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Matteo Varoli¹
“Pythagoreans” and Aristoxenians in Ptolemais’s
Pythagorean Elements of Music

The treatise of Ptolemais in context

Every extant piece of evidence about the only known female musicologist of antiquity, Ptolemais of Cyrene, is transmitted through Porphyry’s *Commentary on Ptolemy’s Harmonics* (1.2.22–26, 1.7.114). Nothing is known on Ptolemais, except that she was from Cyrene and that she was a woman,² a fact that doesn’t appear to be striking to Porphyry.³

The issue at stake in this part of Porphyry’s commentary are the foundations of harmonic science. In the very beginning of Ptolemy’s treatise, the question addressed is “what is the task of the *harmonikos*?” and on which criteria should they build their theorems? Should they take into account reason alone, sensation alone, or rather a combination of both? And how should they combine them (Ptolemy, *Harmonics* 1.1–2)? The issue is very relevant to the history of music theory, because late Hellenistic theorists perceived the history of their own discipline as a dispute between two major schools. An

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² For Ptolemais, see Gabriella Moretti, “Tolomeide di Cirene. Musicologa dell’antichità,” *Kleos* 9 (2004): 123–152; Eleonora Rocconi, “Un manuale al femminile: l’*Introduzione pitagorica alla musica* di Tolomaide di Cirene,” in: *Ars/Techne*, ed. Maria Silvana Celentano (Alessandria: Edizioni dell’Orso, 2003), 99–114; Constantinos Macris, “Ptolémaïs de Cyrène,” *Dictionnaire des Philosophes Antiques* 5b (Paris: CNRS éditions, 2012), 1717–1718.

³ Cf. Moretti, “Tolomeide,” 125; Rocconi, “Un manuale,” 99.

approach commonly defined as “Pythagorean” described musical consonances as *logoi* (numerical ratios), and therefore in terms of quantity: the musicologists belonging to this school represent the musical ratios on an instrument composed by a string divided by a mobile bridge, the *kanōn*, but the scales individuated through this method were often impossible to be applied to musical practice. The followers of Aristoxenus, a pupil of Aristotle who wrote an authoritative work on music theory, thought of the differences between sounds as intervals (*diastēmata*), in terms of quality;⁴ Aristoxenian music theory did admit operations like the halving of the tone, impossible in the Pythagorean scales.⁵ While the “Pythagorean” view was

⁴ I leave aside the question of the historical accuracy of this picture presented by the ancient sources. For the Academic distortion of the sources on ancient Pythagorean antiempiricism, see Walter Burkert, *Lore and Science in Ancient Pythagoreanism*, trans. Edward J. Minar jr. (Cambridge, MA: Harvard University Press, 1972), 369–386. On the label of empiricism attached to Aristoxenus in contrast with his actual theory, see Andrew Barker, “Music and Perception: A Study in Aristoxenus,” *The Journal of Hellenic Studies* 98 (1978): 9–16. For a general survey on this schematization of the ancient debate about harmonics and its origin in the peripatetic school, see Andrew Barker, *Greek musical writings, vol. II: Harmonic and acoustic theory* (Cambridge: Cambridge University Press, 1989), 3–11; Massimo Raffa, “The debate on *logos* and *diastēma* in Porphyry’s Commentary on Ptolemy’s Harmonics,” *Greek and Roman Musical Studies* 1 (2013): 243–52; and Ambra Tocco, “Pensare i suoni, descrivere la musica. Λόγος e αἴσθησις nella scienza armonica di età ellenistica,” in: *Poesia e prosa in età ellenistica. In ricordo di Roberto Pretagostini*, ed. Mauro Tulli (Pisa-Roma: Fabrizio Serra Editore, 2017), 51–60.

⁵ A demonstration that the superparticular ratios of music cannot be halved survives from Archytas, A19 Huffman, and appears in a similar form in [Euclides], *The Division of the Canon*, proposition 3.

embraced by Ptolemy with some reservations (*Harmonics* 1.3–6 and 3.3),⁶ Porphyry engages himself, in the first part of his commentary, in defending a position closer to the peripatetic theory, quoting a huge variety of otherwise lost fundamental texts of ancient music theory from the old Academy and the ancient Peripatos onwards.

Ptolemais is quoted in this very context, along with other musicologists. Porphyry claims to use the work of previous musicologists in his commentary, and says that some of them are also Ptolemy's sources (*Commentary on Ptolemy's Harmonics* prol. 4–5). He also notices that Ptolemy never quotes his sources. As an example of an unacknowledged source of Ptolemy's work, Porphyry explicitly mentions Didymus, the author of a work entitled *On the difference between the Pythagorean and the Aristoxenian music*. This work of Didymus is indeed very close in content to that of Ptolemais, and Porphyry quotes them together in the discussion of the Ptolemaic theory of the criterion. Since this Didymus lived in the age of Nero,⁷ Ptolemais may be placed not too distant in time. As for Düring's guess that Porphyry knew the excerpts of Ptolemais through Didymus,⁸ assuming that her work was

See Carl A. Huffman, *Archytas of Tarentum. Pythagorean, Philosopher and Mathematician King* (Cambridge: Cambridge University Press, 2005), 451–470; Fabio Acerbi, *Euclide, Tutte le opere* (Milano: Bompiani, 2007), 680–682.

⁶ On Ptolemy's position and Porphyry's arguments against him, see Massimo Raffa, *Claudio Tolomeo, Armonica; con il Commentario di Porfirio* (Milano: Bompiani, 2016), 13–28, 54–61.

⁷ According to Barker, *Musical Writings*, 230, he is identified with "Didymus, the son of Heraclides," mentioned in the *Suda* (δ 875).

⁸ I. Düring, *Ptolemaios und Porphyrios über die Musik* (Göteborg: Elanders Boktryckeri Aktiebolag, 1934), 144. This suggestion is widely spread in later literature, despite the weak evidence; see Barker, *Musical Writings*, 239 n. 133.

earlier and that it was used by Didymus as a source, caution is required since Porphyry might have had Ptolemais's original text, or even a miscellaneous scroll with the works of several different authors.⁹

Porphyry seems to quote Ptolemais in an unfinished and unrevised section of his work, as the quotes themselves show. The second extract (*Commentary on Ptolemy's Harmonics* 1.2.23–24), containing an account of the “Pythagorean” position on the criterion of knowledge, is repeated in full at the end of the third extract (1.2.25–26); it is unclear whether the repetition is due to a lack of revision by Porphyry, or to a mechanical error in the process of copying the excerpts in the work.¹⁰ Moreover, it is not always simple to determine where the quotes of Ptolemais end and the commentary of Porphyry begins, especially in the first extract (1.2.22–23).

The form of Ptolemais's work is peculiar: it was arranged according to inquiries (*zētēmata*), presented through questions and answers (*kat' erōtēsīn kai apokrisin*). Such a form seems to suggest that the work of Ptolemais was conceived for didactic purposes. The title reported by Porphyry, *Pythagorean Elements of Music*,¹¹ seems to promise a “Pythagorean” approach to music theory, but, as already noticed by Barker,¹² the doxographic section presents the theory of Aristoxenus in a quite favorable light, while the “Pythagoreans” seem to be the object of a criticism. In order to understand whether Ptolemais criticized, defended, or simply represented “Pythagorean” harmonics within a wider debate on the nature of harmonic

⁹ I share the objections listed in Ambra Tocco, “Ptolemais,” in *Supplementum Grammaticum Graecum* 5, ed. F. Montana (Leiden: Brill, 2021), 202–203.

