, inculcated an aura of arcanum: in prefatory comments to his *Erweiterte und verbesserte Orgel-Probe* (Quedlinburg: Calvisi, 1698), Andreas Werckmeister (1645–1706) observed that his original treatise had drawn the ire of many in the builders’ community who shrank from having their work overseen by the prying eyes of others, an exclusivity that drew Werckmeister’s ire in return for its obstruction of the development of secure craft standards.²⁴ Although there was an ever-increasing interest in probing the alchemy of the organ—even directed at the state level, as Myles Jackson has shown with respect to early nineteenth-century Prussia (Jackson 2006, pp. 105–108)—the uneasy relationship between makers and experimental acousticians would persist, with the former often inclined to eschew the increasingly (and necessarily) complex algorithms developed by the latter, and the latter inclined to employ scientific instrument makers rather than organ makers themselves to scientifically refine a maker’s knowledge tradition.²⁵

Correspondingly, with respect to pipe widths, the apparent maintenance of uniform width in some treatises may simply be an artifact of the calculation of end correction for pipe lengths, to be discussed below in connection with Newton: assuming just one width provided the basis for a consistent factor for end correction.

24. Werckmeister, “Vorrede an den geneigten leser” (no pagination).

25. Jackson notes Wilhelm Weber’s use of precision scientific instrument makers Christian Hoffman (in Leipzig) and J. August D. Oertling (in Berlin)—rather than conventional craftsmen of the trade—for his ground-breaking research into the acoustics of reed pipes (Jackson 2006, pp. 118–20).

The use of the pipe organ or of its component parts as devices for early modern experience or experiment clearly presented a particularly daunting host of challenges, challenges that stood profoundly at odds with the highly disciplined experience of sound made possible by the monochord. Furthermore, these challenges were owing not simply to the considerable number of structural factors that contribute to the production of pitch, but to the complexity of the physical phenomena that were correspondingly engaged: as Sigalia Dostrovsky put it, “in the first half of the seventeenth century, only Mersenne was sufficiently bold (or naïve!) to work on the physics of pipes” (1975, p. 191). Dostrovsky’s perspective is well founded: it was Newton who first clearly identified an association between length of pipe and the concept of wavelength, in the first edition of the *Principia*: “it is likely [*verisimile*] that the breadth of the pulses [*latitudines pulsuum*], in the sounds of all open pipes, equals twice the length of the pipes” (Newton 1687, p. 372.) Though correct in principle, the association is empirically problematic and, in a first edition annotated by Newton himself (Cambridge University Library, Adv.b.39.1), these lines are struck out. Newton was perhaps wary of vain hypothesis here, for, as Dostrovsky further observes, “the dependence of frequency on length, simple in an idealized situation, is not exactly observable with real pipes because of the ambiguity of the boundary conditions” (1975, p. 191). Dostrovsky does not address such conditions here, but they concern above all the phenomenon of “end correction”: that is, the fact that the physical end-point of an open-ended organ pipe (such as Newton specifies) doesn’t precisely correspond to a pressure node; thus, such a node arises some distance outside the pipe, which will vary according to the ratio of width-to-length (amongst other factors). As a general consequence, pipes typically behave as though they are somewhat longer than their physical dimensions would indicate.²⁶

Insofar as the pneumatic physics of pipes was approached in terms of the physical dimensions of pipes themselves—the visible offering some means by which to grasp the invisible—and well in advance of any defined theory of wavelength, one can appreciate the boldness or naïveté of a Vincenzo or a Mersenne with particular immediacy simply by looking at the diversity of pipe lengths and dimensions that all produce the identical pitch (Figure 3).²⁷ Thus, although the organ was a most familiar instrument—particularly in view of its longstanding association with standard liturgical practice²⁸—organ pipes themselves nevertheless presented a kind of pneumatic black box through

26. For further discussion of Newton’s attempts to calculate wavelength in relation to pipe dimensions and the speed of sound, see Gouk 1999, p. 251.

27. The source is Andersen 1969, p. 352, figure 123.

28. Organs are reported as being introduced into churches as early as the tenth century.

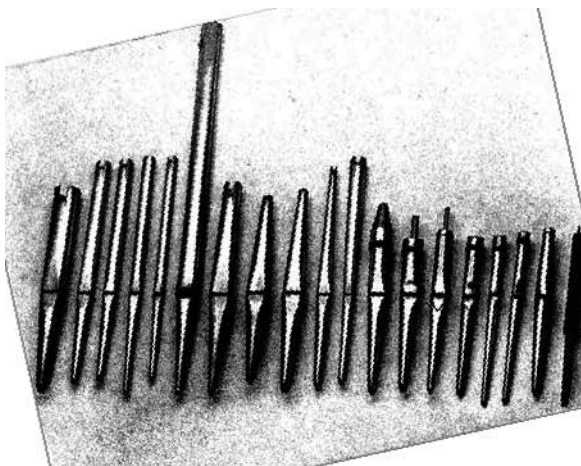


Figure 3. Unison organ pipes. Anderson, *Organ Building and Design*, Figure 123.

which pressurized air was prone to move in mysterious ways, and it is no surprise to find *le Père Mersenne*, O.M. (*ordo minimorum*) keen to divine their secrets. In this sense, the organ was an entirely different sort of experiential beast from the monochord, the workings of which were anything but occult or resistant to precise calibration, thus constituting one of its principal attractions for Mersenne no less than Ptolemy: as Carlos Urreiztieta observes, “for those who enjoy recreating instruments or experiments from the history of science and delight in denying or refuting the results that the scientist arrived at, this experience [i.e., performing actual divisions of the monochord] will defraud them. [By the] 17th century the monochord had more than twenty centuries confirming its truths” (Urreiztieta 2010, p. 82). Urreiztieta’s chronology assumes that Nicomachus’ *monochorda* indeed refers to such a device (Urreiztieta 2010, p. 78), but the essential point remains even if we question the precise meaning of Nicomachus’ reference. The monochord would enjoy several more centuries of use—Wilhelm Weber still employed it to determine the precise pitch produced by his experimental reed pipes (cf. Jackson 2006, p. 121)—and, as we will see, Mersenne appears to have been powerfully inclined to discipline the protean dimensions of organ pipes according to the model of the monochord.

Correspondingly, the organ and its components could prove revealing with respect to yet another black box of sorts: namely, the minds of those intrepid souls who sought to investigate them and define their acoustic properties. Indeed, a thoroughly rigorous experimental analysis of the occult acoustics of organ pipes—a properly Weberian *Entzauberung*—could arguably be said to have begun only in the early nineteenth century, with

the work of the great experimental physicist referred to above, Wilhelm Eduard Weber (1804–1891), who, along with his older brother, Ernst Heinrich Weber (1795–1878), felt that a genuinely scientific acoustics of the organ pipe was unknown before their generation.²⁹ Whatever the reservations of the Weber brothers, Mersenne's examination of the acoustics of organ pipes nonetheless offers a useful tool through which we might assess the corresponding endeavors of Vincenzo. If the requisite experimental, mathematical, and conceptual tools indeed lay so far in the future, what resources did Vincenzo and Mersenne nevertheless bring to investigating the acoustic properties of pipes?

4. Mersenne on Pipe Scaling

When Vincenzo first refers to a case in which pipe volumes are in cubed ratio, he has notably ensured that pipe length is still kept in duple ratio—that is, he reports a situation in which the interval of an octave will likely be produced, according to standard Pythagorean ratios for length:

[in] duple [proportion] also [that is, referring to the conventional ratio of the octave, 2:1] will be the sound of cubic bodies, for example, two organ pipes that are duple in depth, width and height, the volume of which will be octuple.³⁰ And this is the third manner of hearing the octave.³¹

Had Vincenzo been content to stop here, Palisca's line of interpretation—namely that Vincenzo's cubic ratios reflected general scaling practices for organ pipes—remains plausible enough. Along similar interpretive lines, Palisca asserted in his final commentary on the subject that Mersenne's investigation of organ-building practices straightforwardly confirmed Galilei's ratios (Palisca 2006, p. 157): in fact, this only held true where Mersenne likewise insisted upon retaining both traditional Pythagorean ratios of string length for the perfect consonances in corresponding pipe lengths, as well as the simple, whole-number ratios for imperfect consonances that became a staple of Renaissance theory and practice, such as were promoted by prominent theorists like Bartolomé Ramos da Pareja (c.1440–1522) and

29. Cf. Jackson 2006, p. 308n10. Jackson's account of Weber's experimental and theoretical investigation of organ pipes and wave phenomena is taken up in Chapter 5, "Wilhelm Weber, Reed Pipes, and Adiabatic Phenomena" (pp. 111–50).

30. Palisca suggests that Vincenzo had already implicitly reported these dimension in the *Dialogo*, but without directly referring to volumetric ratios: in the *Dialogo*, Vincenzo referred only to cross-sectional areas of the pipes, and thus to squared rather than cubed ratios. Palisca's claim depends upon assuming that Vincenzo likewise adjusts for pipe length without actually stating so, a claim that careful textual analysis does not support (see below).

31. Vincenzo, "Discourse Concerning the Diapason," p. 187.

Gioseffo Zarlino.³² (See Table 1). However, Vincenzo did not in fact restrict himself to such parameters—namely, to retaining traditional ratios of length—in assembling evidence on behalf of his cubic ratios; thus, we need to read Mersenne carefully to properly assess the precise nature of his ostensible confirmation of Vincenzo's claims.

With its relatively close examination of makers' knowledge traditions for the production of musical instruments, Mersenne's *Harmonie universelle* (French edition, 1636; Latin edition, *Harmonicorum libri XII*, 1648),³³ established an early French model of Baconian natural history that was to be taken up with renewed vigor over a century later in the *Encyclopédie* of Diderot and D'Alembert.³⁴ This robust orientation toward a practical empiricism nonetheless rested on key theological, metaphysical, and epistemological premises that no less strongly shaped Mersenne's interpretation of his investigations. In particular, Augustine's profound influence on the *Harmonie universelle*—as well as on Mersenne's particular brand of Platonism or neo-Pythagoreanism³⁵—became well established in English-language

32. *Harmonie universelle*, III, 6, Proposition XIV, p. 334. The "perfect" consonances are the originary, Pythagorean consonances, which have just one fundamental form; the "imperfect" became a part of standard practice during the Renaissance (although they emerged earlier both in theory and in use) and have two basic forms, the larger or "major" consonance and the smaller or "minor," where the former is a semitone larger than the latter: thus, there can be a "major third" or "minor third," but no "perfect third." Dissonances are likewise conventionally characterized in such terms (that is, as imperfect intervals, major or minor).

33. The remarks on Mersenne that follow will rely on the edition in closest temporal proximity to the research of Vincenzo (ie., the *Harmonie universelle*) rather than the Latin edition. I do not know of any extant critical survey of correspondences between these editions.

34. Peter Dear situates this practical orientation in terms of Mersenne's response to Pyrrhonism: though Mersenne understood even mathematical *scientiae* such as optics and music to be vulnerable to skeptical uncertainty due to their assumptions about the nature of light and sound, they were compensated through their engagement with an operational science of appearances. Dear quotes Mersenne, *La vérité des sciences contre les sceptiques ou Pyrrhoniens* (Paris 1625): "It is certain that the artisan must have an idea in order to effect his work, or he would never succeed in it" (Dear 1988, p. 42); and, more aphoristically, *L'usage de la raison* (Paris 1623): "the visible work assures us of the invisible" (Dear 1988, p. 43). As we will see, the practical nature of mathematics was not limited to the mechanical arts, and Mersenne would find music and theology mutually supportive ventures in this respect. For further helpful surveys of Mersenne's musical science, see Dostrovsky 1975, pp. 182–88, 191–96; Cohen 1984, pp. 97–114; Gouk 1999, pp. 170–78; and Pesic 2014, pp. 103–20.

35. We need not take these terms as wholly interchangeable to recognize their often-intimate historical association, such as with the neo-Platonists of late antiquity who tended to view Plato's teachings as an attenuated Pythagorean philosophical tradition awaiting rejuvenation, particularly with respect to the central epistemological and ontological role of mathematics (cf. O'Meara 1989). Mersenne's Platonism is transparent in his characterization of the Good as the source not just of all virtue and but of all intelligibility (*Republic* VII) and to God as divine geometer (the *Timaeus*), but this is as much a reflection of Augustine's Platonism (including its limitations), evidently his key source (see below).

scholarship in the late 1980's through the research of Peter Dear; however, it remains a signal scholarly contribution that has gone noticeably unremarked in accounts of Vincenzo and the reception of his work.³⁶ Dear's scholarship provides an important perspective on the idiosyncrasies of both Vincenzo and Mersenne's engagement with the acoustic mysteries of pipes and pipe organs.