¹⁰ See further, note 15 below.

¹¹ Porphyry, *Commentary on Ptolemy's Harmonics* 1.2.22.

¹² Barker, *Musical Writings*, 230.

science, a closer survey of the meaning of "Pythagorean" in the title and in the extant fragments of Ptolemais¹³ is needed.

Ptolemais on the *kritēron* of harmonics

The longest extant fragment from the work of Ptolemais consists of a piece of doxography used by Porphyry along with a passage from Didymus, offered as a commentary to Ptolemy, *Harmonics* 1.2. Here, the latter explains the approach to the problem of the criterion among the Pythagoreans and the Aristoxenians, claiming that both are wrong about the use of sense perception: the Pythagoreans, since they do not use sensation as a criterion even where it would be needed, obtain results that differ from sensation, and the Aristoxenians, since they go against both reason and sensation by choosing the qualitative approach. Interestingly, Ptolemais's account of the different schools of harmonics and their approach to the problem is much more articulate and complex than those of Ptolemy and Didymus, sketching out a variety of positions.

Porphyry claims to quote the excerpt "altering a few things for the sake of brevity" as follows:

What is the difference between those who are distinguished in the field of music? Some preferred reason by itself, some perception, some both together. Reason was preferred by those of the Pythagoreans who were especially keen on disputing with the *mousikoi*,¹⁴ arguing that perception should be thrown out completely, and that reason should be brought in as an autonomous criterion in itself. These people are

¹³ On the meaning and technical use of "Pythagorean" in Porphyry's work, see Andrew Barker, *Porphyry's Commentary on Ptolemy's Harmonics* (Cambridge: Cambridge University Press, 2015), 28–32.

¹⁴ In fr. 1 Barker (*Porphyry's Commentary* 1.2.23), Ptolemais makes clear that, in its technical meaning, the term *mousikoi* refers to "the theorists who begin from perception."

wholly refuted by their practice of accepting something perceptible at the beginning, and then forgetting that they have done so. The instrumentalists, on the other hand, preferred perception: they gave no thought at all, or only feeble thought, to theory.

Τῶν ἐν τῇ μουσικῇ διαπρεψάντων τίς ἡ διαφορά; οἱ μὲν γὰρ τὸν λόγον προέκριναν αὐτόν, οἱ δὲ τὴν αἴσθησιν, οἱ δὲ τὸ συναμφοτέρων. τὸν μὲν λόγον προέκρινον αὐτόν τῶν Πυθαγορείων ὅσοι μᾶλλον ἐφιλονείκησαν πρὸς τοὺς μουσικοὺς τελέως τὴν αἴσθησιν ἐκβάλλειν, τὸν δὲ λόγον ὡς αὐταρκες κριτήριον καθ' ἑαυτὸν εἰσφέρειν. ἐλέγχονται δ' οὗτοι πάντως τι αἰσθητὸν παραλαμβάνοντες ἐν ἀρχῇ καὶ ἐπιλανθανόμενοι. τὴν δ' αἴσθησιν προέκριναν οἱ ὀργανικοί, οἷς ἡ οὐδαμῶς ἔννοια θεωρίας ἐγένετο ἢ ἀσθενής.

What is the distinction between those who prefer the combination of both? Some accepted both perception and reason in the same way, as being of equal power, others accepted the one as the leader, the other as the follower. Aristoxenus of Tarentum accepted both in the same way. For what is perceived cannot be constituted in itself apart from reason, and neither is reason strong enough to establish anything without taking its *archai* from perception, and delivering the conclusions of its theorizing in agreement with perception once again. In what way does he want perception to be in the lead of reason? In order, not in power. For when the perceptible thing, whatever it may be, has been reviewed by perception, then, he says, we must put reason in the lead, for the theoretical study of this percept.

τῶν δὲ τὸ συναμφοτέρων προκρινάντων τίς ἢ διαφορά; οἱ μὲν ὁμοίως ἀμφοτέρω ἰσοδυναμοῦντα παρέλαβον τὴν τ' αἰσθησιν καὶ τὸν λόγον, οἱ δὲ τὸ ἕτερον προηγούμενον, τὸ δ' ἕτερον ἐπόμενον. ὁμοίως μὲν ἀμφοτέρω Ἀριστόξενος ὁ Ταραντῖνος. οὔτε γὰρ αἰσθητὸν δύναται συστήναι καθ' αὐτὸ δίχα λόγου, οὔτε λόγος ἰσχυρότερός ἐστι παραστήσαι τι μὴ τὰς ἀρχὰς λαβὼν παρὰ τῆς αἰσθήσεως, καὶ τὸ τέλος τοῦ θεωρήματος ὁμολογούμενον πάλιν τῇ αἰσθήσει ἀποδιδούς. τί δὲ μᾶλλον βούλεται προηγῆσθαι τὴν αἰσθησιν τοῦ λόγου; τῇ τάξει, οὐ τῇ δυνάμει. ὅταν γάρ, φησί, ταύτῃ τὸ αἰσθητὸν συναφθῇ ὁποῖον ποτέ ἐστι, τότε δεῖν ἡμᾶς καὶ τὸν λόγον προάγειν εἰς τὴν τούτου θεωρίαν.

†Who are those who treat both together alike? †¹⁵ Pythagoras and his successors. For they wish to accept perception as a guide for reason at the outset, to provide reason with a spark, as it were; but they treat reason, when it has set out from these beginnings, as working on its own in separation from perception. Hence if the *systema* discovered by reason in its investigation no longer accords with perception, they

¹⁵ The text seems to have suffered some corruption here, since one expects to have the exposition about those who admit both criteria, taking reason as a leader (see Barker, *Musical Writings*, 242 n. 148). The question seems out of place and would be much more suitable to introduce the paragraph on Aristoxenus. The error may possibly be attributed to Porphyry himself, since he explicitly claims to shorten and summarize parts of the text, and since a lack of final revision is evident in this part of his work. Perhaps this is not unrelated to the fact that a great part of this fragment, from this exact point to the end of the excerpt, is a repetition of the other fragment of Ptolemais, at 1.2.23–24. See Raffa, *Claudio Tolemeo*, 724–725 n. 100, and Tocco, "Ptolemais", 220–221.

do not retrace their steps, but level accusations, saying that perception is going astray, while reason by itself has discovered what is correct, and refutes perception.

τίνες τὸ συναμφοτέρων ὁμοίως;† Πυθαγόρας καὶ οἱ διαδεξάμενοι. βούλονται γὰρ αὐτοὶ τὴν μὲν αἴσθησιν ὡς ὁδηγὸν τοῦ λόγου ἐν ἀρχῇ παραλαμβάνειν πρὸς τὸ οἰονεῖ ζώπυρά τινα παραδιδόναι αὐτῷ, τὸν δὲ λόγον ἐκ τούτων ὀρμηθέντα καθ' ἑαυτὸν πραγματεύεσθαι ἀποστάντα τῆς αἰσθήσεως, ὅθεν καὶ τὸ σύστημα τὸ ὑπὸ τοῦ λόγου εὑρηθὲν τῆς πραγματείας μηκέτι συνάδη τῇ αἰσθήσει, οὐκ ἐπιστρέφονται, ἀλλ' ἐπεγκαλοῦσι λέγοντες τὴν μὲν αἴσθησιν πλανᾶσθαι, τὸν δὲ λόγον εὑρηκέναι τὸ ὀρθὸν καθ' ἑαυτὸν καὶ ἀπελέγχειν τὴν αἴσθησιν.

Who are in opposition to these? Some of the *mousikoi* who follow Aristoxenus, those who applied themselves to a theoretical science based in thought, while nevertheless setting out from the expertise on instruments. For they treated perception as authoritative, and reason as attending on it, for use only when needed.

τίνες ἐναντίως τούτοις; ἔνιοι τῶν ἀπ' Ἀριστοξένου μουσικῶν, ὅσοι κατὰ μὲν τὴν ἔννοιαν θεωρίαν ἔλαβον, ἀπὸ δ' ὀργανικῆς ἕξεως προκόψαντες. οὗτοι γὰρ τὴν μὲν αἴσθησιν ὡς κυρίαν ἔθεσαν, τὸν δὲ λόγον ὥσπερ ἐπόμενον εἰς μόνον τὸ χρειῶδες.¹⁶

¹⁶ Ptolemais, fr. 3 Barker = Porphyry, *Commentary on Ptolemy's Harmonics* 1.2.25–26), ed. Tocco, “Ptolemais,” 216; the texts of the fragments of Ptolemais are quoted in Tocco’s edition, unless otherwise specified. Trans. Barker, *Musical Writings*, 241–42. All

Ptolemais identifies five different positions: in the first place, she outlines two "extreme" factions that do accept just a single criterion of truth in harmonics. The partisans of pure *aisthēsis* are the so-called *organikoi*. Ptolemais does not give us more clues to understand who these people might be, but it is clear that she has the *Elementa Harmonica* of Aristoxenus in mind. The Tarentine philosopher alludes several times within his work to "those who worked on instruments,"¹⁷ a current of pure empiricists that are accused of "delivering oracular utterances on individual topics, without giving explanations or demonstrations."¹⁸ Didymus, in his own account, compares them to the *phōnaskikoi*, "vocal trainers."¹⁹

The opposite position is held by "some Pythagoreans," who want to eliminate sense perception, and assume reason as a self-sufficient criterion. As we shall see, they share with the "Pythagoreans" the assumption that perception can be deceiving, and that the ultimate object of harmonics is not audible music. Their position is discredited by Ptolemais as untenable because they "forget" that the postulates of their science rely on sense-perception. Interestingly, Ptolemais does not present this radically anti-empirical position as the original Pythagorean one. She explicitly says that these "Pythagoreans" developed their theory later, in opposition to the empiricism of the *mousikoi*.

Having rejected these two extreme positions, Ptolemais passes to an investigation of three positions that do accept a twofold criterion: the first one accords an equal importance to

further translations are from Barker's work, unless otherwise specified. Relevant divergences between Tocco's text and Barker's textual choices will be discussed in footnotes.

¹⁷ Aristoxenus, *Elements of Harmony* 2.35.8; cf. 2.41.25–30.

¹⁸ Aristoxenus, *Elements of Harmony* 2.32.28–31.

¹⁹ Not to be understood as music theorists, see Düring, *Ptolemaios und Porphyrios*, 145; Tocco, "Pensare i suoni," 54–55.

sense perception and reason, while the others assign a leading role respectively to latter. Her approach seems to reflect the Hellenistic debate on the criterion of knowledge, and closely resembles the views of the Peripatetic school in this respect.²⁰ First, she presents the position of Aristoxenus as holding both reason and sense perception in equal importance, with the latter preceding in the order of the cognitive processes, and also ending it by validating the conclusions of reason. Her account of Aristoxenian thought is of the greatest interest: while Aristoxenus did not explain the issue of the criterion in these terms, speaking of *dianoia* and *akoē* rather than *logos* (reason) and *aisthēsis* (sense-perception), Ptolemais seems to refer to what he says in the first part of the second book of the *Elements of Harmony*:²¹

Taken as a whole, our science is concerned with musical melody, both vocal and instrumental. Its pursuit depends ultimately on two things, hearing and reason. Through hearing we assess the magnitudes of intervals, and through reason we apprehend their functions. We must therefore become practised in assessing particulars accurately. While it is usual in dealing with geometrical diagrams to say “Let this be a straight line”, we must not be satisfied

²⁰ See Han Baltussen, “Peripatetic Epistemology after Aristotle: Theorising Knowledge from Theophrastus to Aristocles,” in: *Hellenistic Theories of Knowledge*, eds. F. Verde and M. Catapano (*Lexicon philosophicum* 6, special issue, 2018), 53–75.

²¹ Aristoxenus, *Elements of Harmony*, 2.32–33, cf. 2.41–44. Comparison with the similar account of Aristoxenus’s thought from Didymus (in Porphyry, *Commentary on Ptolemy’s Harmonics*, 1.2.28) shows that these authors might have read the *Elements* in a different arrangement than the one we know, since he explicitly claims to quote the “first book” of the *Elements*.

with similar remarks in relation to intervals. The geometer makes no use of the faculty of perception: he does not train his eyesight to assess the straight or the circular or anything else of that kind either well or badly. [...] But for the student of music accuracy of perception stands just about first in order of importance [...].

Ἔστι δὴ τὸ μὲν ὅλον ἡμῖν <ή> θεωρία περὶ μέλους παντὸς μουσικοῦ τοῦ γιγνομένου ἐν φωνῇ τε καὶ ὀργάνοις. ἀνάγεται δ' ἡ πραγματεία εἰς δύο, εἰς τε τὴν ἀκοὴν καὶ εἰς τὴν διάνοιαν. τῇ μὲν γὰρ ἀκοῇ κρίνομεν τὰ τῶν διαστημάτων μεγέθη, τῇ δὲ διανοίᾳ θεωροῦμεν τὰς τούτων δυνάμεις. δεῖ οὖν ἐθισθῆναι ἕκαστα ἀκριβῶς κρίνειν· οὐ γὰρ ἐστὶν ὥσπερ ἐπὶ τῶν διαγραμμάτων εἴθισται λέγεσθαι· ἔστω τοῦτο εὐθεῖα γραμμὴ,—οὕτω καὶ ἐπὶ τῶν διαστημάτων εἰπόντα ἀπηλλάχθαι δεῖ. ὁ μὲν γὰρ γεωμέτρης οὐδὲν χρῆται τῇ τῆς αἰσθήσεως δυνάμει, οὐ γὰρ ἐθίζει τὴν ὄψιν οὔτε τὸ εὐθὺ οὔτε τὸ περιφερὲς οὔτ' ἄλλο οὐδὲν τῶν τοιούτων οὔτε φαύλως οὔτε εὖ κρίνειν· [...] τῷ δὲ μουσικῷ σχεδὸν ἐστὶν ἀρχῆς ἔχουσα τάξιν ἢ τῆς αἰσθήσεως ἀκρίβεια [...].²²

The parallel that Aristoxenus traces between the geometer and the student of music is of key importance in understanding what Ptolemais is saying: while the *archai* of any demonstration in arithmetic or geometry can be assumed as hypotheses, musical science cannot work like that. Harmonics needs to take its postulates from sense perception, and from the sense of

²² Aristoxenus, *Elements of Harmony*, 2.33, ed. Paul Marquard, *Die Harmonischen Fragmente des Aristoxenus* (Berlin: Weidmann, 1868), 48. Trans. Barker, *Musical Writings*, 150.

hearing in particular, unlike the sense of sight in geometry that provides just a first sparkle to reason through diagrams. In geometry, the validity of a theorem is completely independent from perception and does not need experimental proof, while in harmonics every result obtained by reason into its derivative process that does not fit with the evidence of sense perception, has to be considered as wrong.²³