As the leading experimental investigator of acoustic phenomena in the early seventeenth century, Mersenne was more prepared than most to attempt disentangling the complex variables of pitch production by strings and by pipes. With respect to the latter, as Dostrovsky observed, Mersenne "tried hard to find laws describing the connections between the pitches of pipes and their various properties—length, width, shape, material, blowing pressure. [...] The great variety of pipes in use in organs (especially convenient because each pitch has its own pipe or set of pipes) provided much material for observation" (1975, p. 191). Indeed, organ pipes presented almost too much observational material given the complex interaction of these variables, as well as the production of empirical curiosities such as the creation of partials by overblowing, or the dramatic effect on pitch produced by capping pipes—a common experiential phenomenon that will prove to be of particular consequence.

It was in briefly surveying this lengthy and complex section of the *Harmonie universelle* that Palisca wrote, "Mersenne *also* confirmed Galilei's assertion that the ratio of cubic volume of organ pipes should vary as 8:1 for the octave" (Palisca 2006, p. 157). The first part of Palisca's claim is already problematic, given that "also confirmed" refers to the inverse-square tension ratios that, on his own account, Mersenne could not replicate: the prior 'confirmation' in fact required a further, strictly non-empirical correction, to be addressed later in this discussion. The second part of Palisca's assertion refers to a table produced on p. 335, Proposition XIV ("Table³⁷ de la longueur, & de la solidité des tuyaux"—see Figure 4), in which the cubic ratios are listed for all the consonances,³⁸ as well as the various sizes of tone, a diesis, and a syntonic comma (one of the more common incommensurabilities that arise in tuning

36. Dear is careful to note significant distinctions in neo-Platonic allegiances, observing Mersenne's distaste for the variety associated with Robert Fludd, a distaste shared by Kepler: see Dear 1988, pp. 109–16, 226. Dear's erudite analysis of the Augustinian roots of the *Harmonie universelle* is nowhere addressed in Palisca's account of Mersenne in *Music and Ideas*, nor does it appear in the bibliography.

37. Palisca 2006 cites this as "tablature."

38. Here there is either a misreading or a misprint: Palisca 2006 has 17:8 for the justly-tuned perfect fifth, whereas Mersenne provides the correct ratio, 27:8—that is, $(3^3):(2)^3$. (In the typeface used by Mersenne's printer, the imprint of "2" is not always readily distinguished from "1").

Table de la longueur, & de la solidité des tuyaux.		Il est aysé de trouuer les bates, & les surfaces des mesmes tuyaux, puis qu'elles sont moyennes proportionnelles entre les simples termes des intervalles & leurs cu- bes, c'est à dire qu'elles sont les quarez. Aquoy i'adiouste le Diapason des Orgues en faueur des Facteurs, afin qu'il n'y ayt nul artisan qui ne puisse aysément com- prendre ce liure. Or le commence par C <i>sol ut fa</i> pour m'accommoder à leur façon, & à leur cou- stume; autrement on peut le commencer par F <i>ut fa</i> , ou par telle autre lettre que l'on voudra. La ligne entiere a β re- ff ij
I. Longueurs.	II. Cubes.	
Octaue de 2 à 1.	8 à 1.	
Quinte de 3 à 2.	27 à 8.	
Quarte de 4 à 3.	64 à 27.	
Tierce majeure de 5 à 4.	125 à 64.	
Tierce mineure de 6 à 5.	216 à 125.	
Sexte majeure de 5 à 3.	125 à 27.	
Sexte mineure de 6 à 5.	512 à 125.	
Septiesme majeure de 15 à 8.	3375 à 512.	
Septiesme mineure de 9 à 5.	729 à 125.	
Douzième de 3 à 1.	27 à 1.	
Quinzième de 4 à 1.	64 à 1.	
Ton majeur de 9 à 8.	729 à 512.	
Ton mineur de 10 à 9.	1000 à 729.	
Demiton majeur de 16 à 15.	4096 à 3375.	
Demiton mineur de 25 à 24.	15625 à 13824.	
Demiton moyen de 135 à 128.	2460375 à 2057152.	
Dièse de 128 à 125.	2097152 à 1953125.	
Comma de 81 à 80.	531441 à 512000.	

Figure 4. Mersenne's cubic pipe ratios. Marin Mersenne, *Harmonie universelle* (Paris, 1636–37), "Livres des orgues."

sequences, represented by the ratio 81:80, and corresponding with an appreciably—and, to a musician's ears, grossly—mistuned unison). The title for the table is instructive: it specifies just one of the dimensions that will generate those volumes, giving traditional whole-number ratios for length, before translating these into the concomitant cubic ratios for volume. Although no mention is made of Vincenzo's unpublished findings in this section, it would appear at first sight that Mersenne does indeed testify to an experiential replication in some sense of Vincenzo's pipe dimensions. How closely, then, does this crucial framework established by pipe length correspond with the fuller range of claims made by Vincenzo in his "Discourse on the Diapason"?

Despite initial—and, in certain respects, highly significant—similarities with Mersenne's findings, Vincenzo nonetheless ventured some claims that resemble less pipe experiment than pipe dream, and certainly nothing at all that corresponded with conventions of pipe scaling:

What sort of interval would two pipes make that have the same diameter but duple length? A major third of the intense

tuning of Aristoxenus, which, in fact, is [only] the third *part* of an octave.³⁹

Palisca himself noted that, “depending on the proportions of diameter to length, a pipe [that is] twice another in length will either sound the octave or not sound at all, but it will not sound a third” (Palisca 1989, p. 191n14). On Palisca’s reading, Vincenzo simply “made a slip” when referring to the interval produced by two pipes that have the same diameter but duple length, where Vincenzo should instead have written “the same *length* but duple *diameter*” (1992, p. 151n19), a claim somewhat “closer to the truth” (Palisca 1989, p. 191n14) since it approximates a result of a minor third claimed by Mersenne in Proposition XII of the *Livres des Orgues* (rather than Vincenzo’s major third).⁴⁰ In this proposition, Mersenne examined the feasibility of building a functional organ that kept all pipes fixed at the same length, and varied only pipe diameters to produce distinctions in pitch.⁴¹ He begins the proposition with a framing hypothesis: namely, to find out whether one need only double the width of a pipe, keeping length constant, to produce the same effect in pitch production as doubling the length—that is, for the pitch “to descend an octave like the pipe which is double in length” such “as several believe.”⁴² Mersenne does not identify any of those “several” to whom Proposition XII is addressed in critical response, but we will soon recognize one of the more likely (and most prominent) candidates. Successively doubling the diameter of each pipe, after five pipes Mersenne had arrived at a volume ratio of approximately 256:1—several orders of magnitude beyond Vincenzo’s 8:1 ratio—and was still unable to reach an octave, a minimal requirement for any functional organ (to say the least).⁴³ Based on the results he was able

39. Vincenzo, “Diapason,” pp. 189–91 [emphasis added].

40. Mersenne, *Harmonie universelle*, VI, proposition XII. Mersenne nonetheless reports notable empirical inconsistencies in this finding to which we will return, including the possibility that a major third such as Vincenzo claimed could indeed be produced, “by blowing more forcefully into one than into the other” (“en soufflant plus fort dans l’un que dans l’autre,” p. 332). All translations of this source are my own.

41. “Determiner si l’on peut fair un Orgue qui ayt tous ses tuyaux de memes hauteur, c’est à dire si la seule difference de leurs largeurs peut fair l’estenduë des quatre Octaues qui sont ordinairement sur l’Orgue; & monstrier en quelle raison doiuent estres leurs largeurs pour faire tels sons, & tels interualles que l’on voudra” (*Harmonie universelle*, III, Book 6, p. 331); “to determine if one can make an organ with all its pipes of the same height, that is to say, as if the difference of their widths alone can produce the four-octave range customarily found on organs; and to show in which ratio their widths must be to produce such sounds and such intervals as one would wish.”

42. “plusieurs croyent que si l’on fait un tuyau deux fois plus large, qu’il descendra aussi bas que lors qu’il est deux fois aussi long” (*Harmonie universelle*, p. 331).

43. The pipes at either end of the extreme could barely produce pitch, although Mersenne notes that if the air pressure is reduced in the highest pipe—which can already “barely speak” (“il ne peut quasi parler”)—an octave may be produced (*Harmonie universelle*, p. 332).

to secure, Mersenne calculated that to arrive at the octave, a further pipe would have to be constructed with a diameter of half a foot—that is, matching the length of the pipe itself—and with a volume ratio to the smallest pipe of approximately 575:1. Mersenne concludes: “It is therefore certain that the breadth of pipes [alone] cannot compensate for their lengths, since it is not quite possible to descend or rise up to the octave with pipes of the same height, as experiences teach us.”⁴⁴

Given these results, it had been empirically demonstrated that pipe length played a crucial role in pitch production that could not be accommodated for by pipe width, but to what extent? As with his investigation of pipe width, Mersenne was to find a variety of results in his exploration of pipe length that continued to frustrate any ambition to achieve a clear or straightforward result; nonetheless, the variety of results with respect to pipe length differed in a key way from those for pipe width. Mersenne frames Proposition XIII in the same way that he set up Proposition XII (albeit—unlike the varied widths proposition—not as a critical response to any claim made by other authors), asking whether it would be possible to create all the necessary pitches for an organ simply by varying pipe length without changing pipe width; and, though he comes up with a correspondingly negative result, it is by no means negative to the same extent: where the constraints of Proposition XII produced a pitch range limited to less than an octave, those of Proposition XIII could produce an organ capable of a satisfactory range of “two or three octaves” (*Harmonie universelle*, Prop. XIII, p. 333), such as one would have commonly encountered in organs before their notable expansion in range beginning in the fourteenth century. Pipe length therefore had a demonstrably more incisive role in pitch production than pipe width, but Mersenne was unable to neatly tie that role to simple whole-number ratios for string length on a purely experiential basis—at least, not yet: “one must still consult experience, to observe that the sounds [pitches] do not follow the ratio of the length in pipes of the same thickness [*grosseur*], although nearly so, particularly in the little pipes.”⁴⁵ When working with pipes of greater (but still identical) breadth “which speak better,” Mersenne observes that, “ordinarily, it is by

44. “Il est donc certain que la largeur des tuyaux ne peut recompenser leurs longueurs, puis qu’il n’est pas quasi possible de descendre, ou monter iusqu’à l’octave avec des tuyaux de mesme hauteur, comme enseignent les experiences” (*Harmonie universelle*, p. 332).

45. “il faut encore icy consulter l’experience, afin de remarquer que les sons ne suivent pas aussi la raison de la longueur des tuyaux de mesme grosseur, quoy qu’il s’en faille peu, particulièrement aux petits tuyaux” (*Harmonie universelle*, Prop. XIII, p. 333). Mersenne evidently uses terms like “grosseur” or “largeur” to refer to pipe width, such as one might when referring to the thickness of a string; when clearly referring to pipe volume, as in Proposition XIV, he uses “solidité.”

nearly a semitone or close to a tone that the pipe double in length fails to reach the octave."⁴⁶

What clearly emerges from propositions XII and XIII, therefore, is just how unlike the monochord the organ pipe was inclined to behave; and yet, no less significant was the close *proximity* in pitch production achieved between pipe and string with respect to ratios of length. In this sense, the organ pipe offered a kind of shadow counterpart to the clarity of monochord demonstration, producing experiential phenomena that hovered tantalizingly just beyond the range of clear mathematical focus, like the evasive contours of a figure painted by Leonardo. It is important to recognize Mersenne's awareness of the mathematical shadow realm into which he was delving, and which he characterized in light of Ptolemy's objections both to ratios of tension (as noted earlier in this discussion) and to ratios of pipe length, as represented by the dimensions of flutes:

If we follow Ptolemy's opinion, who maintains in chapter 8 of his first book [that is, of the *Harmonics*] that flutes, and weights suspended from the bottom of strings, are not sufficiently precise to establish the ratios of the consonances, we will find only the monochord to be suitable for that purpose. Now, Ptolemy's reasons are that it is difficult to make flutes sufficiently consistent in structure [*iustes*], and to moderate and direct wind as necessary, which being pushed more forcefully or more feebly makes different sounds [itches], because the same pipe rises a quarter-tone or semi-tone when one pushes the air more forcefully, and if one increases it further it rises an octave or twelfth [that is, a perfect fifth plus octave].⁴⁷ [...] We must therefore conclude with Ptolemy that the monochord is the most appropriate and most precise instrument for regulating pitches and harmony.⁴⁸

46. "Mais, ayant fait l'experience en de plus grostuyaux [*sic*] qui parlent mieux, i'ay remarqué qu'ils s'en faut ordinairement vn demy-ton, ou pres d'un ton, que le tuyaux double en longueur ne face l'Octaue" (*Harmonie universelle*, Prop. XIII, p. 333).

47. Mersenne refers here to the phenomenon of "over-blowing," in which the pipe loses its fundamental and foregrounds a higher partial, such as the second and third partials (1:2 and 1:3) cited by Mersenne.