Indeed, Ptolemais simplifies the position of Aristoxenus on the role of human *logos* in harmonics. Reason, to him, is very important not only in the process of *apodeixis*, but also in the very beginning of the cognitive process, the necessary rational reflection on perception that makes the sensory experience meaningful in the first place.²⁴ Overall, however, her account is quite fair. She also mentions a slightly different position, which she assigns to some followers of Aristoxenus, that reduces the role of reason to that of an “attendant” of perception. In Porphyry’s *Commentary on Ptolemy’s Harmonics*, 1.2.26–27, a very similar view is attributed by Didymus to the musician Archestratus, a shadowy figure also mentioned by Philodemus in his treatise *On Music* (4.137.13–27).²⁵

Ptolemais’s main goal here is to contrast Aristoxenus and the Aristoxenians with another school of theorists, Pythagoras and his pupils, who treated harmonics in the way Aristoxenus later forbade—as a science that relies on sense perception only at the very starting point, like geometry. It is not just a matter of choosing to describe the consonances in terms of quantity rather than of quality, in Ptolemais’s view. According to these philosophers, sounds and notes have, for harmonic science, the

²³ On the theory of knowledge of Aristoxenus, see Barker, “Music and Perception,” 9–16.

²⁴ Barker, *Musical Writings*, 150 n. 13.

²⁵ See Andrew Barker, “Musical Theory and Philosophy: The Case of Archestratus,” *Phronesis* 54.4/5 (2009): 390–422.

same value that the *diagrammata* have for geometry. Both the process of reasoning and the validity of its conclusions must be independent from sense perception; and if their systems and evidence are in disagreement, Ptolemais comments quite sharply, they level accusations against the latter, pointing the finger at the unreliability of sense perception.

Ptolemais appears to be quite hostile to both of the factions of Pythagoreans involved. While completely discrediting the most radical faction, she discusses more attentively the position she attributes to the historical Pythagoras, a position that is therefore credited as the "original" one. Rather than asking whether Ptolemais's account might reflect some historical division between scholars of music, we should rather investigate which core texts she bore in mind, and how she reworked her source material.

One of the earliest accounts of Pythagorean music theory is found in the famous words of Socrates in the seventh book of Plato's *Republic*, the only place in the Platonic *corpus* where the Pythagoreans are explicitly mentioned as a whole. As Socrates and Glaucon finish their long discussion about the reform of the astronomical science, similar problems arise in the case of harmonics:

‘It appears,’ I said, ‘that just as the eyes are fixed to astronomy so the ears are fixed on harmonic motion, and that these two sciences are one another’s sisters, as the Pythagoreans say and we agree, Glaucon. Or what we do?’ ‘As you say,’ he said. [...]

Κινδυνεύει, ἔφη, ὡς πρὸς ἀστρονομίαν ὄμματα πέπηγεν, ὡς πρὸς ἐναρμόνιον φορὰν ὥτα παγῆναι, καὶ αὐταὶ ἀλλήλων ἀδελφαὶ τινες αἱ ἐπιστῆμαι εἶναι, ὡς οἱ τε Πυθαγόρειοί φασι καὶ ἡμεῖς, ὦ Γλαῦκον, συγχωροῦμεν. ἢ πῶς ποιοῦμεν; οὕτως, ἔφη. [...]

‘[...] They measure heard concords and notes against one another, and so labor to no purpose, just like the astronomers.’ ‘Yes, by the gods,’ he said. ‘Their behavior is quite ridiculous, when they name some things *puknomata* and incline their ears as if hunting out a sound from the next door, some of them asserting that they can still just hear a sound in between, and that that is the smallest interval, by which measurement is to be made, while others take issue with them, saying that the notes are already the same, each group putting their ears ahead of their mind.’

[...] τὰς γὰρ ἀκουόμενας αὖ συμφωνίας καὶ φθόγγους ἀλλήλοις ἀναμετροῦντες ἀνήνυτα, ὥσπερ οἱ ἀστρονόμοι, πονοῦσιν. νῆ τοὺς θεούς, ἔφη, καὶ γελοίως γε, πυκνώματ’ ἅττα ὀνομάζοντες καὶ παραβάλλοντες τὰ ὦτα, οἷον ἐκ γειτόνων φωνὴν θηρευόμενοι, οἱ μὲν φασιν ἔτι κατακούειν ἐν μέσῳ τινὰ ἤχην καὶ σμικρότατον εἶναι τοῦτο διάστημα, ᾧ μετρητέον, οἱ δὲ ἀμφισβητοῦντες ὡς ὅμοιον ἤδη φθεγγομένων, ἀμφότεροι ὦτα τοῦ νοῦ προστησάμενοι.

‘You are talking’ I said, ‘about those worthy persons who bully the strings and interrogate them with torture, racking them on the *kollopes*; but I must not spin out the image too long; [...] and so I shall abandon the image and say that I do not mean that people, but those whom we said just now we would question about *harmonia*. They do the same as those concerned with astronomy: they seek the numbers in these heard concords, but do not rise to problems, to investigate which numbers are concordant and which

are not, and why each are so.' 'The task you mention is superhuman,' he said.

Σὺ μὲν, ἦν δ' ἐγώ, τοὺς χρηστοὺς λέγεις τοὺς ταῖς χορδαῖς πρᾶγματα παρέχοντας καὶ βασανίζοντας, ἐπὶ τῶν κολλόπων στρεβλοῦντας· ἵνα δὲ μὴ μακροτέρᾳ ἢ εἰκῶν γίγνηται [...] παύομαι τῆς εἰκόνης καὶ οὐ φημι τούτους λέγειν, ἀλλ' ἐκείνους οὓς ἔφαμεν νυνδὴ περὶ ἀρμονίας ἐρήσεσθαι. ταυτὸν γὰρ ποιοῦσι τοῖς ἐν τῇ ἀστρονομίᾳ· τοὺς γὰρ ἐν ταύταις ταῖς συμφωνίαις ταῖς ἀκουομέναις ἀριθμοὺς ζητοῦσιν, ἀλλ' οὐκ εἰς προβλήματα ἀνίσκιν, ἐπισκοπεῖν τίνες σύμφωνοι ἀριθμοὶ καὶ τίνες οὐ, καὶ διὰ τί ἐκάτεροι. | Δαιμόνιον γάρ, ἔφη, πρᾶγμα λέγεις.

'But it is at any rate useful' I said, 'in the quest for the fine and the good, whereas if pursued in any other way it is useless.'

χρήσιμον μὲν οὖν, ἦν δ' ἐγώ, πρὸς τὴν τοῦ καλοῦ τε καὶ ἀγαθοῦ ζήτησιν, ἄλλως δὲ μεταδιωκόμενον ἄχρηστον.²⁶

This page of Plato's *Republic* has been the object of complex debate.²⁷ For our purpose, we notice that the Pythagoreans are

²⁶ Plato, *Republic* 7.530d–531, ed. John Burnet (Oxford: Oxford University Press, 1903); trans. Barker, *Musical Writings*, 55–56.