48. "Si nous suiuous l'auis de Ptolomée, qui maintient au 8. Chapitre de son I [première] liure que les Flustes, & les poids qui sont pendus au bout des chordes, ne sont pas assez iustes pour establir la raison des consonances, nous treuuerons que le seul Monochorde est propre à cela. Or les raisons de Ptolomée sont, qu'il est difficile de faire les Flustes assez iustes, & de moderer, & gouuerner le vent comme il faut, lequel estant poussé plus fort ou plus foiblement fait des sons differents, car le mesme tuyau monte d'un quart, ou d'un demi-ton, quand on pousse le vent vn peu plus fort, & si on l'augmente dauantage, il monte d'une octaue, ou d'un douzième, comme i'ay dit ailleurs. [...] Il faut donc conclure avec Ptolomée que le Monochorde est l'Instrument le plus propre & le plus exact pour regler les sons & l'harmonie" (*Harmonie universelle*, "Liure premier des instrumens," Proposition IIII [*sic*], pp. 14–5).

Thus, while Mersenne could indeed share in Ptolemy's conviction that the monochord represented "the most appropriate and most precise instrument" for the *scientia* of harmonics—although Mersenne immediately secures himself some measure of critical distance with his prefatory reference to Ptolemy's "*avis*" or "opinion"—the practical world of instrument construction, no less a contributor to *l'harmonie universelle*, correspondingly demanded a pursuit of mathematical harmonies that lurked just beyond the turbulent mirror of experience.

Well into the seventeenth century, then, the pipe organ maintained an umbilical link with *musica speculativa* through the monochord, integrating metaphysical and aesthetic concerns such as Klaus-Jürgen Sachs observed of the *musica fistularum* textual tradition (cf. n23, v.s.). In view of its widely attested clarity of demonstration, Carlos Calderón Urreiztieta asserts that "the monochord as scientific instrument is the only place where this scientific-aesthetical practice could be experienced in both rational and empirical ways" (Urreiztieta 2010, p. 81); however, if organ pipes presented a comparative challenge to the precision expected of (or desired for) a mixed-mathematical *scientia* involving music,⁴⁹ aesthetic considerations were no less constitutive of that scientific challenge, if less baldly characterized—that is, in terms of a simple, categorical distinction between consonance and dissonance. Like all architects of organ design and tuners of organ pipes, Mersenne was as attentive to distinctions in the quality of sound produced by a pipe that contributed to any consonant interval relationship, as to the constituent pitches of the consonance itself: indeed, the extensive cultivation of a craft tradition in ratios between length and width of a single pipe emerged largely in response to concerns about sound quality, whether as matters of type (the "flute"-like versus the "string"-like) or of acceptability (avoiding a shrill or harsh tone). Therefore, well before an "an-aesthetic" science (as Urreiztieta characterizes it [2010, pp. 96, 2016, pp. 109–20]) of partials became possible through the efforts of Joseph Sauveur (1653–1716), it was a scientific-aesthetical approach that drove the investigation of the timbre of organ pipes, and in terms of their physical dimensions. It might therefore be suggested that Mersenne's study of organ pipes represents a scientific-aesthetical project par excellence.

49. Cf. Dear 1988, pp. 62–4, 1995, p. 39. Associated with this term—likewise Aristotelian in origin—are the sciences deemed "subordinate," for which Dear provides Mersenne's account from the *Traité de l'harmonie universelle* (Paris 1627): "if they have a common (*mesme*) formal object, they will be *subordinate* and *subordinating*, although their material objects be different; this happens with mathematics, as when optics and music make use of demonstrations from geometry and arithmetic, for when its demonstrations are joined to sensible matter, it doesn't lose its formal cause (*raison*)" (Dear 1988, p. 64). The particular issue at stake in Aristotelian harmonics is the use of number as formal cause (cf. n7 v.s.).

In view of the decidedly mixed experiential messages conveyed by the interaction of pipe widths and pipe lengths, and the variety of empirical considerations brought forth by Propositions XII and XIII, what can we now make of the summary table and chart of the cubic relationships presented in Proposition XIV?⁵⁰ Mersenne begins the proposition with a brief, wholly general reminder of the variety of options that have been entertained for the tuning of pipes, and of the variables that must therefore be reconciled in any successful tuning:

Since experience has enabled us to see that pipes should be of different lengths and widths, to produce all the sounds of the organ, these two dimensions must be joined together in order to have sounds that are proportional in terms of their pitch, sweetness and harmony. Let us now see what ratio the lengths and widths should have [*doient auoir*].⁵¹

Given the fact that Mersenne had already demonstrated the possibility of mutually compensatory effects between width and length—while yielding no definitive specification in either dimension—Mersenne was in no position to define the only possible solution; thus, his insistence on scaling in cubic ratios—“*doient auoir*”—is plainly incongruous. Moreover, since absolute values for length and width—with absolute values playing a particularly distinctive role in Mersenne’s science, the comparative absence of which he criticized in Galileo’s—had been shown to play an appreciable role in pitch production (even if their particular role remained unknown),⁵² a single solution to be applied across the broader spectrum of

50. Proposition XIV is entitled: “Expliquer la raison que les tuyaux doient avoir entre leurs longueurs, & leurs largeurs pour faire tous les degrez d’un ou plusieurs Octaves: & donner vn diapason tres-iuste” (“To explain the ratio that pipes should have between their lengths, and their widths to produce all the [scale] degrees of one or several octaves: and to provide a very precise [*tres-iuste*] diapason [that is, a perfect octave in just intonation]”—*Harmonie universelle*, III, 6, Proposition XIV, p. 334).

51. “Puis que l’experience nous a fait voir que les tuyaux doient estre de differentes longueurs & grosseurs, pour faire tous les sons de l’Orgue, il faut ioindre ces deux dimensions ensemble, afin d’auoir des sons qui soien proportionnez tant en leur aigu, qu’en leur douceur, & leur harmonie. Voyons donc maintenant quelle raison les longueurs, & les largeurs doient auoir” (*Harmonie universelle*, III, 6, Proposition XIV, pp. 334–35).

52. One of the significant factors left unspecified in Mersenne’s pipe propositions XII and XIII was the means by which vibration was initiated, which can play a great role in determining pitch at a certain scale. For example, when reed pipes are shorter, the frequency of the transverse vibrations of the reed itself has a more pronounced impact on the frequency of longitudinal vibration in the enclosed air that ultimately produces the pitch of the pipe. This was among the many factors addressed by Wilhelm Weber in his pathbreaking investigation of the acoustic properties of reed pipes (cf. Jackson 2006). Irrespective of the effects of scale, Mersenne had observed in Proposition VIII that adjustments to the wire spring or *rasette* holding the reed in place—which would change the functional vibrating length of the reed—correspondingly

scaling offered conveniently practical guidelines, *faute de mieux*, rather than any experiential or experimental confirmation of cubic ratios.

Even then, if we consider professional scaling practices detailed in a standard organ construction manual from the early twentieth century, G. A. Audsley's *Art of Organ Building* (1905), we find that, for wood pipes of substantial dimension (16 feet and 8 feet), the 2:1 ratio in terms of pipe lengths is retained for the octave, while the volumes ratio approximates to 5.32:1—not even remotely close to the numerical purity of the cubic ratio of 8:1 (cf. Audsley 1905, vol. 2, p. 471).⁵³ Thus, to frame his recommendations, Mersenne began by assuming that the ratio of pipe lengths was to be kept in the conventional Pythagorean or whole-number ratios—still confirmed as practical by Audsley nearly three centuries later, although many organ builders find that shortened pipes are fine; but, to ensure that both the tuning and comparative timbre of intervals would be acceptable across the full range of pipe ranks, Mersenne asserted that cubic proportions in volume should be maintained by the corresponding widths—a recommendation not confirmed by Audsley. So, why did Mersenne then insist upon such a precise cubic extension from ratios for length, as if organ pipes were—contrary to experience—‘cubic’ monochords?

5. Galileo, Acoustic Assayer

If Mersenne unwittingly recreated one aspect of Vincenzo's findings—namely, that cubic ratios would suit pipes, so long as traditional whole-number ratios for string length likewise remained the general framework for pipe length—this recreation indeed appears to have been driven by what Peter Dear suggested some decades ago: “for Mersenne, the criterion for a successful mechanical interpretation of music was that it incorporate the simple numerical Pythagorean ratios to express musical consonances” (Dear 1995, p. 141)—ratios whose terms Mersenne expanded exponentially to encompass pipe volume, but for which he had also ‘corrected’ his empirical result for the octave tension ratio from $1:4\frac{1}{4}$ to 1:4, a strikingly neat correction evidently applied by Vincenzo as well.⁵⁴ Mersenne's Pythagoreanism enjoyed a pedigree that he characterized in stolidly patristic terms: speaking

adjusted the pitch of the pipe without recourse to changing the pipe's length or width, a theme to which Mersenne returned in Proposition XI when exploring further techniques employed by tuners (and as prelude to the propositions concerned with variable width and length).

53. The scaling dimensions were provided in Audsley by Edmund Schulze; I refer here to the same example used by Palisca 1989 in his note on Vincenzo's “Discourse Concerning the Diapason” (p.187n10).

54. Newton was likewise inspired by the pure inverse squares ratios, finding in them the clues to a proper interpretation of Pythagorean fable in terms of gravitational attraction (cf. McGuire and Rattansi 1966; Cassini 1984; Gouk 1999, pp. 252–54). Thus, to Gouk's inquiry—“Newton,

of the theological practicality of mathematics in *La vérité des sciences* (Paris, 1625), Mersenne's Christian Philosopher advocated its use "to understand holy scripture [...] and the Church Fathers, particularly those who explain their conceptions by Pythagorean numbers" (quoted in Dear 1988, p. 46). Foremost among those Fathers stood Augustine himself, who rhapsodized at length about the salvific relationship between Christ and humankind in terms of the *harmonian* or ratio 2:1 (*De trinitate* 4.2); thus, it is no surprise to find Augustine intimately associated with the larger ambitions of the *Harmonie universelle*, an association that Dear addressed at length in *Mersenne and the Learning of the Schools*.⁵⁵ Summarizing a rich scholarly assessment both of Mersenne's own principal texts and of his theological and intellectual milieu, Dear observed:

Mersenne could introduce Augustine's stress on number as the characteristic exemplification of God's wisdom manifested in the Creation and reflected in our own minds. God had made the world in accordance with mathematical ratios and proportions, and man could know it by his participation, through divine illumination, in God's wisdom thus expressed. "Universal harmony" existed, and the mathematical sciences served to display it. (Dear 1988, p. 226)

On Dear's account, therefore, Augustine provided Mersenne with a significant conduit of Pythagorean and Platonic traditions concerning the relationship between number, ratio, and the fundamental order of the universe, a source untainted by the countercultural ambience of a Ficino or a Fludd (cf. Dear 1988, p. 82).

This Augustinian framework notwithstanding, if Mersenne may be said to have confirmed any particular aspect of Vincenzo's cubic pipe ratios—or, indeed, of the inverse-square tension ratios—it would appear to be the Pythagorean conceptual framework within which they were evidently conceived, as precise exponential expansions of both the traditional ratios of the perfect consonances and more recent, kindred ratios for the imperfect consonances. Thus, where Cohen (2010) draws a neat distinction between the squared and cubed ratios as, respectively, empirical and "all-too-fanciful" data,⁵⁶

Pythagorean *magus*?"—one might respond that, in this particular respect, he was as much a Pythagorean enthusiast as Mersenne.

55. Cf. "Saint Augustine and Universal Harmony" in particular (Dear 1988, pp. 80–116).

56. "In terms of string tensions [...] the intervals are in a *squared* proportion to the weights; hence, what ratios appear depends on what one decides to measure, and the simplicity of both Pythagoras' *tetraktys* and Zarlino's *senario* dissolves. [...] It is true that, in a vain hope to make his big find symmetrical, Vincenzo spoiled it by pronouncing an insufficiently checked, all-too-fanciful, *cubed* proportion for the volumes of organ pipes" (Cohen 2010, pp. 145–46).

Mersenne's 'correction' of the former and promotion of the latter demonstrate instead a pronounced Pythagorean consistency between the two sets of ratios, with neither set more empirical or fanciful than the other. But before we examine Vincenzo's writings on pipe dimensions more closely, or address whether Mersenne's Pythagorean orientation may indeed be attributed to Vincenzo in quite the same terms, we ought to first consider the response of someone who, unlike Mersenne, would have been highly likely to have known just what claims were made in Vincenzo's unpublished manuscripts, and who could have therefore been consciously motivated to replicate his results, a person who would have been Vincenzo's inheritor in more than one sense: his son, Galileo (1564–1642).

In the *Discourses on Two New Sciences* (1638), Galileo turned to the diversity of ratios of the consonances uncovered by his father, but with one notable omission: as Sagredo recounts,

I had long been perplexed about the forms of the consonances [...]. I saw no reason why wise philosophers should have established the form of the octave as the double [ratio—that is, in terms of string length, 2:1] any more than as the quadruple [that is, in terms of string tension, 1:4]. (Galilei [1633] 1974, pp. 100–02, *passim*)

Galileo fails to make any reference whatsoever to his father's treasured cubic ratio for the octave, even though it clearly would have served the rhetorical purposes of the passage: Sagredo is keen on highlighting how a properly constituted science of motion can harmonize a seeming cacophony of mathematical representations, for which purpose the interrelationships of length, tension, diameter, and density could all be rationalized as components of a single, integrated physical phenomenon—namely, of the motion of strings. Thus, Sagredo continues, "I say that the length of strings is not the direct and immediate reason behind the forms of musical intervals, nor is their tension, nor their thickness, but rather, the ratio of the number of vibrations" (Galilei [1633] 1974, p. 104). One could anticipate a similar prospect at some future date for the motion of structurally constrained columns of air, in terms of length, volume, bore, and the associated phenomena of air pressure and temperature within a pipe; regardless, cubic ratios would have offered yet another dissonant mathematical representation acknowledged by "wise philosophers," to be resolved by a theory of relative rates of motion, a solution similarly pursued by contemporaries such as Giambattista Benedetti (1530–1590), Mersenne, and Isaac Beeckman (1588–1637).⁵⁷

57. On these developments in general, see Cohen 1984 (especially pp. 94–7).

Sagredo's omission of the cubic ratio for the octave does not appear to be a matter of casual oversight. In *The Assayer*, where the optics of the telescope were placed under close scrutiny in relation to the comet controversies of 1618, Galileo pursued an analogy between the effects of length in the optical tube with the length of organ pipes. In section XV, when addressing the relationship between the apparent magnitude of heavenly bodies and telescope length, Galileo invokes analogy with organ pipes, for which he straightforwardly assumes that pitch is determined by functional length. Moving from conventional organ pipes of fixed length, Galileo imagines telescoping pipes of moveable, enjoined sections such that they may produce variable pitch without any corresponding or requisite change in diameter. On what basis does he assume this? Galileo applies further analogy: this is precisely how trombones work, an instrument well known to experience (Galileo [1623] 1977, pp. 90–1). Thus, Galileo found no need whatsoever to entertain cubic ratios for volume, any more than a trombone player would need to rapidly switch between multiple diameters or bores of instrument to play a single scale; and thus, whatever his father might have claimed in his final manuscripts, Galileo found no occasion to confirm the pipe ratios in the *Discourses on Two New Sciences*.

But Galileo could in fact have relied on something more than analogy alone. Indeed, he could have relied on something that everyone who did not have access to his deceased father's manuscripts could have likewise relied upon: his deceased father's well-known published work, the *Dialogo della Musica Antica et della Moderna* (*Dialogue on Ancient and Modern Music* [Florence: Marescotti, 1581]), in which Vincenzo enlisted his leading interlocutor, Bardi, to provide the *coup de grâce* to any claim that a change in pipe circumference was essential to producing a clearly perceived change in pitch. The question will then remain, why didn't Vincenzo himself later acknowledge—let alone, defer to—this experiential refutation? And, confronting the same refutation, on what basis did Mersenne yet seek to 'confirm' cubic pipe ratios?

6. Capping it off: the *Dialogo* on Ratios of Pipe Dimensions

In his own reception of Pythagorean tradition and lore, Vincenzo was notably inclined to skepticism, particularly with respect to accounts that sought a suspiciously schematic representation of ancient theorists. In one of the late, unpublished manuscripts, Vincenzo expressed profound dissatisfaction with the current state of scholarship:

What exactly was the reasoning with numbers that Pythagoras wanted to pursue in his division of the strings? What was the sense of Aristoxenus? And what was the reason and sense that Ptolemy

wanted to harmonize? No one, to my knowledge, has explained these things in a way that can satisfy the intellect (Galilei [c.1589–91] 1989a, p. 165).

Vincenzo then goes on to repeatedly stress the Pythagorean discovery of the “true form” (“*vera forma*”) of the perfect consonances, as expressed in the whole number, epimoric ratios, and furthermore stresses his disbelief that Pythagoras privileged the purity or simplicity of such ratios at the expense of hearing: “nor do I believe [...] that Pythagoras said or believed that such a judgment had nothing to do with the sense of hearing.” (Galilei [c.1589–91] 1989a, p. 167) On Vincenzo’s account, Pythagoras proceeded to “regulate and reorder” (“*regolare et riordinare*”) (Galilei [c.1589–91] 1989a, p. 167) the ancient Greek Greater Perfect System of attunement on the basis of an aesthetic preference for keeping the perfect consonances in a pure, aurally most-pleasurable form that corresponded with the now-conventional ratios, “for all that he heard consonant thirds and sixths [that is, the imperfect consonances] coming from voices singing and instruments playing.” (Galilei [c.1589–91] 1989a, p. 171) On this account, Pythagoras had simply privileged the aural pleasures of one set of consonances at the expense of another, a calculated act of regulation that chose between two sets of equivalent empirical data, and navigated between two sets of aesthetic preferences—namely, the pleasures of the perfect and the imperfect consonances. According to Vincenzo, therefore, “the generosity of considerate Nature” (“*la liberalità della cortese Natura*”) (Galilei [c.1589–91] 1989a, p. 168–69) made both sets of data equally available and equally attractive; it was aesthetic considerations guided by the logical recognition of competing and equally compelling prospects for attunement that would guide further developments in music theory and musical practice. Properly understood—and without misrepresenting the likely intentions of their discoverer—the conventional Pythagorean ratios provided a rational and empirical framework that presented no inherent barriers to further developments in music, whether practical or theoretical, and with their imprint continuing to be found in such reconsiderations.

In the earlier *Dialogo*, Vincenzo had his interlocutor and mouthpiece, Giovanni Bardi—a persona named in honor of his patron, and thus a figure of particular credibility—entertain certain reconsiderations of the Pythagorean ratios, including the introduction of squared ratios of the consonances; however, this first attempt at a ‘squares law’ was in fact drawn from ratios between pipes rather than weight and string tension—and thus, the later discovery of the inverse-square tension ratios would serendipitously free up pipe dimensions for further exponential maneuvers. Initially, Vincenzo’s presentation follows received Pythagorean tradition, for

Bardi accepts the conventional story about the ratios of the weights of the hammers, and then introduces the topic of pipes in a manner that confirms the portrayal by Gaffurio in which all pipes, regardless of length, appear to be roughly of the same diameter:

if you take an organ pipe twelve palms, fingers, or units [*tanti*] long and another of the same size [*grossezza*] and opening [*vano*] but six units long [that is, in the ratio 2:1], and you play the two, you will hear issue from their sounds the consonance that you find between parhypate hypaton and trite diezeugmenon [that is, a perfect octave], the pipe with twelve units making the low sound and that containing six the high. [Likewise,] [i]f you take two strings of the same length, thickness, and goodness, and you stretch them to the unison on a plane surface and cut one of them in half by means of a groove [*scannello*], fret, or bridge, or with a finger of the hand, you will hear [...] a diapason consonance [that is, a perfect octave once again] every time that they are struck together or one after the other.⁵⁸
(Vincenzo [1581] 2003, p. 328)

In this passage, Bardi treats pipes as functionally equivalent to strings, where uniformity of diameter in the former corresponds with uniformity of thickness and “goodness” in the latter, and with a precise correlation between the two in the ratio of the perfect octave, which is therefore determined in both circumstances by relative length. This conceptual framework—the pipe/string analogy—is fundamental to the thread of discussion that immediately follows, in which Bardi elects to change variables in a carefully deployed sequence.

Vincenzo was well aware, of course, that pipes commonly vary in terms of their breadth—that is, in their diameter or circumference—just as strings might vary in thickness and density; thus, having first isolated the factor of variable length (while maintaining constant circumference), Bardi proposes next to isolate variable pipe circumference, considering “ratios of the pipes” in which the pipes are “of the *same* length but *different* breadth” (Vincenzo [1581] 2003, p. 329 [emphasis added]). The term Bardi uses here is “*larghezza*,” which he qualifies in terms of “*circonferenza*” (circumference) and “*vano*” (opening)—but, curiously, not volume

58. Greek scale systems were constructed from what are called tetrachords (from “four strings”): four-pitch segments framed by a perfect fourth, with the framing pitches being fixed—the *bestotes* (literally, “standing” pitches)—and the internal pitches moveable (*keinoumenoi*), subject to different forms of tuning, from which three broad categories of genera were identified (the diatonic, chromatic, and enharmonic—each subject to further distinctions in tuning). Parhypate hypaton and trite diezeugmenon are moveable pitches from the hypaton and diezeugmenon tetrachords, respectively.

(Vincenzo 1581, p. 134). Even when referring to “*capacità*,” which might otherwise seem to welcome such an interpretation, Bardi characterizes it in terms of “*vano*”—that is, once again, “opening” or “entry” as understood in terms of circumference. Here, then, is how the passage begins—the passage for which the conceptual framework is not to maintain constant breadth and vary length (as on the previous page of the *Dialogo*), but to maintain constant length and vary breadth:

I have never seen any written record [*memoria*], but I firmly believe, indeed I am very certain, that when there are two pipes, *both* two braccia long,⁵⁹ if one pipe has a circumference [*circonferenza*] of the opening [*vano*] from which the air passes of a half braccio and the other of three-quarters of a braccio [that is, a ratio of circumferences of 2:3], when played together you will hear the consonance of the diapente [that is, a perfect fifth, ratio 2:3]. [...] In the same way, you can obtain most of the musical intervals from pipes of *equal* length and *unequal* capacity [*capacità*] of the opening [*vano*] [...]. But here you have to note that the capacity of the pipes considered in this second way does *not* have between them the same relation as in the first instance, where we observed the *difference of length* [but] of the *identical capacity* [*capacità*]. (Vincenzo [1581] 2003, p. 329; Vincenzo 1581, p. 134; emphasis added, with emendations drawn from this source to be discussed below)

Given the specific context provided in the final sentence, *capacità* plainly cannot refer to volume (where any difference in length would entail a change in volume); thus, the term *capacità* in itself is not used by Vincenzo to refer to volume, though a change in some aspect of *capacità* may entail a change in volume. Note moreover how Bardi emphatically reminds his interlocutor precisely what is being varied in this portion of their discussion—namely, *capacità* (that is, in terms of circumference or “opening”)—and what is not being varied—namely, length. The discussion will proceed in light of this careful distinction in variables of dimension; but, before following those developments, we need to assess the premise of Bardi’s claims.

What is particularly striking in this passage is the way in which Bardi now appears to imagine pipe circumference as a correlate of string length: the ratio of the perfect fifth—3:2 for string length, or 2:3 for pipe circumference—involves the same numbers for the circumference of the pipe as for the

59. The *braccia* is derived from an “arm’s length”—that is, approximately a meter. Bardi therefore proposes considering pipes of a medium size of rank, and therefore reasonably substantial in dimension.

divided string (or, indeed, as for pipe length, the fundamental string/pipe analogy with which Bardi began), and is represented as producing the same effect on pitch production. Ratios of circumference are thus conceptualized in terms of ratios of string length, as though Bardi intuited Kepler's physical analogy in the *Harmonices mundi* (1619), in which Kepler likened the proportions of arcs of the circle inscribed by regular polygons to divisions of a string with its ends joined in a circle (Kepler [1619] 1997, p. 144).⁶⁰ Though Bardi is now varying a further, characteristic feature of pipes—their circumference—Vincenzo thus portrays him, favorably, as continuing to think in terms of familiar Pythagorean ratios.

What Bardi does not represent himself as varying in any way is volume, and here Palisca's translation becomes problematic at times for being wishful. When Vincenzo had the opportunity to address matters of volume earlier in the *Dialogo*, it was rather in terms of the weight of liquid-bearing capacities, in keeping with the visual logic of Gaffurio's illustration: he had Bardi refer to "vases" into which "you put [...] the number of pounds of water equivalent to the iron of the hammers and you hit the vases with a rod of iron" to produce intervals (Vincenzo [1581] 2003, p. 327), like harmonious blacksmiths striking their anvils, but with the anvils themselves effectively becoming the resonating bodies (with the corresponding ratios of weight). In his own version, Palisca had conceived the demonstration in terms of volumetric ratios of volume ("two glasses, one filled, the other half-filled with water"),⁶¹ as would likely any of us attempting to replicate Gaffurio's visual demonstration of the ratios of liquid-filled glasses. Perhaps it was thinking along these lines that prompted Palisca to translate "*capacità*" as "volume" in the first part of the final sentence in the passage quoted above ("But here you have to note that the volume [*capacità*] of the pipes considered in this way second way"), which I have directly translated as "capacity," in keeping with Vincenzo's consistent use of this term in the rest of the passage, and which Palisca is demonstrably otherwise content to use. Far from allowing the consideration of volume to enter the discussion as mere happenstance, Vincenzo will save its introduction for a particular role in the further sequence of investigation.

60. Cf. Johannes Kepler, *The Harmony of the World*, edited and translated by E. J. Aiton, A. M. Duncan, J. V. Field (Philadelphia: American Philosophical Society, 1997), Book III, Axiom I: "A string stretched out straight can be divided in the same way as when it is bent round into a circle [...] divided by the side of an inscribed figure." This is not to suggest that Vincenzo is making any kind of Keplerian argument, as Kepler's sharp critique of the nature of Vincenzo's claims will demonstrate.