²⁷ The problems are many, from the identification of the various theorists alluded to in the passage to the relationship with Pythagorean literature, since the reference to the "sister sciences" echoes the famous fragment 1 Huffman of Archytas, and finally to the correct interpretation of the radical anti-empirical view on music expressed in the passage. For an exhaustive overview, see Angelo Meriani, "Teoria musicale e antiempirismo," in *Platone*,

evoked by Socrates as the best interlocutors for the science of harmonics since they relate it to astronomy. Eyesight and hearing are keen and “sisters” because they are the only senses that seem to entertain a privileged tie not only to motion, but also to intellect and mathematical beauty, as Ptolemy also underlines (*Harmonics* 3.3.93–94), probably bearing in mind these words of Plato. The experts that Socrates compares in what follows might comprehend, of course, but they are not necessarily limited to the Pythagoreans; Socrates is clearly speaking about mathematical harmonics in general. Glaucon’s misunderstanding gives Socrates the chance to direct his irony towards empirical musicians who try to find the *puknomata* through perception. But, as Socrates makes clear, it is not a matter of empiricism against reason here: he intends to direct his criticism toward the mathematical harmonics of his time, which do not elevate themselves to posing problems. Coherently with their program mathematical science reform,²⁸ Socrates and Glaucon aim to give to harmonics a foundation completely independent from perceptible objects and sense perception. It is unsatisfactory, in Plato’s view, to individuate numbers in heard consonances. The real question would rather be which numbers are concordant and which not, and why.

This view of Plato on the scope and object of harmonics was challenged by the Peripatetics, as is evident in the passage of Aristoxenus quoted above, within the frame of the dispute between the Academics and Peripatetics on the object and method of the mathematical sciences—in particular, what

“*Repubblica*”, *Libri VI–VII*, ed. Mario Vegetti (Napoli: Bibliopolis, 2003), 565–602; see also Ferruccio Franco Repellini, “Astronomia e armonica,” in the same volume, 541–563.

²⁸ See Elisabetta Cattanei, “Le matematiche al tempo di Platone e la loro riforma,” in: *Platone, “Repubblica”, Libri VI–VII*, ed. Mario Vegetti (Napoli: Bibliopolis, 2003), 473–493.

Aristotle says in the *Posterior Analytics* in response to Plato's *Republic* VII, concerning the objects of sciences. He claims that the *dioti* (the "why", i.e. demonstrative reason) is the field of study of mathematical sciences such as harmonics, while it is the scope of perception to gain knowledge of the facts (the *hoti*):

There is another way, in which the *hoti* and *dioti* differ, viz., in each being studied by a different science. This is true of all subjects which are so related that one is subordinate to the other, as in the relation of optical problems to plane and of mechanical problems to solid geometry and of harmonical problems to arithmetic and of the study of phenomena to astronomy. Some of these sciences have practically the same name; e.g., [...] both mathematical harmonics and acoustic harmonics are called harmonics. In these cases, it is for the collectors of data to know the *hoti*, and for the mathematicians to establish the *dioti*. The latter can demonstrate the causes, whereas they are often ignorant of the *hoti* [...]. Of this kind are all objects which, while having a separate substantial existence, yet exhibit certain specific forms. For the mathematical sciences are concerned with forms; they do not confine their demonstrations to a particular substrate.

Ἄλλον δὲ τρόπον διαφέρει τὸ διότι τοῦ ὅτι τῷ δι' ἄλλης ἐπιστήμης ἐκάτερον θεωρεῖν. τοιαῦτα δ' ἐστὶν ὅσα οὕτως ἔχει πρὸς ἄλληλα ὥστ' εἶναι θάτερον ὑπὸ θάτερον, οἷον τὰ ὀπτικά πρὸς γεωμετρίαν καὶ τὰ μηχανικά πρὸς στερεομετρίαν καὶ τὰ ἀρμονικά πρὸς ἀριθμητικήν καὶ τὰ φαινόμενα πρὸς ἀστρολογικήν. σχεδὸν δὲ συνώνυμοί εἰσιν ἔναι τούτων τῶν ἐπιστημῶν, οἷον [...] ἀρμονικὴ ἢ τε μαθηματικὴ καὶ ἡ κατὰ τὴν

ἀκοήν. ἐνταῦθα γὰρ τὸ μὲν ὅτι τῶν αἰσθητικῶν εἰδέναι, τὸ δὲ διότι τῶν μαθηματικῶν· οὗτοι γὰρ ἔχουσι τῶν αἰτίων τὰς ἀποδείξεις, καὶ πολλάκις οὐκ ἴσασι τὸ ὅτι [...]. ἔστι δὲ ταῦτα ὅσα ἕτερόν τι ὄντα τὴν οὐσίαν κέχρηται τοῖς εἶδεσιν. τὰ γὰρ μαθήματα περὶ εἶδη ἐστίν· οὐ γὰρ καθ' ὑποκειμένου τινός.²⁹

Aristotle, discussing the role played by demonstration in different sciences, focuses on the forms of knowledge that refer to separate, perceptible objects, such as harmonics, astronomy, optics and mechanics. By highlighting their subordination to sciences such as arithmetic (as in the case of harmonics) and geometry, Aristotle shows that, also for these mathematical sciences that describe the phenomena, the real object of study is not the *hoti*, the thing or the phenomenon itself, but rather the *dioti*. Actually, in some cases the study of the phenomenon and the mathematical knowledge of the reasons are one science in name: this is precisely the case with harmonics.

Aristotle outlines the distinction between mathematical harmonics and harmonics as the observation of acoustic phenomena that resurfaces in Ptolemais's treatise. In his opinion, these two branches of harmonics simply have a different scope: while knowledge of the notes and consonances of music falls within the realm of harmonics *kata tēn akoēn*, mathematical harmonics is able to investigate the causes and reasons behind them, without even considering them in their physical, acoustic aspect. This is why mathematical sciences deal with the forms, and not with the objects. According to Aristotle, who defended this view against Plato and the Academics, the objects of mathematics are neither separate from the sensory, nor perceptible things themselves. The

²⁹ Aristotle, *Posterior Analytics* 78b 34–79a8, trans. Hugh Tredennick (Cambridge MA: Harvard University Press, 1960), modified.

mathematician must exercise an abstraction of properties from the objects, not considering them as belonging to a substrate.³⁰ This is evident for "hard sciences" such as arithmetic and geometry, but it is true also in the case of sciences such as harmonics, which apparently have their own, physical object.

Ptolemais's *kanonikē* and Hellenistic harmonics

Ptolemais, very aware of this debate, ties her discussion of harmonics to it. The example Plato chose of a mathematical truth that needs investigation on the ground of principles, that the concords consist of numerical ratios, is confronted by Ptolemais in another fragment. She claims that, while the existence of consonances and dissonances is a postulate assumed through perception by the musicians, the fact that the *diastēmata* consist of numerical ratios is one of the objects of the investigation of mathematicians, which the experts of *kanonikē* study in their own, "special" way:

The theory that uses the *kanōn*—of what does it consist? Of the things postulated by the *mousikoi* and those adopted by the *mathēmatikoi*. | The things postulated by the *mousikoi* are all those adopted by the *kanonikoi* on the basis of perceptions, for instance that there are concordant and discordant intervals, and the octave is compounded by a fourth and a fifth, and that the excess of a fifth over a fourth is a tone, and similar things. Those adopted by the *mathēmatikoi* are all those which the *kanonikoi* study theoretically in their own special way, only beginning from the starting points given by perception, for instance that the intervals are in ratios of numbers, and that a note

³⁰ Cf. Elisabetta Cattanei, *Enti matematici e metafisica* (Milano: Vita e Pensiero, 1996), 178–188.

consists of numbers colliding at once,³¹ and other things of the same sort. Hence one might define the postulates of *kanonikē* as lying both within the science concerned with music, and within that concerned with numbers and geometry.