61. Palisca 2008, p. 236. A glass filled with twice as much water would of course double the weight of water, but what is being conceived of here is actual volume rather than weight.

Squared expressions of the string-length ratios for consonances emerge in the continuation of this passage in the *Dialogo*, as measured by ratios of areas of pipe cross-sections. To make certain that the connection with the preceding discussion is clear, we begin by repeating the final sentence of the passage quoted above, retaining Vincenzo's original "*capacità*":

But here you have to note that the capacity of the pipes considered in this second way does *not* have between them the same relation as in the first instance, where we observed the difference of length of the identical capacity. For those that answer at the octave, the opening [*vano*] of the low pipe of whatever form *necessarily* will contain four times that of the high pipe. Each of the dimensions [*lati*]*—*the diameter and the outer surface [*d'intorno*]*—*have a duple ratio, however. Thus, those played together that yield a diapente [a perfect fifth in string length ratio 3:2] will have openings in the 9:4 ratio [*dupla sesquiquarta*] and those that sound the diatessaron [a perfect fourth in string length ratio 4:3] will be in the 16:9 ratio [*supersette partiente noue*]. (Vincenzo [1581] 2003, p. 329–30; emphasis added)

To begin with, Bardi reinforces the precise framework for the demonstration: "in this *second* way," it is circumference, not vertical pipe length, that is being varied, to see what effect this variable might have on pitch. Bardi has already claimed that, with the vertical length of the pipes left unchanged, a 2:3 ratio of *circumferential* lengths will produce the diapente or perfect fifth, just like the string length ratio of 3:2; the circumferential ratio of 1:2 for the perfect octave employs the same logic. Thus, pipes may be understood as functional lengths in two distinct ways—vertical and circumferential—ways that Bardi explores in two distinct discussions, the second resulting in squared ratios for cross-sectional areas: in this manner, one geometrical dimension—the line—begets another—a surface—by means of enclosure, a kind of geometric scaffolding that Vincenzo would later attempt to extend, in determining the "true form" of the octave. Thus, exponential expansion of the traditional Pythagorean ratios—in this case, as the squares of the original terms—are represented by Vincenzo as a seemingly physical rather than purely mathematical phenomenon, a phenomenon for which organ pipes offered an empirical framework that Vincenzo had not yet found in the dimensions or behavior of vibrating strings themselves, but which could yet be understood in terms of string lengths reconceived as lengths of pipe circumference. Furthermore, we can now plausibly identify Vincenzo as at least one of those "several" who Mersenne identified as believing that doubling the breadth of a pipe produced the same result as doubling its length: thus, when Mersenne advocated for the consultation of experience to assess belief, Mersenne's

reported experience in Proposition XII of the *Livre des orgues* explicitly contradicted rather than confirmed Vincenzo's belief as published in the widely read *Dialogo*.

Next, Palisca is clearly correct in translating "*vano*" in this passage in the sense of "opening" or entry, rather than in terms of a space or compartment of some kind (implying a volumetric interpretation),⁶² as Bardi unmistakably refers here to the relationship between diameter and cross-sectional area of the pipe: if "diameter" is in duple ratio, so too will be the corresponding radii or circumferences, with the corresponding cross-sectional areas in approximately squared ratios (4:1, or "four times that of the high pipe"). Thus, Bardi returns to the diapente he discussed immediately above—where vertical pipe length was explicitly kept unchanged ("when there are two pipes, *both* two braccia long")—and notes that the corresponding ratio of cross-sectional areas will be 9:4 (and likewise squared for the diatesseron or perfect fourth, 16:9, and he continues on with squared values for the major third, 25:16, and minor third, 36:25) (Vincenzo [1581] 2003, p. 329–30). However, Bardi notably does not proclaim a discovery of squared ratios that thereby defeat or destroy Pythagorean tradition: on the contrary, Vincenzo plainly frames them as the product of conventional Pythagorean ratios, deployed in novel ways. Since Bardi has already confirmed that, as in the Gaffurio illustration, pipe circumference may be kept equal when varying vertical length, when Bardi subsequently introduces the squared ratios for cross-sectional areas, there is no apparent necessity for this procedure to produce distinctions in pitch if vertical pipe length can likewise be changed; however, if vertical length is kept unaltered, as Bardi emphasizes at the outset of the passage, another form of length may evidently be varied to produce a corresponding variation in pitch. Thus, Bardi aims to demonstrate that pipe dimensions make it possible to apply conventional Pythagorean ratios for length in two orientations, perpendicular to each other. Each corresponds to a 'surface,' but only circumference in combination with vertical length defines the area of that surface: vertical length must be explicitly correlated with circumferential length—which may be visualized as the enfolding 'sheet' or surface area of a tube opened up to form a regular polygon—to define an area and, by entailment, the volume enclosed by that area. This explicit correlation will be introduced to the dialogue in due course, but not before Bardi understands it to have been suitably prepared.

62. Palisca notes that "*vano*" may be clearly understood in volumetric terms only in the late discourse on the diapason—see Palisca, in Vincenzo [1581] 2003, p. 329n691. The careful distinction he observes there is crucial to the meaning of the passage.

Having introduced the squared ratios for cross-sectional areas, Bardi reminds his interlocutor of the proportional dimensions or *lati* that occasioned it: “Each of the dimensions—the diameter and the outer surface [*d'intorno*—have a duple ratio” (Vincenzo [1581] 2003, p. 329). Bardi’s method of isolating each of the available pipe lengths—that is, both vertical and circumferential length, with its corresponding diameter (and radius)—depends upon keeping the other length fixed in his demonstrations. Thus, Palisca’s suggestion that “the outer surface [*d'intorno*],” as he translates it here, “evidently consists of two dimensions, circumference and length,” does not fit the context established by Bardi, and is likewise problematic with respect to the way in which Bardi expresses himself. Palisca correctly observes that, “the third dimension, length, would also have to be doubled” to produce a justly-tuned perfect octave, “something that is not clearly expressed here,”⁶³ but Bardi employs “*d'intorno*” as a single, circumferential dimension for his interlocutor, Strozzi, to consider: configuring the specific dimensions of the sheet of metal used to make the walls of the pipe is an idea that will be explicitly raised only later in the dialogue.

With Gaffurio having evidently served as a reference for key portions of the *Dialogo*—not just in the passages we have considered here, but throughout the work—it is instructive to examine the illustration that Vincenzo himself employed to represent Bardi’s demonstration of the squared ratios. Vincenzo’s illustration frames its own distinctive visual logic, but relies nonetheless on familiar forms of thought and representation (Figure 5). Pythagorean and Platonic traditions conventionally referred to composite, regular polygon figures to represent numbers, and squared composite figures to represent squares of numbers or areas corresponding to the sides of regular polygons, as in the familiar demonstration of the ratios of areas formed by sides of the right angle triangle (later expressed algebraically as $A^2 + B^2 = C^2$), or in the ‘nuptial number’ of Plato’s *Republic* (546c).⁶⁴ Of the seven figures represented on this page of the *Dialogo*, the first frames the essential physical, rather than geometric, idea of proportional cross-sections of pipe, as if representing organ pipes viewed from above: when doubling the diameter (and, correspondingly, the circumference) of the smaller pipe, the cross-sectional area is made to appear as though two of the smaller pipes would fit inside, forming a “*quadrupla*

63. See Palisca, in Vincenzo [1581] 2003, p. 329n691.

64. “The first harmony is a square, the product of equals, so many times 100. The second harmony is of equal length one way, but a rectangle. One side is the square of the rational diagonal of a five-by-five square, minus one, times 100, or the square of the irrational diagonal of a five-by-five square, minus two, times 100. The other side is three cubed times 100. Taken as a whole, this geometrical number is master of this domain—of better and worse births” (*Rep.*VIII, 546c [Plato 2010]).

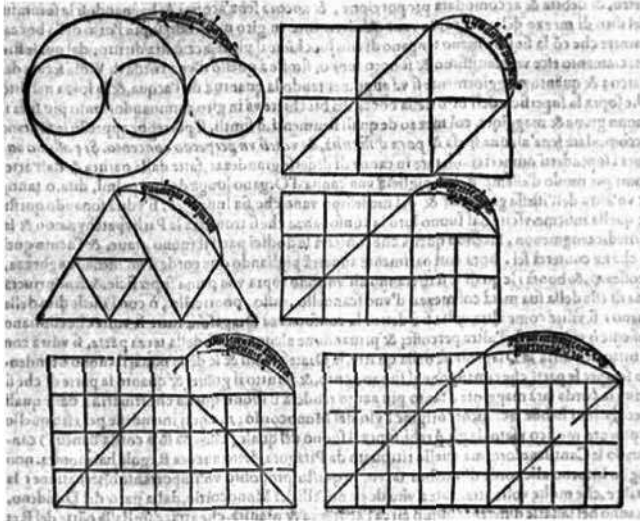


Figure 5. Proportions of cross-sectional pipe areas. Vincenzo Galilei, *Dialogo Della Musica Antica, Et Della Moderna* (Florence, 1581).

diapason”—that is, a diapason or perfect octave, but in the cross-sectional area ratio of 4:1. To demonstrate this quadruple proportion, a geometric representation of the physical illustration in cross-section is offered in the second figure, to its immediate right: Vincenzo represents his claim in terms of segments of diameter—the term he finally introduces just before the illustration, which effectively replaces “*circunferenza*”⁶⁵—where the ratio is expressed through the duple proportion of such segments. These segments of diameter, 2:1, are treated as the diagonals of squares that represent ratios of area; thus, a single diagonal segment corresponds to one square, while the doubled diagonal segment cuts through a larger, composite square. The two figures—one a single square, the other a composite of four—together represent the area ratio of 4:1, and are again labeled as “*quadrupla diapason*,” so that the geometric ‘translation’ of the first figure is transparent. In the third figure (immediately below the pipe cross sections), Vincenzo treats the proportions of segments of length in terms of the sides of an equilateral triangle, which may be tessellated

65. The shift in terminology possibly serves as a visual aid or convenience—that is, as a proportional ‘length’ that requires no straightening to be grasped with visual immediacy. However, Bardi will soon turn to areas of the sheets of metal that form the pipe walls, for which circumference provides a key dimension.

to form a composite triangle in the same way that the composite squares were formed, employing the same visual rhetoric: one can immediately see the quadruple proportion in areas, even if neither squares nor triangles have anything to do with the actual cross-sectional shape of the pipes concerned.⁶⁶

Perhaps this is how the younger Galilei first encountered the great book of the universe (“questo grandissimo libro... [io dico l’universo]”) that he was to later envisage in *The Assayer*, “its characters [...] triangles, circles, and other geometrical figures” (Galileo [1623] 1977, p. 33)—a book that was evidently no less harmonic than geometric, bound by ratios. Vincenzo then proceeds to represent the squared ratios of cross-sectional areas for the remaining intervals identified in his text (that is, the perfect fifth, fourth, and major and minor thirds), as portrayed by the same ‘geometric numbers’ familiar to Pythagorean or Platonic usage. Further anticipating the geometric rhetoric of his late discourse on the diapason, to which we will soon return, Vincenzo here derives geometric relationships from lines, with squares or triangles emerging as constructions from discrete, arithmetic lengths—in this case, segments of diameter. There is, however, no attempt to portray volume, and thus no implication of any distinction in proportions of vertical pipe length. In his accompanying commentary, Palisca claims that the squared ratios of cross-sectional areas are enroute to the claim first announced in Vincenzo’s published *Discorso* of 1589—namely, that “pitch will vary inversely as the cube of the volume of pipes”⁶⁷—because such a claim would indeed be an approximate volumetric consequence of doubling both pipe length and pipe diameter. If this were a consideration of dimension that Vincenzo had in mind—namely, if Vincenzo were indeed integrating rather than isolating variables of vertical and circumferential length, without having Bardi bother to announce such an intention—it is curious that Bardi also fails to observe or to articulate such an implicit cubic ratio, given the tremendous importance it will assume in Vincenzo’s writings before the decade is out.

66. Here I depart significantly from the account given by Palisca in “Mathematic and Geometry in Galilei’s *Dialogo*” (Palisca 2008), which adopts a literalistic reading that furthermore asserts without textual support that the figures also refer to segments of vertical length of the corresponding pipes, even though this reading would remove entirely the correlation of diameter and cross-sectional area with distinctions in pitch production that the figure aims to demonstrate: “The ratios in these diagrams compare the areas of the sections of cylindrical, square, and triangular pipes whose lengths are in duple proportion” (Palisca 2008, p. 250). Palisca likewise does not discuss or identify the conventional Pythagorean or Platonic composite, regular polygon shapes used to represent numbers and their squares, his insistence on actual “square, and triangular pipes” precluding such recognition.

67. Palisca, in Vincenzo [1581] 2003, p. 329n691.

It is after this figure and its accompanying discussion—the figure itself on page 134 of the original publication, with discussion continuing on to the top of page 135—that Bardi's companion, Strozzi, is at last compelled to ask for an integration of these factors, as if impatient to finally take this step. He does so by explicitly considering the circumferential surface of a pipe—and, implicitly, the enclosing of ratios of volume. "All of this is fine," he begins—referring to the squared ratios of cross-sectional areas, where vertical lengths of pipe are invariant—

But tell me, please, if we took two sheets [*falde*] of lead—the kind that is used in making organ pipes—after pounding, planing, polishing them thoroughly, and cutting them not to the shape of a perfect square but so that each of the longer sides is, for example, a braccio and the shorter sides four-fifths of a braccio, do we think that if we joined and soldered together with due diligence the longer sides of one and the shorter sides of the other and then played them together, they would respond at the unison? (Vincenzo [1581] 2003, p. 333)

The imagination of the procedure is neatly done, and corresponds with the approach of pipe makers themselves, who attended to the dimensions of sheets of metal in the same way that a sometime cloth merchant like Vincenzo measured his textiles:⁶⁸ for the first time, both vertical and circumferential lengths of the respective pipes will be correspondingly varied, and it is volume that will now be invariant; and, having at last introduced this final variable, Bardi responds to Strozzi's question with a simple, "*Indubitatamente*" (Vincenzo 1581, p. 135). The corresponding vertical and circumferential lengths are in proportions of 5:4, or what would normally be associated with a major third in just tuning; but, deployed in tandem with each other, Strozzi and Bardi imagine them to be sufficiently compensatory by enclosing the same volume to produce a perfect unison—and, given the innumerable ways in which organ pipes may be adjusted for pitch, they may well be right: after all, Strozzi is insistent that what he has in mind are organ pipes, and no pitch-producing solid body has proven more protean in this very capacity.

We glimpse here the way in which Vincenzo later came to envisage pipe volume: namely, as though it were its own protean beast, a kind of mathematical gas capable of retaining its essential characteristics or identity even while its particular dimensions respond to the structural and physical constraints imposed on them. In line with this conception, Strozzi's

68. The cloth trade was one of the means by which Vincenzo sought to maintain his household (the Galilei family were of lesser nobility and fallen in their fortunes). On Vincenzo's *vita*, see Canguilhem 2001 and Orsini 1988a/b.

demonstration keeps volume invariant, rather than framing a circumstance—for example, the production of any interval other than a unison—in which pipe volumes would be in squared or cubic proportion to each other. But perhaps Strozzi anticipates the next stage of the dialogue, for Bardi recognizes that, in response to Strozzi's attention to invariant volume, long-established maker's practice demands that he go yet one step further in this very capacity. If we have been correct in this reading of Bardi's method (further assisted by Strozzi's curiosity) of isolating and varying distinct dimensions of pipes, a clear sequence of demonstrations emerges:

1. vary length, maintain circumference (volume varies between the pipes)
2. vary circumference, maintain length (volume varies between the pipes)
3. vary circumference *and* length (volume kept invariant)

Given this trajectory of demonstrations, a fourth option presents itself, which is to vary neither length nor circumference nor, seemingly, volume, but to probe what else might produce a notable change in pitch.

Here, Bardi retreats from the domain of the thought experiment and, unmistakably, re-enters the empirical arena, for Bardi now reports a phenomenon well-known to organ builders:

We will say [...] this last word. If you take two organ pipes of *equal* length and the *same* capacity of opening⁶⁹ [*il vano loro della medesima capacità*], and you *close* the mouth of one of them on top [...] you will hear when you play them together the same consonance as you get singing hypate [meson] and nete [diezeugmenon—that is, producing a perfect octave], the closed pipe sounding the low pitch and the open one the high.⁷⁰

What Bardi accurately reports was the well-established practice of the “capping” of pipes, by which organ builders (as we now understand it) effectively doubled the wavelength of the column of air in the pipe. Though a conventional practice, pipe capping nonetheless serves as the crucial experience with which to cap this entire discussion, since the only variable clearly affected is in some sense the functional length of the pipe (in some sense, because the effect on wavelength could be judged only in perceptual terms, by pitch production), without even bothering to make

69. (Palisca inserts “volume” here.)

70. Vincenzo [1581] 2003, p. 333; emended and with emphasis added; Vincenzo 1581, p. 135. Hypate meson and nete diezeugmenon are fixed pitches of the hypaton and diezeugmenon tetrachords, respectively.

adjustments for circumference or volume.⁷¹ Bardi's exploration of pipe dimensions has, in a sense, come full circle: having kept pipe circumference unchanged in his first demonstration while doubling length to successfully produce a perfect octave, he has now found a way to produce the same result, not only by failing to change the ratio of vertical pipe length, but without varying circumference or evident volume either. Any potential cubic volumetric ratios have thereby become an empirical irrelevance.

One of the singular ironies of the modern scholarly literature that has sought to enlist Vincenzo as unalloyed empiricist is its failure to address or even recognize Vincenzo's demonstrated reliance here on an organ maker's knowledge of pipe capping, the distinctive conceptual challenges presented by this phenomenon, or that Vincenzo's knowledge of the practice considerably predates composition of the texts in which he emphasizes the cubic ratios of pipe volume.⁷² In view of references to the *Dialogo* by the likes of Kepler in the *Harmonices mundi* and Mersenne in the *Harmonie universelle*, the published record of Vincenzo's familiarity with pipe-capping that was readily accessible to leading figures of early modern science suggests yet another reason why Galileo passed over the cubic pipe ratios in silence when he had occasion to address the pitch production of pipes in the *Assayer*, or ratios of the consonances in the *Discourses on Two New Sciences*: here was a cautionary tale in the mathematical interpretation of acoustic phenomena that was evidently better left not retold. The concern was perhaps less a matter of propriety as of due wariness before the complexity and conundrums confronting any science of pneumatic phenomena.

Thus, the well-established phenomenon of pipe capping leaves modern scholars with the perplexing issue signaled earlier: if Vincenzo himself already had well-established empirical proof that volume does not in fact play a defining role in the production of pitch by a pipe, then why did Vincenzo subsequently press on to concoct the 'cubes law' scenario, with its protean system of measurement determinedly targeted at a "true form" of the octave, 8:1? Likewise, we must inquire about Mersenne's curious, corresponding insistence on cubic ratios for pipes when he also found himself obliged to address the phenomenon of capped organ pipes and the dramatic change in pitch they produced. To address these conundrums, we

71. If anything, enclosing a space would suggest its functional reduction rather than enlargement, with the capped pipe thereby presumed to produce the higher rather than the lower pitch—the opposite of the empirical result.

72. Cf. Palisca 2008, pp. 247–50 in particular.

will find a helpful guide in their contemporary, Johannes Kepler, though Kepler's more immediate concerns lay with Robert Fludd.

7. The Mother of all Ratios of Consonance

It is time to reconsider Vincenzo's outlandish claim that two organ pipes with a ratio of length of 2:1 will produce "a major third of the intense tuning of Aristoxenus": (Galilei [c.1589–91] 1989b, pp. 189–91; emphasis added) here, as both Palisca and D. P. Walker confirmed, Vincenzo could not have performed "even the most rudimentary empirical check."⁷³ And yet, such gross empirical inaccuracy was no accident. Vincenzo's pipe dream is recounted in the "Discourse Concerning the Diapason," where his goal is to show that the Pythagorean ratio of string lengths for the octave, 2:1, is actually the ratio for the major third if volume has not been correctly adjusted according to the cubes law; therefore, the ratio 2:1 cannot be the true ratio of the octave, if it may be thus singly conceived. Although he has ostensibly arrived at the octuple ratio through consideration of the length and breadth of pipes, Vincenzo's cubic ratios nonetheless seek to transcend the particular dimensions of any particular pneumatic resonating body.

Rather than building functional organs, therefore, Vincenzo ultimately appears to have had an entirely different project in mind. As we heard at the outset of this discussion, Vincenzo proclaimed that,

the true form of the octave [*la uera forma dell'Ottava*] is the octuple and not the duple [ratio], considering that the octave shows itself to be *truly the mother* [of the other ratios] [...] in that it has a form that is capable of including any other consonant interval in its smaller terms. (Galilei [c.1589–91] 1989b, p. 189; emphasis added)

To translate: there is a ratio of the octave, 8:1, which delimits a sequence of numerical terms from 1 to 8, and from which the other ratios of consonance may be constructed only if they are kept in their traditional ratios: that is, if they are kept in those very ratios which Vincenzo has demonstrated to be true only in terms of string length, the ad hoc constituents of a Frankensteinian body of evidence. Vincenzo's favored polemical target, Zarlino, had asserted that the number 6 defined the limits of consonance, a numerical framework identified in *Le istituzioni harmoniche* (*The Institutions of Harmony* [Venice: Franceschi, 1558]) as the *senario*, but Zarlino plainly got the number

73. Walker 1978, p. 24. D. P. (Daniel Pickering) Walker was one of the principal cautionary scholarly voices to effectively question both the broader claims and key details of Palisca's account of Vincenzo: cf. "Vincenzo Galilei and Zarlino," pp. 14–26, in particular.

wrong: the new, true sonorous number had been revealed to be 8, the basis of an *ottonario*.⁷⁴

The *ottonario* held many attractions, particularly in light of its ostensibly cubic nature. Returning to the merely representational use of ‘geometrical numbers’ in the *Dialogo*, Vincenzo now proposed to grant them strikingly new ontological dimensions. He portrayed a series of geometrical analogies or correspondences between conventional ratios and his squared and cubed numerical relationships:

We may first hear and consider the diapason contained in the duple [that is, the duple proportion, 2:1] by means of numbers [presumably, as units of linear measure], and this corresponds to lines. Secondly we can hear and consider it as contained in the quadruple [the proportion 1:4] by means of weights, and this corresponds to a surface. Lastly, we can hear and consider the content of the octuple [the proportion 8:1] by means of measurement, and this corresponds to solids [*corpi*]. (Galilei [c.1589–91] 1989b, p. 181)

These geometric correspondences immediately precede a distinction that Vincenzo draws between “*numeri numerati*,” which we can understand as cardinal numbers (as Palisca’s translation recommends), and “*numeri numeranti*,” which we are to understand as numbers representing some form of physical magnitude, identified in the immediate context as “numbers measuring only those portions of the strings capable, when struck, of producing a pitch.” (Galilei [c.1589–91] 1989b, pp. 182–3) Therefore, any connection between Vincenzo’s geometrical scheme quoted above and actual material phenomena is entirely speculative—in the fullest sense, a *musica speculativa*. Vincenzo himself draws close attention in the late discourses to the dimensionality of the single vibrating string when he experiments with varying thicknesses of it, a crucial component, furthermore, of one of Mersenne’s laws of the vibrating string (affecting mass per unit of length), not to mention suggestive of an analogy between thickness of string and width/volume

74. As a highly educated priest and the music director of San Marco in Venice—among the most prestigious of sixteenth-century ecclesiastical posts in music—it is no surprise that Zarlino’s choice of the number 6 enjoyed a distinctive Augustinian pedigree, as featured in the *De trinitate* (4.4–6), *Civitate dei* (11.30), and the *De Genesis ad litteram* (4.13–14)—the latter text, Augustine’s third and most extensive commentary on Genesis (a genre to which Mersenne also contributed, with his *Quaestiones celeberrimae in Genesim* [Paris 1623]), which furthermore served as Galileo’s principal theological source for the “Letter to the Grand Duchess Christina” (cf. McMullin 2005). This theological context has not been observed in extant scholarship on Zarlino or his relationship to Vincenzo, despite Augustine’s ranking as Zarlino’s most cited patristic source in a seminal work such as *Le istituzioni harmoniche* (a quantifiable preeminence that Augustine correspondingly enjoys in Mersenne’s *Harmonie universelle*).

of pipe that appears to have stimulated Vincenzo's initial attention to such details in his own *Dialogo*. Furthermore, with respect to squared ratios and 'surfaces,' what had formerly been a meaningful geometrical relationship within the *Dialogo* drawn between the circumference and surface area of a pipe opening, has instead become a physically unintelligible analogy drawn between weight and extension. Nevertheless, as the product of such allegorical scaffolding, Vincenzo insists upon the literal fecundity of the number 8 and the ratio it defines as the "mother" of all consonant ratios. Why?

At least part of Vincenzo's idiosyncratic treatment of number comes from a rather simple association with interval spans: namely, that most of the conventional consonances lie 'within' the span of the octave. He correspondingly advances a rather bizarre critique, claiming that, for Zarlino, any number that stood above 2—and, therefore, "outside" the ratio for the perfect octave, 2:1—was, in this arithmetic sense, "outside the octave," which meant that all the ensuing perfect and imperfect consonances smaller than the compass of the octave could in fact only be found "outside the octave" on Zarlino's purported terms.