Ἡ κατὰ τὸν κανόνα θεωρία, ἐκ τίνων σύγκειται; ἐκ τῶν παρὰ τοῖς μουσικοῖς ὑποτιθεμένων καὶ ἐκ τῶν παρὰ τοῖς μαθηματικοῖς λαμβανομένων. Ἰ Ἔστι δὲ τὰ παρὰ τοῖς μουσικοῖς ὑποτιθέμενα, ὅσα παρὰ τῶν αἰσθήσεων λαμβάνουσιν οἱ κανονικοί, οἷον τὸ εἶναι τινα σύμφωνα καὶ διάφωνα διαστήματα καὶ τὸ εἶναι σύνθετον τὸ διὰ πασῶν ἔκ τε τοῦ διὰ τεσσάρων καὶ τοῦ διὰ πέντε καὶ τὸ εἶναι τόνον τὴν δ' ὑπεροχὴν τοῦ διὰ πέντε παρὰ τὸ διὰ τεσσάρων καὶ τὰ ὅμοια. τὰ δὲ παρὰ τοῖς μαθηματικοῖς λαμβανόμενα, ὅσα ἰδίως οἱ κανονικοὶ τῷ λόγῳ θεωροῦσιν ἐκ τῶν τῆς αἰσθήσεως ἀφορμῶν μόνον κινήθεντες, οἷον τὸ εἶναι ἐν ἀριθμῶν λόγοις τὰ διαστήματα καὶ τὸ εἶναι ἐξ ἀριθμῶν συγκρουστῶν τὸν φθόγγον καὶ τὰ παραπλήσια. τὰς ὑποθέσεις οὖν τῆς κανονικῆς διορίσειεν ἂν τις ὑπάρχειν τῇ τε περὶ τὴν μουσικὴν ἐπιστήμῃ καὶ τῇ περὶ τοὺς ἀριθμοὺς καὶ τὴν γεωμετρίαν.³²

Ptolemais outlines the principles of the science she calls *kanonikē*. These are, on the one hand, the *hypotheses* of the

³¹ As for the text, here I follow Tocco “Ptolemais” in maintaining the *lectio* of the manuscripts τὸ εἶναι ἐξ ἀριθμῶν συγκρουστῶν τὸν φθόγγον. For Barker’s conjecture συγκρούσεων instead of συγκρουστῶν, see Barker, *Porphry’s Commentary on Ptolemy’s Harmonics*, 117 n. 82.

³² Ptolemais, fr. 1 Barker = Porphyry, *Commentary on Ptolemy’s Harmonics* 1.2.23), trans. Barker, *Musical Writings*, 240, with modifications.

mousikoi, postulates drawn by perception in the Aristoxenian way, to be eventually confirmed through reason. On the other hand, the objects of inquiry of the *mathematikoi* are not evident to sense perception, but achieved through demonstration. Ptolemais declares that the *kanonikoi* have a special method that consists of taking only the very starting point from perception, and relying on reason for the rest of the process. The "special method" that the *kanonikoi* use to establish the premises of their demonstrations is none other than that of the "Pythagoreans": its peculiarity consists of applying to harmonics the method of geometrical demonstration, using perception as a spark for reason, as Ptolemais makes clear in the last, surprising, and quite unparalleled claim, that the postulates of *kanonikē* lie between music, arithmetic, and geometry.³³

It has been correctly noticed by Barker that all the examples of postulates and propositions brought by Ptolemais have a precise correspondence in the ps. Euclidean *Division of the Canon*. Indeed, Ptolemais might have this text in mind, since the postulate that some intervals are concordant while others are not is mentioned in the introduction. The demonstrations of the postulates rooted in the phenomena, such as the fact that the octave is compounded by a fifth and a fourth, and that the difference between a fifth and a fourth correspond to a tone, are reported in propositions 6 and 8. The two theorems that are not rooted in perception, are more interesting. In particular Ptolemais's statement that "a note consists of numbers colliding at once" is relevant, since it implies that the *frequency* of impacts of the air produced by a string produces the pitch, and not the speed of the movement or the force of the impact³⁴.

³³ The peculiarity of this assertion has been noticed by David Creese, *The Monochord in Ancient Greek Harmonic Science* (Cambridge: Cambridge University Press, 2010), 223–224.

³⁴ See Barker, *Greek Musical Studies*, 240 n. 139.

For Ptolemais, the quantitative nature of pitch seems to find proof not only in the fact that the intervals of music can be expressed as numerical ratios, but also in the fact that every single note consists of a pitch produced by a specific number of impacts. The texts of ancient music theory that hold this specific view are very scarce: the closest example is, once more, the introduction of the *Division of the Canon*.

Since all sounds occur when some impact occurs, and since it is impossible for an impact to occur unless movement has occurred beforehand—and since of movements some are closer packed, others more widely spaced, those which are closer packed producing higher notes and those which are more widely spaced lower ones—it follows that some notes must be higher, since they are composed of closer packed and more numerous movements, and others lower, since they are composed of movements more widely spaced and less numerous. [...] We must therefore assert that notes are composed of parts, since they attain what is required through addition and subtraction. Now all things that are composed of parts are spoken of in a ratio of number to one another, so that notes, too, must be spoken in a ratio of number to one another.

Ἐπειδὴ πάντες οἱ φθόγγοι γίνονται πληγῆς τινος γινομένης, πληγὴν δὲ ἀμήχανον γενέσθαι μὴ οὐχὶ κινήσεως πρότερον γενομένης, τῶν δὲ κινήσεων αἱ μὲν πυκνότεραι εἰσιν, αἱ δὲ ἀραιότεραι—καὶ αἱ μὲν πυκνότεραι ὀξυτέρους ποιοῦσι τοὺς φθόγγους, αἱ δὲ ἀραιότεραι βαρυτέρους, ἀναγκαῖον τοὺς μὲν ὀξυτέρους εἶναι, ἐπεὶπερ ἐκ πυκνοτέρων καὶ πλειόνων σύγκεινται κινήσεων, τοὺς δὲ βαρυτέρους ἐπεὶπερ ἐξ ἀραιωτέρων καὶ ἐλασσόνων

σύγκεινται κινήσεων. [...] διόπερ ἐκ μορίων τοὺς
φθόγγους συγκεῖσθαι φατέον· ἐπειδὴ προσθέσει
καὶ ἀφαιρέσει τυγχάνουσι τοῦ δέοντος. πάντα δὲ
τὰ ἐκ μορίων συγκείμενα ἀριθμοῦ λόγῳ λέγεται
πρὸς ἄλληλα ὥστε καὶ τοὺς φθόγγους, ἀναγκαῖον
ἐν ἀριθμοῦ λόγῳ λέγεσθαι πρὸς ἀλλήλους.³⁵

It has been rightfully observed³⁶ that there are some Archytan echoes in the text of the introduction to the *Sectio*, but this particular explanation of the origin of pitch is unparalleled in Archytas and Plato, who identified it with the speed³⁷ of the movements rather than with frequency of the impacts. The author, however, does not make any attempt to give proof, or to demonstrate in any "special" way, his theory of pitch. A possible origin of his view, and an interesting example of what Ptolemais might have had in mind when she speaks of the nature of pitch as an object of study of the *mathematikoi*, can be found, once more, in the Old Academy. In a fragment of a work of Xenocrates quoted by Porphyry through Heraclides,³⁸ the relationship between the number of impacts of a string and the pitch is argued for in a peculiar way:

Pythagoras, as Xenocrates says, discovered that the intervals in music, too, do not arise in separation from

³⁵ [Euclides], *The Division of the Canon*, 148–149, ed. André Barbera (Lincoln: University of Nebraska Press, 1991) 114–116. Trans. Barker, *Musical Writings*, 191–192.

³⁶ Huffman, *Archytas of Tarentum*, 130, also remarks on the differences between the two.

³⁷ On the acoustic theory of Archytas, see Archytas fr. 1 Huffman and the commentary in Huffman, *Archytas of Tarentum*, 129–148.

³⁸ It is uncertain whether or not this Heraclides is to be identified with the Academic and Peripatetic philosopher Heraclides of Pontus. See Margherita Isnardi Parente, *Senocrate-Ermodoro, Frammenti* (Napoli: Bibliopolis, 1982), 314.

number: for they are a blending of quantity with quantity. He therefore investigated the conditions under which there arise concordant and discordant intervals, and everything that is well-attuned and ill-attuned. And turning to the way in which sound comes about, he said that if something concordant is to be heard, arising from an equality, some movement must have occurred. Now, movement does not arise without number, neither does number without quantity. [...]³⁹

Πυθαγόρας, ὥς φησι Ξενοκράτης, εὕρισκε καὶ τὰ ἐν μουσικῇ διαστήματα οὐ χωρὶς ἀριθμοῦ τὴν γένεσιν ἔχοντα· ἔστι γὰρ σύγκρισις ποσοῦ πρὸς ποσόν. ἐσκοπεῖτο τοίνυν, τίνος συμβαίνοντος τὰ τε σύμφωνα γίνεται διαστήματα καὶ τὰ διάφωνα καὶ πᾶν ἡρμωσμένον καὶ ἀνάρμοστον. καὶ ἀνελθὼν ἐπὶ τὴν γένεσιν τῆς φωνῆς ἔφη· "ὥσει μέλλει τι ἐκ τῆς ἰσότητος σύμφωνον ἀκουσθήσεσθαι, κίνησιν δεῖ τινα γενέσθαι." ἡ δὲ κίνησις οὐκ ἄνευ ἀριθμοῦ γίνεται, ὁ δ' ἀριθμὸς οὐκ ἄνευ ποσότητος. [...]

The impact, he says, is in no [duration of] time, but in a boundary between time past and time to come [...]. Just as when a line cuts a plane, he says, the line is neither of the two planes but is a boundary of both of them, so the impact is at the "now", and is in neither of the times, neither the time past nor the time to come. But the impact appears to occur, he says, in some time that is imperceptible because of the weakness of the hearing, like things that we see to be

³⁹ In the following sentences Xenocrates provided commentary on the many forms of movements, probably bearing in mind Plato's *Timaeus* 40a–b, and *Republic*, 530d.

the case with respect to sight. For often when a cone is in motion, and there is on the cone one white or black spot, it appears that there is a circle on the cone, of the same color of the spot. [...]

Ἡ πληγὴ δέ φησιν ἐν οὐδενὶ χρόνῳ ἐστὶν ἀλλ' ἐν ὄρῳ χρόνου τοῦ παρεληλυθότος καὶ τοῦ μέλλοντος. [...] καθάπερ γὰρ φησιν εἰ γραμμὴ τέμνοι τὸ ἐπίπεδον, ἐν οὐδετέρῳ ἐπιπέδῳ ἐστὶν ἡ γραμμὴ, ἀλλ' ὅρος ἀμφοτέρων ἐστὶ τῶν ἐπιπέδων ἡ γραμμὴ. οὕτω καὶ ἡ πληγὴ οὕσα κατὰ τὸ νῦν ἐν οὐδετέρῳ τῶν χρόνων ἐστὶ τοῦ παρεληλυθότος καὶ μέλλοντος. φαίνεται δέ φησιν ἡ πληγὴ ἐν χρόνῳ τινὶ γινομένη ἀνεπαισθήτῳ διὰ τὴν τῆς ἀκοῆς ἀσθένειαν, καθάπερ καὶ ἐπὶ τῆς ὄψεως ὁρῶμεν γινόμενον. πολλάκις γὰρ κώνου κινουμένου, στιγμῆς ἐπούσης μιᾶς ἐπὶ τοῦ κώνου λευκῆς ἢ μελαίνης, φαίνεσθαι συμβαίνει κύκλον ἐπὶ τοῦ κώνου ὁμόχρουν τῇ στιγμῇ. [...].

The hearing is in even greater confusion than the sight. For, if one stretches a string, he says, and plucks it, and allows it to resound, the result will be that notes are heard, while the string continues in its vibratory movement, bending back and forth in the same place, in such a way that movement is more clearly perceived by sight than by hearing. [...] But if it is so, he says, it is plain that each of the strings emits several notes. Then if each note occurs in the impact, and if it is the case that an impact occurs not in a time but in a boundary of time, it is clear that in between the impacts that correspond to the notes there must be silences, which exist in a time.

καὶ μάλλον ἐν ταράχῳ ἐστὶν ἡ ἀκοὴ ἢ περ ἡ ὄψις. εἰ γὰρ τις φησὶ χορδὴν κατατείνας καὶ κρούσας ἔαση

αὐτὴν ἀπηχεῖν, συμβήσεταιί τινων μὲν ἀκηκοέναι
φθόγγων, τὴν δ' ἔτι κινεῖσθαι σειομένην καὶ ἐπὶ τὸν
αὐτὸν τρόπον ἀνακάμψεις ποιεῖσθαι, ὥστε τῇ μὲν
ὄψει τὴν κίνησιν τῆς χορδῆς φανεράν μᾶλλον ἢ τῇ
ἀκοῇ γίνεσθαι. [...] εἰ δὲ τοῦτό φησιν, οὕτως ἔχει,
φανερόν ὅτι ἐκάστη τῶν χορδῶν πλείους προῖεται
φθόγγους. εἰ οὖν ἕκαστος φθόγγος ἐν τῇ πληγῇ
γίνεται, πληγὴν δ' εἶναι συμβέβηκεν οὐκ ἐν χρόνῳ
ἀλλ' ἐν ὄρῳ χρόνου, δηλὸν ὅτι ἀνὰ μέσον τῶν κατὰ
φθόγγους πληγῶν σιωπαὶ ἂν εἴησαν ἐν χρόνῳ
ὑπάρχουσαι.⁴⁰

As argued by Dillon,⁴¹ the content of this fragment, whose Xenocratean paternity is controversial,⁴² is consistent with the approach of Xenocrates to the study of mathematical sciences. It is well known that he considered the idea of a continuum as illusory in the realm of geometry, since he postulated the existence of indivisible lines.⁴³ He seems to have conceived a very similar theory in order to explain acoustic phenomena. He claims, then, that perception must be even more deceitful in the

⁴⁰ Xenocrates, fr. 87 Isnardi Parente = Porphyry, *Commentary on Ptolemy's Harmonics* 1.3.30–31: Ed. Düring, *Ptolemaios und Porphyrios*, 30–31. Trans. Barker, *Ancient Greek Musical Writings*, 235–236.

⁴¹ John Dillon, *The Heirs of Plato, a Study of the Old Academy* (Oxford: Oxford University Press, 2003), 117–118.

⁴² Barker, *Musical Writings*, 235 n. 113 remains skeptical about the possibility that anything but the first sentence in the Heraclides fragment can be attributed to Xenocrates. Most scholars, however, seem to agree that the fragment's content has to be traced back to Xenocrates, cf. Isnardi Parente, *Xenocrate-Ermodoro*, 314–319.