He [Zarlino] believed that the octuple [ratio, 8:1] necessarily contained every place three octaves and never one, and that the octave is always contained in the duple [ratio, 2:1], as if he considered it only in terms of lines. He did not find the perfect consonances [specifically, the ratios 3:2 and 4:3] except outside the octave, since there was no room to admit them except outside the octave within the fifteenth [the double perfect octave, ratio 4:1, encompassing the numbers 3 and 4] and the imperfect consonances [the ratios 5:3 and 8:5] outside the latter [but] within the twenty-second [the triple perfect octave, ratio 8:1, further encompassing 5 and 8]—that is, in terms of lines, as I said—whereas in terms of solids [*corpi*] they are all within the octave [that is, 'within' the ratio 8:1]. (Galilei [c.1589–91] 1989b, p. 193)

This was Vincenzo's bootless strategy for portraying Zarlino as the victim of his own paralogic: how can an interval that is smaller than the octave be found only "outside" of it, since the numbers used to create consonances other than the unison and octave—that is, 3, 4 and 5—are beyond the numbers 1 and 2? Unlike Zarlino, however, Vincenzo has abandoned all pretense of addressing his argument to actual ratios of the consonances; instead, Vincenzo abstracts their numerical terms, which he then treats as though they exist purely as the constituents of an arithmetic sequence. This sequence of abstract numbers is encompassed by the first cubic numbers, 1 (1^3) and 8 (2^3)—a numerical corpus from which he extracts, as needed, the numerical terms that can supply ratios for the remaining

perfect and imperfect consonances in traditional ratios. The maternal character of that corpus recalls the Pythagorean decad,⁷⁵ the spectre of which is summoned by Vincenzo with his direct reference to the Pythagorean *tetraktys*.⁷⁶

Thus the octave is produced and generated [*prodotta et generata*], for example, by lines; it is pregnant in its perfection [*gravidata dalla sua perfettione*] with regard to length and width of surface, so that it is capable [of containing] at one time the ratios of the fifth and of the fourth—but in terms of lines—as befits the order of things of nature in this progression 1, 2, 3, 4. (Galilei [c.1589–91] 1989b, p. 187)

Such numerical speculation was entirely familiar to a near contemporary, Kepler, who had clearly come across the line of reasoning that so enchanted Vincenzo. In the introduction to Book III of his *Harmonices mundi libri V* (1619), Kepler reconstructs Pythagorean confidence in the ontological efficacy of number on the foundation of such relationships:

But the number 8 is the cube of 2, and the number 9 is the square of 3. Then the following numbers were already before them: 1, 2, 3, 4, 8, 9 [that is, the Pythagorean *tetraktys* plus the first distinct squared and cubic numbers that may be derived from them]. However, since Unity is the same as its square and its cube, whereas the binary had as its square 4 and its cube 8, to the ternary they also added its cube 27 as well as its square 9, because they supposed that it was right always [to] go as far as the cubes on account of the fact that the whole world, and everything that gives [musical] notes, consisted not of empty surfaces but of solid bodies. Eventually from that beginning such a strong opinion grew up about these numbers, on account of the fact that they were Primes, and their squares and their cubes, that the Pythagoreans resolved that the whole of Philosophy should be composed of them.⁷⁷

75. Multiple early commentators claim an etymology for the decad, δεξάδα, that derives the term from δεχάδα—“dechad” or ‘receiver,’ as in Philo of Alexandria’s *On the Decalogue* 23: “those who first established names seem to me reasonably to call it the decad (δεξάδα), as being the dechad (δεχάδα), in so far as it receives and makes room for all of the numbers” (cf. Huffman, *Philolaus*, pp. 352–53). Vincenzo’s account appears to supplant the decad in this conceptual role with the ‘ogdoad’ (ογδοάδου [dative], as given in the *Theologumena arithmeticae* 74.10 [cf. Huffman, p. 357]), though without the supportive etymology.

76. The numbers of the Pythagorean *tetraktys*, 1 through 4, add up to 10; thus 10 may be conceived as a form of numerical ‘container’ for the *tetraktys*.

77. Johannes Kepler, “On the Origin of the Harmonic Proportions, and on the Nature and Differences of Those Things which are Concerned with Melody” (*Harmonice mundi*, III), p. 132.

While his direct knowledge of Vincenzo's late discourses, their content, and line of argument is highly unlikely, Kepler nevertheless demonstrates a ready familiarity with the attractions of a supposedly 'geometric' derivation of the 8:1 ratio that had drawn Vincenzo into temptation, moving from "lines," through "surfaces" (squared ratios), to "solid bodies" (the cubic ratios)—and its 'containment' or 'completion' of the ratios for string length. Observing the all-too-convenient association of the number 8 with both the generic identity of the interval (*octavus*) and the cubic ratio 8:1, Kepler castigated those who would attempt to harmonize diverse ratios by such pseudo-Pythagorean means:

Do people vainly philosophize at this point about the number [8], that is to say about why the eighth note completes them all, and returns to the same [pitch-class]? For in truth the answer must be through a circular argument, because [...] it comes about naturally that the interval of double proportion [2:1], which [...] is identical in sound, is divided into seven melodic intervals, which are bounded by eight notes [...]. They think that comes about because the number 8 is the first cubic number and the first cubic shape. But what has the division of a string to do with solids?⁷⁸

One could indeed ask Vincenzo no more pertinent a question than "what has the division of a string to do with solids?" Vincenzo arrived at an answer to Kepler's rhetorical question via the labyrinthine passageways of pipe dimensions; and, having reached the end of his acoustic odyssey, Vincenzo appears before us resembling nobody so much as Kepler's ultimate target in these passages of the *Harmonices mundi*: Robert Fludd.

8. Re-Enchanting the Pythagorean Universe?

If the book of the universe were truly written in the "language of mathematics," then Vincenzo's diverse assortment of ratios—an even greater diversity than that which provoked Sagredo's concern in Galileo's *Discourses*—threatened the deconstruction of that text: how could mathematics provide a useful tool for natural philosophy if a coherent set of perceptual phenomena—the consonant musical intervals—could not be read with a correspondingly coherent mathematical representation? Vincenzo strove to reconcile his potentially contradictory sets of ratios through idiosyncratic juxtapositions of data that he understood to be framed by his ratios for pipe dimensions and specifically by his cubic ratio for the perfect octave, which provided a convenient mathematical enclosure for the ratios of the monochord: consistent with the line of argument initially pursued in the earlier *Dialogo*, this framework

78. Kepler [1619] 1997, "On the Natural Division of Consonant Intervals into Melodic Intervals, and their Designations which Arise from That" (*Harmonices mundi* 3.5, pp. 185–86).

ultimately served to uphold and extend rather than overthrow the traditional Pythagorean ratios, thus saving the appearance of a mathematically coherent universe depicted by Gaffurio. And yet, however idiosyncratic in strategy, was Vincenzo's ambition thereby *sui generis*?

In "The Hermetic Tradition," France Yates sought to distinguish the mathematical orientation of Mersenne's *Harmonie universelle* from that encountered in the *De harmonia mundi* of the Franciscan, Francesco Giorgi (1466–1540), a text that had been a seminal influence on Fludd:

Mersenne will have nothing to do with Francesco Giorgi, of whom he sternly disapproves. Mathematics replaces numerology in Mersenne's harmonic world; magic is banished; the seventeenth century has arrived [...]. It is perhaps somehow in these transitions from Renaissance to seventeenth century that the secret might be surprised, the secret of how science happened. (Yates 1967, pp. 271–72)

In response to such a neat partition, Peter Dear cautioned, "the suggestion that an unproblematic 'mathematics' replaces a 'numerology' requiring comprehension in terms of magical beliefs takes rather too much for granted about the character of mathematical approaches to the study of nature."⁷⁹ Whether we refer to it as "numerology," "arithmology," or "numerical formalism" (Dear's preferred term, possibly seeking to evade the evaluative), the investigation of pipe dimensions by both Mersenne and Vincenzo reveals a robust, evolving Pythagoreanism still intimately connected with the monochord and its division that continued to play a productive, if at times rather quirky role in the mathematical disciplining of experience in early modern science.

To what extent were these Pythagorean pipe dreams shared, in latent if not in manifest content? For Mersenne, the dream was as much Augustinian as Pythagorean, and animated by anxieties over Pyrrhonian skepticism; for Vincenzo, on the other hand, the critical concern appears rather more like his son's objections to the dogged, uncritical devotion of Simplicio to Aristotle in the *Dialogue on the Two World Systems*, for the more orthodox Pythagoreans had constricted a growing *scientia* of music within the boundaries of unmodified tradition, "solely because Pythagoras said it" (Vincenzo 1589, p. 105). In this sense, the limitations of the traditional Pythagorean ratios lay not so much in the ratios themselves, as in those proponents who left them inflexible to the demands of increasing empirical exploration. Thus, whether in Vincenzo's or Mersenne's Pythagorean disciplining of experience, we might recognize Peter Pesic's provocative suggestion in *Music and the Making*

79. Dear 1988, p. 98. Dear was to subsequently take up such historical challenges directly in *Discipline and Experience: The Mathematical Way in the Scientific Revolution* (Dear 1995).

of *Modern Science* that a “[Scientific] ‘revolution’ may more nearly have been a phase in the restoration and augmentation of the ancient project of musicalizing the world than a change in the basic project of natural philosophy” (Pesci 2014, p. 5). Correspondingly, we may yet discern the string of that cosmic monochord, albeit retuned by the hand of a figure who has been threaded through various aspects of this discussion:

Newton was able to make the ultimate connection between the behaviour of vibrating strings, wave dynamics, and the laws of gravity, all of which were governed by the same harmonic principles. The musical string served as a model of the heavens in both a scientific and an allegorical sense.⁸⁰

As Ian Hacking has observed, the lure of Pythagoras or of Pythagorean thought is not solely intelligible in doctrinaire terms, constrained by a tightly wound thread of tradition or textual transmission (Hacking 2012).⁸¹ Correspondingly, our considerations here do not entail that we subscribe to any simplistic portrait of either Vincenzo or Mersenne as “Pythagorean,” or as thereby disinterested in empirical measures: these findings recommend instead that the Pythagorean and the empirical be recognized as terms framing a rich spectrum of both consonant and dissonant relationships with one another, rather than a polarity of the enchanted and the disenchanting in early modern physico-mathematics. In this respect, it might be suggested that the complexities of the phenomena under investigation corresponded with the idiosyncracies of the minds that grappled with them, as the investigators themselves could be moved to observe. When attempting to give an account of the mechanics of the capped pipe and its dramatic drop in pitch—reporting a uniquely troubling phenomenon in which Mersenne still found no occasion to reconsider the cubic Pythagorean ratios for pipe dimensions he had promoted just ten propositions before⁸²—Mersenne invoked analogy with wave and wind action observed in nature, “as [when] the waves in the sea hold back those that come from the opposite side, and as the southern wind is slowed down by that which

80. Gouk 1999, p. 256. Gouk further reflects: “The correspondence between macrocosm and microcosm that Newton argued for, coupled with his invocation of occult forces, was rightly associated with the unorthodox philosophies of the natural magicians of the sixteenth century” (Gouk 1999, p. 256)—and thereby is her chapter on Newton affirmatively titled, “Isaac Newton, Pythagorean *magus*.”

81. Hacking 2011 takes up similar concerns with respect to a Pythagoreanism/pythagoreanism distinction.

82. The critical point is not that capped pipes could not be in cubic relationships with one another, but that the same pipe, once capped, produces an octave with itself, without the cubic change in volume.

blows from the north.”⁸³ If the analogy from nature provided Mersenne with no clear answer to an empirical conundrum produced by human artifice—for, along with wavelength, an account of wave interference awaited the arrival of Newton’s *Principia*—it nonetheless inspired Mersenne to reflect further upon human nature:

One can also compare this to different passions and reasons by which man is so agitated that he often can neither resolve himself nor choose any part of all those which he considers, the more the different motives which push him, as the waves [that] push the ship, are in equilibrium.⁸⁴

That the ship of the soul is subject to multiple, potentially conflicting sources of inspiration or motivation is familiar even to those who, unlike Mersenne, have never heard confession. To recognize that the early modern ship of physico-mathematics itself might have been driven by winds or waves no less “Pythagorean” than “empirical” perhaps offers some orientation in historical waters less frequently navigated, witness to a persistent Pythagorean hope that would tame the wide seas of experience.

References

- Andersen, Poul-Gerhard. 1969. *Organ Building and Design*. Translated by Joanne Curnutt. London: George Allen and Unwin Ltd.
- Audsley, George Ashdown. 1905. *The Arts of Organ Building: A Comprehensive Historical, Theoretical and Practical Treatise on the Tonal Appointment and Mechanical Construction of Concert-Room, Church and Chamber Organs*, two volumes. New York: Dodd, Mead and Company.
- Barbera, André. 1984. “The Consonant Eleventh and the Expansion of the Musical Tetractys: A Study of Ancient Pythagoreanism.” *Journal of Music Theory*, 28: 191–223.
- Braun, Christoph. 1992. *Max Weber’s “Musiksoziologie.”* Laaber: Laaber-Verlag.
- Burkert, Walter. 1972. *Love and Science in Ancient Pythagoreanism*. Cambridge: Harvard University Press.
- Canguilhem, Philippe. 2001. *Fronimo de Vincenzo Galilei*. Collection “Épitome musical,” no. 9: 17–44. Paris: Minerve.
- Cassini, Paolo. 1984. “Newton: The Classical Scholia.” *History of Science* 22: 1–58.