⁴³ On this controversial theory, which raised a dispute between the pupils of Plato and Aristotle, see Elisabetta Cattanei, "Apologie des défenseurs des lignes insécables," *Cahiers du Centre d'Études sur la Pensée antique* "Kairos kai Logos" 79 (2006): 1–30.

case of notes produced on a string, than in the case of sight: the sense of hearing makes us believe we hear but a single note, whereas multiple strikes of the air must produce a number of notes. The acoustic theory reported here is slightly different than that of the *Division of the Canon*, but the ties are evident.

What is more, the fact that physical sound is discrete and made of numbers despite what we hear is argued for by Xenocrates by means of a geometrical argument that assumes the impact to be a discrimen of time and movement, which belongs to neither of the parts it divides. Impact is for sound, to Xenocrates, what the point is for the line, or the line for a plane surface. Interestingly, Xenocrates seems to have ascribed at least the principles of the theory to Pythagoras. This "special" harmonics, then, looks like an attempt to save acoustic phenomena without abandoning the Platonic project; to give mathematical sciences a unitary, common theoretical foundation. It appears to be consistent with the position that Ptolemais identifies as the "orthodox Pythagorean" one.

Conclusion

Who then were the Pythagoreans involved and why did Ptolemais criticize them? It is hard to state whether she did or did not give a radically anti-empirical reading of the passage on harmonics of Plato's *Republic*, putting the latter amidst the "radical" faction of Pythagoreans. I think the possibility that she did should be taken into account.⁴⁴ Ptolemais does not seem to adopt an Aristoxenian, qualitative approach to music theory, since she admits that the *diastēmata* consist of numbers, and that the notes are made of numbers: she is "Pythagorean" in this respect. She blames, in the Platonic approach, the lack of reference to perceptible objects in the first place. In the "orthodox Pythagorean" view, she sees arbitrary selection of the initial data of perception, as well as a failure to refer the

⁴⁴ See Barker's commentary in *Musical Writings*, 56 n. 5.

conclusions to perception—that is exactly the contrary of what she praises in the Aristoxenian approach.⁴⁵ Ptolemais's position appears shaded, and quite optimistic on the possibility of building a mathematical harmonic science able to account for the phenomena, as is evident in the curious note, historically incorrect, on the origin of the name *kanonikē*:

The science of *kanonikē*—of whom is it mainly characteristic? In general, of the Pythagoreans; for what we now call *harmonikē*, they used to name *kanonikē*. From what do we derive the term *kanonikē*? Not, as some people think, by transference from the instrument called the *kanōn*,⁴⁶ but from straightness, on the grounds that it is through this science that the *logos* discovers what is correct, and discovers the regulative marks [*parapēgmata*] of what is well attuned [...].

ἡ οὖν κανονικὴ πραγματεία, κατὰ τίνας μᾶλλον ἐστι; καθόλου κατὰ τοὺς Πυθαγορικούς· ἦν γὰρ νῦν ἁρμονικὴν λέγομεν, ἐκεῖνοι κανονικὴν ὠνόμαζον. ἀπὸ τίνος κανονικὴν αὐτὴν λέγομεν;

⁴⁵ Perhaps Ptolemais adopted some arguments similar to those of Ptolemy on the limits of the Pythagorean approach: for instance, considering only the epimoric and multiple ratios, they only pick some as fundamental, not considering as consonants some other intervals like one octave plus a fourth, or one octave plus a fifth (cf. Ptolemy, *Harmonics* 1.6; see Rocconi, “Un manuale,” 105–106). Not by chance, the last reference to Ptolemais in Porphyry (*Commentary on Ptolemy's Harmonics*, 1.7.114) consists of a notice on the difference between *isotonia* and *homophonia*, i.e., “between notes with an identical sound and notes that produce a single sound when played together, regardless of acuteness and gravity. She seems to argue that the distinction was of little importance” (cf. Tocco, “Ptolemais,” 224–225).

⁴⁶ On the etymology of the canon, see also Panaetius minor, in Porphyry, *Commentary on Ptolemy's Harmonics*, 1.66.

οὐχ ὥς ἔνιοι νομίζουσι ἀπὸ τοῦ κανόνος ὄργάνου
παρονομασθεῖσαν, ἀλλ' ἀπὸ τῆς εὐθύτητος ὥς διὰ
ταύτης τῆς πραγματείας τὸ ὀρθὸν τοῦ λόγου
εὐρόντος καὶ τὰ τοῦ ἡρμοσμένου παραπήγματα.
κανονικὴν γέ τοι καλοῦσι [...].⁴⁷

Apparently, Ptolemais reversed the etymology of the name *kanōn*, considering the one-stringed instrument as a help for reason, able to provide a spark of perception in order to build geometrical theorems that might be useful for harmonics.⁴⁸ *Kanonikē* comes to be a science of "straightness" that measures and determines the correctness of the reasonings. The unusual word *parapēgmata* apparently refers to calendars inscribed on stone, on which movable pegs were inserted to count the days. This is a metaphor for the divisions of the *kanōn*'s string, divided by its mobile bridge, stretched on a table where the ratios are reported as marks.⁴⁹ It also shows that Ptolemais's *kanonikē* is deeply connected to the theory of proportion: the *kanōn* as an instrument was probably not known to Plato and Aristotle,⁵⁰ but it first appeared in the 4th century, according to a report of Duris of Samos (in Porphyry, *Life of Pythagoras*, 3),⁵¹

⁴⁷ Ptolemais, fr. 1 Barker = Porphyry, *Commentary on Ptolemy's Harmonics* 1.2.22, trans. Barker, *Musical Writings*, 240, modified.

⁴⁸ I agree with Tocco, "Ptolemais," 208–209, that such an etymology speaks in favor of a late date for Ptolemais, since the technical use of the term *kanonikē* for the study of musical proportions appears to be common in her text as well as in late antiquity: Ptolemais appears to live in a time when the term was desemanticized, and seems to openly ignore that the *kanōn* was an instrument purposely created for the study of music theory, since in what follows she reports, as a proof for her claim, that the Pythagoreans also called some wind instruments "canonic."

⁴⁹ Creese, *Monochord*, 76; see also Barker, *Musical Writings*, 239 n. 135.

⁵⁰ Creese *Monochord*, 131–136.

⁵¹ See Creese *Monochord*, 97–104.

which suggests that it was used as an auditory and geometrical representation of proportional means.

Ptolemais's work presents the science of the *kanōn* as a form of mathematical investigation able to answer not only technical, but also philosophical questions. Her "critically Pythagorean" approach is likely to have influenced later authors, and especially the position of Ptolemy himself.⁵² Ptolemais attempted, on a philosophical ground, to build a mathematical harmonic science whose results do not contradict perception, and might be compared to Aristoxenian scales, thus "saving the phenomena." Although our sources for harmonics as a science in the Hellenistic age are quite scarce, we know that such an approach was likely adopted by her countryman Eratosthenes.⁵³ Even if a link between the two, as suggested by West,⁵⁴ cannot be easily proven, it is clear that Ptolemais's treatise provides a precious insight into Hellenistic musicology and philosophy of science, and gives an all-around interpretation of a debate, whose origins are rooted in the dispute between the Peripatetics and the Old Academy on the nature of reason and sensation in the process of knowledge.

⁵² Creese *Monochord*, 228–233.

⁵³ Despite the almost complete loss of Eratosthenes's work, some scholars have tried to reconstruct his harmonic theory, in which the use of the monochord was of some importance. He seems to have made an attempt to reconcile Aristoxenian qualitative theory with Platonic and Pythagorean musical ratios: see the highly hypothetical reconstruction in Creese, *Monochord*, 196–209.

⁵⁴ Martin L. West, *Ancient Greek Music* (Oxford: Clarendon Press, 1992), 239.