83. *Harmonie universelle*, III, 6, Proposition XXIV, p. 359.

84. “Ce que l’on peut aussi comparer aux différentes passions, & raisons dont l’homme est tellement agité, qu’il ne peut souvent se résoudre, ny choisir aucun party de tous ceux qu’il se propose, d’autant que les différents motifs qui le poussent, comme les flots poussent le nauiere, se treuuent en équilibre.” (*Harmonie universelle*, III, 6, Proposition XXIV, p. 359.)

- Chua, Daniel K. L. 1999. *Absolute Music and the Construction of Meaning*. Cambridge: Cambridge University Press.
- Chua, Daniel K. L. 2001. "Vincenzo Galilei, Modernity and the Division of Nature." Pp. 17–29 in *Music Theory and Natural Order from the Renaissance to the Early Twentieth Century*. Edited by Suzannah Clark and Alexander Rehding. Cambridge, UK: Cambridge University Press.
- Cohen, H. F. 2010. *How Modern Science Came Into The World: Four Civilizations, One 17th-Century Breakthrough*. Amsterdam: Amsterdam University Press.
- Cohen, H. F. 1984. *Quantifying Music: The Science of Music at the First Stage of the Scientific Revolution, 1580–1650*. Dordrecht/Boston: D. Reidel Publishing Company.
- Creese, David. 2010. *The Monochord in Ancient Greek Harmonic Science*. Cambridge: Cambridge University Press.
- Dear, Peter. 1988. *Mersenne and the Learning of the Schools*. Ithaca and London: Cornell University Press.
- Dear, Peter. 1995. *Discipline and Experience: The Mathematical Way in the Scientific Revolution*. Chicago: The University of Chicago Press.
- Dostrovsky, Sigalia. 1975. "Early Vibration Theory: Physics and Music in the Seventeenth Century." *Archive for History of Exact Sciences*, 14(3): 169–218.
- Drake, Stillman. 1970. "Renaissance Music and Experimental Science." *Journal of the History of Ideas*, 31(4): 483–500.
- Fend, Michael. 2010. "Witnessing a 'Process of Rationalisation'? A Review-Essay of Max Weber's Study on Music." *Max Weber Studies* 10(1): 101–120.
- Fludd, Robert. [1617–1624] 2011. *The Temple of Music by Robert Fludd*. Edited by Peter Hauge. Farnham and Burlington, Vt: Ashgate.
- Gaffurio, Franchino. [1492] 1993. *Theorica musica*. Translated, with Introduction and Notes, by Walter Kreyszig. Edited by Claude V. Palisca. New Haven: Yale University Press.
- Galilei, Galileo. [1638] 1974. *Two New Sciences, Including Centers of Gravity and Force of Percussion*. Translated by Stillman Drake. Madison, Wisconsin: The University of Wisconsin Press.
- Galilei, Galileo. [1623] 1977. *Il Saggiatore*. Turin: Giulio Einaudi.
- Galilei, Vincenzo. 1581. *Dialogo della Musica Antica et della Moderna*. Florence: Marescotti.
- Galilei, Vincenzo. [1581] 2003. *Dialogue on Ancient and Modern Music*. Edited and translated by Claude V. Palisca. New Haven & London: Yale University Press.
- Galilei, Vincenzo. 1589. *Discorso intorno all'opere di Messer Gioseffo Zarlino, et altri importanti particolari attenenti alla musica*. Florence: Marescotti.
- Galilei, Vincenzo. [c.1589–1591] 1989a. "A Discourse Concerning the Various Opinions that the Three Most Famous Sects of Ancient Musicians

- had Concerning the Matter of Sounds and Tunings" (*Discorso intorno a diuersi pareri che hebbono le tre sette piu famose degl'antichi musici, intorno alla cosa de suoni, et degl'acchordi*). Pp. 164–179 in *The Florentine Camerata: Documentary Studies and Translations*. Edited by Claude V. Palisca. New York: Hamilton Printing Company.
- Galilei, Vincenzo. [c. 1589–1591] 1989b. "A Special Discourse Concerning the Diversity of the Ratios of the Diapason" (*Discorso particolare intorno alla diuersita delle forme del diapason*). Pp. 180–197 in *The Florentine Camerata: Documentary Studies and Translations*. Edited by Claude V. Palisca. New York: Hamilton Printing Company.
- Gouk, Penelope. 1999. *Music, Science and Natural Magic in Seventeenth-Century England*. New Haven: Yale University Press.
- Gozza, Paolo. 2000. "Introduction." Pp. 1–63 in *Number to Sound: The Musical Way to the Scientific Revolution*. Dordrecht: Kluwer Academic Publishers.
- Hacking, Ian. 2011. *Why Is There Philosophy of Mathematics At All?* Cambridge, U.K. and New York: Cambridge University Press.
- Hacking, Ian. 2012. "The Lure of Pythagoras." *Iyyun: The Jerusalem Philosophical Quarterly* 61: 1–26.
- Heilbron, John L. 2010. *Galileo*. Oxford: Oxford University Press.
- Hopkins, E. J., and E. F. Rimbault. [1877] 1965. *The Organ: Its History and Construction*, 3rd edn. Hilversum: Frits Knuf.
- Huffman, Carl A. 1993. *Philolaus of Croton, Pythagorean and Presocratic: A Commentary on the Fragments and Testimonia with Interpretive Essays*. Cambridge: Cambridge University Press.
- Huffman, Carl A. 2005. *Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King*. Cambridge: Cambridge University Press.
- Huffman, Carl A. (Ed). 2014. *A History of Pythagoreanism*. Cambridge: Cambridge University Press.
- Jackson, Myles. 2006. *Harmonious Triads: Physicists, Musicians, and Instrument Makers in Nineteenth-Century Germany*. Cambridge, Mass.: The MIT Press.
- Joost-Gaugier, Christiane L. 2009. *Pythagoras and Renaissance Europe: Finding Heaven*. Cambridge: Cambridge University Press.
- Kahn, Charles H. 2001. *Pythagoras and the Pythagoreans: A Brief History*. Indianapolis: Hackett Publishing Company, Inc.
- Kepler, Johannes. [1619] 1997. *The Harmony of the World*. Edited and translated by E. J. Aiton, A. M. Duncan, J. V. Field. Philadelphia: American Philosophical Society.
- Kepler, Johannes. 1937. *Gesammelte Werke*, Vol. VI: *Ioannis Kepleri Harmonices mundi libri V*. Edited by W. von Dyck and M. Caspar. Munich: C. H. Beck.
- Machamer, Peter. 1998. "Galileo's Machines, His Mathematic, and His Experiments." Pp. 53–79 in *The Cambridge Companion to Galileo*. Edited by P. Machamer. Cambridge: Cambridge University Press.

- McGuire, J. E., and P. M. Rattansi. 1966. "Newton and the Pipes of Pan." *Notes and Records of the Royal Society of London*, 21: 108–43.
- McMullin, Ernan. 2005. "Galileo's Theological Venture." Pp. 88–116 in *The Church and Galileo*. Edited by Ernan McMullin. Notre Dame: University of Notre Dame Press.
- Mersenne, Marin. (1636–1637). *Harmonie universelle, contenant la theorie et la pratique de la musique*. Paris: Cramoisy.
- Moyer, Ann E. 1992. *Musica Scientia: Musical Scholarship in the Italian Renaissance*. Ithaca and London: Cornell University Press.
- Newton, Isaac. 1687. *Philosophiæ Naturalis Principia Mathematica*. London: Jussu Societatis Regiæ ac Typis Joseph Streater. First edition with Newton's annotations, accessed via Cambridge Digital Library; physical location: Cambridge University Library, Adv.b.39.1.
- Nicomachus of Gerasa. 1989. *Enchiridion*. Pp. 245–269 in *Greek Musical Writings II: Harmonic and Acoustic Theory*. Edited by Andrew Barker. Cambridge: Cambridge University Press.
- O'Meara, Dominic J. 1989. *Pythagoras Revived: Mathematics and Philosophy in Late Antiquity*. Oxford: Clarendon Press.
- Orsini, Chiara. 1988. "Vincenzo Galilei (1520?–1591): La professione di un 'musicista pratico e teorico, tra aspirazioni e realtà." Pp. 89–105 in *Vincenzo Galilei: Atti del convegno di studi svoltosi nell'Aprile (1987) presso la Biblioteca Comunale*. Ed. Donata Bertoldi and Renzo Cresti. Pontedera: Bandecchi & Vivaldi.
- Orsini, Chiara. 1988. "Vincenzo Galilei." *Il Fronimo* 16 (January): 7–28.
- Palisca, Claude V. 1961. "Scientific Empiricism in Musical Thought." Pp. 91–137 in *Seventeenth Century Science and the Arts*. Edited by H. H. Rhys. Princeton: Princeton University Press.
- Palisca, Claude V. (Ed). 1989. *The Florentine Camerata: Documentary Studies and Translations*. New York: Hamilton Printing Company.
- Palisca, Claude V. 1992. "Was Galileo's Father an Experimental Scientist?" Pp. 143–151 in *Music and Science in the Age of Galileo*. Edited by Victor Coelho. Dordrecht/Boston/London: Kluwer Academic Publishers.
- Palisca, Claude V. 2006. *Music and Ideas in the Sixteenth and Seventeenth Centuries*. Edited by Thomas J. Mathiesen. Urbana and Chicago: University of Illinois Press.
- Palisca, Claude V. 2008. "Applications of Mathematic and Geometry in Galilei's *Dialogo* of 1581." Pp. 235–251 in *Music and Mathematics*. Edited by Phillippe Vendrix. Turnhout: Brepols Publishers.
- Pesic, Peter. 2014. *Music and the Making of Modern Science*. Cambridge, Mass.: The MIT Press.
- Plato. 2000. *The Republic*. Edited by G. R. F. Ferrari. Translated by Tom Griffith. Cambridge: Cambridge University Press.

- Prins, Jacomien. 2015. *Echoes of an Invisible World: Marsilio Ficino and Francesco Patrizi on Cosmic Order and Music Theory*. Leiden: Brill.
- Sachs, Klaus-Jürgen. 1970, 1980. *Mensura fistularum: Die Mensurierung der Orgelpfeifen im Mittelalter*, 1: Edition der Texte; 2: Studien zur Tradition und Kommentar der Texte. (Schriftenreihe der Walcker-Stiftung für orgelwissenschaftliche Forschung, 2/1–2.) Stuttgart (1) and Murrhardt (2): Musikwissenschaftliche Verlags-Gesellschaft.
- Steege, Benjamin. 2012. *Helmholtz and the Modern Listener*. Cambridge: Cambridge University Press.
- Stephenson, Bruce. 1994. *The Music of the Heavens: Kepler's Harmonic Astronomy*. Princeton, NJ: Princeton University Press.
- Tomlinson, Gary. 1993. *Music in Renaissance Magic: Toward a Historiography of Others*. Chicago and London: Chicago University Press.
- Turley, Alan C. 2001. "Max Weber and the Sociology of Music." *Sociological Forum* 16(4): 633–653.
- Urreiztieta, Carlos E. Calderón. 2010. "The Monochord according to Mersenne: Bits, Atoms, and some Surprises." *Perspectives on Science* 18(1): 77–97.
- Urreiztieta, Carlos E. Calderón. 2016. *El Beso y El Mordisco: Un museo imaginario con vestigios de armonía, consonancia y monocordios*. Barcelona: Huygens Editorial.
- Walker, D. P. 1985. *Music, Spirit and Language in the Renaissance*. Edited by Penelope Gouk. London: Variorum.
- Walker, D. P. 1978. *Studies in Musical Science in the Late Renaissance*. London: The Warburg Institute, The University of London; Leiden: E. J. Brill.
- Walker, D. P. 1958. *Spiritual and Demonic Magic*. London: The Warburg Institute, The University of London.
- Weber, Max. 1958. *The Rational and Social Foundations of Music*. Translated by Don Martindale, Johannes Riedel, and Gertrude Neuwirth. Carbondale: Southern Illinois University Press.
- Weber, Max. 2004. *Zur Musiksoziologie: Nachlaß 1921*. Edited by Christoph Braun, Ludwig Finscher. *Max Weber Gesamtausgabe*. Part I: *Schriften und Reden*. Vol. XIV. Tübingen: J. C. B. Mohr.
- Werckmeister, Andreas. 1698. *Erweierte und verbesserte Orgel-Probe*. Quedlinburg: Calvisi.
- Yates, Frances A. 1967. "The Hermetic Tradition in Renaissance Science." Pp. 255–274 in *Art, Science, and History in the Renaissance*. Edited by C. S. Singleton. Baltimore: The Johns Hopkins Press.
- Zarlino, Gioseffo. 1558/rev. 1573. *Le institutioni harmoniche*. Venice: Franceschi.
- Zhmud, Leonid. 2012. *Pythagoras and the Early Pythagoreans*. Translated by Kevin Windle and Rosh Ireland. Oxford: Oxford University Press.